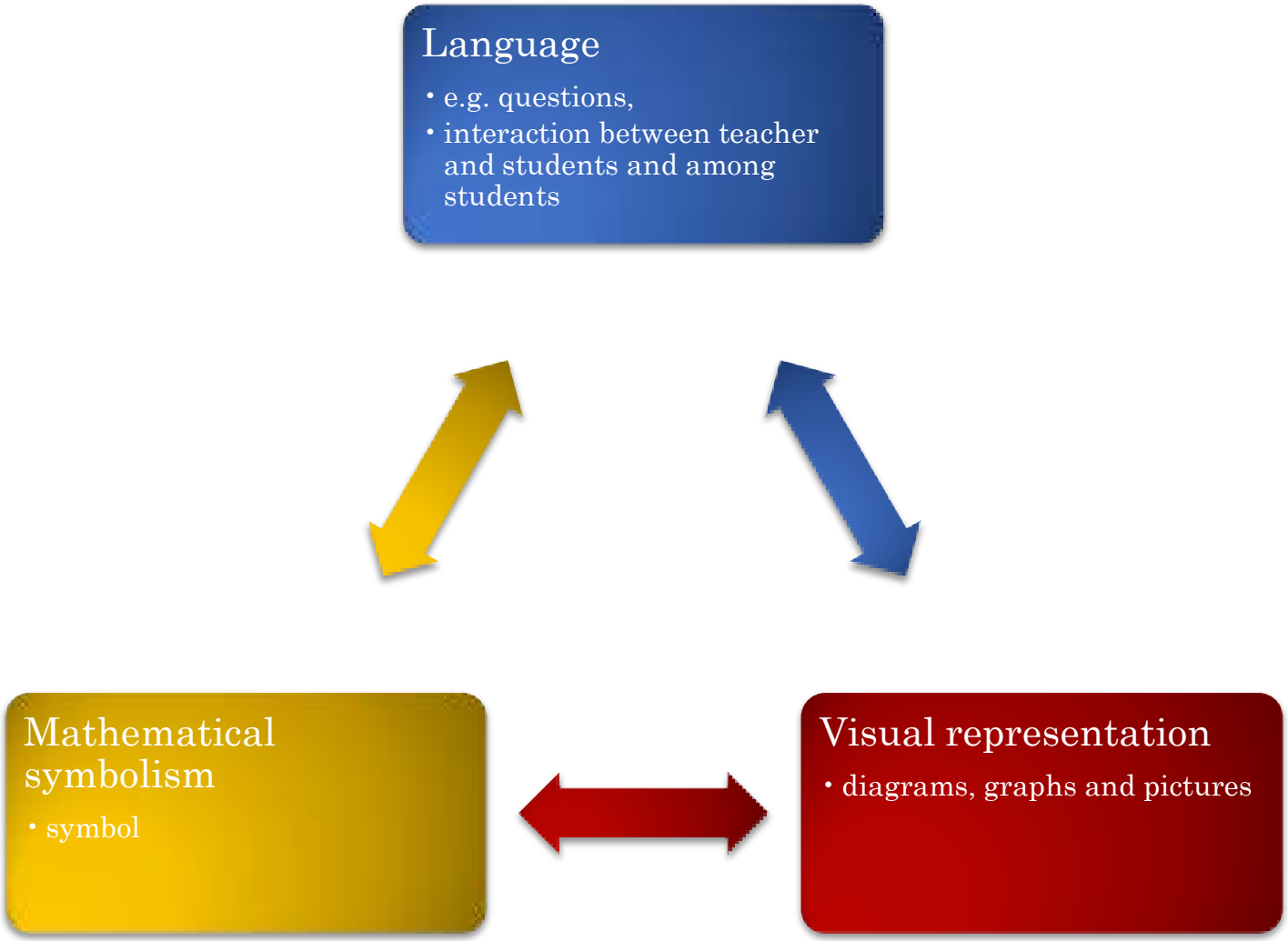
Exploring How Meanings are made in Mathematics: Simultaneous Linear Equations in Two Unknowns

CUHKFAA Chan Chun Ha Secondary School
Mr. Lai Yat To & Miss Ho Suk Yin

School & Students Background

- CMI school
- Form 3 students
- Enrichment Class
- Lesson conducted in English

Making meaning in Mathematics



Language

Nominal Group

Nominal Group

- Nominal groups are built up around a key noun.
- Making meaning through nominal group
  - Understanding definitions
  - Understanding the meaning in mathematical problems

