S2 Topic 2

Irrational Numbers

Level: Key Stage 3

Dimension: Number and Algebra
Module: Number and Number Systems
Unit: Rational & Irrational numbers

Student ability: Average

Content Objectives:

After completing the activity, students should be able to identify rational numbers and irrational numbers.

Language Objectives:

After completing the activity, students should be able to

- understand the English terms for the concepts related to discussions of rational numbers and irrational numbers (e.g., *square, square number, square root, positive square root, negative square root, non-negative number, denoted by, rational numbers, finite numbers, integers, ratio, simple fraction, irrational numbers, infinite, non-recurring decimal, approximation, not accurate*);
- understand the English expressions for explaining the relations between square numbers and square roots, e.g.,
  - *If a is a non-negative number, then the* ***square root*** *of a is a number x such that $x^2 = a$, usually, the non-negative square root of a is denoted by $\sqrt{a}$.*
  - For example, 25 has two square roots, 5 and –5, since $5^2 = 25$ and $(-5)^2 = 25$.

    $5=\sqrt{25}$ is the ***positive square root*** of 25.

    $-5=-\sqrt{25}$ is the ***negative square root*** of 25.

    25 is the square of an integer, it is a ***square number***.

    The ***positive square root*** of 25 is 5.

- understand the English expressions for explaining the differences between rational numbers and irrational numbers, e.g.,
  - *Integers, fractions, finite decimals and recurring decimals are* ***rational numbers***.

- *A **rational number** can be written as a ratio of two integers (i.e., a simple fraction). For example,*

  1.5 is rational, because it can be written as the ratio 3/2

  7 is rational, because it can be written as the ratio 7/1

  0.317 is rational, because it can be written as the ratio 317/1000

- *An **irrational number** is a real number that cannot be written as a simple fraction, for example, π is an irrational number.*

  *π = 3.14159265358979323846264338327950 (and more...). We **cannot** write down a simple fraction that equals π because it is an infinite and non-recurring decimal. The popular approximation of $^{22}/_{7}$ = 3.14 is only an approximation.*

- *Other examples of irrational numbers are:* $\sqrt{10}$, $\sqrt[3]{9}$, $\sqrt{\frac{2}{3}}$

• follow English instructions on solving problems concerning this topic and work on related problems written in English.

Materials required:

PowerPoint file, computer and projector

Prerequisite knowledge:

Students should have learned about Pythagoras' Theorem in Chinese.

Time: 1 lesson (40 minutes)

Procedure:

1. The teacher should first distribute the worksheet to students. Then he/she should remind students of what they have learned about Pythagoras' Theorem.
2. After that, the teacher should use Example 1 to teach the terms "square number" and "square root", making sure to provide a clear model of the pronunciation of the terms for students to imitate and learn.

3. The teacher should then ask the students to do the exercise. Students should answer in English.
4. Then, the teacher should use Example 2 in the worksheet to illustrate the concept of irrational numbers, again making sure that a clear model of pronunciation is provided for students to learn.
5. The teacher should then introduce the game.
6. The teacher should form groups of 3 students. A group leader should be given the responsibility for raising his or her hand, the other group-mates should be responsible for answering the questions in English and writing the answers on the board.
7. The teacher should then show the PowerPoint slides prepared beforehand.
8. When the teacher shows the PowerPoint examples one by one, the group leaders are expected to raise their hands if their group members can identify the terms.
9. When the teacher calls out the group number, group-mates will call out the answer (if the pronunciation is incorrect, the group will get no mark). If the pronunciation is correct, the teacher will ask another group-mate to come out and write the words on the blackboard (if the spelling is incorrect, the group will get no mark).
10. Students should answer as quickly as possible and need not wait for all the examples to be provided.
11. The group with the greatest number of correct answers will be the winner.
12. The teacher can give a prize to the winners (e.g. a candy or a bonus mark)

1. Explanation notes for teachers:

1. The introduction of irrational numbers through the examples of “finding the hypotenuse of a right-angled triangle” helps students to learn the names of new terms more easily. They should feel a practical need to learn the concept of irrational numbers.
2. The teacher may explain to the students that integers, fractions, finite decimals and recurring decimals can be expressed as ratios of two integers with no common factor other than 1 and hence rational numbers are also known as “可比數” in Chinese.
3. The teacher should help to explain why the square root of 52 is not a rational number in Example 2 if the students find it too difficult.
4. The teacher should set rules for the students before the start of the game (e.g. if the students are too noisy, the teacher could stop the game).
5. Grouping the students into threes aims to create greater involvement of the whole class.
6. The idea of giving examples and guessing the keywords is not limited to any topic. It can be carried out just after the teaching of the topic or near a test or examination.
7. The questions in the PowerPoint file are examples only. The teacher can add as many questions as appropriate.
8. There can be more than one answer to the PowerPoint examples. For example, if students answer “rational number” to the slide of “whole number”, the teacher can still say that they are correct and give marks to the group. Then the teacher can say that there can be more than one answer and invite other groups to continue guessing the second answer.

