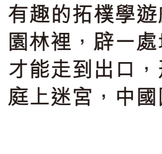
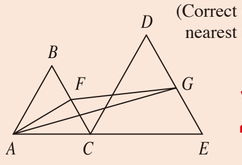
| SUN 日 | MON | TUE 二 | WED 三 | THU 四 | FRI 五 | SAT 六 |
|---|---|---|---|---|---|---|
| Find the remainder $r$ when $6^{84}$ is divided by 13.<br>**1** 廿一 | Given that the real numbers $x$ and $y$ satisfy the relation $y = \frac{x}{2x-1}$.<br>Find the minimum value of $\frac{1}{x^2} + \frac{1}{y^2}$.<br>**2** 廿二 | In the figure, $ABCD$ is a rectangle with $AB = \sqrt{\frac{8+\sqrt{64-\pi^2}}{\pi}}$ and $BC = \sqrt{\frac{8-\sqrt{64-\pi^2}}{\pi}}$. $\overset{\frown}{BE}$ and $\overset{\frown}{BF}$ are the arcs of circles with centres at $C$ and $A$ respectively. If $b$ is the total area of the shaded parts, find $b$.<br><br>**3** 廿二 | Birthday of **Sir William Rowan Hamilton**. Hamilton taught himself mathematics at the age of 17. In 1833, he devised a version of rational mechanics (based on so-called conjugate momenta) which helps clarify modern formulations of quantum mechanics. He invented quaternions in 1843.<br>**4** 廿四 | What is the last digit of<br>$1^{2010} + 2^{2010} + 3^{2010} + 4^{2010} + 5^{2010}$?<br>**5** 廿五 | 鴨和豬共有48隻腳，如果鴨和豬的數目對調，便有42隻腳，求鴨原有的數目。<br>**6** 廿六 | How many prime numbers less than 100 have 3 as the unit digit?<br>**7** 立秋 |
| 3 balls are chosen with replacement from an urn containing 3 red, 4 white and 5 blue balls. The probability that all the 3 chosen balls are of the same colour is $\frac{1}{a}$, find $a$.<br>**8** 廿八 | Let $f(1) = c + 1$ and $f(n) = (n-1) f(n-1)$, where $n > 1$. If $f(5) = 240$, find $c$.<br>**9** 廿九 | Imagine 100 light bulbs with light switches numbered 1 through 100, all in a row, all off. Suppose you do the following: toggle all switches that are multiples of 1, then toggle all switches that are multiples of 2, then toggle all switches that are multiples of 3, etc. By the time you are finished, how many light bulbs are on?<br>**10** 七月 | In the figure, a square with area equal to 25 cm$^2$ is divided into 25 small squares with area equal to 1 cm$^2$. If the total number of different squares in the figure is $5k$, find $k$.<br>  <br>**11** 初二 | Find the maximum value of the function $f(x) = \lvert x^2 - 4 \rvert - 6x$ (where $-2 \le x \le 5$).<br>**12** 初三 | Given the sequence 1, 2, 3, 5, 8, $x$, ..., find $x$.<br>**13** 初四 | If $\log_2 Q + \log_4 Q + \log_8 Q = 7$, find $Q$ (Correct to the nearest integer).<br>**14** 初五 |
| Given that the area and the perimeter of a rectangle are 60 cm$^2$ and 38 cm respectively. Find the longer length, in cm, of this rectangle.<br>**15** 初六 | Given that $a$, $b$, $c$ are positive integers and $a < b < c = 10$. Find the number of triangles formed with sides equal to $a$ cm, $b$ cm and $c$ cm.<br>**16** 初七 | Birthday of **Pierre de Fermat**. He was a mathematician who is given credit for early developments that led to modern calculus. In particular, he is recognized for his discovery of an original method of finding the greatest and the smallest ordinates of curved lines, which is analogous to that of the then unknown differential calculus, as well as his research into the theory of numbers. He also made notable contributions to analytic geometry, probability, and optics.<br>**17** 初八 | Take any four digit number (whose digits are not all identical):<br>1. Rearrange the string of digits to form the largest and smallest 4-digit numbers possible (insert leading zeroes for any numbers with fewer than 4 digits.)<br>2. Subtract the smaller number from the larger<br>3. Use the number you obtain and repeat the above process.<br>Find the sum of digits of the number obtained after 7 repetitions.<br>**18** 初九 | There are ten old machines in a factory. Suppose the probability that any of them malfunction in a month is 10%. What is the probability (in %) that any of two malfunction on the same month?<br>**19** 初十 | $ABCDE$ is a positive 5-digit integer divisible by 9. $A + B = 7$. What is the maximum value of $C + D + E$?<br>**20** 十一 | Birthday of **Augustin Louis Cauchy**. Cauchy wrote 789 papers in all areas of the mathematics and theoretical physics of his time. In 1821, his Cours d'analyse at Polytechnique put analysis on a rigorous footing. He originated the calculus of residues (1826) and complex analysis (1829).<br>**21** 十二 |
| Find the coefficient of the term $n$ for $(2n+1)^{11}$.<br>**22** 十三 | How many people do you need in a group to ensure that the probability of getting 2 people in the group share a birthday is not less than 0.5?<br>**23** 十四 | A two-digit number is 4 times the sum of the digits, and the number formed by reversing the digits exceeds 5 times the sum of the digits by 12. What is the original two-digit number?<br>**24** 十五 | Find the second positive integer to show that the following conjecture is not true: integer which is larger than one can be written in the form $p + a^2$, where $p$ is either a prime or 1, and $a$ is a positive integer.<br>**25** 十六 | Let $n$ be a positive integer. The area of the triangle bounded by the line $nx + (n+1)y = \sqrt{2}$ and the two coordinate axes is $S_n$.<br>Find $27(S_1 + S_2 + ... + S_{26})$.<br>**26** 十七 | Let $f(n)$ be a non-negative integer function defined on the set of natural numbers. For all natural numbers $m$ and $n$, we have $f(m+n) = f(m) + f(n) + (0 \text{ or } 1)$. If $f(2) = 0$, $f(3) > 0$ and $f(9999) = 3333$, find $f(81)$.<br>**27** 十八 | There are 8 persons in the classroom. If each person in the classroom shakes hands exactly once with each other in the classroom and there are altogether $n$ handshakes, find $n$.<br>**28** 十九 |
| In the figure, $C$ lies on $AE$, $\Delta ABC$ and $\Delta CDE$ are equilateral triangles. $F$ and $G$ are midpoints of $BC$ an $DE$ respectively. If area of $\Delta ABC$ is 8 cm$^2$, area of $\Delta CDE$ is 40 cm$^2$, find the area of $\Delta AEG$ in cm$^2$ (Correct to the nearest integer).<br>  <br>**29** 二十 | Two dice are rolled, one green and one red. How many different outcomes are there with different values on the two dice?<br>**30** 廿一 | A number is selected at random from the integers between 1 and 77 inclusively such that it is exactly divisible by 4 or 5.<br>If the probability is $\frac{x}{77}$, what is $x$?<br>**31** 廿二 | | | | |

## 拓樸學

拓樸學源於數學家Euler用來解決「七橋問題」的方法，就是在連續變形下保持不變的性質。而迷宮就是一種有趣的拓樸學遊戲，同時亦可應用於建築學上。如在園林裡，辟一處地面，建出一幅幅牆壁，要左拐右拐才能走到出口，形成一個迷宮，在建築學上來說是庭上迷宮，中國圓明園便是一個典型的例子。

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