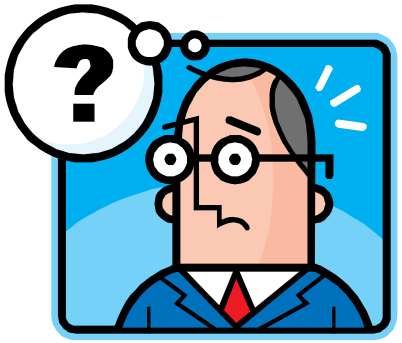
Understanding definitions

- Definitions are constructed through the nominal group

Word being defined	relating process =	class or generalisation	[embedded clause or phrase] *particular part*
A decimal point	is	a mark	which separates the whole number part from the fractional part
The perimeter of a closed figure	is	a measurement	of the distance around the boundary of the figure
Polygons	are	closed plane figures	with line sides which do not cross over themselves

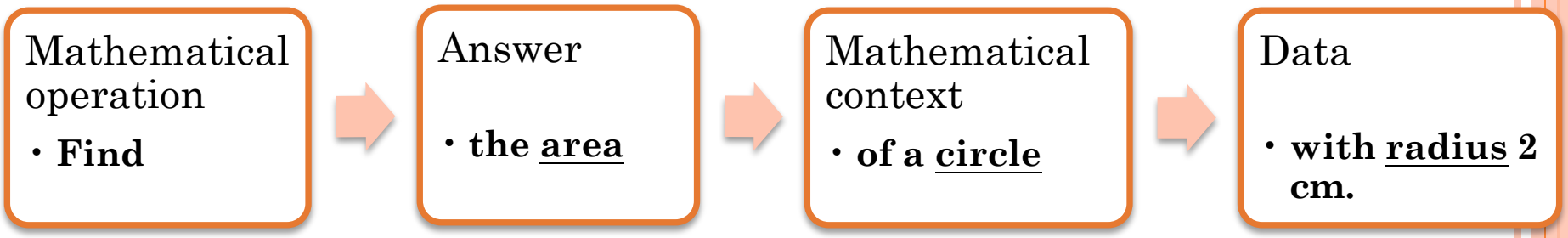
Strategies for solving mathematical word problems

- Read a question in a systematic way
  - In between the "." or "and" there would be a piece of information
  - Every piece of information may be represented in a symbolic way
  - Focus on some important words or phrases in order to write an equation
- How to find keywords for mathematical word problems?



Understanding the meaning in mathematical problems

- Identify the keywords in the problems through nominal group
  - Find the area of a circle with radius 2 cm.

