S2 Mathematics

Trigonometry

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Background

- CMI school up to 2009-2010
  - Students fear to use English

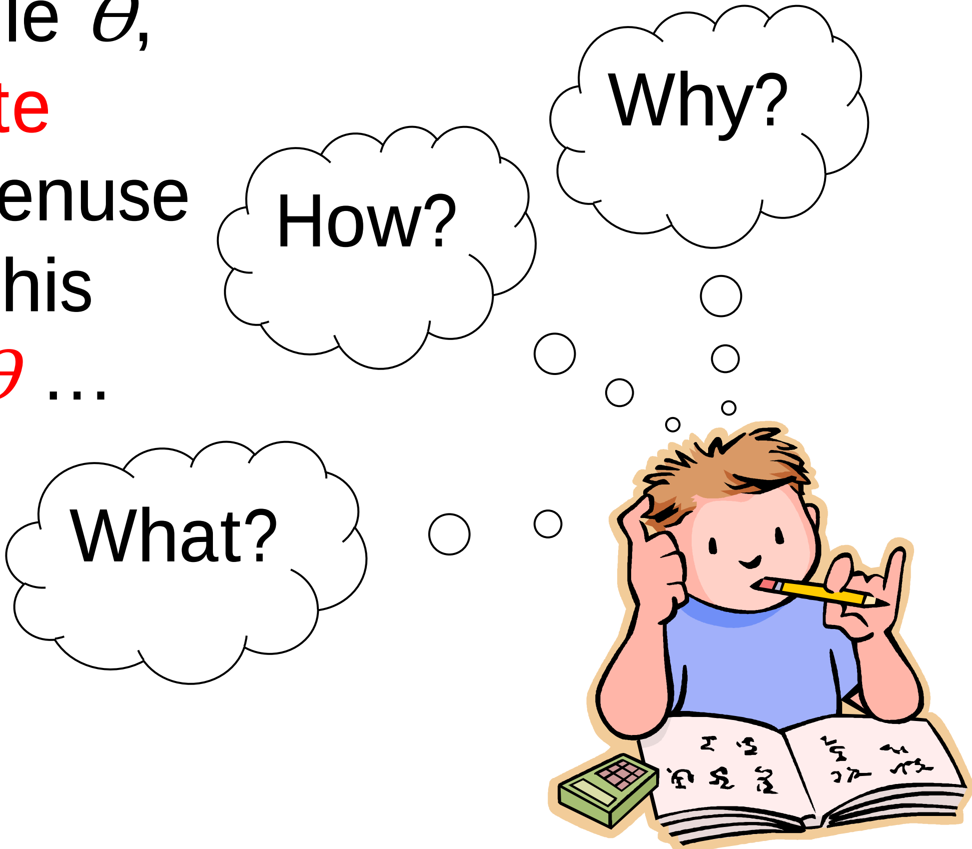
- Problems of direct instruction
  - Students find it difficult to develop concepts

