SCHOOL MATHEMATICS NEWSLETTER

2

OCTOBER, 1978

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Please ensure that every member of your mathematics staff has an opportunity to read this Newsletter.

MATHEMATICS SECTION

EDUCATION DEPARTMENT

HONG KONG

P R E F A C E

The principal objective of the School Mathematics Newsletter (S.M.N.) is to improve the teaching of school mathematics.

You will find a variety of articles in S.M.N. expounding views, theories, experiences, critiques together with extensive assortment of information written by those people directly responsible. We hope to provide a veritable pool of ideas for teachers to use, including recreational material and also hope to create the challenge of a puzzle and problem corner.

An important aspect of S.M.N. is the correspondence page. We wish to encourage people to express their views freely and hope to establish a forum in this respect. So if you have something to say or something to argue about, whatever your field in education, put your pen to paper and forward your correspondence to the Editor, School Mathematics Newsletter, Mathematics Section, Advisoty Inspectorate, Education Department, Lee Gardens, Hong Kong.

We extend our thanks to all who have contributed to this month's issue.

F. Parkin

First Joint Schools Mathematics Exhibition

The First Joint Schools Mathematics Exhibition sponsored by the Mathematics Teaching Centre, Advisory Inspectorate, Education Department will be held in St. Paul's Secondary School, Ventris Road, Happy Valley, Hong Kong from 7th to 10th October, 1978.

The 10 participating schools are :

1. Caritas St. Francis Prevocational School
2. King's College
3. Kwun Tong Maryknoll College
4. Moral Training English College
5. Queen's College
6. Raimondi College
7. St. Francis Xavier's School
8. St. Louis School
9. St. Paul's Secondary School (Organizer)
10. Ying Wa College

The exhibition will include many interesting and thought-provoking topics on mathematics such as :

1. Linkage drawing and curve stitching
2. The 4-colour problem, both plane and 3-D
3. Geometric patterns built up from circles
4. Probability experimental material, sampling boxes, coin tosser, galton guincunx, dices, etc.
5. Topology and related topics
6. Number and related topics
7. Mathematical games and puzzles
8. Machines or tools in mathematics
9. Statistics and related topics
10. Challenge in mathematics

Teachers and pupils are cordially invited to the exhibition as it will give some new ideas or insight in the teaching and learning of mathematics. More details will be announced in due course through posters and press.

MTC

New Mathematics Teaching Centre (Kowloon)

The Mathematic **Teaching** Centre (Kowloon) mainly for secondary school mathematics teachers has been removed from Ma Tau Chung Government Primary School to Wong Tai Sin Government Primary School. The full address of the new centre is

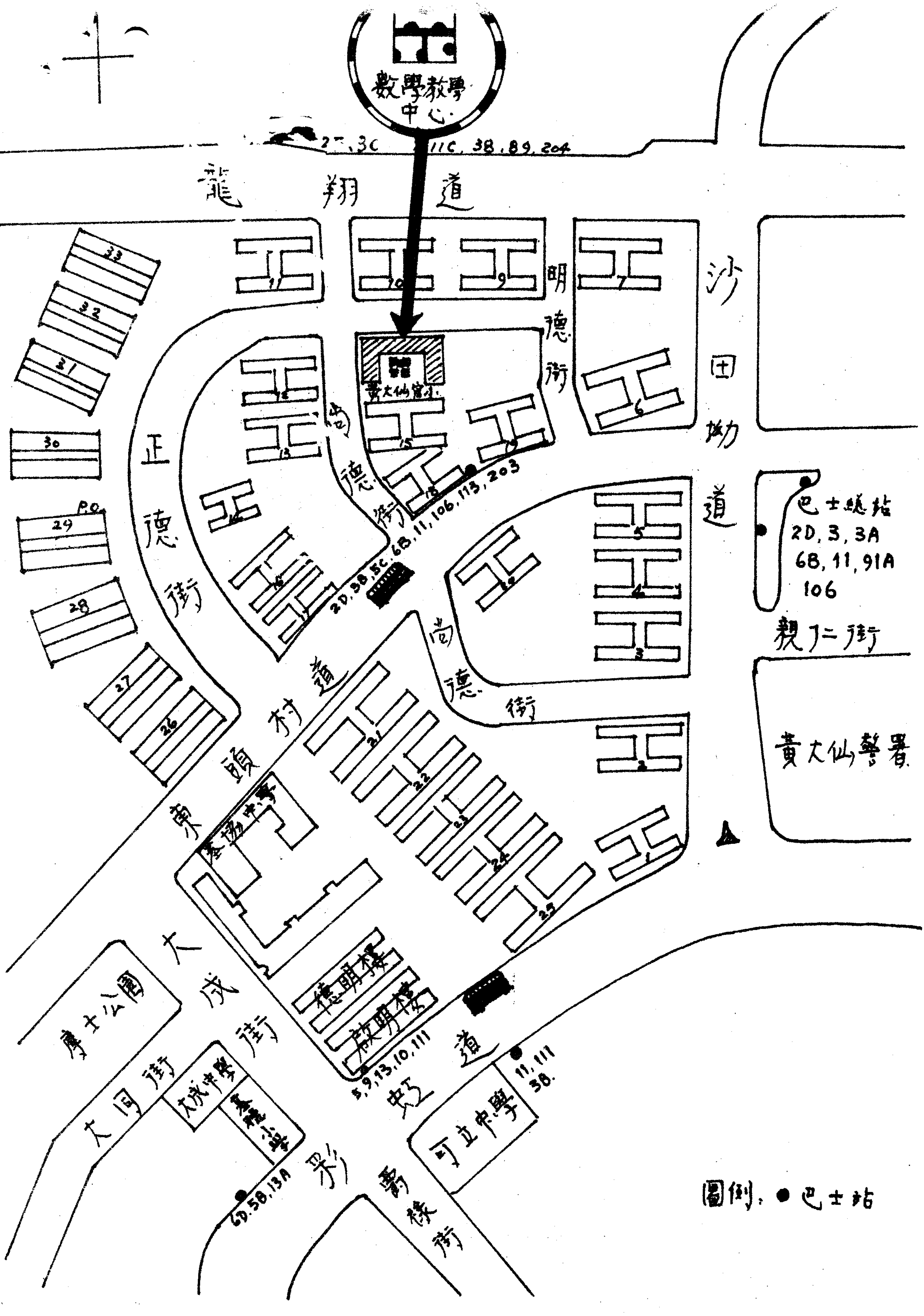
> Rooms 25 and 26, Second floor,
> Wong Tai Sin Government Primary School,
> 100 Ching Tak Street,
> Wong Tai Sin, Kowloon.