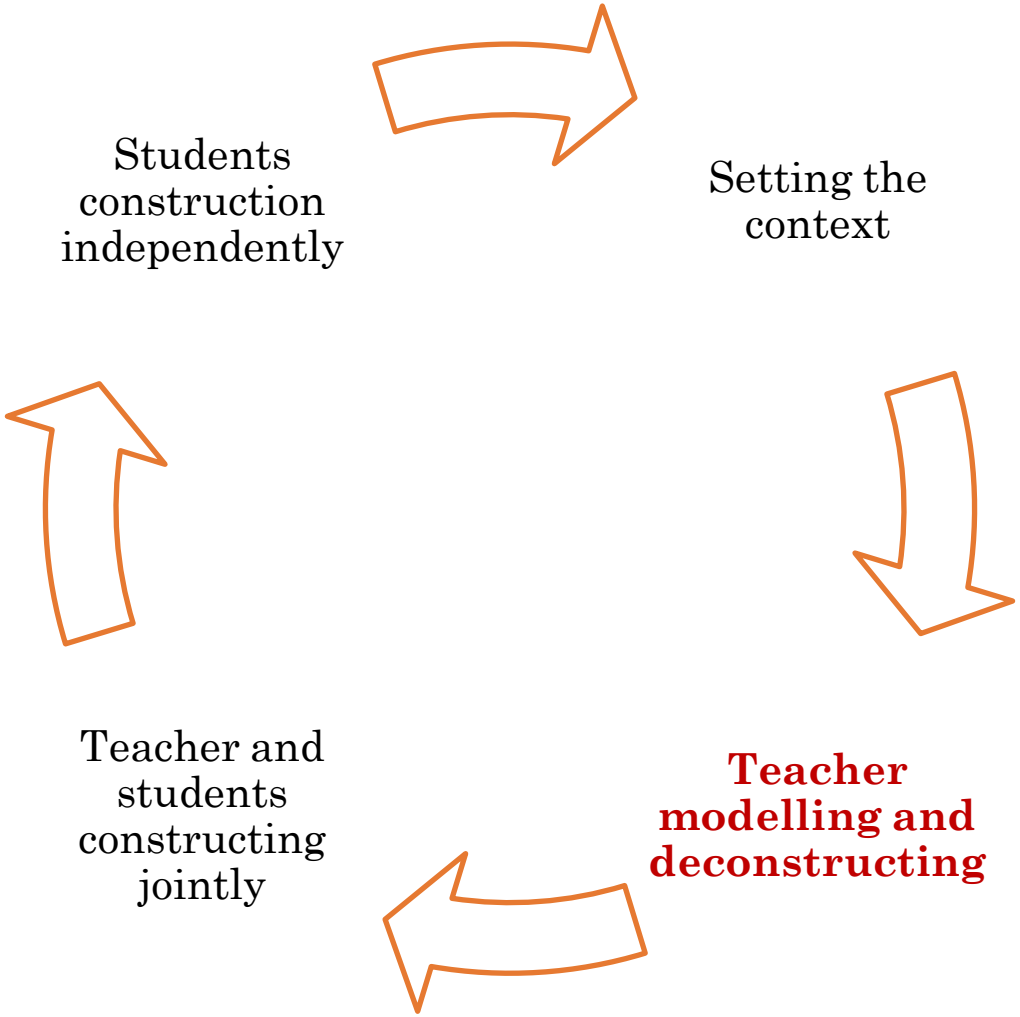


Remarks: [nominal groups are indicated in red and the key noun in each nominal group is underlined]

4 Types of Typical Questions in Simultaneous Linear Equations

- Linear problems
- Numbers & Digits problems
- Age problems
- Time, Speed, Distance problems

TEACHING AND LEARNING CYCLE



Linear Problems

- Language:
  - The total cost of 4 apples and 3 oranges *is* $23, while the total cost of 7 apples and 4 oranges *is* $34. How much does each apple and orange cost?
- Visual representation

Total cost of apples ($)		Total cost of oranges ($)		Total cost ($)
number of apples	cost of an apple ($)	number of oranges	cost of an orange ($)	
4	x	3	y	23
7	x	4	y	34

- Mathematical symbol

$$\begin{cases} 4x+3y=23 & \text{total cost of 4 apples and 3 oranges } \textit{is} \text{ \$23} \\ 7x+4y=34 & \text{total cost of 7 apples and 4 oranges } \textit{is} \text{ \$34} \end{cases}$$

Understanding the meaning in mathematical problems

- Numbers problem
  - The sum of two numbers *is* 40 and their difference *is* 8. Find the two numbers.
- Digits problem
  - The sum of the digits of a two-digit number *is* 8. If we exchange the positions of the two digits, the number obtained *is* greater than the original number by 36. Find the original number.

Remarks: [nominal groups are indicated in red and
the key noun in each nominal group is underlined]

Sentence Pattern

Language	→	Mathematical Symbol
1. The sum of *x* *and* *y*		$x + y$
2. The difference of *x* *and* *y* (for $x > y$)		$x - y$
3. The product of *x* *and* *y*		$x \times y$
4. Three times *x*		$3 \times x$

Sentence Pattern

Language	→	Mathematical Symbol
5. x is greater than y by c (x is c greater than y)		$x = y + c$
6. x is smaller than y by c (x is c smaller than y)		$x = y - c$
7. *The two digit number* (*unit digit: x, tens digit: y*)		$10y + x$

Understanding the meaning in mathematical problems

- Numbers problem
  - The sum of two numbers *is* 40 and their difference *is* 8. Find the two numbers.

$$\begin{cases} x + y = 40 \\ x - y = 8 \end{cases}$$

- Digits problem
  - The sum of the digits of a two-digit number *is* 8. If we exchange the positions of the two digits, the number obtained *is* greater than the original number by 36. Find the original number.

Let the unit digit be x and tens digit be y

$$\begin{cases} x + y = 8 \\ 10x + y = (10y + x) + 36 \end{cases}$$

Remarks: [nominal groups are indicated in red and the key noun in each nominal group is underlined]

Age Problems

- Language:
  - Ten years ago, a father's age *was* four times his son's.
    Ten years later, the father's age *will be* twice his son's.
    How old are they now?
- Visual representation

Year	Son's age	Father's age
Now	**x**	**y**
Ten years ago	**x-10**	**y-10**
Ten years later	**x+10**	**y+10**