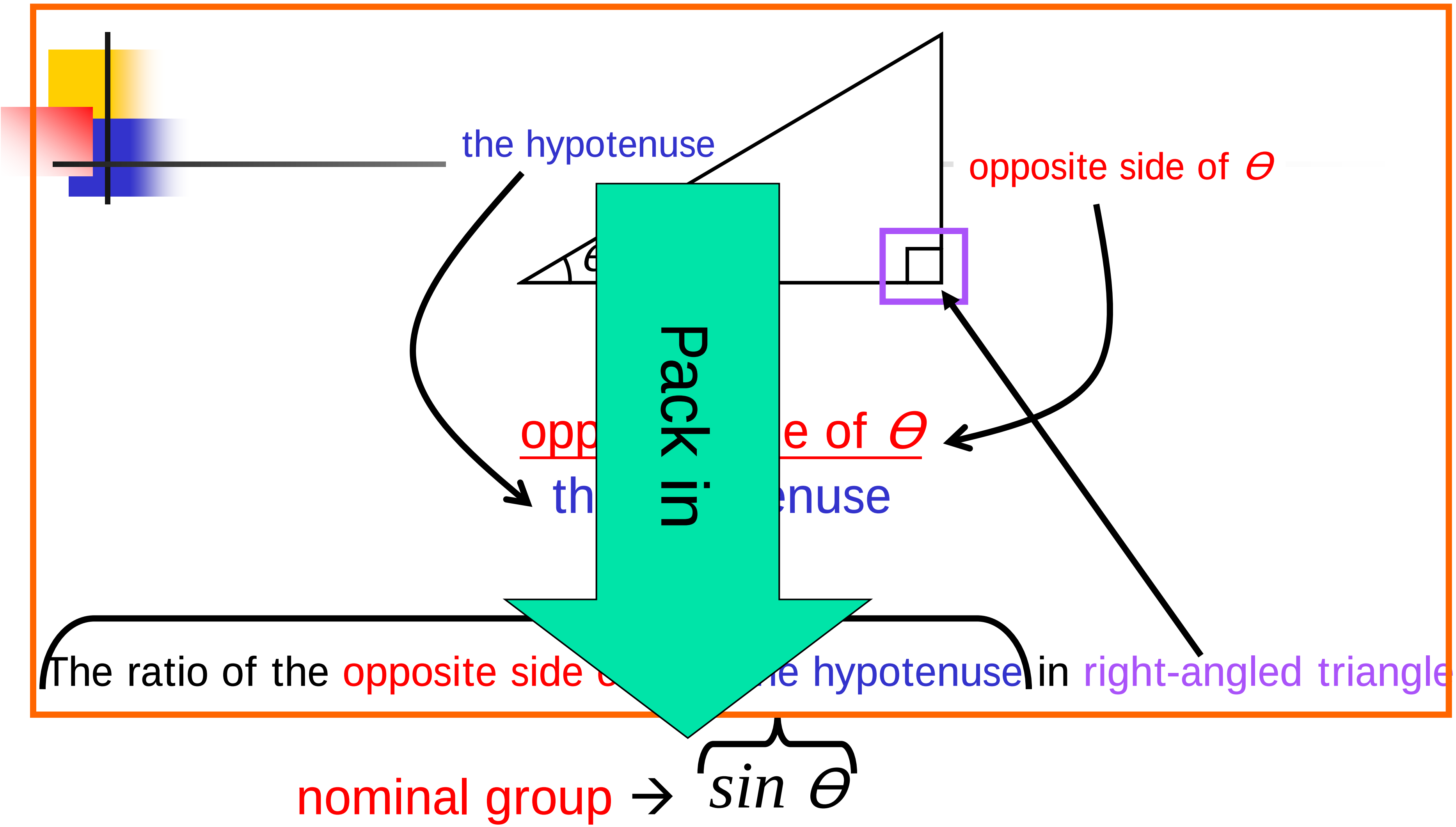
Main ideas

- Abstraction through nominalisation
- Making meaning in mathematics through: language, visuals & the symbolic
- The Teaching Learning Cycle

Direct instruction

For a right-angled triangle with a given acute angle θ, the ratio of the opposite side of θ to the hypotenuse is a constant. We call this ratio the sine ratio of θ …





the hypotenuse
opposite side of θ
θ
Pack in
opp e of θ
th enuse
The ratio of the opposite side o e hypotenuse in right-angled triangle
nominal group →
sin θ

Problems

Some students :

