Constructing Congruent Triangles

Level: Key Stage 3

Dimension: Measures, Shape and Space
Module: Learning Geometry through an Intuitive Approach
Unit: Congruence and Similarity

Student ability: Average

Content Objectives:

After completing the activity, students should be able to recognize the properties and conditions for congruent triangles and the notation of corresponding sides and angles in congruent triangles.

Language Objectives:

After completing the activity, students should be able to:
- recognize and understand the key English terms related to congruent triangles, e.g., *congruent figures, same size, same shape, equal length, equal width, equal radii, congruent triangles, fit into each other, rotate, overlap, coincide, corresponding sides, corresponding angles, symbol, position.*

- understand the English expressions for explaining the properties and conditions for congruent triangles (e.g., *If two triangles can fit into each other perfectly, they are congruent triangles. When you cut the triangles out and rotate them, they can exactly overlap/coincide with each other. The overlapping sides are called 'corresponding sides' and the overlapping angles are called 'corresponding angles'.*

- use correct notations to indicate the conditions for two triangles to be congruent, e.g.,
    - *SSS (<u>Two triangles are congruent if</u> their three pairs of corresponding sides are equal.)*
    - *ASA (<u>Two triangles are congruent if</u> two pairs of corresponding angles and their included sides are equal.)*
    - *AAS (<u>Two triangles are congruent if</u> two pairs of corresponding angles and one pair of corresponding sides are equal.)*
    - *SAS (<u>Two triangles are congruent if</u> two pairs of corresponding sides and the included angles are equal.)*
    - *RHS (<u>Two right-angled triangles are congruent if</u> their hypotenuses and another pair of their sides are equal.)*
- follow English instructions on solving problems concerning this topic and work on related problems written in English.

Prerequisite knowledge: Nil

Materials: rulers, compasses, protractors, pencils and visualizer

Time: 4 lessons (4x40 minutes)

Procedure:

Lesson 1

1. The teacher should introduce the idea of congruence by using circles, squares and rectangles as examples.

2. The teacher should then ask the students to give the conditions which make 2 triangles congruent.

3. Using a visualizer, the teacher should demonstrate how to construct triangles with compasses, protractors and rulers.

4. The teacher should ask students to work in groups of four to construct triangles of given sides. (Each group will be assigned different sides)

5. The teacher should then ask students to compare their triangles and see if they can draw triangles with different shapes but with the same sides.

6. The teacher should then help the students to draw the conclusion: two triangles are congruent if their corresponding sides are equal (SSS).

Lesson 2

1. After students have understood the idea of "SSS", the teacher should ask them to draw triangles with two given angles and one given side between the two angles. Different groups will be responsible for drawing triangles with different sides and angles.

2. Students should then compare their triangles with their group-mates to see if the triangles are congruent. Each group should report their finding to the rest of the class.

3. The teacher should then draw the conclusion: two triangles are congruent if 2 corresponding angles and the included side are equal (ASA).

4. The teacher should then ask the students to draw triangles with two fixed angles and one fixed side (not included). Different groups will be responsible for drawing triangles with different sides and angles.

5. Students should compare their triangles with those of their group-mates to see if the triangles are congruent. Each group should report their finding to the rest of the class.

6. The teacher should then draw the conclusion: two triangles are congruent if 2 pairs of corresponding angles and one pair corresponding side are equal (AAS).

Lesson 3

1. The teacher should continue to ask students to draw triangles with two given sides and a given included angle. Similar to the previous lesson, students should compare their triangles with those of their group-mates and the teacher should guide students to come to the conclusion that SAS is a sufficient condition for two triangles to be congruent.

2. The teacher should continue to ask students to draw right-angled triangles with given hypotenuse and one given leg. Students should compare their triangles with those of their group-mates. The teacher should help the students to discover that RHS is a sufficient condition for two triangles to be congruent.

3. Then the teacher should give a summary of all the conditions for two triangles to be congruent.

Lesson 4

1. Using the examples in the worksheet, the teacher can explain to the students what corresponding sides and angles are. The teacher should stress that it is important to write the name of the congruent triangles correctly (according to the corresponding sides and angles).

2. In order to check students' understanding, the teacher should ask the students to practise doing the exercise at the end of Part A (if the students are not very familiar with the concept, the teacher can provide more examples and practise on the blackboard).

