Distance between two points

Level: Key Stage 3

Dimension: Measures, Shape and Space
Module: Learning Geometry through an Analytical Approach
Unit: Coordinate Geometry of Straight Lines

Student ability: Average

Content Objectives:

After completing the activity, students should be able to

- gain further practice to find the distance between two points using Pythagoras' Theorem'
- derive the Distance formula, and
- calculate the distance between two points by using the formula.

Language Objectives:

After completing the activity, students should be able to

- understand and use the key English terms (e.g., distance, rectangular coordinates, Pythagoras' Theorem, right-angled triangle, and surd) for explaining how to calculate the distance between two points,
- state in English the coordinates of point, using the sentence pattern: The coordinates of __ are (__, __).
- answer in English the questions of "What is the distance between point __ and point __?" by using the sentence pattern: *The distance between point __ and point __ is __.*
- State in English the distance formula: The distance between two points with coordinates (x_1, y_1) and (x_2, y_2) equals to $\sqrt{(x_2 - x_1)^2 + (y_1 - y_2)^2}$
- follow English instructions on solving problems concerning this topic and work on related problems written in English.

Material required: geometry software such as *GeoGebra* or *Sketchpad*

Prerequisite knowledge:

Students should have gained an understanding through the medium of Chinese and an ability to use the rectangular coordinate systems to describe the positions of points in a plane and be able to locate a point in a plane by means of an ordered pair in the rectangular coordinate system.

Time: 2 lessons (2 x 40 minutes)

Procedure:

Lesson 1

1. The teacher should set up the computer and projector to show the rectangular coordinate plane on the board.
2. The teacher should then distribute the worksheet to students.
3. The teacher then introduces the terms to be used (Table 1).
4. The teacher should then discuss with the students how to read the coordinates of points P, Q, R, S and T (Question 1).
5. The teacher then shows students how to plot different lines on the coordinate plane and find the distance between points with the same x- or y- coordinates (Question 2).
6. The teacher should then remind students of what they have learnt before about Pythagoras' Theorem. Then, the teacher can ask students to complete Question 3.
7. The teacher should then ask students to report and explain what they have done in Question 3.
8. Finally, students should be asked to complete Exercise I and II as class exercises to check that they can find the distance between two points using Pythagoras' Theorem.

Lesson 2

1. The teacher should ask students to complete Question 4 in 5-8 minutes.
2. The teacher then asks the students to present and explain what they have done and discovered in Question 4.
3. The teacher should then ask the students to complete Question 5 after which the teacher can explain to students that the result (distance) is the same even though they use the coordinates of point 1 minus those of point 2, or vice versa.

4. The teacher should finally ask students to finish Exercise III.

Explanatory Notes for Teachers:

1. Usually, teachers will add extended learning activities (ELA) at the very end of a sequence of teaching when students have gained a certain understanding of the topic. Actually, we can also select some small, independent and easier units and use English only to teach these units at any time. The unit “Distance between two points” was selected because it is an independent not dependent upon other content topics. At the same time, as students have some previous knowledge about coordinate planes and Pythagoras’ Theorem, the topic should be relatively easy to understand in English.
2. The use of visual tools (e.g. the projector, the graph software) can facilitate student learning and help the students to follow the teacher’s explanations. The software can help teachers to show the points and lines more flexibly but the PowerPoint file provided can be a substitute if the software is not available. Teachers can also use the diagrams in the worksheet for the same purpose.
3. The distance formula relates to two items of prerequisite knowledge, i.e. the rectangular coordinate plane and Pythagoras’ Theorem. The teacher should link up students’ past learning experience with the new knowledge.
4. The teacher should emphasize that the formula could be derived by students themselves. Compared with the learning of formulae which only involves memorization, this could increase students’ confidence.

S3 Mathematics

Coordinate Geometry

Distance between two points

Name: _______________   Class: ___________ (   )

Vocabulary (Table 1):

distance 距離	rectangular coordinates 直角坐標
Pythagoras' Theorem 畢氏定理	right-angled triangle 直角三角形
Surd 不盡根	

Question 1:

In a rectangular coordinate plane (Fig 1):

What are the coordinates of points P, Q, R, S and T?