- $\sin \theta = 1/2$

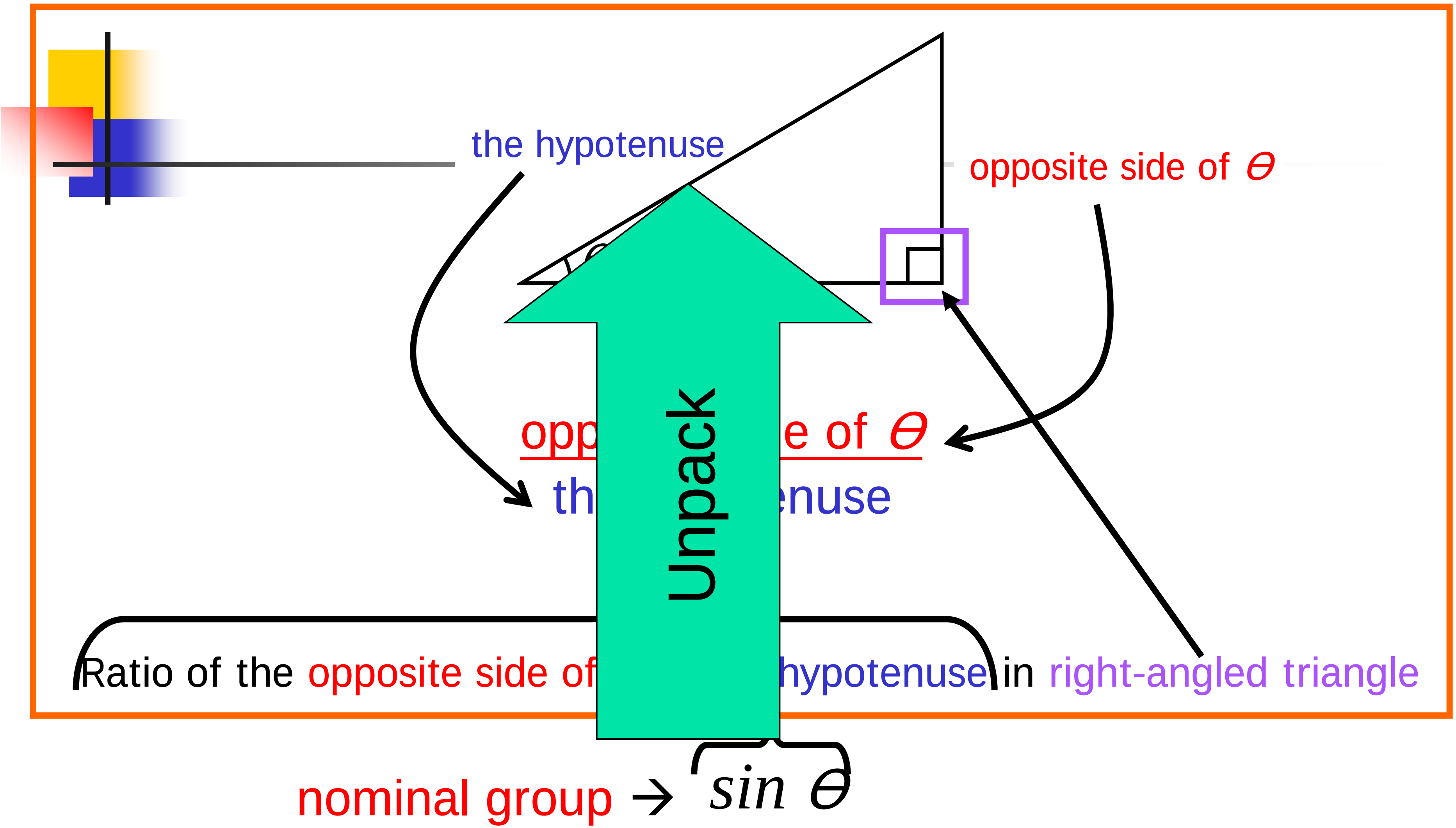
  $= 30^{o}$

- $\sin (\theta/2) = 1/3$

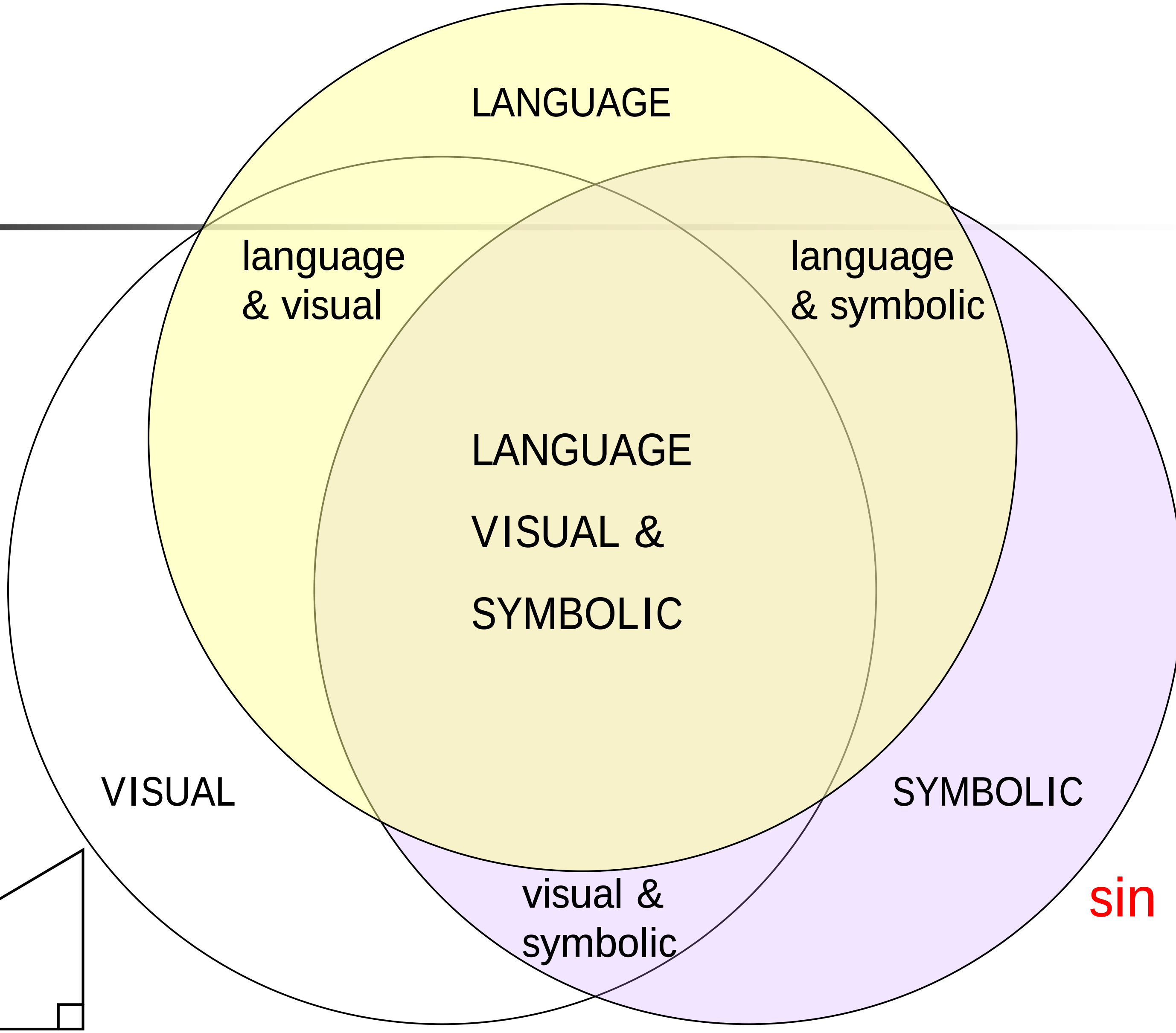
  $\sin \theta = 1/3 * 2$

  $\sin \theta = 2/3$

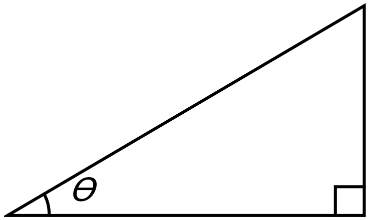
- … …


the hypotenuse
opposite side of θ
opp e of θ
th enuse
Unpack
Ratio of the opposite side of hypotenuse in right-angled triangle
nominal group → sin θ