A map showing the exact location of the new centre is printed overleaf.

Principals and mathematics teachers are always welcome. Those who wish to visit the Centre or make use of its facilities, please do not hesitate to contact the Mathematics Section, Advisory Inspectorate, Education Department at 5-774001 Ext. 47.

MTC

University lecturers, college of education lecturers and mathematics teachers who wish to contribute articles for publication are more than welcome. Contributions need not be typed. For further information, please contact the Editor, School Mathematics Newsletter at 5-774001 ext. 36.

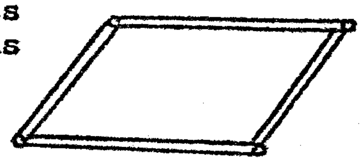
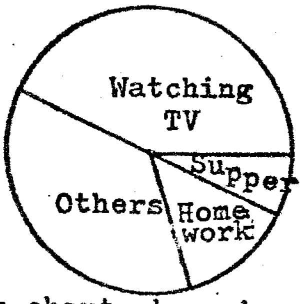


數學教學中心
11C, 38, 89, 204
龍 翔 道
明德街
沙田坳道
黃大仙官小
正德街
尚德街
6B, 11, 106, 113, 203
2D, 3B, 5C, 6B
巴士總站
2D, 3, 3A
6B, 11, 91A
106
親仁街
尚德街
黃大仙警署
大成街
大同街
德明樓
啟明樓
大成中學
可立中學
道
11, 111
3B.
6D, 5B, 13A
圖例：● 巴士站

OPEN-ENDED QUESTIONS/ACTIVITIES

Y.H. Yang

An open-ended situation, be it a question or an activity, is one to which there is no single answer or solution, but there can be several correct responses. It is opposite to a situation which is specifically directed. To appreciate the difference between the two, let us look at some examples:

Specifically directed	Open-ended*
1. A parallelogram has a base of 6 cm and an area of 27 cm^2. What is its height ?	1. Use cardboard strips (10 cm and 15 cm long) and paper fasteners to make a parallelogram. When you change its shape, what is changing and what remain the same ?  Investigate the relationship between the area and the height as you change the shape. Illustrate with a graph. What is the shape when the area is the greatest? Least?
2. Find the time it takes for 20 swings of pendulums of lengths 1·2 m, 1·0 m, 0·8 m, 0.6 m, ...,. Draw a graph of your results.	2. How can you vary the swing of a pendulum ? Comment on the results you obtain.
3. Measure the length and width of the Hall, giving your answers to the nearest 5 cm.	3. Without using a ruler, find the ratio of the length to the width of the Hall in as many ways as you can. Compare your results and comment on them.
4.  This chart shows how a boy divides up his time between arriving home at 5pm after school and going to bed at 10pm. (a) How long does the boy spend on each part ? (b) What fraction of the total time is spent on each part ?	4. (The same chart) Discuss possible deductions from this chart. Discuss whether other sorts of diagram could serve the same purpose equally well or even better. OR (The same chart with no description of the situation) Devise, in as much detail as you like, a situation which could be diagrammatically represented by this chart.

* There are different degrees of openness in the four questions.

