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數聞 IMOment

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笑一笑

LAUGH OUT LOUD

2016 年第五十七屆國際數學奧林匹克籌備委員會
Organising Committee of the 57th International Mathematical Olympiad 2016

2016 年第五十七屆
國際數學奧林匹克籌備委員會

香港將於 2016 年 7 月主辦第五十七屆國際數學奧林匹克（IMO），迎接來自超過 100 個國家和地區的中學生數學精英。希望《數聞》可在我們邁向 2016 年 IMO 期間帶動同學和公眾對數學的興趣，更希望這種氣氛歷久不衰。

歡迎讀者向《數聞》投稿。文章須為原著，以中文或英文寫成（或兩種文本兼備），長度為一至四頁（就一種語言而言），並以電郵附件方式傳送至 info@imohkc.org.hk，或郵寄至 九龍油麻地 405 號九龍政府合署 4 樓 403 室 教育局數學教育組《數聞》編輯，標題為「Submission to IMOment」。

Hong Kong is proud to be hosting the brightest secondary school mathematics talents from over 100 countries and regions at the 57th International Mathematical Olympiad (IMO) in July 2016. We hope that IMOment will promote interest in mathematics among students and the public in this period leading up to IMO 2016, and beyond.

Readers are welcome to submit articles on mathematics and/or mathematical Olympiad to IMOment. Submissions should be original, one to four pages in length in either Chinese or English (or both), and should be sent by attachment to an email to info@imohkc.org.hk, or be mailed to Rm . 403, 4/F, Kowloon Government Offices, 405 Nathan Road, Kowloon, titled "Submission to IMOment."

Organising Committee of the 57th International Mathematical Olympiad 2016

第三期內容
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IMO 2014 後記 (下)
IMO 2014 AND BEYOND (II)

/梁達榮 LEUNG TAT-WING

（**上文提要**：梁博士以香港隊領隊的角度，分享他對 IMO 2014 行程及選題的感想...）

協調

協調工作的過程非常嚴謹。當領隊大會（由 101 個國家的領隊組成）挑選出 6 道題目後，首席協調員會指引 6 名試題隊長把領隊提供的不同答案歸納於詳細的評分準則內。每位試題隊長只會負責一條題目，所以他會清楚了解該題的有關資料，包括問題的原創性及不同的解題方法等等。接着，評審委員會便會正式通過各評分標準，他們在比賽後會把所有學生的答題簿進行電腦掃描。領隊取回自己學生的答題簿後便會評核各題得分。中途還發生了一個小插曲，由於電腦無法識別塗改液的痕跡，我多次被問到為何我學生的答題簿上會有塗改液。當然，我們絕對不曾在答題簿內作任何改動。

由於大會為領隊提供了詳盡的時間表，因此我們十分清楚行程。事實上，協調的工作在兩天內已經完成。我想，基於語言溝通及其他理由，所以授任的協調員都是一些來自不同國家的前領隊及資深的解難專家，甚至有一些協調員與我們十多年前已碰過面。他們經驗豐富，不論是學生的小失誤（不扣分）、小錯（扣 1 至 2 分）或大錯（至少扣 4 至 5 分）都能一一指出。在此，我要衷心感謝副領隊程德永──他是我們以前的學生，也是 IMO 的金牌得主。他細閱學生的答案後，發現他們已經努力嘗試解答問題的部分，於是便與協調員討論並成功說服他們，從而為港隊爭取了更多得分。整體而言，我認為改卷過程公平公正，而協調工作亦進行得相當順利。

(**Recap of last time**: As the Hong Kong team leader, Dr. Leung shares his thoughts on the itenary and problem selection of IMO 2014...)

Coordination

The process of coordination was done seriously and rigorously. After the six problems were selected by the Jury (composed of leaders of 101 countries), I believed the chief coordinator then instructed the six problem captains to write up detailed marking schemes, incorporating various solutions supplied by leaders. Each problem captain was responsible for only one specific problem, he knew essentially everything concerning that problem, originality, various solutions, etc. The marking schemes were then formally approved by the Jury. After the two contests, they scanned all the answers scripts of the students. We leaders then got back answer scripts of our students and tried to allocate suitable points for our students. A minor mishap was, the scanner could not scan marks of correcting fluid, and thus I was asked several times why there were correcting fluids found on my students' scripts. Of course, we did not amend anything in the scripts.