Ratio of two sides



sin θ

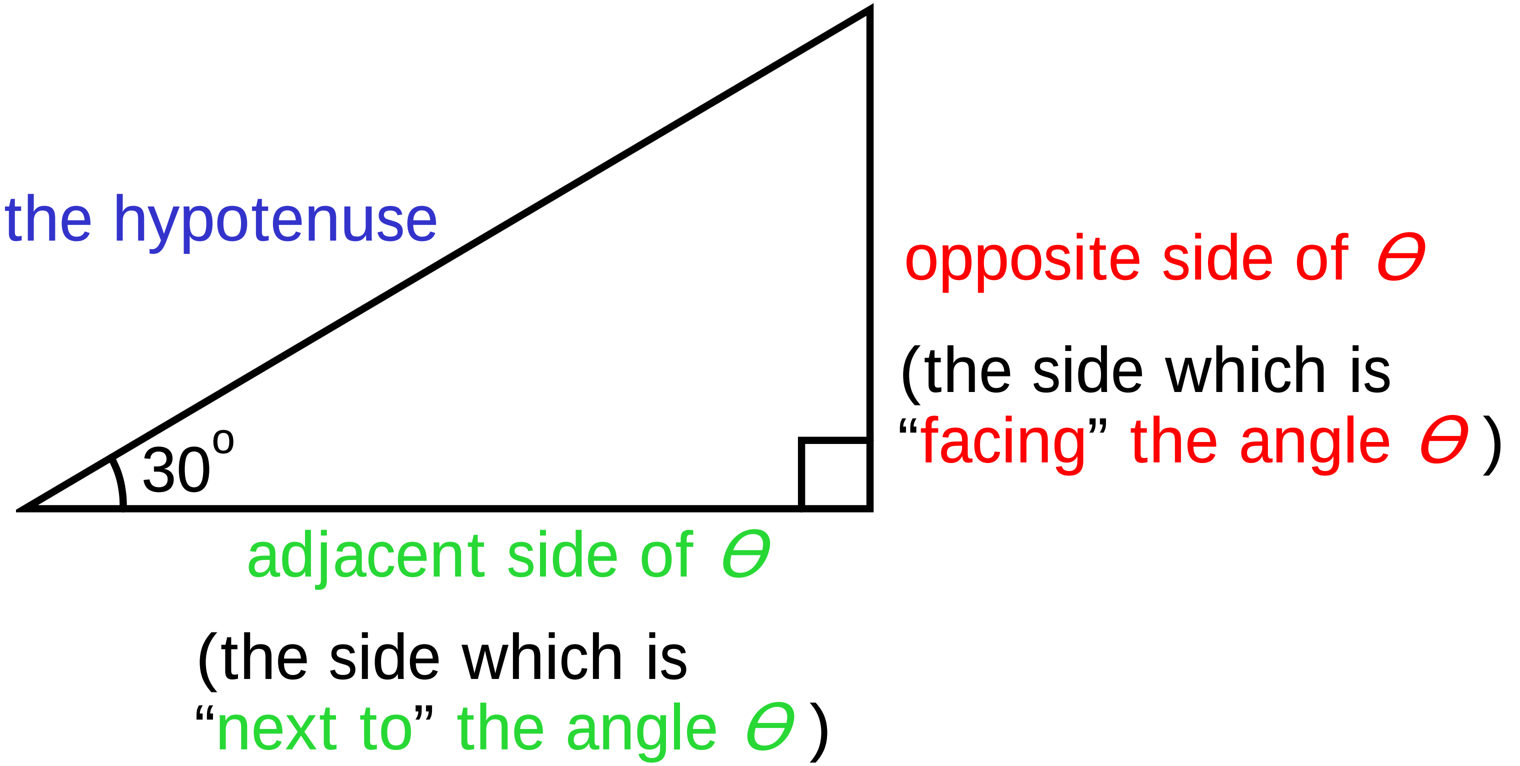


Unpack the meaning of sinθ

through

similar triangles.

Define the side of a right-angled triangle.