We can thus see a specifically directed question, which is found in most textbooks and used in most lessons, is essential in drilling pupils towards a destined end, e.g., the acquisition of mathematical knowledge or skills. Such types of question are 'closed' in the sense that there is little scope for investigation and discovery. On the other hand, when a pupil is confronted with an open-ended situation, he has to apply his knowledge and skill to search for all possibilities; he has to plan a strategy in tackling the problem, and select the appropriate mathematical tools to solve it. Thus an open-ended question gives the pupils greater freedom to explore, and it encourages active thinking and develops creativity in the children. Besides, each pupil can work according to his ability and at his own pace. However humble his level, he is a 'mathematical thinker' and not merely a 'mathematical doer'.

It may not seem easy to introduce an open-ended situation into the classroom for the first time. An uninitiated pupil, when meeting such a problem, usually feels a certain degree of insecurity which is often expressed in a question like "But what do you want me to do?" It takes time to build up pupils' confidence so that they can make good use of the freedom which is theirs in an open-ended context. Nevertheless, an open-ended problem, like an open door, can always be closed when a pupil shows the problem is too difficult for him. A teacher has to determine, by judging from the pupil's difficulty, whether to close the door to a greater or smaller extent. He can do this by re-framing the question so that it becomes more directed, or by giving an appropriate amount of clues and assistance to the pupil. On the other hand, a closed question in the first instance may well give the pupil a clue he does not need, thus depriving him of the opportunity of thinking and discovering for himself. A closed door has never the chance to face openness.

For a change of classroom climate towards a more thought-provoking way of learning mathematics, it is worthwhile to amalgamate the traditional types of question with some open-ended ones.

遊 戲 與 數 學 學 習

鄭肇楨博士
香港中文大學教育學院

今天我想討論的問題，是數學學習和遊戲的關係。由此而探討一下遊戲對學習數學有沒有幫助。如果有的話，則這種幫助是在那一方面？在確定了要達成的目標後，教師便可以選擇或創作出配合學習的遊戲。

首先，我想看看遊戲和數學有些甚麼共同的地方。最少這些共有的地方有下列數點：

㈠ 結構上的相類

數學必具的條件，就是必須有一個元素集。不論是甚麼也好，是自然數、有理數、複數、點、綫、面、矢量等等。然後必須有運算的規則。例如加、減、乘、除、變換、射影等等的運算。而且運算的進行，又規定了一些法則，例如結合性，可易性，分配性等等。

遊戲的構成，本質上和數學也一樣，須具有一個元素集，這一個集可能是棋子、籌碼、彈子、號數、紙牌，甚至可能是一群人。然後必須有遊戲的法則，好像數學的運算一樣，清清楚楚地加以定義的。沒有法則的遊戲，不算是正式的遊戲。例如足球比賽，如果沒有規定如何進行，則球賽便被破壞了。

要舉出一個例，來看它們是如何相似是很容易的。例如小孩子常常玩「包、剪、鎚」的遊戲。如果這裡用 p 、 c 、 t 來分別代表這個集的元素，則遊戲法則可以表來表示：

o	p	c	t
p	p	c	p
c	c	c	t
t	p	t	t

這個玩法是猜拳淘汰。例如「包」與「剪」相遇，「剪」便淘汰了「包」；同元素相遇，大家都沒有被淘汰，故此「包」與「包」相遇，仍保留「包」。

這一個集 $\{p, c, t\}$，和定義的運算「 o 」，便是數學了。在這裡，我們的數學教師，當然還可以看出更多的數學來的。例如：

① o 是一個閉合的運算，

② 不存在中性元，當然也不存在逆元，

③ 這個運算並不滿足結合律，例如：

$(p o c) o t = c o t = t$

但 $p o (c o t) = p o t = p$

④ 這個運算具有可易性，因為從表可看到表對稱主軸，

⑤ $\forall x \in \{p, c, t\}, x^n = x$, n 為自然數，

由這個例，可見遊戲是數學，而數學也是遊戲。

㈡　過程重於內容

遊戲與數學同是着重於過程的事（ Process – oriented ）。遊戲產生的樂趣，是在遊戲的過程中獲得的。遊戲的內容（ Content ）反而不重要。例如棋子、紙牌，本身產生不出甚麼樂趣。其內容也無關重要，基本上它們已成爲某些事物的抽象代表。數學也是重於過程的。被運算的元素，並不須有甚麼特殊的意義，甚至在學校裡被運算的元素，通常是有明確意義的實數。但是這些數本身，和數學學習並不一一定有甚麼關係。例如我們教加法，以 3 ＋ 5 爲例也好， 7 ＋ 6 爲例也好，都不是一樣達到目標嗎？我們要使學生明白的是加的意義，這就是運算過程。又例如教以三爲底的數的運算，內容對於我們是不重要的，我們不用三而用四、五、七，甚麼也一樣。事實上三底的數字運算結果（內容），和我們日常生活絶少關係，我們也不須知道這種知識。但是我們從以三爲底的數的運算中，便可明白數底的意義，由此而明白數構成的概念。

