

Sequences and Functions

Level: Key Stage 3

Dimension: Number and Algebra

Module: Observing Patterns and Expressing Generality

Unit: Arithmetic and Geometric Sequences

Student ability: Average

Content Objectives:

After completing the activity, students should be able to recognize patterns of arithmetic and geometric sequences.

Language Objectives:

After completing the activity, students should be able to

- better understand the English terms and expressions related to sequence (e.g., *sequence, pattern, follow, classify, state, arithmetic sequences, geometric sequences, common difference, common ratio, the first term of a sequence*);
- state in English how the pattern of an arithmetic or a geometric sequence is formed, using the following sentence pattern: Each term *is formed by* + *present participle* (e.g., *Each term is formed by adding 2 to the previous term. Each term is formed by multiplying the previous terms by 3.*);
- define an arithmetic sequence and a geometric sequence in English, using the sentence pattern: ___ *is formed by* + *present participle*, for example, *An arithmetic sequence is formed by adding a fixed number, or a common difference, to one term to form the following term. A geometric sequence is formed by multiplying a fixed number, or a common ratio, to one term and form the following term.*
- follow English instructions on solving problems concerning this topic and work on related problems written in English.

Prerequisite knowledge: nil

Time: 2 lessons (80 minutes)

Procedure:

The teacher should ask the students to fill in the missing numbers and guide students to investigate the pattern of the sequences in part (i) of Activity 1. The teacher may assign this activity for group discussion.

1. After the discussion, the teacher should check the answers with students.
2. The teacher should discuss part (ii) of Activity 1 with the students and introduce the terms, such as *arithmetic* and *geometric sequences*, *common differences* and *common ratio*.
3. Students complete exercise 1 as classwork.
4. The teacher should discuss the answers for exercise 1 with the class.
5. The teacher should then assign exercise 2 to the students and then discuss the answers with the class.
6. The teacher should then ask students to complete exercise 3 and invite students to provide the answers in English.

Explanatory Notes for Teachers:

1. Through the inductive approach, students can observe the relations between the numbers in different types of sequences.
2. The suggested answers are provided on the last page.

Name: _____ Class: _____ ()

Title: Sequence

Activity 1:

i) Observe the following sequences and find the missing terms. What pattern do the numbers follow?

	Sequence	Pattern
a)	{1, 2, 3, _____, 5, 7}	Each term is formed by adding 1 to the previous term.
b)	{2, 4, 6, _____, 10}	Each term is formed by _____ _____
c)	{1, 2, 4, _____, 16, 32}	Each term is formed by _____ _____
d)	{10, 5, 0, _____, -10}	Each term is formed by _____ _____
e)	{3, _____, 9, 12, _____, 18}	Each term is formed by _____ _____
f)	{64, 32, 16, _____, _____, 2}	Each term is formed by _____ _____

ii) Classify the above sequences into groups:

Group 1: Including (a), _____, _____, and _____. The sequences are formed by _____. This type of sequence is called an arithmetic sequence.

The common number added to each term to form the next term is called the _____ (_____).

Group 2: Including _____ and _____. The sequence is formed by _____. This type of sequence is called a geometric sequence.

The common number multiplied to each term to form the next term is called the _____ (_____).

Exercise 1:

The following sequences are either arithmetic sequences or geometric sequences. For question numbers 1 to 5, state the type of the sequence. If it is an arithmetic sequence, state the common difference. If it is a geometric sequence, state the common ratio.

	Sequences	Type of sequence	Common difference / ratio
1.	$\frac{2}{9}, \frac{2}{3}, 2, 6, 18$		
2.	3, 11, 19, 27, 35		
3.	1, 2, 4, 8, 16		
4.	95, 88, 81, 74, 67		
5.	81, 27, 9, 3, 1		

Exercise 2:

- 8, 4, 16, 28, 40, ... is an arithmetic sequence. Find the 8th term of the sequence.
- 128, 64, 32, 16, 8, ... is a geometric sequence. Find the 6th term of the sequence.
- 10, 10.3, 10.6, 10.9, 11.2, ... is an arithmetic sequence. Find the 9th term of the sequence.
- 176, 88, 44, 22, 11, ... is a geometric sequence. Find the 7th term of the sequence.

Exercise 3:

- i) Given that the first term of a geometric sequence is 6 and the common ratio is 2.5, what is the fourth term of the sequence?
- ii) Given that the fifth term of an arithmetic sequence is 100 and the common difference is -18, what is the first term of the sequence?

Suggested answers:

Activity 1:

- (i) a) 4, 6 b) 10 each term is formed by adding 2 to the previous term
c) 8 each term is formed by multiplying the previous term by 2
d) -5 each term is formed by adding -5 to the previous term
e) 6, 15 each term is formed by adding 3 to the previous term
f) 8, 4, each term is formed by multiplying the previous term by $\frac{1}{2}$

(ii) Group 1: b, d and e

adding a fixed number to one term to form the following term

Common difference, d

Group 2: c and f

multiplying a fixed number to one term to form the following term

Common ratio, r

Exercise 1:

1. Geometric sequence, common ratio 3 2. Arithmetic sequence, common difference 8
3. Geometric sequence, common ratio 2 4. Arithmetic sequence, common difference -7
5. Geometric sequence, common ratio $\frac{1}{3}$

Exercise 2:

1. 76 2. 4 3. 12.4 4. 2.75

Exercise 3:

- i) 93.75 ii) 172