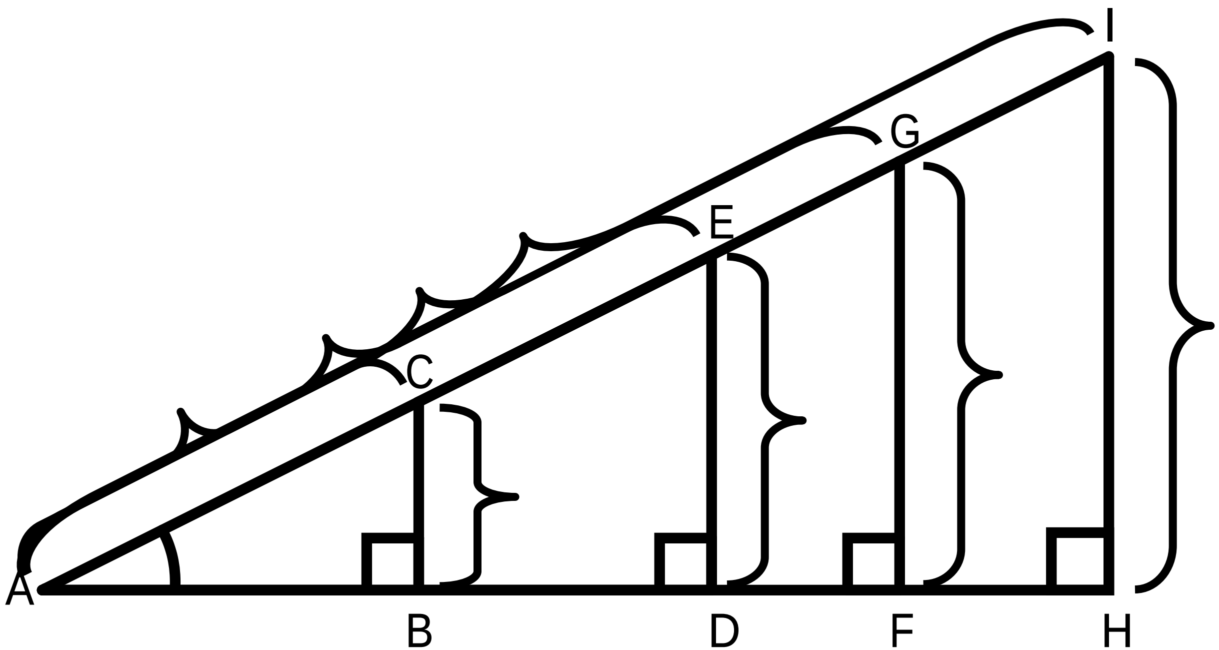


Activity

- There are different types of right-angled triangles
- Measure the opposite side of a given angle and the hypotenuse
- Find the ratio of the opposite side of a given angle to the hypotenuse

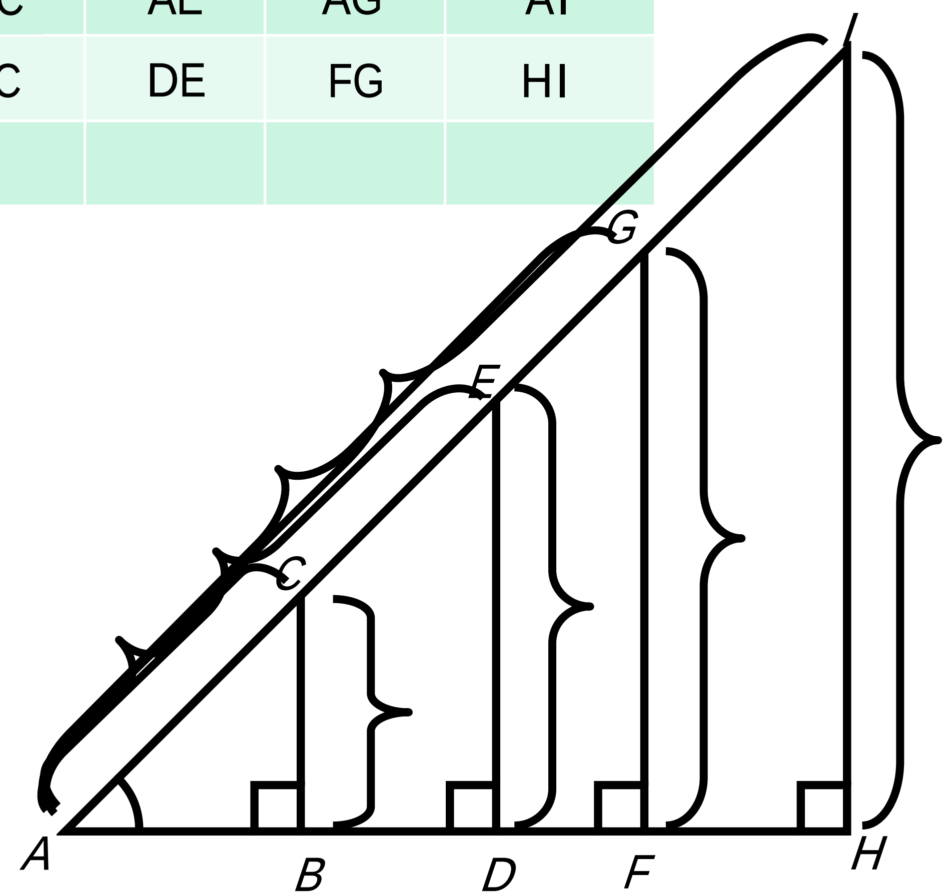
Activity (Example – 30^o)

	1	2	3	4
Hypotenuse (A)	AC	AE	AG	AI
Opposite side of θ (B)	BC	DE	FG	HI
Ratio of (B / A)				



Activity (Example – 45^o)

	1	2	3	4
Hypotenuse (A)	AC	AE	AG	AI
Opposite side of θ (B)	BC	DE	FG	HI
Ratio of (B / A)				



Findings from the table ……

- The ratio of the opposite side of θ to the hypotenuse of a right angled triangle, which has the same acute angle (θ), **is very close**.
- The values of the ratio of the opposite side of θ to the hypotenuse from a different acute angle are different
- WHY ???? Is there any relationship ??