Detailed schedules were given to us, so leaders knew when and where to go. The process of coordination was done formally within two days. I believe because of language issue and other reasons, coordinators were recruited internationally. They were composed of former leaders, experienced problem solvers etc. Some we met more than 10 years ago. They were very experienced and were able to spot errors made by students, whether an error is trivial (no point deducted), minor (1 or 2 points deducted) or major (at least 4 to 5 points deducted). I thank my deputy leader, Ching Tak Wing, our former trainee and IMO gold medalist, who helped us to go through the many convoluted arguments of our team, and we were able to discuss (or argue) with our coordinators, to convince them that our team did do something of certain parts of a problem or so, and thus got a few extra points. On the whole, I think our papers were fairly marked and the process of coordination was done well.

賽果

港隊今年取得四銀二銅的佳績，在 101 個國家和地區中（非官方）排名為 18。三名銀牌得主各答對 4 道題目，另外一名則答對了 3 題。其餘兩名銅牌得主基本上成功解答 3 條問題，其實他們與銀牌的距離已十分接近。我不會責怪學生沒有盡力，其實近年來，他們已不斷學習不同的技巧去完善自己的答題。我觀察到他們解答第二題及第五題（難度適中題）時會習慣先搜集數據，然後用各種組合、簡化、歸納等方法來解決問題。可惜，他們用的方法太冗贅，加上解答過程有錯漏，因此被扣了幾分。由於他們在第二題及第五題上花了太久，導致不夠時間解答第三題及第六題，所以沒有人能答對最難的這 2 道題。我們的六名參賽隊員中有四名是「老將」，他們即將升上大學離開我們。我想我們大概還需要兩至三年時間去培訓另一支實力相若的隊伍。

換一個角度看，如果我們要保持世界排名，那麼爭取幾面銀牌和銅牌是必須的。如果我們想打入世界頭十位，便需要兩至三面金牌及幾面銀牌和銅牌。這完全取決於我們的目標。但對我來說，能夠在過程中培育一羣訓練有素、勇於挑戰、積極學習的新人便已足夠。拿下 IMO 的金牌只是一個過程和訓練的一部分，它並不是我們的終極目標（不同於足球的世界杯）。至今，約有十名菲爾茲獎（Fields Medal）得主曾參加 IMO，但並非每位得主都獲得金牌（金牌得主佔其中一半左右）。甚至連陶哲軒在首次參加 IMO 時也只得銅牌，後來他才相繼摘下銀牌和金牌。當然，我明白有些行政人員對於派代表隊出戰 IMO 有另一套想法。

我聽過很多人探討為甚麼香港不能產生一支更強的隊伍，例如我們的學生在文憑試上花太多時間，尤其是校本評核；加上香港不像越南和新加坡設有專科學校；香港培訓學生太少，教練不夠專業，訓練時間又不足......雖然以上的分析似乎難以反駁（沒有反例？），但卻又無從論證。即使這是事實，我們又可以如何應對？近年，我們已經強化培訓過程，不斷進行更多的考核，鼓勵學生積極表達自己對題目的觀點並加以論證。而且，很多建議都是我們以前的學生所提出的。

Result of our Students

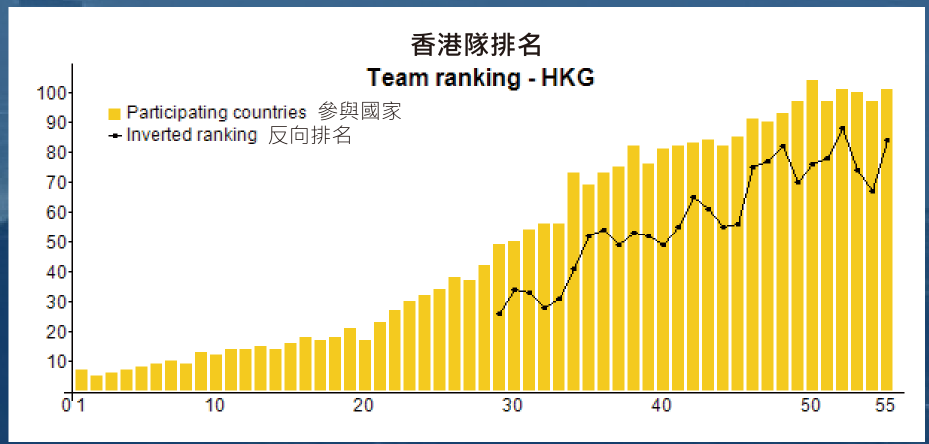
We got 4 silvers and 2 bronzes, ranked (unofficially) 18 out of 101 countries and regions. Indeed 3 of our 4 silver medalists solved essentially 4 problems and the other silver medalist got 3 problems correct. Also our 2 bronze medalists essentially got 3 problems correct and were really close to silver. I don't think I can blame our students for not trying hard. Indeed they picked up a lot of techniques in these few years, learned (and are still learning) to face a problem fairly and squarely. I observed when they were doing problem 2 and 5 (medium problems), they had generated the habit of gathering data and information, using various grouping and simplification methods, induction and other techniques to solve them, even though their approaches were later found to be a bit clumsy and there were a few gaps (thus few points deducted). Because a lot of time were spent on problem 2 and 5, no one could do problem 3 and 6, thus no one could tackle the hard problems. Four of our six members were old-timers, and they are leaving us for universities. I think we need 2 to 3 years to have another group of members of this caliber.

