

Hong Kong Mathematics Olympiad (2015 / 2016)

Heat Event (Individual) Sample Paper

香港數學競賽 (2015 / 2016)

初賽項目(個人) 模擬試卷

Part A

甲部

1. 除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
2. 本部各題估分相等，每題 1 分。
All questions in this section are of equal marks, each question carries 1 mark.

1. 整數 x 減去 12 後是一個整數的平方。將 x 加上 19 後則是另一個整數的平方。求 x 的值。
An integer x minus 12 is the square of an integer. x plus 19 is the square of another integer. Find the value of x .
(2010/11 Heat Event (Individual) Qu. 5)

2. 已知 $(10^{2015})^{-10^2} = 0.\underbrace{000\dots01}_{n \text{ 個 } 0}$ ，求 n 的值。

Given that $(10^{2015})^{-10^2} = 0.\underbrace{000\dots01}_{n \text{ times}}$. Find the value of n . (2014/15 Heat Event (Individual) Qu. 2)

3. 如圖一所示， $ABCD$ 為圓內接四邊形，其中 $AD = 5$ 、 $DC = 14$ 、 $BC = 10$ 及 $AB = 11$ 。求四邊形 $ABCD$ 的面積。

As shown in Figure 1, $ABCD$ is a cyclic quadrilateral, where $AD = 5$, $DC = 14$, $BC = 10$ and $AB = 11$. Find the area of quadrilateral $ABCD$.

(Modified from 2013/14 Heat Event (Individual) Qu. 5)

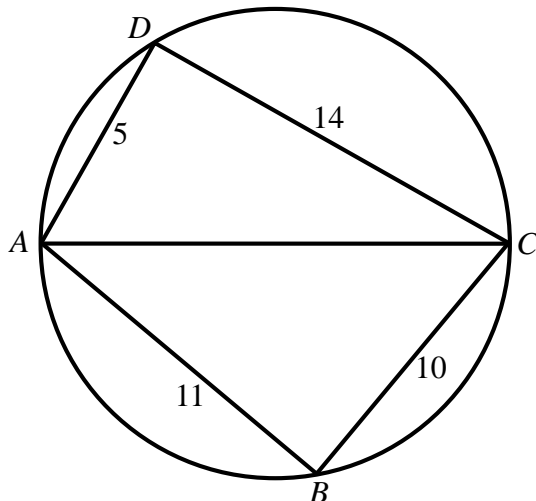
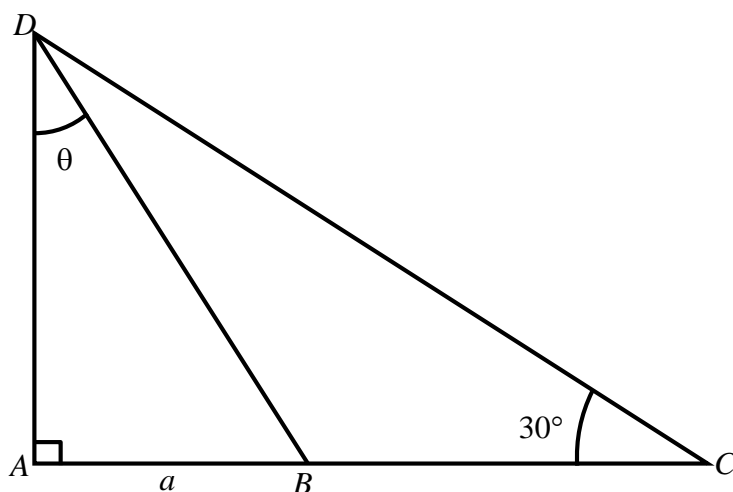


Figure 1
圖一

4. 圖二所示為一直角三角形 ACD ，其中 B 是 AC 上的點且 $BC = 2AB$ 。已知 $AB = a$ 及 $\angle ACD = 30^\circ$ ，求 θ 的值。

Figure 2 shows a right-angled triangle ACD where B is a point on AC and $BC = 2AB$. Given that $AB = a$ and $\angle ACD = 30^\circ$, find the value of θ .

(Modified from 2012/13 Heat Event (Individual) Qu. 3)



圖二

Figure 2

5. 學校推出每張面值為 \$10、\$15、\$25 及 \$40 的四種賣物券。甲班用若干張 \$100 紙幣買了 30 張賣物券，包括其中兩種賣物券各 5 張及另外兩種賣物券各 10 張。問甲班共用了多少張 \$100 紙幣購買賣物券？

A school issues 4 types of raffle tickets with face values \$10, \$15, \$25 and \$40. Class A uses several one-hundred dollar notes to buy 30 raffle tickets, including 5 tickets each for two of the types and 10 tickets each for the other two types. How many one-hundred dollar notes does Class A use to buy the raffle tickets? (2010/11 Heat Event (Individual) Qu. 8)

6. 求 2^{2011} 除以 13 的餘數。

Find the remainder when 2^{2011} is divided by 13. (2010/11 Heat Event (Individual) Qu. 1)

7. $2^{20} \times 25^{12}$ 是一個多少個位的數？

Find the number of places of the number $2^{20} \times 25^{12}$. (2011/12 Heat Event (Individual) Qu. 4)

8. 甲、乙及丙三人互相傳球。甲首先將球傳出。有多少不同方案使得經過 5 次傳球後，球會回傳給甲？

A , B and C pass a ball among themselves. A is the first one to pass the ball to the other one. In how many ways will the ball be passed back to A after 5 passes?