The coordinates of point P are (______, ______).

The coordinates of point Q are (______, ______).

The coordinates of point R are (______, ______).

The coordinates of point S are (______, ______).

The coordinates of point T are (______, ______).

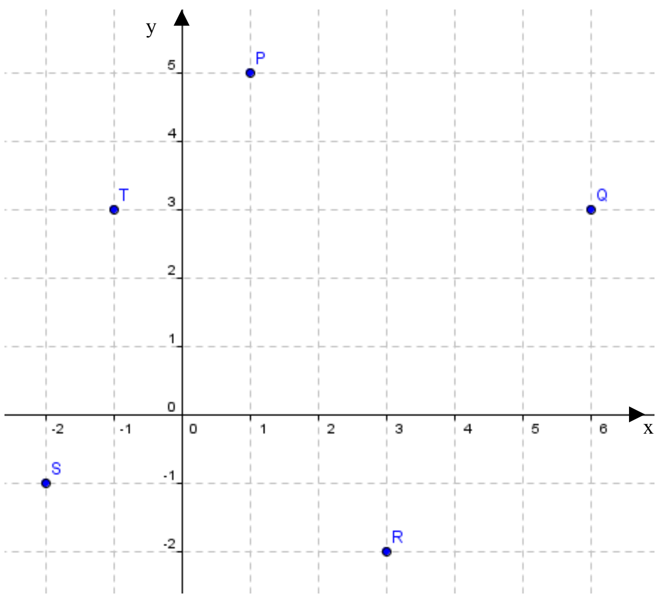


Fig. 1

Question 2:

In Figure 2,

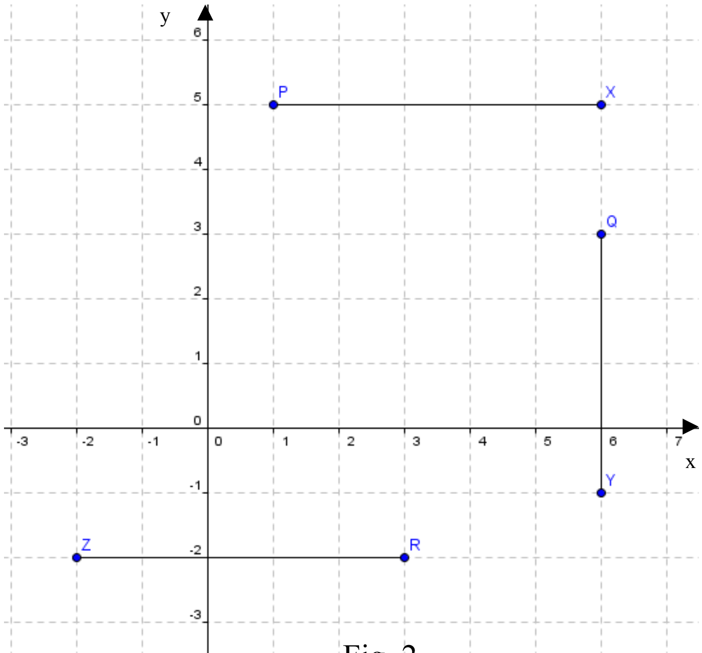


Fig. 2

a) What is the distance between point P and point X?

The distance between point P and point X is:

$(\quad) - (\quad)$

= ________.

b) What is the distance between point Q and point Y?

The distance between point Q and point Y is:

$(\quad) - (\quad) =$ ________________.

c) What is the distance between point R and point Z?

The distance between point R and point Z is: $(\quad) - (\quad) =$ ____________.

Question 3:

In Figure 3,

a) What are the coordinates of C?

The ______________________

of C are (_____, _____).

b) Find the length of AC and BC.

