Basic Mathematics and Numbers – Directed Numbers

Level: Key stage 3

Dimension: Number and Algebra
Module: Number and Number Systems
Unit: Directed Numbers and the Number Line

Student ability: Low to average

Content Objectives:
After completing the activity, students should be able to understand the key concept of positive and negative numbers and master the addition and subtraction of directed numbers.

Language Objectives:
After completing the activity, students should be able to

- understand the English terms related to the arithmetic operations with positive and negative numbers (e.g., represent, quantities, positive, negative, symbols, arithmetic operation, addition, subtraction, number line, to the right, to the left, forward, backwards);
- understand and use the English expressions for explaining how to
  - represent directed numbers on a number line (e.g., We can represent directed numbers by the positions of points on a number line. The numbers to the right of zero are positive numbers. The numbers to the left of zero are negative numbers);
  - do addition on a number line (e.g., In the arithmetic operation 2+3 =5, we start at 2 and move 3 units forward to the right and reach 5. We can consider +3 as moving 3 units forward);
  - do subtraction on a number line, (e.g., In the arithmetic operation 3−2 = 1, we start at +3 and move 2 units backwards to the left and reach +1. We can consider −2 as moving 2 units backwards); and
- follow English instructions on solving problems concerning arithmetic operations and work on related problems written in English.

Prerequisite knowledge:
Students should have gained foundation knowledge related to arithmetic operations and numbers which they have learned from key stage 2 through the medium of Chinese.

Time: 2 lessons (80 minutes)
Procedure:

1. The teacher should first ask student to complete part A(i) of the worksheet.

2. The teacher should then discuss part A(ii) with students showing them that some quantities need to be represented in directed numbers.

3. The teacher should then show the class the horizontal number line in part B, and demonstrate to the students how to represent directed numbers on a number line.

4. Students should be asked to complete the exercises of part B of the worksheet.

5. The teacher then shows the class how to do addition and subtraction using the number line in part C.

6. Students complete the exercises in part C.

7. The teacher then discusses with the class the rules of addition and subtraction for directed numbers in part D.

8. Finally, the teacher should assign the exercise as classwork to check on students’ understanding while they are working. The teacher may assign the exercise as a group activity.

Explanatory Notes for Teachers:

1. The students have learned calculation, addition and subtraction in key stage 2. However, the calculations did not involve negative numbers. The teacher should check students’ understanding while introducing the rules of sign.

2. The suggested answers of the exercises can be found on the last page.
A) Basics with Numbers - Positive and Negative Numbers

i) Use numbers to present the following quantities:
1. Your height in cm. 
2. The number of students in your class. 
3. The price in dollars of a can of soft drink. 
4. Your score in the last mathematics test. 
5. The length of a swimming pool in metres. 

ii) Represent the following quantities in symbols:
1. A profit of $5000 
2. A loss of $5000 
3. 500 m above sea level 
4. 500 m below sea level 
5. A $2000 salary increased 
6. A decrease in salary by $2000 
7. A temperature of 20°C 
8. A temperature of 20°C below 0°C 
9. 5 steps forward 
10. 5 steps backward

In daily life, we find that we will have difficulty in representing quantities by just using a number. To represent the quantity clearly, we need to have more information. In the cases in section (ii) above, besides numbers, we need different symbols to represent the situations. In Mathematics, we use “+”(positive, 正) and “−”(negative, 负) to provide additional information about numbers.

For example, we use +300 m to represent 300 m above sea level and −300 m to represent 300 m below sea level.

Numbers with “+” or “−” sign are called directed numbers.

Now use directed numbers to represent the quantities in (ii).
B) Representing directed numbers on a number line:

We can represent directed numbers by the positions of points on a number line.

![Number line diagram]

The numbers to the right of 0 (zero), e.g. +1, +2, +2.5 ..., are positive numbers. The numbers to the left of 0, e.g. −1, −2, −2.3..., are negative numbers.

Usually, the “+” sign of a positive number may be omitted. For example, “+5” is the same as 5.

Example: The numbers, 0.5, 2.5, −4.5 and −1.2 are marked on the number line.

![Example number line diagram]

Exercise:

Draw a number line and mark down the numbers, −3, −2.5, −1.3, 1, 1.5 and 5 on the number line.
C) Addition and Subtraction on the number line:

Addition of positive numbers:

e.g. $2 + 3 = 5$

On the number line, we start at 2 and move 3 units to the right and reach 5. We can consider $+3$ as moving 3 units to the right.

e.g. $-2 + 6 = 4$

We start at $-2$ and move 6 units forward to the right and reach 4. Therefore, $-2 + 6 = 4$.

Subtraction of positive numbers:

e.g. $3 - 2 = 1$

On the number line, we start at $+3$ and move 2 units to the left and reach $+1$. We can consider $-2$ as moving 2 units to the left.

e.g. $-1 - 3 = -4$

We start at $-1$ and move 3 units to the left and reach $-4$. Therefore, $-1 - 3 = -4$. 
Exercise:

Using a number line, find the values of the following:

1. 3+2

2. 4+5

3. −2+1

4. −6+8

5. 7−2

6. 10−4

7. −1−7

8. −3−4
D)  Addition and Subtraction of directed numbers:

To add or subtract directed numbers, we need to use the following rules of sign.

(i) \( x + (+y) = x + y \)

(ii) \( x - (+y) = x - y \)

(iii) \( x + (-y) = x - y \)

(iv) \( x - (-y) = x + y \)

Examples:

1. Find the value of \(-3 + (+4)\)
   
   Solution: \(-3 + (+4) = -3 + 4\)
   
   \[= 1\]

2. Find the value of \(2 - (+4)\)
   
   Solution: \(2 - (+4) = 2 - 4\)
   
   \[= -2\]

3. Find the value of \(3 + (-4)\)
   
   Solution: \(3 + (-4) = 3 - 4\)
   
   \[= -1\]

4. Find the value of \(2 - (-3)\)
   
   Solution: \(2 - (-3) = 2 + 3\)
   
   \[= 5\]
Exercise:

Find the value of the followings:

1. $(+3) - (-3)$
2. $(+4) + (-7)$

3. $(-5) - (-2)$
4. $4 - (-5)$

5. $(+6) + (-10)$
6. $(-6) - (-9)$

7. $(-2) + (-4)$
8. $(+18) - (+7)$

9. $(-16) + (-8)$
10. $(+17) - (-22)$
Suggested answers:

Part A:

(i) 1. 1XX cm  2. XX students  3. $2.X  4. XX marks
    5. 50 m

(ii) 1. +$5000  2. −$5000  3. +500 m  4. −500m  5. +$2000
    6. −$2000  7. +20°C  8. −20°C  9. +5 steps  10. −5 steps

Part B exercise:

[Graph showing a number line from -6 to 6 with marked points at -2.5, -1.5, 1.5, and 2.5.

Part C exercise:

1. 5  2. 9  3. −1  4. 2
   5. 5  6. 6  7. −8  8. −7

Part D exercise:

1) 6  2) −3  3) −3  4) 9
   5) −4  6) 3  7) −6  8) 11
   9) −24  10) 39