說到這裡，我覺得特別要引起教師的注意，希望教師注意到，數學教材的本身並不是學習的主要目的，甚至它可以是完全不是目的。因爲教材的內容雖然也會帶給學習者一些知識，但是這些知識可能並無大用的。例如要學生砌一個圖形，這個知識本身是不須有的。但是在砌的過程中，學生的思考分析便是學習的目的了。

現在我再從另一方面來探討遊戲與學習的問題。這裡先看一個分析：

數學學習技巧識知層分析

層	識知技巧佔百分比
1. Knowledge	35.9
2. Comprehension	39.9
3. Application	8.3
4. Analysis	11.0
5. Synthesis	4.6
6. Evaluation	0.3

Blooms 等人把識知層次分爲六層，企圖把人的思想運用技巧，由簡單而至複雜，由較具體而至抽象，區分出來。研究者根據每層所運用的技巧的定義，來審覘現在的數學教學，得出了以上的一個統計表。（根據 G. Cheung 1974）。這一個表，是由分析中學數學教本中，對學生要求的識知活動而獲得的。在這一分析中，顯見學生的活動，集中在最低的第一、第二層次，這兩種思考活動已佔了全部的 75.8% 。高層次的思考活動，如分析、綜合與評鑑所佔比重甚少。所以假如 Blooms 氏的分類爲對，而人類的識知技巧的培養訓練，應普及於六個層面的話，則顯然，現有的數學教育尙有待改進的地方。

要把分析、綜合與評鑑的思維活動增加，當然可以由數學教學方面來補救。例如教師多鼓勵學生分析問題，找尋關係，綜合觀察結果等。另一方面是用遊戲來增加這些高層次思維活動的機會。

遊戲，特別是策略性遊戲，如下棋、玩橋牌等，所用的識知技巧，往往是在高層的。因爲最低層的知識所佔比重極少．只須認識全部不多的棋子及其走法，或各種牌的不同，便是所需的知識。但在策略性遊戲時，遊戲的人必須綜觀全局，分析每一步的可能後果，從而決定如何行動。

有人認爲決定行動，是遊戲的最大優點。因爲決定行動，是先要對事物認識清楚，分析佈局，衡量各種後果，才能獲得最佳的決策。這一種能力，是最可貴的。它比死知識珍貴得多，能夠培養學生有這種能力，實在比灌注一大堆知識給他們更重要．其實學生在學習中，他們往往是被動的一群，一切事情，教師及其他人早已爲他們作出安排。他們實在並無獨立作出決策的需要及機會，所以如果多鼓勵學生作策略性的遊戲活動，可能是矯正偏差的一個辦法。

利用遊戲教學，當然是早有人提出過，而且在某一程度上實行。不過這裡要區別一下的是，利用遊戲方式教學與遊戲教學是不相同的。譬如我們見到小學上算術課時，分組比賽計算，這便是利用遊戲的競賽一個因素來進行計算而已。實質上這不是我這裡所講的遊戲。因爲它沒有具備其他作爲遊戲的因素。

談到遊戲的因素，它們應是如下的幾點：

1. 遊戲的目的。
2. 程序。
3. 法則。
4. 參加規定。
5. 報酬。
6. 具備下列若干能力與技巧：
   (a)識知方面；(b)情緒域；(c)肌體功能。
7. 交互作用模式。
8. 環境及器具。

遊戲是一個過程而非一堆內容，藉助以上的因素使過程完成。而在這過程中，識知等活動便得以進行。遊戲的一個特點是：卽管在進行中的環境是假的，一個投入於遊戲的人是應該有眞正的心理重現的現象。例如棋類遊戲可能是沿於摹仿戰爭的氣氛，但是下棋者必須具有決戰的心境。如果沒有欲勝對方的企圖的話，則這個棋賽是沒有意義的了．

利用遊戲教學，近來漸多人作研究．其中最爲人知的是 Prof. James Coleman 。他是 Johns Hopkins 的教授。特別是他領導下研究用於社會科學教學的遊戲，現在已大量地爲人所應用。

使用遊戲於數學學習，較大規模的有美國佛羅連達州的 Nova 學校。它並舉行一年一度的 Nova Academic Olympics 。努華中學已使用十五個遊戲，來幫助學生學習科學、數學及社會。

在數學科中，它以「求証」（ Wff'n Proof ）來教邏輯。在一九六四年使用這個遊戲。以四十三個學生的試驗組，經過三個星期後，學生竟能提高智商達 20.9點（在非語言部份）。而同期控制組的學生，智商則提高爲 6.6 點。

又在同時八十四個第九班的學生，使用「等式」（ Equation ）的一個遊戲，此遊戲是爲教數學的基本演算而設計的。經過四個月後，試驗組的學生平均提高算術推理能力達 1.3 年，而同時的控制組，只提高 0.6 年。