AC =

BC =

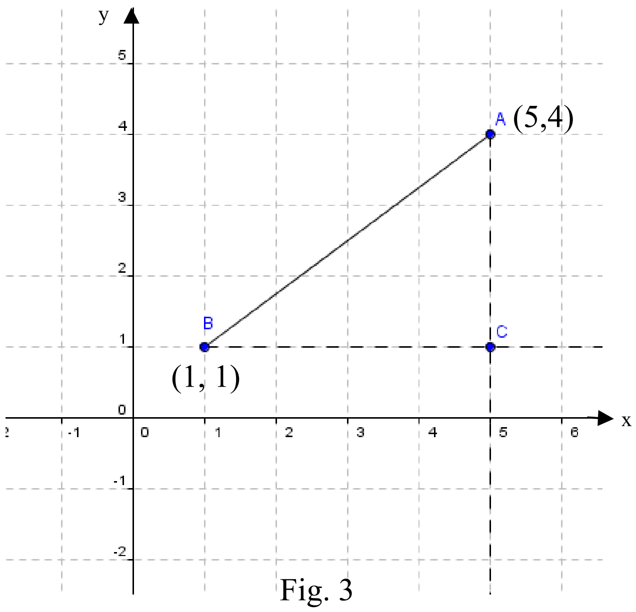


Fig. 3

c) $\triangle$ABC is a ______________________ triangle.

d) We can use __________________ Theorem to find the length of AB.

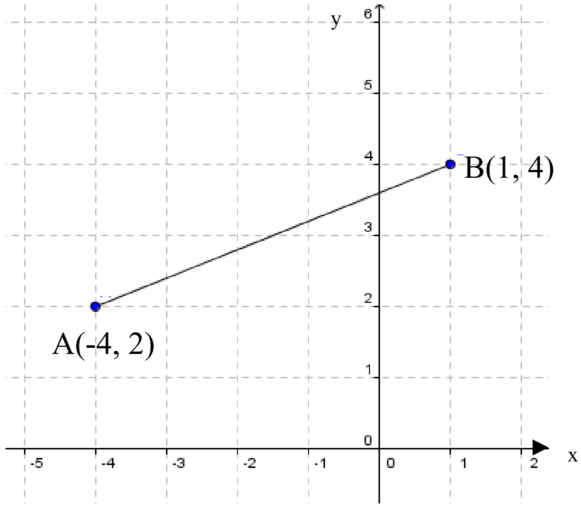
$$AB = \sqrt{(\quad)^2 + (\quad)^2}$$

$=$

$=$

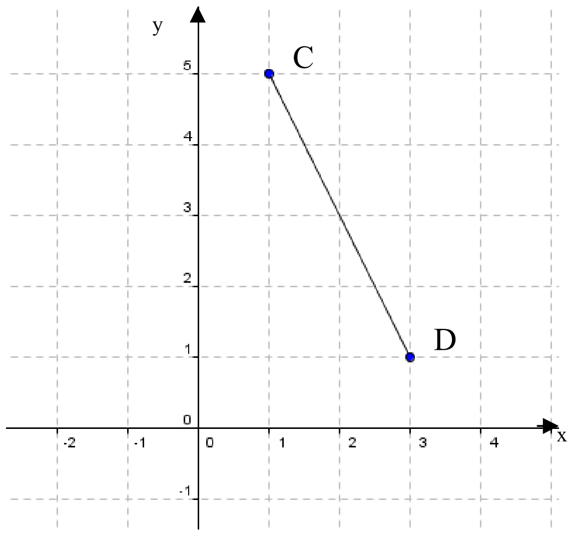
Exercise I:
(Express your answer in surd form if necessary.)

Find the distance between A and B.



Exercise II:
(Express your answer in surd form if necessary.)

Find the distance between C and D.



Question 4:

In Figure 4, how can we find the distance between P and Q?

a) What are the coordinates of R?