Think of this issue the other way. If we want to keep our ranking, surely several silver and bronze medals are required. If we want to be ranked within top 10, for instance, we need two or three gold medals, and some silvers and bronzes. It depends on really what we want. For me, I think it is fine if we can produce a bunch of well-trained students, good and brave to face problems and are ready to pick up necessary skills and other things in the process. Getting a gold medal in an IMO is a process, is part of the training, but not necessarily is an end (not like getting a World Cup in football). So far, about 10 Fields' medalists did participate in IMOs, but not everyone was a gold medalist (about half of them are). Even Terry Tao, he got bronze in his first year, then silver, then gold. Yes, of course I realize some administrators may think otherwise and have different ideas of what it means by sending a team to an IMO.

I heard many theories why we cannot produce even stronger team: that our students have to devote too much time to DSE, in particular SBA, that we have no specialized schools, not like Vietnam or Singapore, that our pool too small, that our trainers no good, that our training time not enough, etc. All these are hard to refute (no counter-examples?), and yet not sure how to verify. They may well be so and so what can we do? Indeed in these few years we have strengthened our training process, conducted more tests, asked our members to present and substantiate their views, etc. And indeed many suggestions came from our former trainees.

其實只是觀看賽果也能有所得着。例如烏克蘭隊不受政局動盪影響仍有高水準的表現，在 101 個國家和地區中排第 6。以色列隊則與港隊不相伯仲（並列第 18 位）。兩韓一如以往有傑出的成績，雖然相比去年稍為遜色；南韓排第 7，而朝鮮排第 14。二十多年來，朝鮮曾兩度因不誠實行為而被取消參賽資格，因此錯過了十年的賽事，但他們比賽所得的成績尚算不俗。雖然我們的表現沒有兩個人口大國優秀──中國（排第 1）、美國（排第 2），但我們比印度（排第 40）及印尼（排第 30）更勝一籌。香港亦稍勝泰國（排第 22），泰國將主辦 IMO 2015，聽說他們在籌劃比賽及訓練隊員上投放了大量資源。另外，今年港隊比那些傳統強隊的表現更好，例如波蘭（排第 28）、伊朗（排第 21）及保加利亞（排第 37）。保加利亞舉辦數學競賽的歷史悠久，他們的比賽材料一向很吃香。不過我們近幾年的排名不及新加坡（排第 8），當仔細分析他們的賽果時，我發現港隊的銀牌選手與他們金牌得主的距離並不遙遠。想必香港也能達到他們的水準吧？總而言之，觀察不同國家在這些年來的賽果是一件趣事，我們或能從中得到一些啟發，思考如何令隊員取得長足的進步。

We observed a few things by simply looking at the overall results. For instance, despite political trouble in the east, the Ukraine team still did very good. They ranked 6 out of 101. The Israelites did as well as us (ranked 18). The Koreans, as usual, did very good, but not as formidable as last year. Indeed Republic of Korea was ranked 7 and the PRK (Democratic Republic of Korea) was ranked 14. During these 20 or so years, the North Koreans were not honest twice and were disqualified and missed the contest altogether for 10 years, but during the times they were around, they did reasonably well. Although we were not as good as the two most populous countries, China (ranked 1), USA (ranked 2), we did better than India (ranked 40) and Indonesia (ranked 30). We did better than Thailand (ranked 22). The country will host IMO 2015, they have been good, and I was told they put a lot of money into the event and in training their team. We also did better than several traditionally strong countries such as Poland (ranked 28), Iran (ranked 21) and Bulgaria (ranked 37). Indeed Bulgaria has a long tradition of mathematical competitions, and their competition materials are often highly sought-after. As in the last few years, we still did not do as well as Singapore (ranked 8). However when looked closely their results, I found their gold medalists were not really much better than our silver medalists and I think we can do as well? In short, it is very interesting by simply looking at the results of countries during the years, and we may gather some idea how we should train our team.