- Mathematical symbol

$$\begin{cases} y-10=4(x-10) \\ y+10=2(x+10) \end{cases}$$

father's age *was* four times his son's

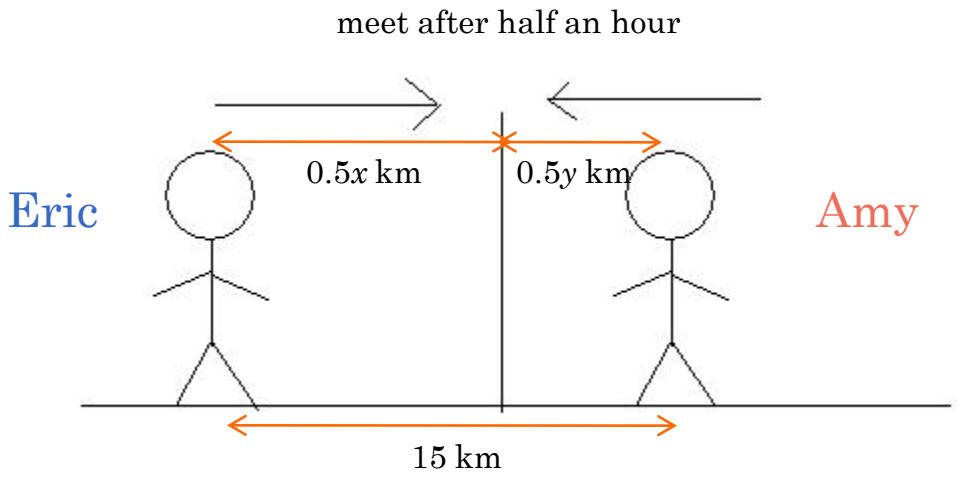
father's age *will be* twice his son's

Time, Speed, Distance Problems

Language

Eric and Amy are riding bicycles and they are 15 km apart.
If they go towards each other, they will meet after half an hour.
If they travel in the same direction, Eric will catch up with Amy after 3 hours.
Find their speeds.

- Visual Representation



- Mathematical Symbol

0.5x + 0.5y =15

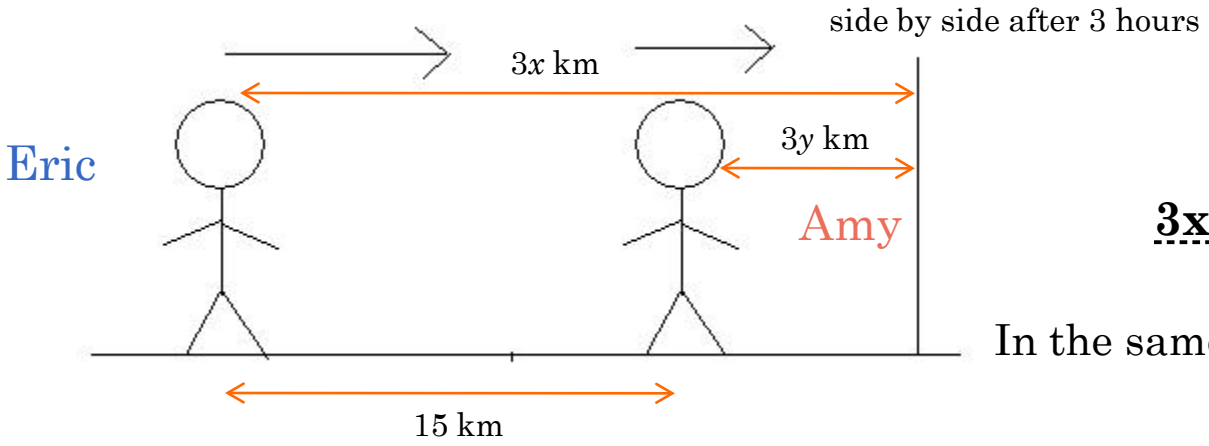
Towards each other

Time, Speed, Distance Problems

Language

Eric and Amy are riding bicycles and they are 15 km apart.
If they go towards each other, they will meet after half an hour.
If they travel in the same direction, Eric will catch up with Amy after 3 hours.
Find their speeds.

- Visual Representation



- Mathematical Symbol

3x – 3y = 15

In the same direction

Sharing

- Feedback from students
- Applications to the other forms
  - S1 Bridging Course
  - S4 Topic: Solving Quadratic Equations in One Unknown

Secondary 1 (Bridging Course)

- Language → Mathematical Symbol
- Addition
  The sum of 3 and 9 → 3 + 9
  3 plus 9
- Subtraction
  The difference between 13 and 5 → 13 – 5
  5 is subtracted from 13
  13 minus 5
- Multiplication
  The product of 2 and 7 → 2 x 7
  Multiply 2 by 7
  2 times 7
- Division
  30 is divided by 6 → 30 ÷ 6
  Divide 30 by 6

Secondary 1 (Bridging Course)

- Integration

1. 5 is subtracted from 3 times y
2. The sum of p and q is multiplied by 4
3. $3x$ times 5, and then multiply the difference by y
4. Divide the sum of 3 times a and 4 times b by the sum of a and b
5. Subtracted y from the product of x and one-fourth of z
6. Multiply h by 2 and then divide the product by k to the power of 3

Secondary 4 (Solving Quadratic equations in one unknown)

- Language → Mathematical Symbol

The **sum** of the square of x & y → $x^2 + y^2$

The square of the **sum** of x & y → $(x + y)^2$

The End

Thank you!