The ______________________

are (_____, _____).

b) Similar to Question 3,

PR =

QR=

PQ=

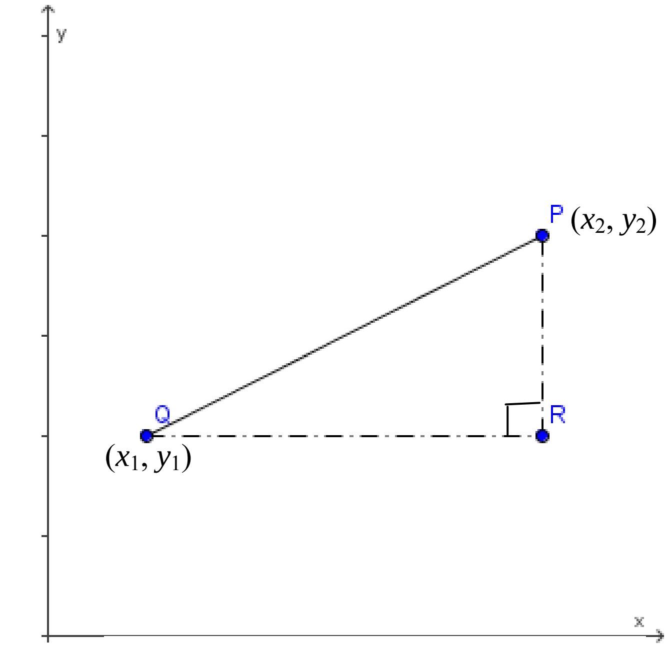


Fig. 4

$\therefore$ The distance between two points with coordinates (x_1, y_1) and (x_2, y_2) is:

Question 5:
In Figure 5, how can we find the distance between P and Q?

a) What are the coordinates of R?

The ______________________

are (_____, _____).