其餘尚有 Allegheny County, Penn 的 Bethel Park School ，在一九六六年，四、五、六年的一百零二個學生，玩「等式」的一個遊戲共九個星期，結果有如下的收獲：

數學概念	5.5 月
解問題能力	5.9 月

這個與不用遊戲的一組學生比較，其收獲大約爲兩倍。

當然，這些個別的事例，還未能証明遊戲對學習的長期效果，因爲短期的使用，由於新奇的刺激，學生是會有較好的表現的。不過無論如何，在衡量各種跡象與支持的理由之下，使用這遊戲於數學學習，是教學上值得探討與試行的一個方向。

香港中學數學課程十年來的演變(1966-75)

潘智添

第一章 緒言

數學課程編訂的原則，一般為配合數學理論的概念，與適應客觀環境對數學之要求。由於數學的理論和技術的改進，社會環境的變遷，因此數學的課程亦應作合理的改變。

在中學數學的目標中，數學的主要目的在介紹形與數之基本觀念，使學生充分了解其相互之關係，熟習數學說理，推證之性質和方法。在供給研究各學科所必需的數學基本知識，以充實其體驗自然現象及社會現象之能力。灌輸歸納方法，函數概念，以培養分析能力及啟發其探討之興趣。數學課程的設計與編排，是基於此而建立。

在六十年代的十年中，是人類科技史上光輝燦爛的一頁，可媲美文藝復興與工業革命等對人類生活的影响。在這短短的十年中，人造衛星發射的成功，使人類踏足月球從夢想成為事実。電腦的發明和它的廣泛應用，差不多可代替人的腦子，解答困難的問題，完成繁複的工作。因此人類在學習與知識的運用，產生了很大的變化。在欧美先進的國家中，數學技術因心理學家和教育專家不斷的研究，在四十多年以前，已逐漸在改進，但課程的內容，仍舊沒有重大的改變。由於教育的普及，學校已不是貴族和富有人士的專利品。在這一大群由社會各階層而來的學生中，教師們發現大部份的學生——除少數的天才外——都感到數學這一門是特別困難，枯燥和乏味。為着提高學習興趣，使學生能獲得足夠的基本知識，以適應現在和將來的社會，教師們因此感到課程的內容，有改進的必要。在六十年代的初期，經過很多學者，專家和教師們的努力，一個全新的數學課程，便

由此而產生。

由此觀之，數學課程，除符合一般課程設計之概念，更須要適合時間與環境的要求。社會不斷的改變，課程亦應隨之而改變。

第二章　課程之改進經過

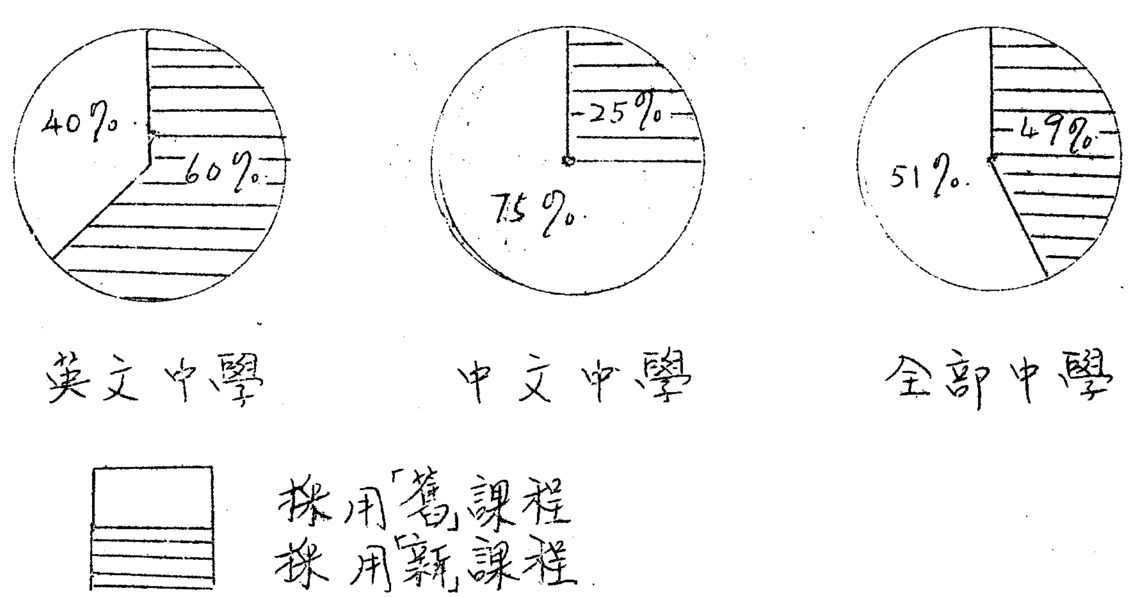
香港開埠以來，人口不斷增加，尤以第二次世界大戰後的廿多年為然。在這段期間，香港從一個細小的商港，蛻變成為一個擁有四百多萬人口的大工業城市。市民的生活水準，不斷的提高和改進，教育漸漸的普及，小學已實行六年免費教育。最近，政府又提議施行三年的補助中學教育，使每一個適齡接受教育的兒童，都可以得到九年教育的機會。又由於工業急速的發展，造成熟練技工的缺乏，當局和有關人士大力提倡工業教育，訓練大量的工業人材。同樣發生在歐美的問題，亦在這裡出現。