結語

最後我想說，舉辦 IMO 可能沒有甚麼大不了，但我們定要積極推動，認真辦好這項活動。值得借鏡的是，即使南非為 IMO 投放了極大努力，都未能做到十全十美。

Conclusion

One last point I want to mention, admittedly perhaps organizing an IMO is not really a very big deal, but to do it right, we still have to do it seriously and vigorously and to make it complete. We have seen how South Africa did it with so much effort and dedication, and the work was still not perfectly done.

數學歸納法：不一樣的歸納法

Mathematical Induction: A Different Kind of Induction

/盧安迪 ANDY LOO

我的中學數學老師曾說：「數學歸納法不是歸納法。」此話充滿智慧。的確，數學歸納法跟自然科學中的歸納法不同，後者是藉著把觀察到的現象推廣，以找出規律。數學歸納法其實是一種演繹法，從若干前提出發，用邏輯推導出結論。

要證明命題 $P(n)$ 對所有正整數 n 都成立，我們只需證明兩點：
(i) $P(1)$ 成立。
(ii) 對任何正整數 k，若 $P(k)$ 成立，則 $P(k+1)$ 亦成立。

我們一旦知道這兩點，便可作以下推理：由 (i) 可知 $P(1)$ 成立。於是由 (ii) 可知 $P(2)$ 也成立。再用 (ii)，我們可知 $P(3)$ 也成立，如此類推。我們因而得知 $P(n)$ 對所有正整數 n 都成立。這就是數學歸納法原理，跟骨牌的運作原理相似。

My secondary school math teacher once said, "Mathematical Induction is not induction." There is a lot of truth to this remark. Indeed, Mathematical Induction is different from the inductive reasoning in natural sciences, which seeks to obtain patterns by generalizing observed phenomena. Mathematical Induction is properly a deductive method, a logical way of deriving a conclusion from premises.

To prove that a statement $P(n)$ is true for all positive integers n, we only need to prove two statements:
(i) $P(1)$ is true.
(ii) For any positive integer k, if $P(k)$ is true, then $P(k+1)$ is also true.

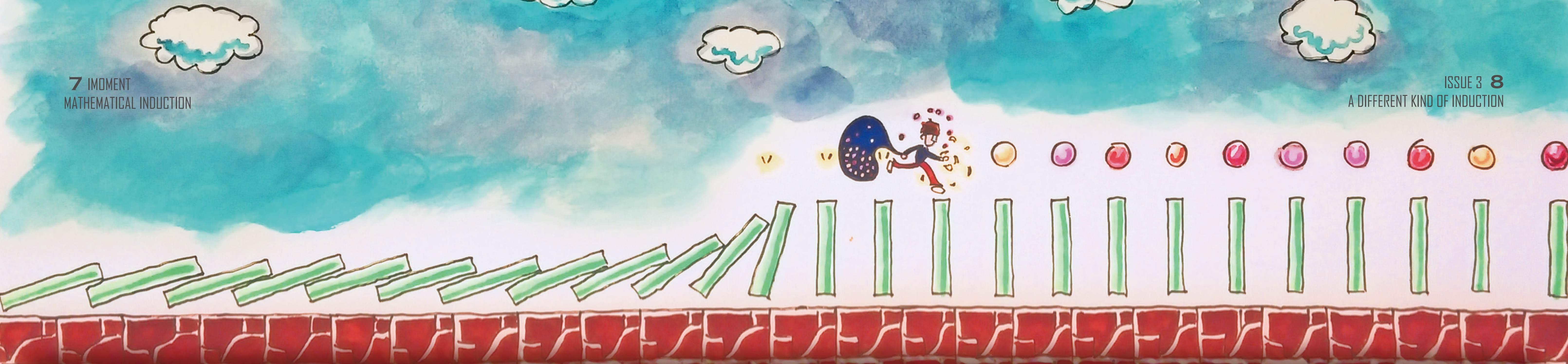
Once we prove these two statements, we can apply the following reasoning: By (i), $P(1)$ is true. Thus by (ii), $P(2)$ is also true. Using (ii) again, we know that $P(3)$ is also true, so on and so forth. We therefore conclude that $P(n)$ is true for all positive integers n. This is the principle of Mathematical Induction, and is similar to the workings of dominoes.