b) Similar to Question 4,
PR =

QR=

PQ=

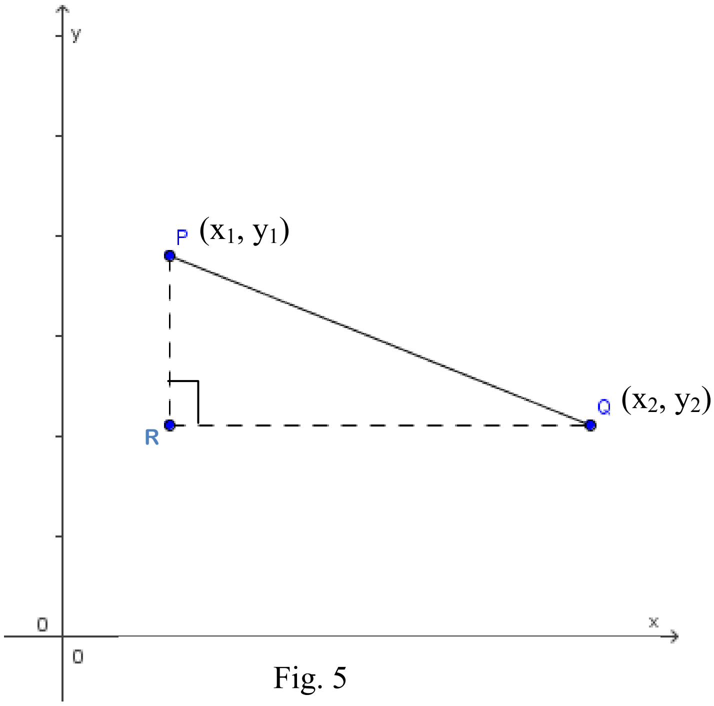


Fig. 5

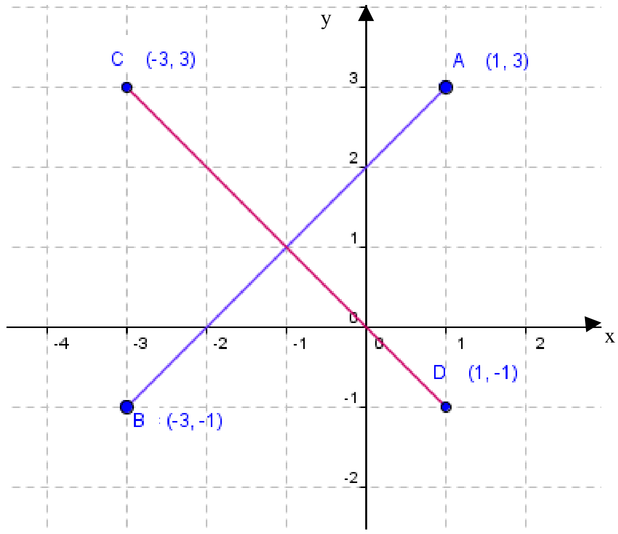
∴ The distance between two points with coordinates (x_1, y_1) and (x_2, y_2) is:

Comparing the results of Question 4 and Question 5, we can find that the distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by ______________________________. This is called the "**distance formula**".

Exercise III:

a) Use the distance formula to find AB.

AB =



b) Use the distance formula to find CD.

CD =

c) Use the distance formula to find the distance between P (-8, -5) and Q (-3, -3).

d) Use the distance formula to find the distance between R (-5, 4) and S (-1, -2).

e) If the coordinates of A, B and C are (5,4) , (-1, 2) and (3,-2) respectively, find the perimeter (周界) of triangle ABC.

Answers

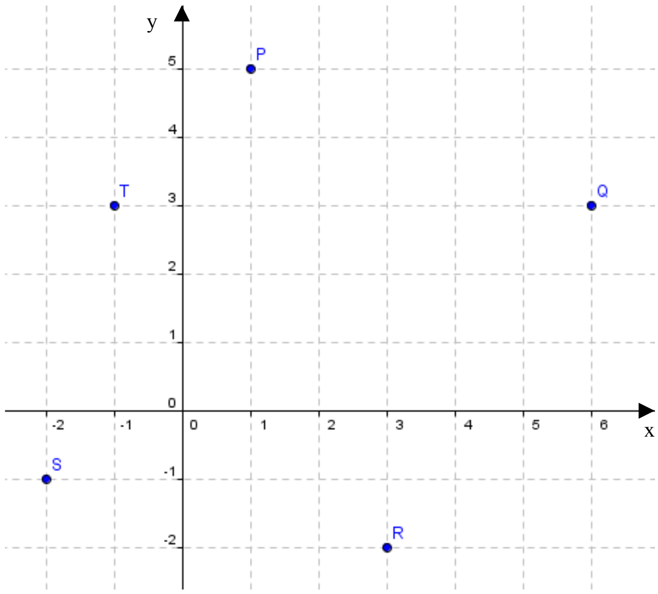
1. The coordinates of point P are (__1__, __5__).

The coordinates of point Q are (__6__, __3__).

The coordinates of point R are (__3__, __−2__)

The coordinates of point S are (__−2__, __−1__).

The coordinates of point T are (__−1__, __3__).



2. In Figure 2:

a) What is the distance between point P and point X?

The distance between point P and point X is:

(6) – (1)

= ____5______.

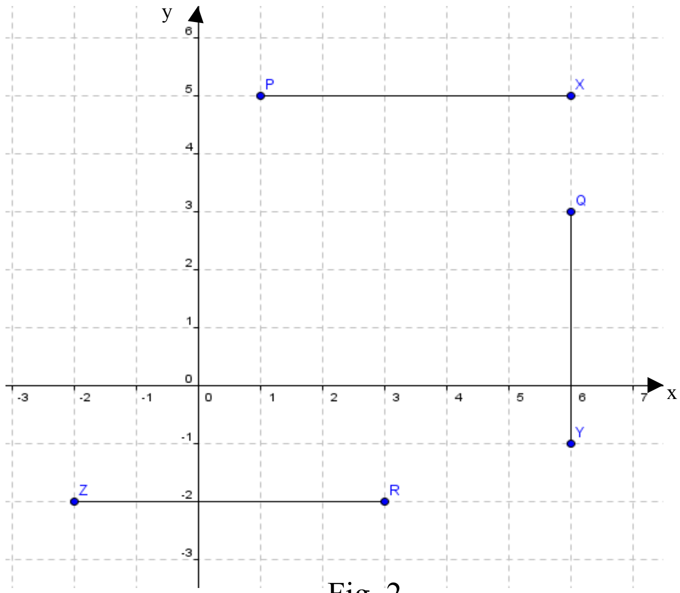


Fig. 2

b) What is the distance between point Q and point Y?