3. When the teacher thinks that students have understood the idea, he/she can ask the students to finish the classwork in Part B, which is an exercise to check their understanding of this lesson and also the previous one.

Explanatory Notes for Teachers:

1. When teaching about ASA and SAS, it is important for the teacher to emphasize the sequence of angles and sides (i.e. whether the angle is included between the two sides or not)..

2. The teacher can ask students to put their drawings on the visualizer. They can demonstrate that two triangles are congruent by drawing on a transparency or by cutting the triangles out.

3. Besides rotation, transformations like translation and reflection also preserve congruence. However, transformations like expansion and contraction do not preserve congruence.

Constructing Congruent Triangles

Name: _____   **Class:** _____ (   )

Part A: Congruent (全等) figures

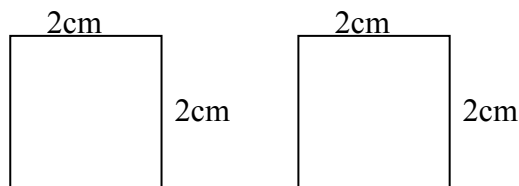If two figures can fit into each other, that is, they have the same size and shape, they are congruent figures.
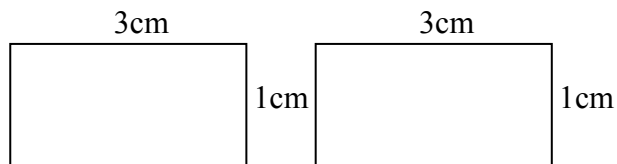
Example:
1. *Circles*
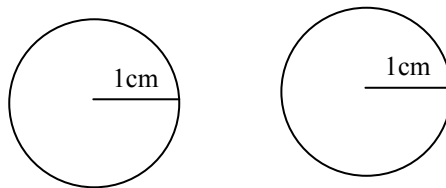
Their radii are equal.
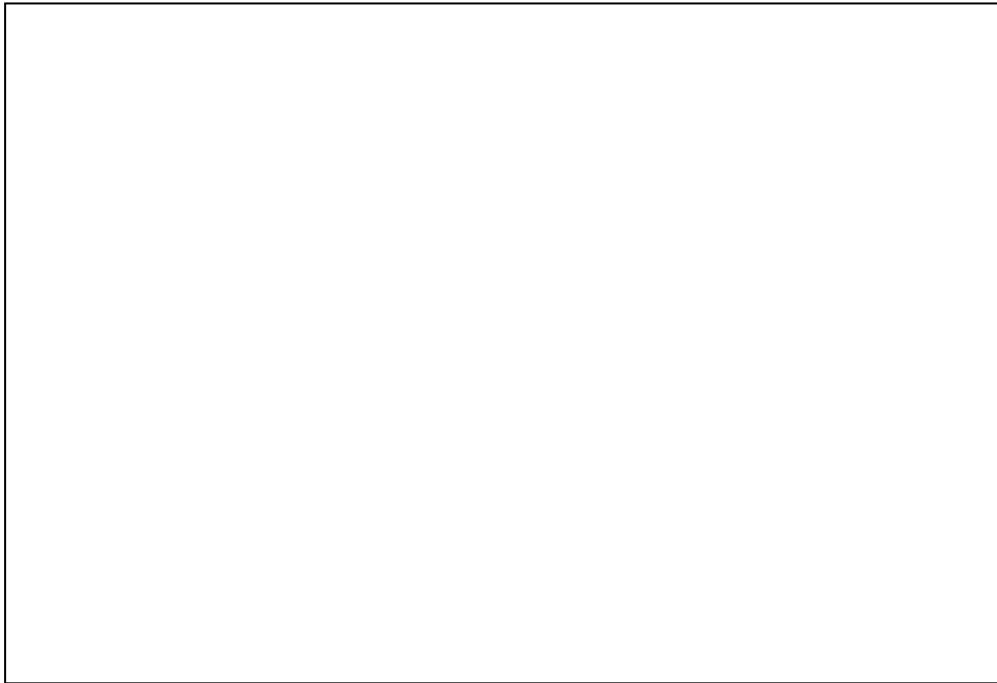
2. *Squares*

Their sides are equal.

3. *Rectangle*

Their lengths and widths are equal.

4. *Triangles*

What about triangles?