現在，讓我們看看運用數學歸納法的一個例子，這題出自 *Mathematical Excalibur (2012), Vol. 16, No. 3*：

設 n 為正整數。現有 2^n 個球，被分成若干堆。對任何兩堆 A、B，若它們分別有 p 個球和 q 個球，而 $p \geq q$，則可把 q 個球從 A 堆轉移到 B 堆。求證：總存在一種方法，能經過有限次這類轉移，令所有球都在同一堆中。

Now let us look at an example of using Mathematical Induction, adapted from *Mathematical Excalibur (2012), Vol. 16, No. 3*:

Let n be a positive integer. There are 2^n balls divided into a number of piles. For every two piles A and B with p and q balls respectively, if $p \geq q$, then we may transfer q balls from pile A to pile B. Prove that it is always possible to make finitely many such transfers so as to have all the balls ended up in one pile.



讀者可先驗證這個命題對 $n = 1$ 成立。現在假設命題對 $n = k$ 成立。我們怎樣證明命題對 $n = k+1$ 成立呢？以下是一個概略的證明：

由於 2^{k+1} 是偶數，所以有球數為奇數的堆的數目必為偶數。因此我們可把這些堆兩兩配對。在每對中，我們可像題中所述般，把球從其中一堆轉移至另一堆，使得兩堆都變成有偶數個球。現在，每一堆都有偶數個球。我們可把每一堆的球兩個兩個放進袋子，於是情況變為共有 2^k 個袋分成若干堆。由於我們假設命題對 $n = k$ 成立（這稱為歸納假設），我們可通過若干次轉移，把所有袋都放到同一堆中。證畢！

讀者或許留意到，運用數學歸納法時，在證明 $P(1)$ 後，我們要做的，基本上就是證明原命題，但多了一個假設：命題對前一個正整數成立。

我們再看數學歸納法的一個進階例子，這題來自 2013 年普林斯頓大學數學比賽：

> 設 $a_1 = 2013$，且對於所有正整數 n，$a_{n+1} = 2013^{a_n}$。設 $b_1 = 1$，且對所有正整數 n，$b_{n+1} = 2013^{2012b_n}$。求證：對於所有正整數 n，都有 $a_n > b_n$。

乍看之下，自然會想到用數學歸納法解決此題。然而，我們很快便會發現，$a_k > b_k$ 這個前設，並不導致 $a_{k+1} > b_{k+1}$（讀者可試試看！）。我們是否走投無路？

The reader may first wish to check that the statement is true for $n = 1$ Now supposing the statement is true for $n = k$, how can we prove that the statement is true for $n = k+1$? Here is a sketch of an argument:

Since 2^{k+1} is an even number, the number of piles that have an odd number of balls must be even. So we put them into pairs of piles. In each pair, we can transfer balls from one pile to the other in the manner indicated in the problem, so that both piles end up having an even number of balls. Now, every pile has an even number of balls. We can put every two balls in a sack in every single pile so that we have a total of 2^k sacks. Since we are assuming that the statement is true for $n = k$ (this is called the inductive hypothesis), we can make finitely many transfers so that all the sacks end up in one pile. We are done!

The reader may notice that with Mathematical Induction, after proving $P(1)$, we essentially need to prove the original statement but are equipped with an additional assumption: that the statement is true for the preceding positive integer.

We turn to a more advanced example of Mathematical Induction, taken from the 2013 Princeton University Mathematics Competition:

> Let $a_1 = 2013$ and $a_{n+1} = 2013^{a_n}$ for all positive integers n . Let $b_1 = 1$ and $b_{n+1} = 2013^{2012b_n}$ for all positive integers n. Prove that $a_n > b_n$ for all positive integers n.

At first sight, one natural reaction to this problem would be to do Mathematical Induction. However, we would quickly realize that the assumption $a_k > b_k$ does not imply $a_{k+1} > b_{k+1}$ (check it!). Are we doomed?