The distance between point Q and point Y is:

(3) – (–1) = __4_____.

c) What is the distance between point R and point Z?

The distance between point R and point Z is: (3) – (–2) =______5_________.

3. In Figure 3:

a) What are the coordinates of C?

The ______coordinates______

of C are (__5___, __1___).

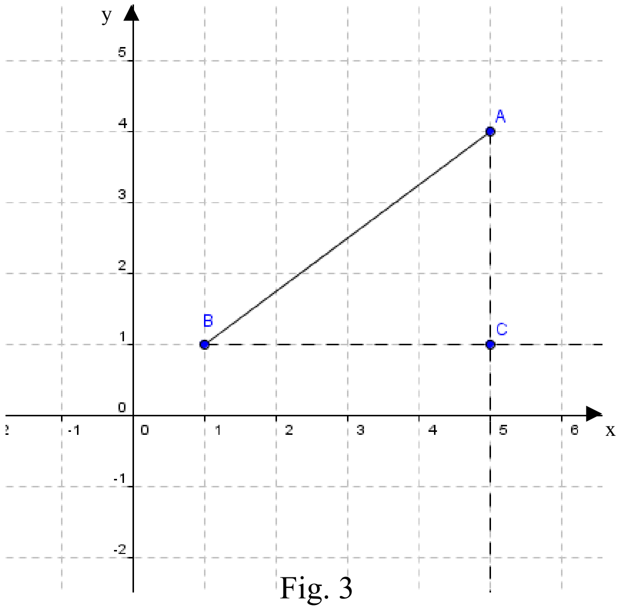


Fig. 3

b) Find the length of AC and BC.

AC = 4 − 1

= 3

BC = 5 − 1

= 4

c) $\triangle$ABC is a __right-angled_____________ triangle.

d) We can use __Pythagoras'________ Theorem to find the length of AB.

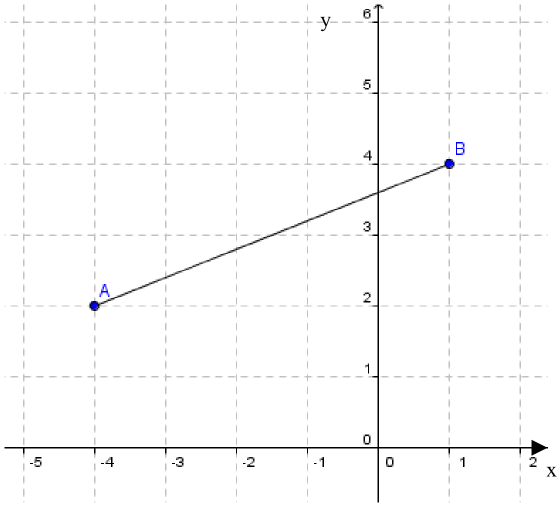
$AB = \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16}$

= 5 (units)

Exercise I:

Find the distance between A and B.

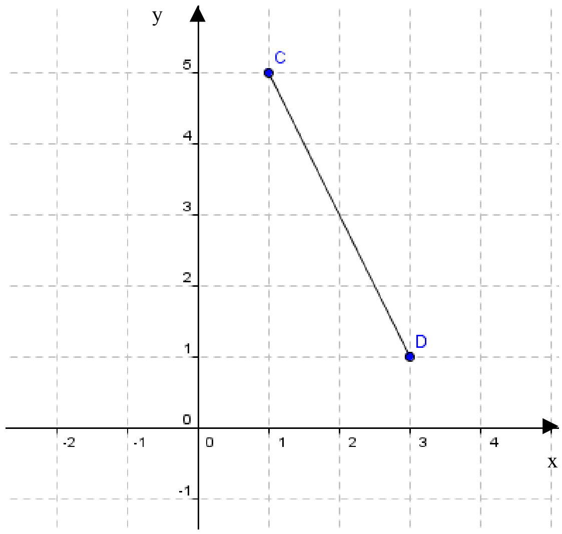


$$AB = \sqrt{(2^2 + 5^2)}$$

$$= \sqrt{29}$$

Exercise II:

