Different Ways of Understanding - the Case of Fractions in Mathematics (Primary 4)

It is crucial for teachers to be sensitized to pupils' different ways of understanding an object of learning. Only when teachers know the prior conceptions and knowledge of their pupils can they help them to build on it. In what follows, a learning activity on the topic 'fractions' is used to illustrate how different pupils may have different ways of understanding the same object of learning - fractions.

How different pupils may have different ways of understanding fractions

In a P.4 Mathematics class, the teacher wished to test the pupils' understanding of 'fractions' learnt in P.3 before developing it further. He gave the pupils the following diagram and asked them to give the fraction represented by the shaded part.

The different answers elicited from the pupils were 1, 1/3 and 1/4.

What did pupils think?

Pupils	Perception of the shaded part in relation to 'fractions'		Problem identified	Understanding of 'fractions'
Pupils whose answer was 1		They saw the shaded part as one whole unit in itself.	The concept of fractions was not present.	No understanding
Pupils whose answer was 1/3		They saw the circle as divided into three parts, and the shaded part as one part out of the three parts.	They missed one of the critical features in learning about fractions: each part must be divided evenly.	Partial understanding
Pupils whose answer was 1/4		They realized that if the circle is divided into 4 equal parts, the shaded part represents one of the four equal parts.	No problem identified.	They had a better grasp of the idea of fractions because they had captured more critical features.

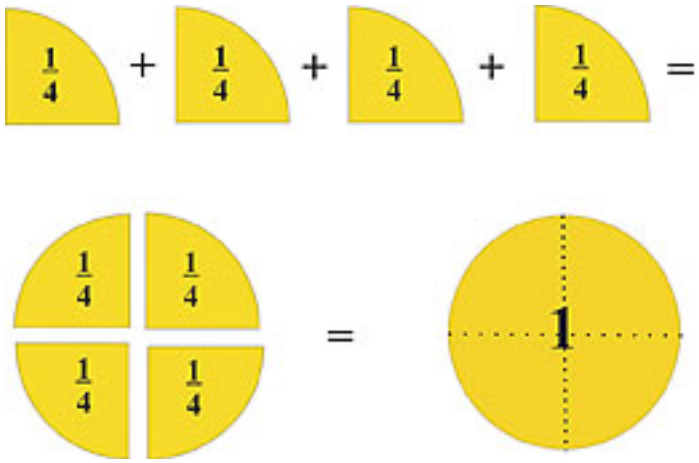
What did the teacher do?

➢ Knowing the different ways in which the pupils understood 'fractions', the teacher determined a suitable entry point for learning and teaching before developing pupils' understanding of proper fractions, improper fractions and mixed numbers :

• the concept of 'one whole'

• the concept of 'equal parts'

➢ The teacher used pictures / diagrams to contrast 'one whole' with 'a part'.



➢ Putting together the cuttings of each 1/4 part of one whole unit and comparing the shape and size of each of them, pupils commented on what they found: each 1/4 part of one whole unit got (i) the same shape, and (ii) the same size.

➢ In a series of subsequent lessons, the teacher introduced proper fractions with different denominators.

(i) Pupils were asked what 1/4, 1/5, 1/6 represented. With teacher support, pupils commented on the values of the representations: they represented one 1/4 part, one 1/5 part and one 1/6 part respectively.

(ii) In groups, pupils were given a number of clips to find out what 1/3, 1/4, 1/6, and 1/8 of the clips were. They were asked to compare and comment on the values of fractions with a fixed numerator but a different denominator: given a fixed numerator, the larger the denominator, the smaller the size of the fraction.

(iii) Pupils were assigned a task in the application of fractions with different denominators. They were asked to compare fractions given in the form of diagrams. They worked in groups to find ways to decide which fractions were greater.

Task description

In this beginning mathematical task, iconic representation of fractions is employed as a way to tackle the comparison of fractions of different denominators. By varying the pattern of dissection, both figures can be divided into a different number of congruent parts (corresponding to the new

denominators). The pupils can then compare the two fractions by counting the number of congruent parts (corresponding to the new numerator) that the initial parts have been turned into.

How pupils solve the problem

There are a limited number of different approaches used by the pupils to solve the problem. (Please refer to Appendix 1 for the pupils' group works).

In the case of Group 1, each figure is dissected into six parts. The fractions can then be compared by the number of parts. In the case of Group 2, the first figure is regrouped into three parts instead of two. It can then be seen that in the first figure, there are less than two parts while in the second figure, there are two parts. Thus the fractions can also be compared. In the case of Group 3, both figures are divided into 12 congruent parts. Again, it is possible to compare the fractions by counting the number of new congruent parts.

How learning and teaching is enhanced

- ♦ All the methods are viable and very good ones as they are created by pupils who have never learned the proper method beforehand.
- ♦ The contrast between these different methods provides an opportunity to discuss and see why the first case (using L.C.M.) will be better than the others. In addition, the first case can be used as a basis and an iconic representation of the new algorithm which they will learn very soon.
- ♦ The third method that incorporates the concept of finding a common denominator, but does not recognise the idea of the L.C.M. should be aware of.

What did the pupils benefit from sharing one another's understanding of 'fractions'?

- When the teacher elicited answers from the pupils on what they knew about fractions, varied understandings emerged. For example, from the pupils' variation in understanding 'one whole', 'a part' and 'equal parts', the teacher grasped the entry point for learning and teaching. In return, the learning point could be highlighted. With more focussed practice, the differences among pupils' understandings were decreased.
- Pupils valued the collective success that resulted from each individual contributing what features he / she could observe in each 1 / 4 part of one whole unit and also in the comparison of fractions.

Appendix 1

A P. 4 mathematics lesson on 'comparing fractions' -the pupil group works

Group 1
第一組

數學科課業（一）
分數大小的比較

你怎樣比較下面分數的大小？試利用下面的圖形試試看。

2/3 比 1/2 大（ 1/6 ）

我的方法是：將他們的分母主廣大變變樣，再作比較。

Group 2
第二組

數學科課業（一）
分數大小的比較

你怎樣比較下面分數的大小？試利用下面的圖形試試看。

2/3 比 1/2 大（ 1/6 ）

我的方法是：先用分母來找出最小公倍數，然後分母變少，分子也乘多少

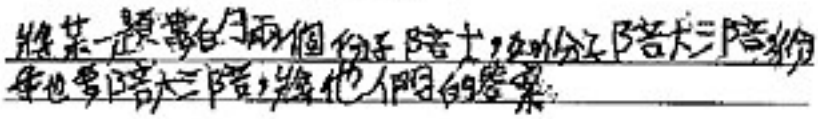
* Group 3
第三組

數學科課業（一）
分數大小的比較

你怎樣比較下面分數的大小？試利用下面的圖形試試看。

$\frac{2}{3}$ 比 $\frac{1}{2}$ 大（ $\frac{1}{6}$ ）

我的方法是：



* Note that the pupils' explanations in words do not necessarily reflect what they actually did in dissecting the figures.

Source:

The contents of the exemplar are extracted from the learning package on “Catering for Individual Differences -- Building on Variation” designed by the Research Team of the Department of Curriculum Studies, Faculty of Education, the University of Hong Kong.