與其證明 $a_n > b_n$，我們倒不如證明對所有正整數 n，都有 $a_n \geq 2013b_n$。當 $n = 1$，這顯然成立。如果對於某正整數 k，有 $a_k \geq 2013b_k$，則

$$a_{k+1} = 2013^{a_k} \geq 2013^{2013b_k} = 2013^{b_k} \times 2013^{2012b_k} \geq 2013b_{k+1}$$

（在最後一步，我們使用了 $b_k > 1$ 這一事實。）根據數學歸納法原理，對所有正整數 n，都有 $a_n \geq 2013b_n$！

這個證明貌似簡單，卻耐人尋味：如果我們連原本的結果都證明不了，為何能奇蹟地證明一個更強的結果呢？

解答這個疑惑的關鍵，在於歸納假設。當命題變得更強時，我們需要證明的結果變得更難，但我們可用的歸納假設也變得更強。在這題的情況，歸納假設的變強，蓋過了證明所需結果的難度，所以我們能順利解決問題。

香港隊在數學比賽中利用數學歸納法解決過無數問題，更往往需動用到更複雜的數學歸納法技巧。例如，數學歸納法原理中的第 (ii) 點可以改為：

對任何正整數 k，若 $P(1)$、$P(2)$、...、$P(k)$ 皆成立，則 $P(k+1)$ 亦成立。

快到本期「挑戰園地」用數學歸納法一展身手吧！

Instead of $aa_n > b_n$, we shall prove $a_n \geq 2013b_n$ for all positive integers n. This is clearly true for $n = 1$. If $a_k \geq 2013b_k$ for some positive integer k, then

(In the last step we used the fact that $b_k > 1$.) By the principle of mathematical induction, $a_n \geq 2013b_n$ for all positive integers n!

There is something intriguing about this seemingly easy proof: if we cannot even prove just the original result, how come we can miraculously prove a stronger result?

The key to answering this puzzle lies in the inductive hypothesis. If the statement is strengthened, what we need to prove becomes more demanding but the inductive hypothesis that we can use also gets strengthened. In the case of this problem, the upgrade of the inductive hypothesis outweighs the increase in difficulty of proving the desired result, so we end up being able to solve the problem.

Mathematical Induction has helped the Hong Kong team solve numerous problems in mathematics competitions, and more sophisticated varieties of Mathematical Induction are often employed. For example, statement (ii) in the principle of mathematical induction may be changed to:

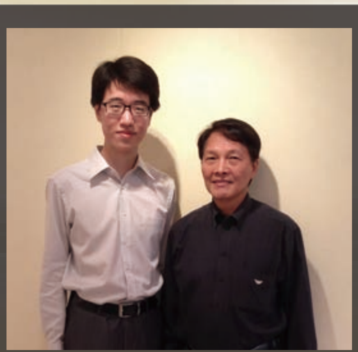
For any positive integer k, if $P(1)$, $P(2)$, ... $P(k)$ are true, then $P(k+1)$ is also true.

Turn to Challenge Corner to try Mathematical Induction yourself!

鄺其志的數學心得

THE MATHEMATICAL INSIGHTS OF KWONG KI-CHI

/盧安迪 ANDY LOO



鄺其志，人稱「神童志」，前庫務局局長、資訊科技及廣播局首任局長、前香港交易所行政總裁。你又是否知道，鄺先生是一位數學高材生？

（圖: 筆者與鄺先生 (右) 合照 ）

Kwong Ki-chi, known as the "Prodigy," is former Secretary for the Treasury, the first Secretary for Information Technology and Broadcasting, and former Chief Executive of Hong Kong Exchanges and Clearing. Did you know that Mr. Kwong is a math whiz?

(Photo: Mr. Kwong (right) and the author)

二百多年前，當時是小學生的「數學王子」高斯發現了一個把首 100 個正整數相加的快速算法：

設
$$S=1+2+3+\dots+100$$

則
$$S=100+99+98+\dots+1$$

兩式相加，可得
$$2S=\underbrace{101+101+101+\dots+101}_{100}$$
$$S=\frac{101\times100}{2}=5050$$

鄺先生讀小學時也發現了一個把首 100 個正整數相加的快速算法，但跟高斯的算法稍有不同：

設
$$S=1+2+3+\dots+99+100$$

則
$$S=99+98+\dots+1+100$$

兩式相加，可得
$$2S=\underbrace{100+100+100+\dots+100}_{99}+200$$
$$S=\frac{100\times101}{2}=5050$$

另一個例子，對於個位數乘以個位數的乘數表，我們都十分熟悉。但鄺先生發現，形如「$a5$」的數的平方也有一個有趣的規律，例如：

$$35^2=1225 \quad 45^2=2025 \quad 55^2=3025$$

More than two hundred years ago, Carl Friedrich Gauss, "the Prince of Mathematicians," as a primary school student, discovered a swift way of summing the first 100 positive integers:

