

JUNE

六月 2010

Find the remainder of 3^{2010} divided by 26.

If $6^b + 6^{b+1} = 2^b + 2^{b+1} + 2^{b+2} + 2^{b+3} + 2^{b+4} + 2^{b+5}$, find b .

If c is the largest slope of the tangents from the point $A(\frac{3}{2}, \frac{3}{2})$ to the circle $x^2 + y^2 = 1$, find c (Correct to the nearest integer).

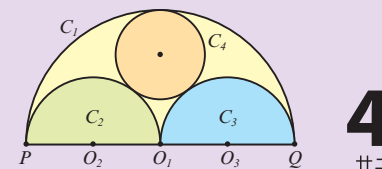
In the figure, PQ, PO_1, O_1Q are diameters of semi-circles C_1, C_2, C_3 with centres at O_1, O_2, O_3 respectively, and the circle C_4 touches C_1, C_2 and C_3 . If $PQ = 24$, find the radius of circle C_4 .

Let $a = \overline{m04}$ be a 3-digit number. If the 6-digit number formed by putting a at the end of the number 504 is divisible by 7, 9 and 11, find m .

1
十九

2
二十

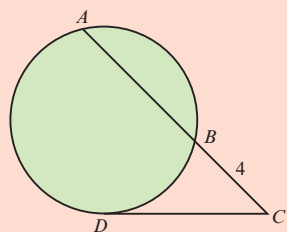
3
廿一



4
廿二

5
廿三

Given $AB = 5, BC = 4$ and DC is a tangent to the circle, find DC .



6
芒種

If $(\frac{11\dots1}{n})^2 = 1234567654321$, find n .

7
廿五

If $\frac{4a}{(1-x^{16})} = \frac{2}{(1-x)} + \frac{2}{(1+x)} + \frac{4}{(1+x^2)} + \frac{8}{(1+x^4)} + \frac{16}{(1+x^8)}$, find a .

8
廿六

If the letter 'S' represents 4, what digit does the letter 'A' represent?

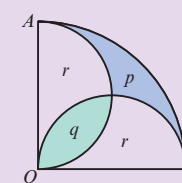
$$\begin{array}{r} S \quad E \quad E \\ S \quad E \quad E \\ S \quad E \quad E \\ + \quad Y \quad E \quad S \\ E \quad A \quad S \quad Y \end{array}$$

9
廿七

If $\int_0^{10} \int_0^9 \int_0^8 \dots \int_0^1 1 dx_1 \dots dx_8 dx_9 dx_{10} = n!$, find n .

10
廿八

In the figure, OAB is a quadrant of a circle and semi-circles are drawn on OA and OB . If p, q denote the areas of the shaded regions, where $p = 11\text{cm}^2$ and $q = c\text{cm}^2$, find c .



11
廿九

12
五月

A man has \$4.55 in change composed entirely of dimes (10 cents) and quarters (25 cents). When the number of dimes equals the number of quarters, find the number of quarters.

13
初二

Birthday of **Andrei Andreyevich Markov**. Markov is a Russian mathematician, best known for his work on theory of stochastic processes. His research was later known as Markov chains. He and his younger brother Vladimir Andreevich Markov proved Markov brothers' inequality.

14
初三

The banker pays the gambler \$120 if all heads or tails are obtained when 3 coins are tossed, and gets \$60 from the gambler if otherwise. What is the expected gain of the banker?

$$\frac{7x+5}{(x+1)^2(x-1)} \equiv \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1}$$

where A, B, C are constants. Find $7B - 2A + C$.

15
初四

端午節

16
初五

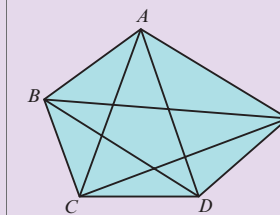
Consider the sequence 5, 12, 10, x , 15, 22, 20, 27, 25, ..., find x .

17
初六

Consider the sequence 1, 1, 2, 3, $a, 8, b, 21, 34, 55$, find $a + b$.

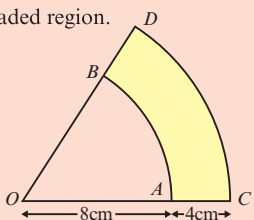
18
初七

In the figure, $AC = AD = AE = ED = DB$ and $\angle BEC = c^\circ$. Given that $\angle BDC = 26^\circ$ and $\angle ADB = 46^\circ$, find c .



19
初八

In the diagram, arcs AB and CD are arcs of concentric circles centred at O . If OA is 8cm, AC is 4cm and the area of sector AOB is 16cm^2 , find the area of the shaded region.



20
初九

Birthday of **Siméon Denis Poisson**. Among his many mathematical contributions, it is a very abstract construct in analytical mechanics (Poisson Brackets, 1809) which helped Dirac formulate a precise correspondence between classical and quantum mechanics.

21
夏至

Find the smallest positive integer k such that $5^k \equiv 1 \pmod{23}$.

22
十一

Mr. Lee is a years old. If the product of a and his month of birth is 253, find a .

23
十二

有一個兩位數，十位數是個位數的 2 倍。將原數的個位數及十位數互換得一新數，原數減去這新數得 18。求新數。

24
十三

Mr. Lee has a sweets. If he divides them equally among 10 persons, 5 sweets will remain. If he divides them equally among 7 persons, 3 more sweets are needed. Find the minimum value of a .

25
十四

An unknown virus attack H.K. with estimated infection rate at t (in month) is given by $T'(t) = 100te^{-t}$. Given $T(0) = 0$, find $T(1)$, the number of people suffered from the virus in the first month (correct to the nearest integer).

26
十五

How many ways can three boys leave a room with three doors?

27
十六

Birthday of **Henri Léon Lebesgue**. Lebesgue is a French mathematician most famous for his theory of integration, which was originally published in his dissertation *Intégrale, longueur, aire* ("Integral, length, area") at the University of Nancy in 1902.

28
十七

Given that $111111222222 = (3333y3)(3333y4)$, find $y + 2$.

29
十八

A bag of sweets is distributed to three persons A, B and C in the ratio of 1:2:3 respectively. If the sweets are re-distributed to A, B, C according to the ratio 2:3:5 respectively, then one of them would get 1 more sweets than his original number. Find the total number of sweets in the bag.

30
十九

Reflection

Two 2-dimensional characters are obtained by a reflection. To construct the line of reflection, choose any pairs of points from the characters such that they are mutual images under the reflection. Use a ruler and a compass to draw the perpendicular bisector of the line segment between the two chosen points. That perpendicular bisector is the line of reflection we sought.