香港的每一個中學生，在完成五年中學教育後，都要參加一個公開的考試——香港教育文憑考試。經過這個考試後，每一個參加考試的學生都獲頒給文憑，列出該生所考取的科目及成績。將來考生們的升學和就業，都受到這次考試的影響，因為大學預科班是以學生在中學會考中所考得的成績作為取錄的標準。同時香港大學入學試報考的先決條件中，數學是必須在中學會考中獲得E級或以上的成績的。在找尋工作方面，有很多的大机構，商行，甚至政府聘請新職員時的條件中，都是以在中學會考所得的成績來決定。所以中學各科的課程是完全被這個考試所支配。主理這個考試的部門，每年都訂定一個考試的課程，給各參加這項考試的學校作為參照。負責編訂考試課程的人士包括有大學講師，教育司署視學官，中學校長和有經驗的教師們。

課程的編訂又受到兩間大學所頒佈的入學試課程所影响。因為中學生在完成五年課程後，部份優秀的學生，繼續攻讀預科班，兩年後參加大學入學試。所以中學的課程，必須與大學入學試的課程，互相配合。另一方面，課程的編訂，亦受到社會需求所左右。這幾年來，因為需要大量半熟練技工，提倡工業教育，最近又推行三年補助中學教育，使所有適齡的兒童都有機會接受教育。課程內容的安排，便要作出適當的改變，以滿足社會和個人的要求。

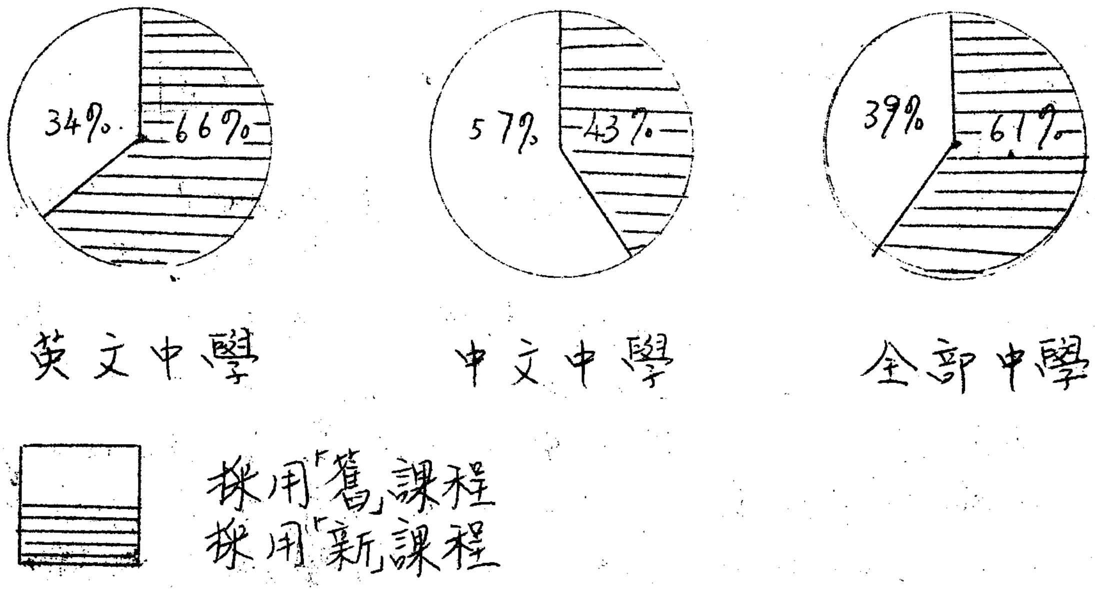
一九六二年的夏天，首由周紹棠博士在香港大學內，舉辦教師暑期研究會，將歐美各國的新數學課程，介紹到香港。其後，他又聯合有關方面人士，組成一個數學研究社，在聯合書院出版了一套供本港英文中學採用的「新數學」課本。在一九六四年的秋季，本港的伊利沙伯中學首先試行採用「新數學」的課程授課。翌年有更多的中學採用「新數學」課程講授。但亦有很多學校沿用舊有的課程。教育當局在一九六九年——即第一屆採用「新數學」課程的學生參加中學會考——曾在三百一拾二間中學作一次統計調查，獲得下列的資料。