Result

The ratios are the same !! (**Informal language**)

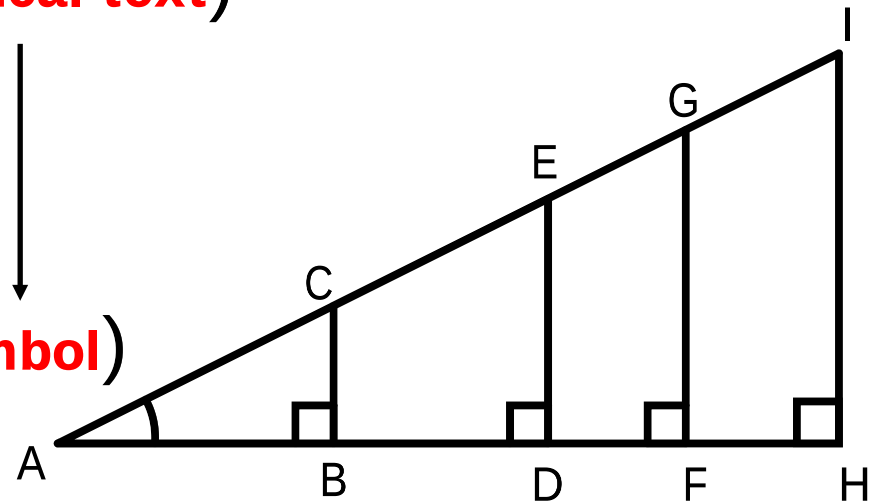
$$\frac{BC}{AC} = \frac{DE}{AE} = \frac{FG}{AG} = \frac{HI}{AI}$$

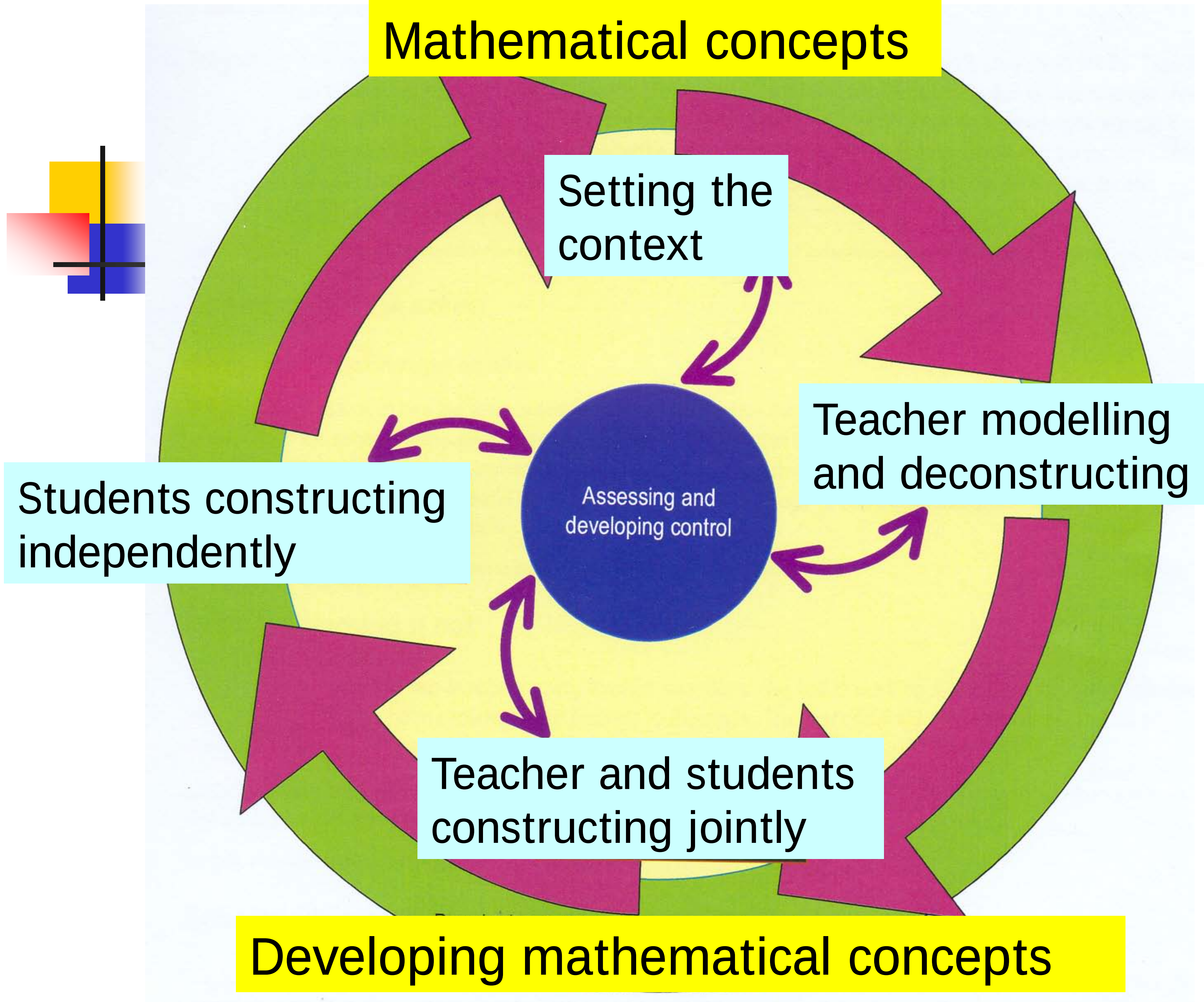
(**Mathematical text**)

$$\frac{BC}{AC} = \frac{DE}{AE} = \frac{FG}{AG} = \frac{HI}{AI} = x \ \text{(constant)}$$

(**Symbol**)

$$\sin 30° = x = \text{constant}$$




Mathematical concepts
Setting the context
Teacher modelling and deconstructing
Assessing and developing control
Students constructing independently
Teacher and students constructing jointly
Developing mathematical concepts

Findings

- For a right-angled triangle with a given acute angle θ, the ratio of the opposite side of θ to the hypotenuse is a constant.