Let
$$S=1+2+3+\dots+100$$

Then
$$S=100+99+98+\dots+1$$

Adding the two equations, we get
$$2S=\underbrace{101+101+101+\dots+101}_{100}$$
$$S=\frac{101\times100}{2}=5050$$

When Mr. Kwong was a primary school student, he also found a way to sum the first 100 positive integers, but slightly different from Gauss's:

Let
$$S=1+2+3+\dots+99+100$$

Then
$$S=99+98+\dots+1+100$$

Adding the two equations, we get
$$2S=\underbrace{100+100+100+\dots+100}_{99}+200$$
$$S=\frac{100\times101}{2}=5050$$

Let's look at another example. We are all familiar with the multiplications of one-digit numbers. But Mr. Kwong discovered an interesting pattern about the squares of numbers of the form "$a5$":

$$35^2=1225 \quad 45^2=2025 \quad 55^2=3025$$

其中 $12=3\times4$、$20=4\times5$、$30=5\times6$。那麼 65^2、75^2 等等，是否也有類似規律呢？讀者又能否解釋這個規律呢？

鄺先生認為，學習數學的竅門正正在於多思考，既要天馬行空地大膽猜想，然後也要小心求證。

鄺先生以優異成績考入香港大學攻讀數學和物理，一級榮譽畢業。由於當時課堂上的數學對他來說太過簡單，沒有挑戰性，所以鄺先生畢業後選擇加入政府工作。但當鄺先生當上庫務司（回歸後稱為庫務局局長），他發現學生時期學習數學所培養的邏輯思考和條分縷析的能力，在掌管港府千億儲備這盤「大數」的時候大有幫助。後來鄺先生自告奮勇成為資訊科技及廣播局首任局長，多年來累積的數學、科學、電腦等知識正好大派用場。因此，即使你從事的職業看似與數學無關，可能有一天，數學會跟你的工作變得息息相關！

事實上，鄺先生認為數學跟其他領域密不可分。不但物理等自然科學跟數學互相依賴，社會科學亦如是。當亞當．斯密寫下《國富論》時，他只用定性的經濟原則進行推導，所以只能得到大約的結論。隨著數學在經濟學中的應用逐漸增加，我們便可通過計算得出越來越精準的結果。

同時，鄺先生忠告有志學習數學的同學，也要多留心和思考其他範疇的問題，不單因為一個人應該多瞭解自己專業以外的知識，全面發展，以免鑽牛角尖，更因為其他方面的思想，隨時可能為你的專業工作帶來靈感！舉例說，《道德經》的「道生一，一生二，二生三，三生萬物」之說，不正是幾千年後提出的宇宙大爆炸理論的寫照嗎？

where $12=3\times4$, $20=4\times5$ and $30=5\times6$. Does this pattern also hold for 65^2, 75^2, etc.? Can the reader explain this pattern?

Mr. Kwong thinks that the gist to learning mathematics is to think more. We need bold conjectures that may even seem like pie in the sky, as well as careful justification afterward.

Mr. Kwong matriculated at The University of Hong Kong with flying colors, specialized in mathematics and physics, and graduated with First Class Honours. Finding the mathematics in school too easy and not challenging, Mr. Kwong chose to join the government after graduation. However, when Mr. Kwong became Secretary for the Treasury, he realized how helpful the logical and analytic thinking ability instilled by his mathematical upbringing was in handling Hong Kong's hundreds of billions of reserves, a bigger sum than any he had previously dealt with. Mr. Kwong would later volunteer to become the first Secretary for Information Technology and Broadcasting, a position in which his knowledge in mathematics, physics and information technology proved tremendously useful. So, even if you are not looking into a mathematical career, maybe mathematics will enter your job one day!

In fact, Mr. Kwong notes that mathematics is inseparable from other fields of knowledge. Not only are the natural sciences (such as physics) inextricably interdependent with mathematics, but so are the social sciences. When Adam Smith wrote The Wealth of Nations, he was only able to derive general conclusions from qualitative economic principles. Yet, as more and more mathematics was applied in economics, we are able to obtain more and more accurate results from calculation.

At the same time, Mr. Kwong also advises students passionate about mathematics to pay attention to and think about issues in other areas too. For one, we should learn more knowledge outside of our profession to foster all-round development. As importantly, ideas in other aspects can bring inspirations for your professional work! For instance, the Tao Te Ching states, "Tao breeds one, one breeds two, two breeds three, three breeds all." Is this not a depiction of the Big Bang theory proposed thousands of years later?