一九六九年採用
「新」、「舊」課程之中學數目比較



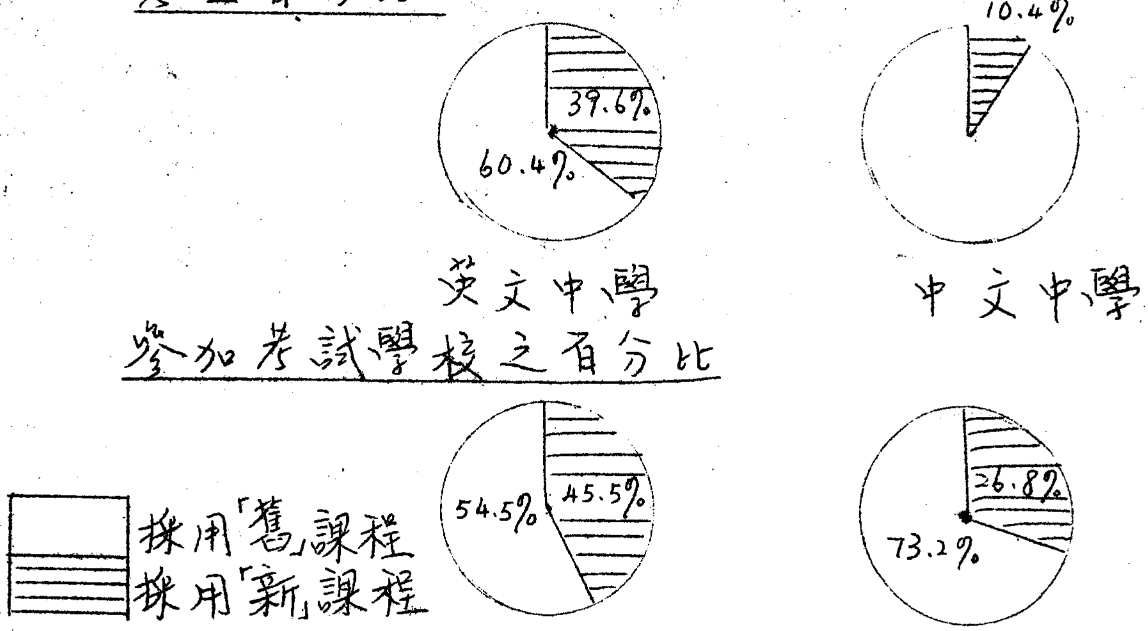
而在一九七二年三月的第二次統計調查得到的資料中，顯示採用「新」課程講授的學校，續有增加。