Express that constant mathematically

Sinθ

Abstraction through nominalisation

Ratio of
the opposite side of θ
to the hypotenuse

Abstraction →

$\sin\theta$

Examples

Find the value of θ in the following question.

Q.1 – $\sin\theta = 0.7$

Q.2 – $\sin(\theta /2) = 0.5$

Q.1 – sinθ = 0.7

sinθ = 0.7

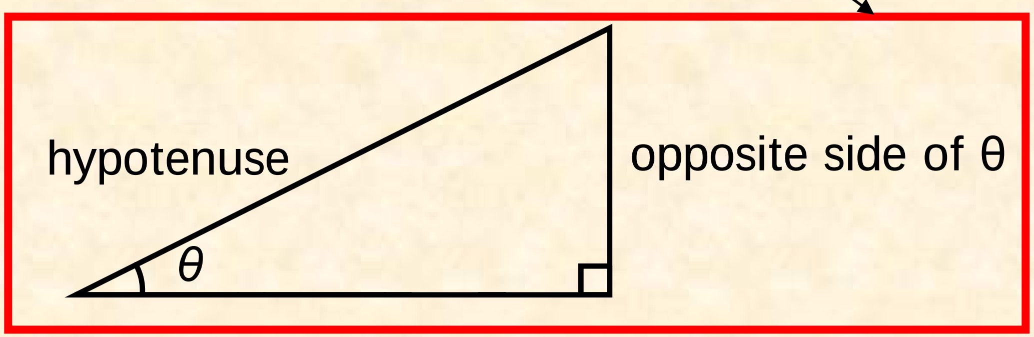
Symbol

Unpack the nominal group

the ratio of the opposite side of θ to the hypotenuse

Language

is 0.7



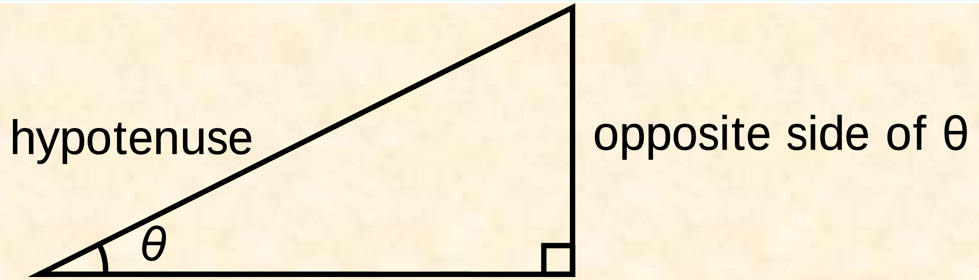
Visual

Q.1 – sinθ = 0.7

From the table ,
when the acute angle is 45°, the value of sin45° ~ 0.7

$$\sin\theta = 0.7$$
$$\sin\theta \approx \sin 45^\circ$$
$$\theta \approx 45^\circ$$

45°	1	2	3	4
Hypotenuse (A)	9.3	11.1	13.3	16
Opposite side of θ (B)	6.5	7.8	9.4	11.3
Ratio of (B / A)	0.699	0.702	0.707	0.706

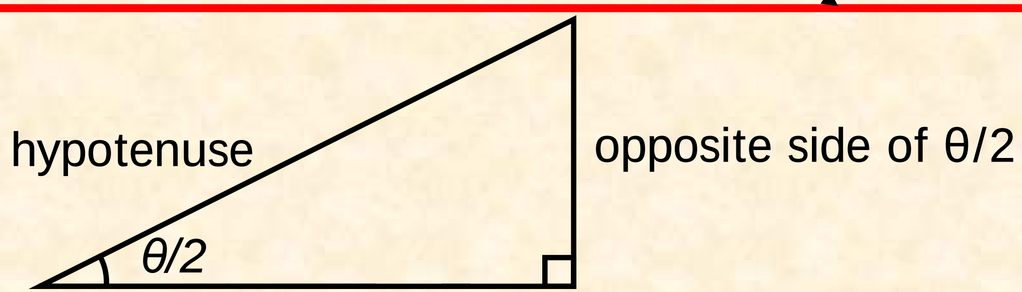


Q.2 – sin(θ/2) = 0.5

$\sin(\theta/2) = 0.5$

the ratio of the opposite side of θ/2 to the hypotenuse

is 0.5



Symbol

Language

Visual

Unpack the nominal group

Q.2 – sin(θ/2) = 0.5

From the table ,
when the acute angle is 30°, the value of sin30° = 0.5

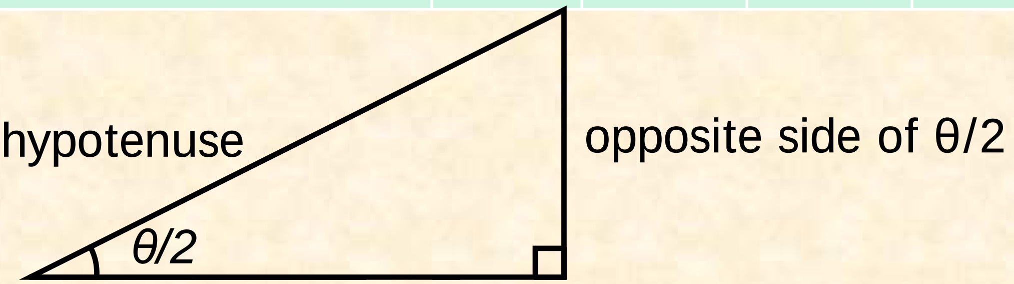
$$\sin(\frac{\theta}{2}) = 0.5$$

$$\sin(\frac{\theta}{2}) \approx \sin 30^\circ$$

$$(\frac{\theta}{2}) \approx 30^\circ$$

$$\theta \approx 60^\circ$$

30°	1	2	3	4
Hypotenuse (A)	10.6	12.7	15.2	18.3
Opposite side of θ/2 (B)	5.3	6.4	7.6	9.1
Ratio of (B / A)	0.5	0.504	0.5	0.497