Part B: Congruent Triangles

1. In the space provided, and with the help of a ruler, compasses and a pencil, draw a triangle ABC with sides: AB =_____ cm, BC = _____ cm and CA= _____cm. (the value will be provided by the teacher for different groups).

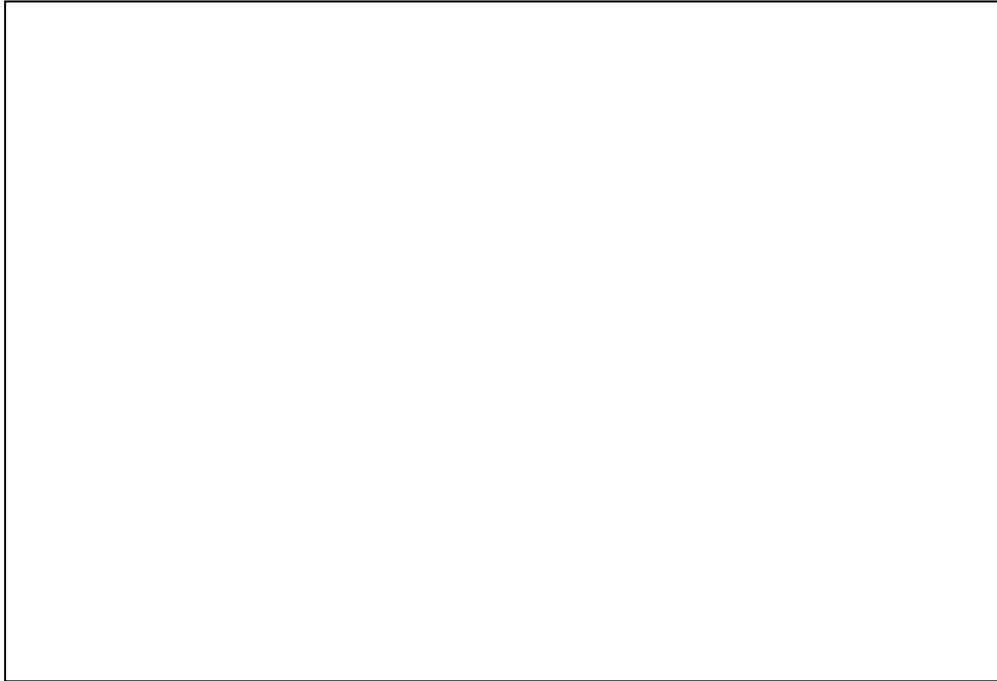Compare your triangle with those of your group-mates, are they different or are they the same?

    They are _____ .

Two triangles are congruent if _____

_____

        We can use _____ to represent this situation.

2. In the space provided, and with the help of a ruler, protractor and pencil, draw a triangle

   DEF in which: $\angle$DEF= _____°, EF= _____ cm and $\angle$EFD=_____° (the values will be

   provided by the teacher for different groups).

Compare your triangle with those of your group-mates, are they different or are they the
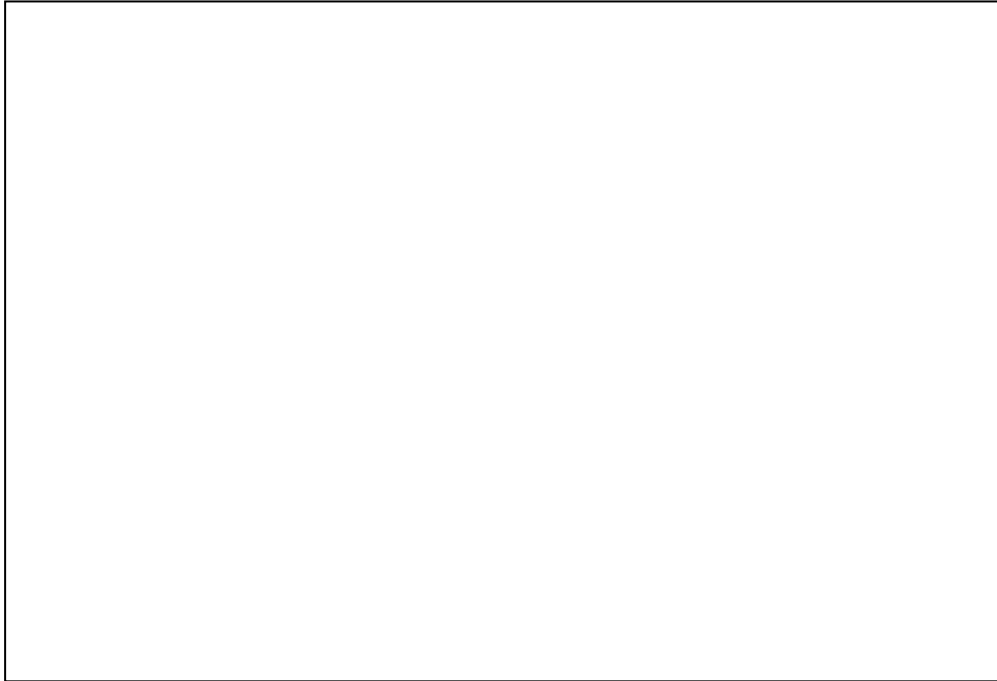
same?