笑一笑 Laugh Out Loud

挑戰園地 Challenge Corner

過往挑戰園地的解答及得獎名單，可見:
For the solutions and list of awardees of the Challenge Corner of the past issues, please see:
http://www.edb.gov.hk/tc/curriculum-development/kla/ma/IMO/IMOment.html

1. 設 $a_1, a_2, a_3, \ldots$ 為一數列。已知 $a_1 = 2$，及對所有正整數 n，均有 $a_{n+1} = a_n + 2^n + 1$，求 a_n 的值，答案以 n 表示。

Let $a_1, a_2, a_3, \ldots$ be a sequence of numbers. Given $a_1 = 2$ and for all positive integers n, $a_{n+1} = a_n + 2^n + 1$, find a_n in terms of n.

2. 若一個正整數的數位由 1 和 8 梅花間竹地組成，它便稱為一個「牛數」。例如 1、8、18、181、8181 是牛數，而 79、11、1881 則不是牛數。求所有既是牛數，亦是平方數的正整數。

A positive integer is said to be a "cow number" if its digits consist of alternate 1's and 8's. For example, 1, 8, 18, 181, 8181 are cow numbers, whereas 79, 11 and 1881 are not cow numbers. Find all cow numbers that are perfect squares.

3. 設 $ABCD$ 為等腰梯形，其中 AB // DC，$AD = BC$，且梯形內有一個與四邊都相切（即與每邊都剛好交於一點）的圓。設這個圓跟 BC 交於 E，跟 AD 交於 F。求證：直線 AC、BD、EF 交於一點。

Suppose $ABCD$ is an isosceles trapezium with AB // DC and $AD = BC$, and there is a circle in the trapezium that is tangent to all four sides (i.e. that touches each side at exactly one point). Suppose the circle touches BC at E and touches AD at F. Prove that the lines AC, BD and EF meet at one point.

4. 以下是一個「所有人的身高相同」的「證明」：「設 $P(n)$ 代表以下命題：『無論怎樣選出 n 人，他們的身高總是相同。』$P(1)$ 顯然成立。假設 $P(k)$ 成立。那麼，每當選出 $k+1$ 人，由歸納假設可知，第 1、2、...、k 人的身高相同，亦可知第 2、3、...、$k+1$ 人的身高相同。故此，這 $k+1$ 人的身高都相同。所以 $P(k+1)$ 亦成立。根據數學歸納法原理，$P(n)$ 對所有正整數 n 都成立。」這個證明有什麼問題？（可參考本期關於數學歸納法的文章。）

Here is a "proof" that all people have the same height: "Let $P(n)$ be the statement that whenever n people are chosen, they always have the same height. Clearly, $P(1)$ is true. Suppose $P(k)$ is true. Then whenever $k+1$ people are chosen, the inductive hypothesis tells us that the 1st, 2nd, ..., kth people have the same height, and also that the 2nd, 3rd, ..., $(k+1)$th people have the same height. Hence, all the $k+1$ people have the same height. So $P(k+1)$ is also true. By the principle of mathematical induction, $P(n)$ is true for all positive integers n." What is wrong with this proof? (You may refer to the article on mathematical induction in this issue.)

歡迎香港中學生讀者電郵至 info@imohkc.org.hk 提交解答（包括證明），並於電郵中列明學生中英文姓名、學校中英文名稱及學生班級。每一名學生只可發送一份電郵。首 20名 答對最多題目的同學將獲贈紀念品，但每間學校最多有 3 名同學得獎。解答可以中文或英文提交。打字及掃描文件皆可接受。得獎者將於下一期公布。2016 年第五十七屆國際數學奧林匹克籌備委員會對本活動安排有最終決定權。如有疑問，可電郵至 info@imohkc.org.hk 查詢。

Hong Kong secondary school student readers are welcome to submit solutions (with proofs) via email to info@imohkc.org.hk, specifying the student's name in Chinese and in English, the school's name in Chinese and in English, and the student's class in the email. Each student may send at most one email. Souvenirs will be awarded to the first 20 students solving the most questions on the condition that each school can have at most 3 awardees. Solutions can be submitted in Chinese or English. Both typed and scanned files are acceptable. The awardees will be announced in the next issue. The decision of the Organising Committee of the 57th International Mathematical Olympiad on any matter of this activity is final. Enquiries may be emailed to info@imohkc.org.hk.