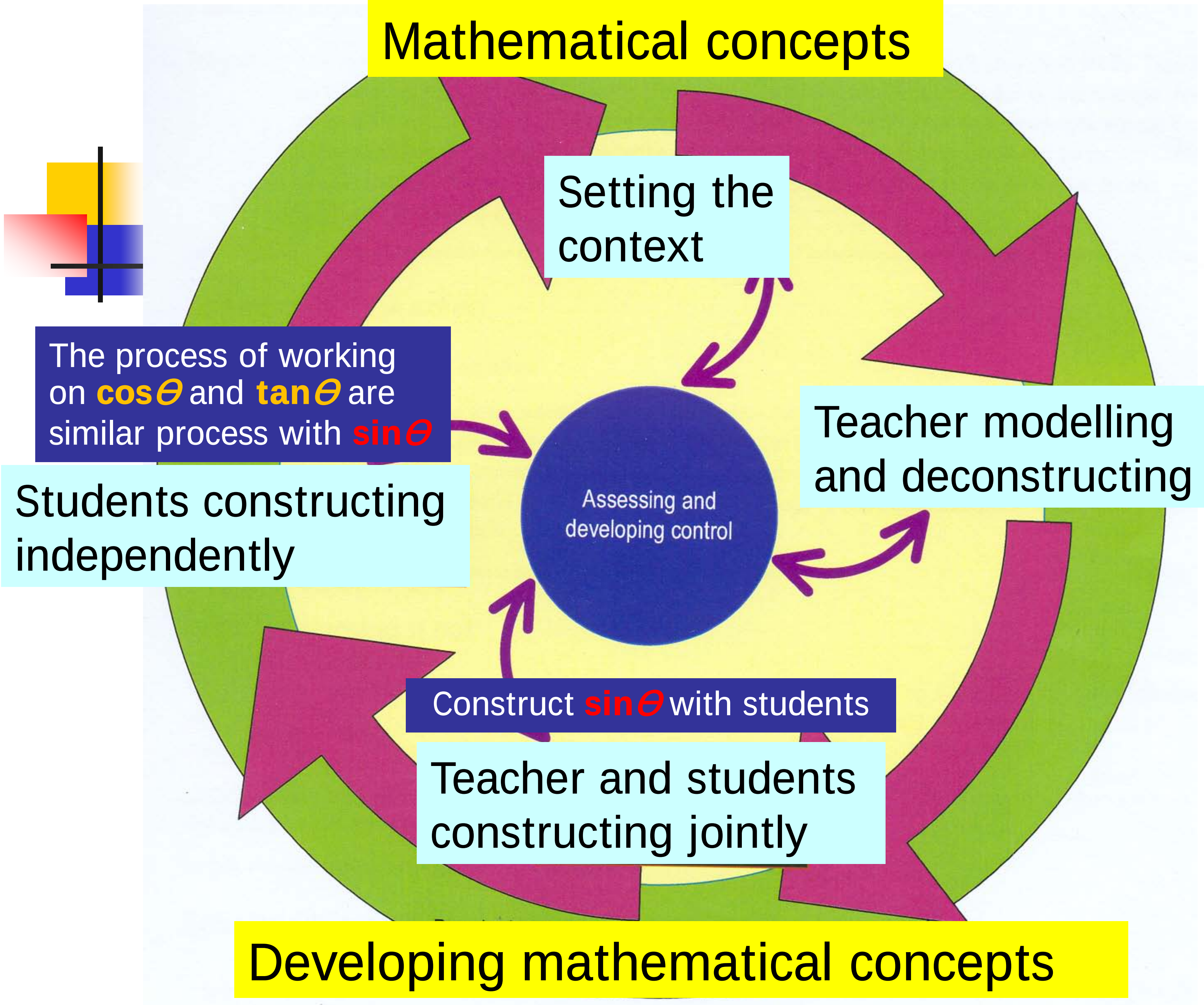
Conclusion

Indentify and unpack the nominal groups

- Experience the process of abstraction

- Make use of the three meaning-making systems in mathematics

- Scaffolding : The teaching learning cycle


Mathematical concepts
Setting the context
Teacher modelling and deconstructing
Assessing and developing control
The process of working on cosθ and tanθ are similar process with sinθ
Students constructing independently
Construct sinθ with students
Teacher and students constructing jointly
Developing mathematical concepts

Self Reflection

- More confidence to use the three meaning making systems in Mathematics
- Spend too much time on these activities?
  - Hands-on experience VS Direct Instruction
- Student centred Vs Teacher centred
- Teachers' understanding VS Students' understanding
  - Denaturalize ourselves – starting from students
- Preparing is better than repairing

Thank you!