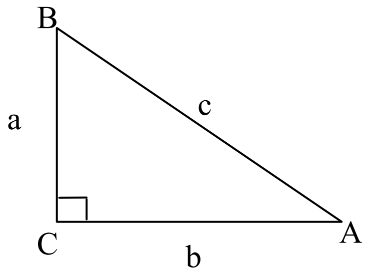
Irrational Numbers

Name:_______________ Class:________ (   )

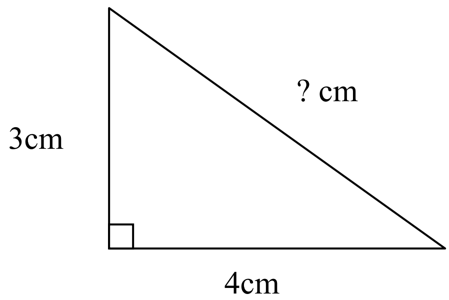
We have learned from Pythagoras' Theorem (畢氏定理) that:

In a right-angled triangle, the sum of the squares of the two legs is equal to the square of the hypotenuse (斜邊).

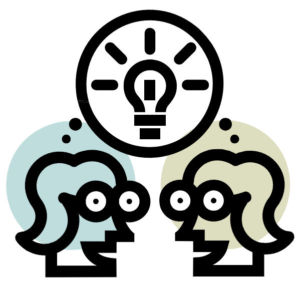
If C = 90° in ΔABC, $a^2 + b^2 = c^2$,



Example 1:



If the lengths of the 2 legs of a right-angled triangle are 3 cm and 4 cm, what is the length of the hypotenuse?
The square of the length of hypotenuse equals to: (3^2+4^2) cm^2 = 25 cm^2
The square of 5 is 25. The length of the hypotenuse = 5 cm



If a is a non-negative number, then the ***square root*** of a is a number x such that $x^2 = a$. Usually, the non-negative square root of a is denoted by $\sqrt{a}$.

For example, 25 has two square roots (平方根), 5 and –5, since $5^2 = 25$ and $(-5)^2 = 25$. $5=\sqrt{25}$ is the ***positive square root*** of 25$-5=-\sqrt{25}$ is the ***negative square root*** of 25.

25 is the square of an integer, it is a "***square number***" (平方數).
i.e. the "***positive square root***" (平方根) of 25 is 5.

Exercise 1:

1. Give 3 examples of square numbers.

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2. Write down their corresponding positive square roots.

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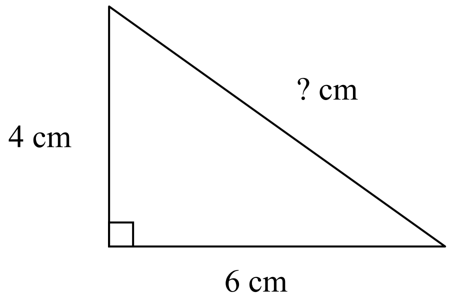
Example 2:



If the lengths of the 2 legs of a right-angled triangle are 4 cm and 6 cm, what is the length of the hypotenuse?

The square of the length of hypotenuse is equal to: (4^2+6^2) cm^2 = 52 cm^2

The length of the hypotenuse = $\sqrt{52}$cm = ? cm

We have learned about integers (整數), fractions (分數), finite decimals (有盡小數) and recurring decimals (循環小數) before. We call these numbers "rational numbers" (有理數, 可比數).

Now, can you find a rational number whose square is 52?

As 52 is not a square number its square root cannot be a whole number.

Can the answer be a finite or recurring decimal, or a fraction?

Why?/Why not?

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∴ $\sqrt{52}$ cannot be expressed as a fraction or a decimal.

We call this kind of number an "***irrational number***" (無理數).

Other examples are: $\sqrt{10}$, $\sqrt[3]{9}$, $\sqrt{\frac{2}{3}}$

Note:

When you find the value of $\sqrt{52}$ using a calculator, it gives you a decimal number. This is only an approximate value (近似值). In fact, $\sqrt{52}$ is an infinite and non-recurring decimal.

Besides numbers with radical sign, π is also an irrational number. $\frac{22}{7}$ is also an approximate value of π.

Exercise 2:

1. Give 3 examples of irrational numbers.

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2. Give 3 examples of rational numbers.

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Suggested answers

Exercise 1:

1. 9, 100, 625

2. 3, 10, 25

No. It is because a decimal or fraction, not a whole number, can be expressed in the form $\frac{p}{q}$, where p and q are integers and q is not equal to 1. p and q do not have a common factor other than 1. The square of $\frac{p}{q}$ is $\frac{p^2}{q^2}$. p^2 and q^2 do not have a common factor and q^2 is not equal to 1. $\frac{p^2}{q^2}$ cannot be an integer and therefore, the square of a decimal or a fraction cannot be 52.

Exercise 2:

1. Any 3 irrational numbers.
2. Any 3 rational numbers.