They are _____ .

Two triangles are congruent if _____

_____

We can use _____ to represent this situation.

3. In the space provided, and with the help of a ruler, protractor and pencil, draw a triangle GHI in which: $\angle$GHI= _____°, IG= _____ cm and $\angle$HIG=_____° (the values will be provided by the teacher for different groups).

Compare your triangle with those of your group-mates, are they different or are they the same?

They are _____ .

Two triangles are congruent if _____

_____

We can use _____ to represent this situation.

4. In the space provided, and with the help of a ruler, protractor and pencil, draw a triangle PQR in which: $\angle$PQR= _____°, PQ= _____ cm and QR= _____ cm (the values will be provided by the teacher for different groups).

Compare your triangle with those of your group-mates, are they different or are they the same?

They are _____ .

Two triangles _____

_____

We can use _____ to represent this situation.

5. In the space provided, and with the help of ruler, protractor and pencil, draw a triangle

   XYZ in which: $\angle$XYZ= 90°, YZ= _____ cm and ZX=_____ cm (the values will be

   provided by the teacher for different groups).

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   Compare your triangle with those of your group-mates, are they different or are they the

   same?

   They are _____ .

   _____

   _____

   We can use _____ to represent this situation.

Summary:

The conditions for two triangles to be congruent include:

1) _____   2) _____   3) _____

4) _____   5) _____

Notation of Congruent Triangles

Name: _____   **Class:** _____ (   )

Part A:

Congruent Triangles

If two triangles can fit into each other perfectly, they are congruent triangles.

ΔABC, ΔDEF and ΔPQR are congruent triangles.

When you cut the triangles out and rotate (轉) them, they can exactly overlap/coincide (重疊) with each other.
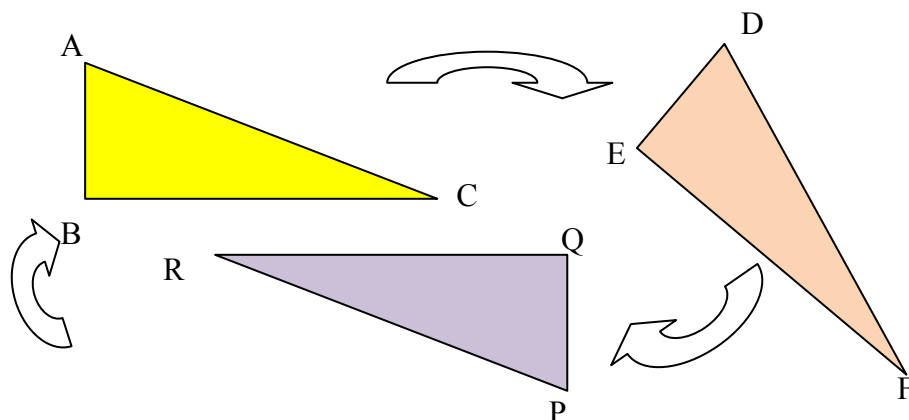
The overlapping sides are called "**corresponding sides**" (對應邊) and the overlapping angles are called "**corresponding angles**" (對應角).

From the above diagram, ΔABC and ΔPQR are congruent triangles. Then ∠**A** and ∠**P,** ∠**B** and ∠**Q,** ∠**C** and ∠**R** are corresponding angles. AB and PQ, BC and QR, CA and RP are corresponding sides.

We use the symbol "≅" to indicate that two triangles are congruent.

i.e. ΔABC ≅ ΔDEF

* It is very important that the name of the corresponding angles must be in the same position.