(2010/11 Heat Event (Individual) Qu. 6)

9. 已知 a 及 b 為不相同質數，且 $a^2 - 19a + m = 0$ 及 $b^2 - 19b + m = 0$ ，求 $\frac{a}{b} + \frac{b}{a}$ 的值。

Given that a and b are distinct prime numbers, $a^2 - 19a + m = 0$ and $b^2 - 19b + m = 0$. Find the value of $\frac{a}{b} + \frac{b}{a}$. (2011/12 Heat Event (Individual) Qu. 6)

10. 已知 $a_1, a_2, \dots, a_n, \dots$ 為一正實數序列，其中 $a_1 = 1$ 及 $a_{n+1} = a_n + \sqrt{a_n} + \frac{1}{4}$ 。求 a_{2015} 的值。

It is given that $a_1, a_2, \dots, a_n, \dots$ is a sequence of positive real numbers such that $a_1 = 1$ and $a_{n+1} = a_n + \sqrt{a_n} + \frac{1}{4}$. Find the value of a_{2015} . (2014/15 Heat Event (Individual) Qu. 5)

Part B

乙部

1. 除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
2. 本部各題佔分相等，每題 2 分。
All questions in this section are of equal marks, each question carries 2 marks.

11. 若方程 $(k^2 - 4)x^2 - (14k + 4)x + 48 = 0$ 有兩個相異的正整數根，求 k 的值。

If the quadratic equation $(k^2 - 4)x^2 - (14k + 4)x + 48 = 0$ has two distinct positive integral roots, find the value(s) of k .
(2011/12 Heat Event (Individual) Qu. 8)

12. 已知 $y = (x+1)(x+2)(x+3)(x+4) + 2013$ ，求 y 的最小值。

Given that $y = (x+1)(x+2)(x+3)(x+4) + 2013$, find the minimum value of y .
(2012/13 Heat Event (Individual) Qu. 5)

13. 在 1 至 2015 之間（包括 1 及 2015 在內）有多少對相異整數的積是 5 的倍數？

How many pairs of distinct integers between 1 and 2015 inclusively have their products as multiples of 5?
(2014/15 Heat Event (Individual) Qu. 1)

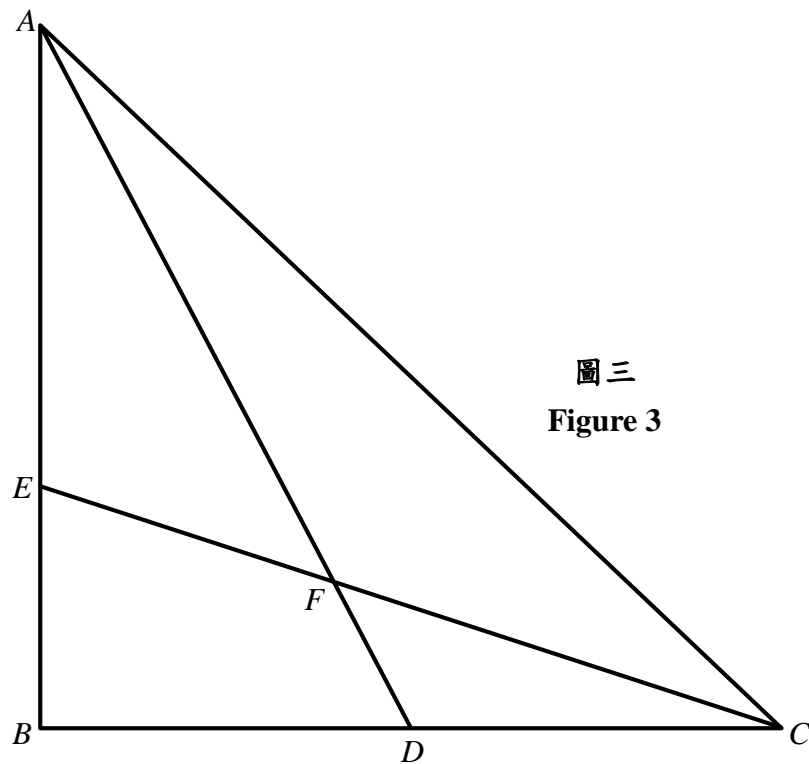
14. 設 x 為實數。求 $\sqrt{x^2 - 4x + 13} + \sqrt{x^2 - 14x + 130}$ 的最小值。

Let x be a real number. Find the minimum value of $\sqrt{x^2 - 4x + 13} + \sqrt{x^2 - 14x + 130}$.
(2014/15 Heat Event (Individual) Qu. 9)

15. 如圖三， $AE = 14$ ， $EB = 7$ ， $AC = 29$ and $BD = DC = 10$ 。求 BF^2 。

In Figure 3, $AE = 14$ ， $EB = 7$ ， $AC = 29$ and $BD = DC = 10$. Find BF^2 .

(Modified from 2011/12 Heat Event (Individual) Qu. 10)



圖三
Figure 3

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END