Find the distance between C and D.



$$CD = \sqrt{4^2 + 2^2}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$

4. In figure 4, how can we find the distance between P and Q?

a) What are the coordinates of R?

The coordinates of R

are $(x_2,\ y_1)$.

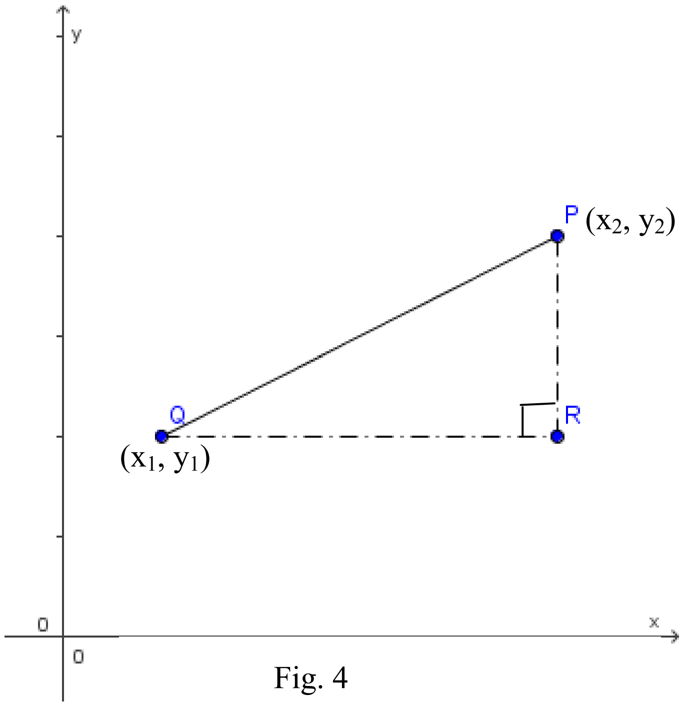


Fig. 4

b) Similar to question 3,

$PR = y_2 - y_1$

$QR = x_2 - x_1$

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\therefore$ The distance between two points with coordinates (x_1, y_1) and (x_2, y_2) is:

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

5. In figure 5, how can we find the distance between P and Q?

a) What are the coordinates of R?

The coordinates of R are

(x_1, y_2).