Think about:

From the above diagram, which
one is correct? (Put ✓ or ×)
1) ΔEDF ≅ ΔPQR   (   )
2) ΔEDF ≅ ΔQPR   (   )
3) ΔDEF ≅ ΔRPQ   (   )

Exercise:
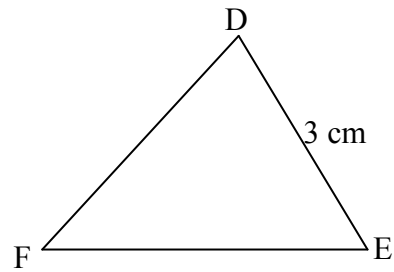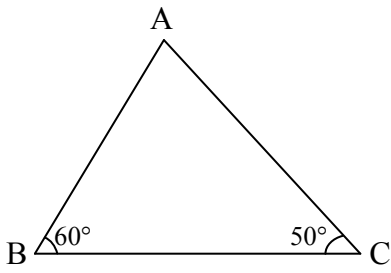
If ΔABC ≅ ΔDEF, find ∠E, ∠F and the length of AB.

Part B: classwork

Determine which pairs of triangles in the following are congruent (Name the triangles and give reasons).

Answer:

1) Δ _____ ≅ Δ _____   (   )
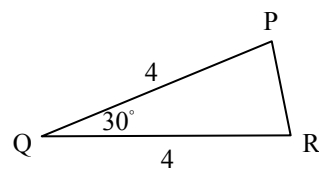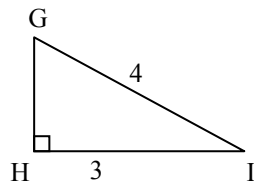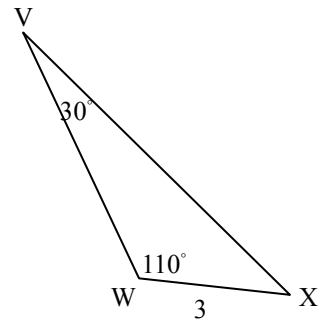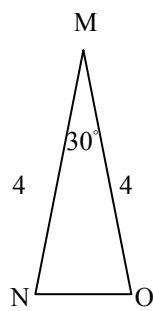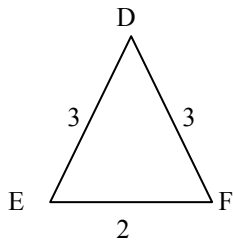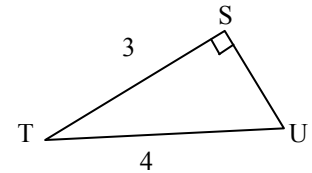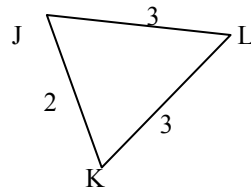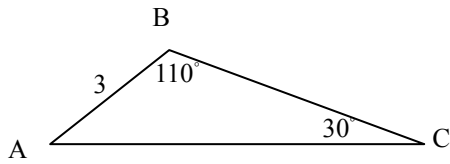
2) _____ ≅ _____   (   )

3) _____   (   )

4) _____

Suggested Answers

Constructing congruent triangles

Part B:

1. They are the same.   Two triangles are congruent if their three pairs of corresponding sides are equal.   SSS

2. They are the same.   Two triangles are congruent if two pairs of corresponding angles and their included sides are equal.   ASA

3. They are the same.   Two triangles are congruent if two pairs of corresponding angles and one pair of corresponding sides are equal.   AAS

4. They are the same.   Two triangles are congruent if two pairs of corresponding sides and the included angles are equal.   SAS

5. They are the same.   Two right-angled triangles are congruent if their hypotenuses and another pair of sides are equal.   RHS

Summary:

1) SSS   2) SAS   3) ASA

4) AAS   5) RHS

Part A:

Think about: 1) ×   2)✔   3)×

Exercise: ∠E=∠B=60°

  ∠F=∠C=50°

  AB=DE=3cm

Part B:

1) △ MNO ≅ △ QRP   (SAS)

2) △ DEF ≅ △ LJK/LKJ   (SSS)

3) △ ABC ≅ △ XWV   (AAS)

4) △ GHI ≅ △ UST   (RHS)