b) Similar to question 4,

$PR = y_1 - y_2$

$QR = x_2 - x_1$

PQ

$= \sqrt{(x_2 - x_1)^2 + (y_1 - y_2)^2}$

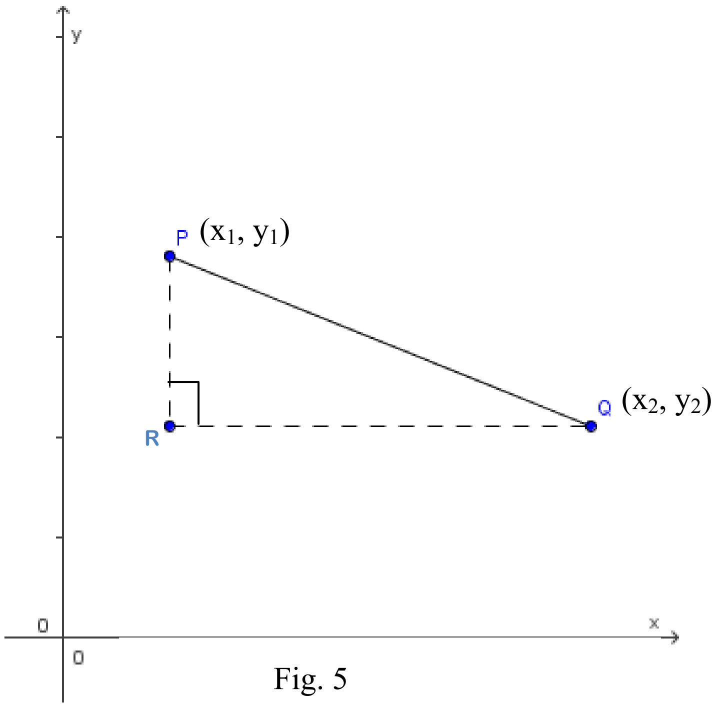


Fig. 5

$\therefore$ The distance between two points with coordinates (x_1, y_1) and (x_2, y_2) is:

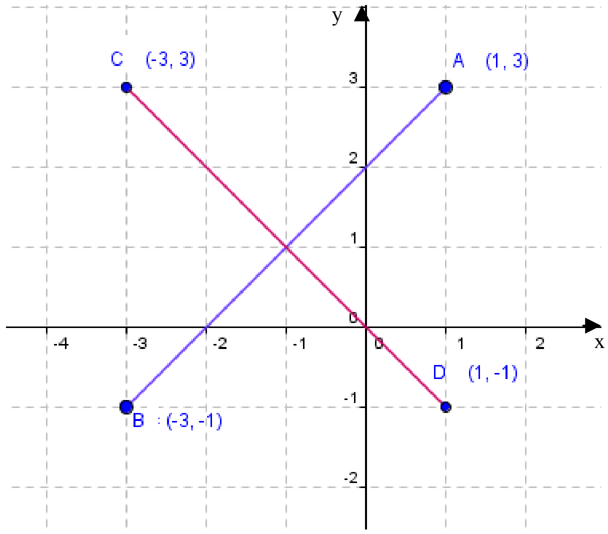
$$\sqrt{(x_2 - x_1)^2 + (y_1 - y_2)^2}$$

Comparing the results of question 4 and question 5, we can find that the distance between P(x_1, y_1) and Q(x_2, y_2) is given by $\underline{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$.

This formula is called the "**distance formula**".

Exercise III:

a) Use the distance formula to find AB.



$$AB = \sqrt{(-3-1)^2 + (-1-3)^2}$$

$$= \sqrt{(-4)^2 + (-4)^2}$$

$$= \sqrt{32}$$

$$= 4\sqrt{2}$$

b) Use the distance formula to find CD.

$$CD = \sqrt{[1-(-3)]^2 + (-1-3)^2}$$

$$= \sqrt{4^2 + (-4)^2}$$

$$= \sqrt{32}$$

$$= 4\sqrt{2}$$

c) Find the distance between P (-8, -5) and Q (-3, -3).

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{[-3-(-8)]^2 + [-3-(-5)]^2}$$

$$= \sqrt{5^2 + 2^2}$$

$$= \sqrt{29}$$

d) Find the distance between R (-5, 4) and S (-1, -2).

$$RS = \sqrt{[-1-(-5)]^2 + (-2-4)^2}$$

$= \sqrt{4^2 + 6^2}$

$= \sqrt{52} = 2\sqrt{13}$

e) If the coordinates of A, B and C are (5,4) , (-1, 2) and (3,-2) respectively, find the perimeter (周界) of triangle ABC.

PQ = $\sqrt{[5-(-1)]^2 + (4-2)^2}$

$= 2\sqrt{10}$ (units)

QR= $\sqrt{[2-(-2)]^2 + (-1-3)^2}$

$= 4\sqrt{2}$ (units)

RP= $\sqrt{[4-(-2)]^2 + (5-3)^2}$

$= 2\sqrt{10}$ (units)

The perimeter of triangle ABC = $2\sqrt{10} + 4\sqrt{2} + 2\sqrt{10}$ (units)

$= 4(\sqrt{10} + \sqrt{2})$ (units)