一九七二年採用
「新」、「舊」課程之中學數目比較



又從中學會考的「新」數學與「舊」數學課程之考生數目與參加考試的學校數目作一比較。英文中學不論在人數和學校數目方面之百分比，均遠較中文中學為高。

一九七二年中學會考選修
「新」、「舊」數學課程之比較



要知道數學的課程，能否滿足以上所談到的需求。從這十年來的會考課程的改進，可以得到一個答案。

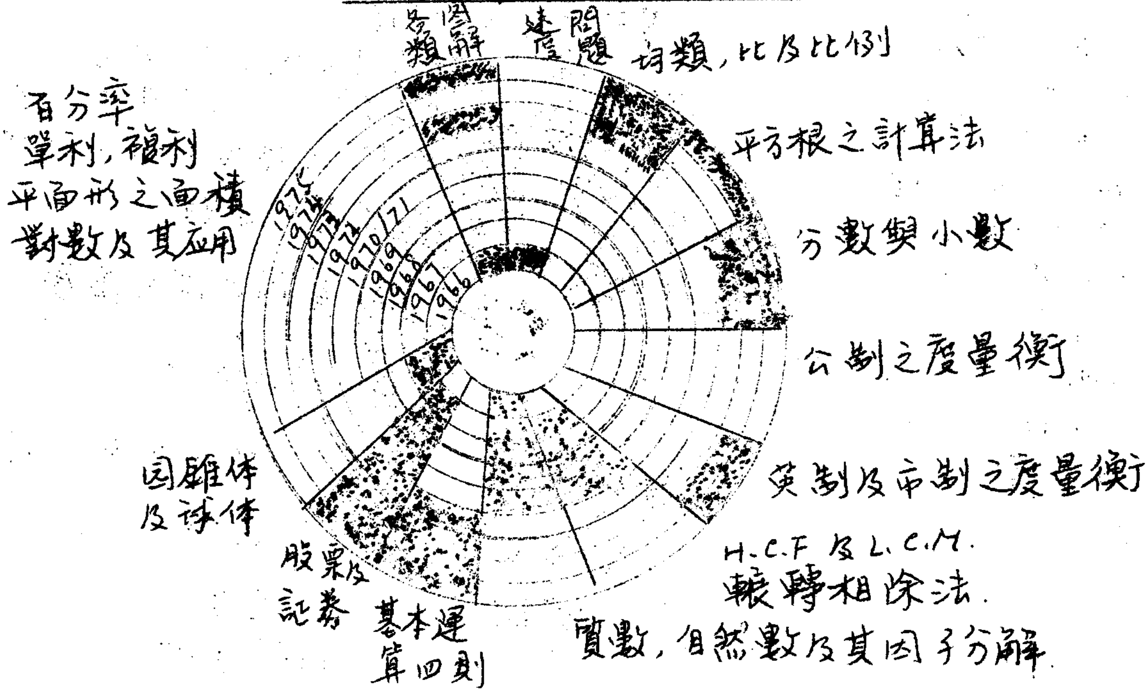
一九六五年至一九七五年
英文中學會考數學課程興替表

1965年以前	1965－1966	1967－1968	1969－1973	1974－1975
初級數學	／	／	／	／
普通數學	普通數學	普通數學	數學課程甲	同等課程乙
／	／	／	數學課程乙	數學課程乙
／	／	附加數學	附加數學	附加數學(乙)

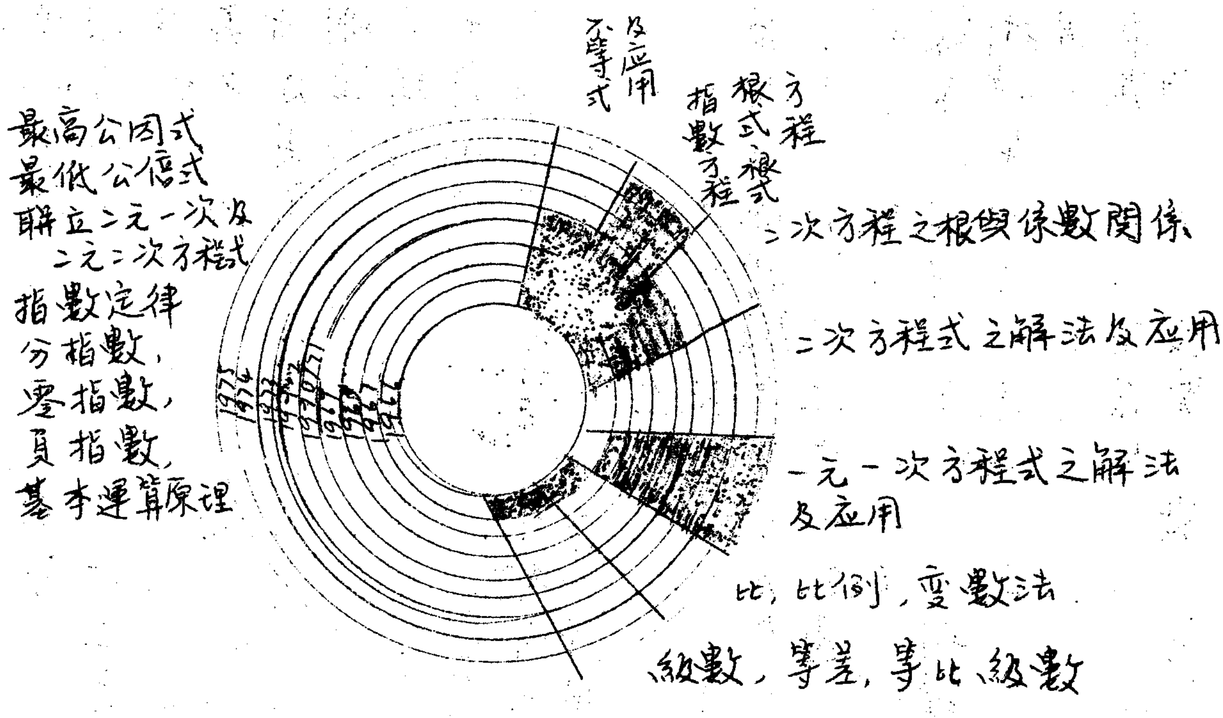
從上表看，在一九六五年以前，中學會考的數學課程是分為兩部份，一部份稱為「初級數學」(Elementary Mathematics)另一部份稱為「普通數學」(Mathematics)。前者的內容只是後者的一部份，它的目的是讓一些對數學感到困難的學生能學習一些簡易而實用的數學知識。「普通數學」的課程，注重平面幾何的說理，求証和應用。利用它來訓練學生的歸納，推理和判斷能力。後來因為學生們的數學程度日漸提高，「初級數學」課程被廢棄了。代之而起的課程是從一九六七年所設的「附加數學」課程。至於「普通數學」課程，在一九六九年起分為兩部份。一部為原有的課程，稱為「課程甲」，於一九七四年又改稱為「同等課程乙」。另一部為「新數學」課程，內容與舊有的有很大差別，稱為「課程乙」。

「普通數學」課程甲的內容，可分為算術，代數，三角和幾何等科。其中的三角學內容沒有太多的改變，而算術、代數和幾何科等，從下面各圖中，顯示出在一九六六年至一九七五年的一段期間中有很大的改變。

算術科課程演變圖

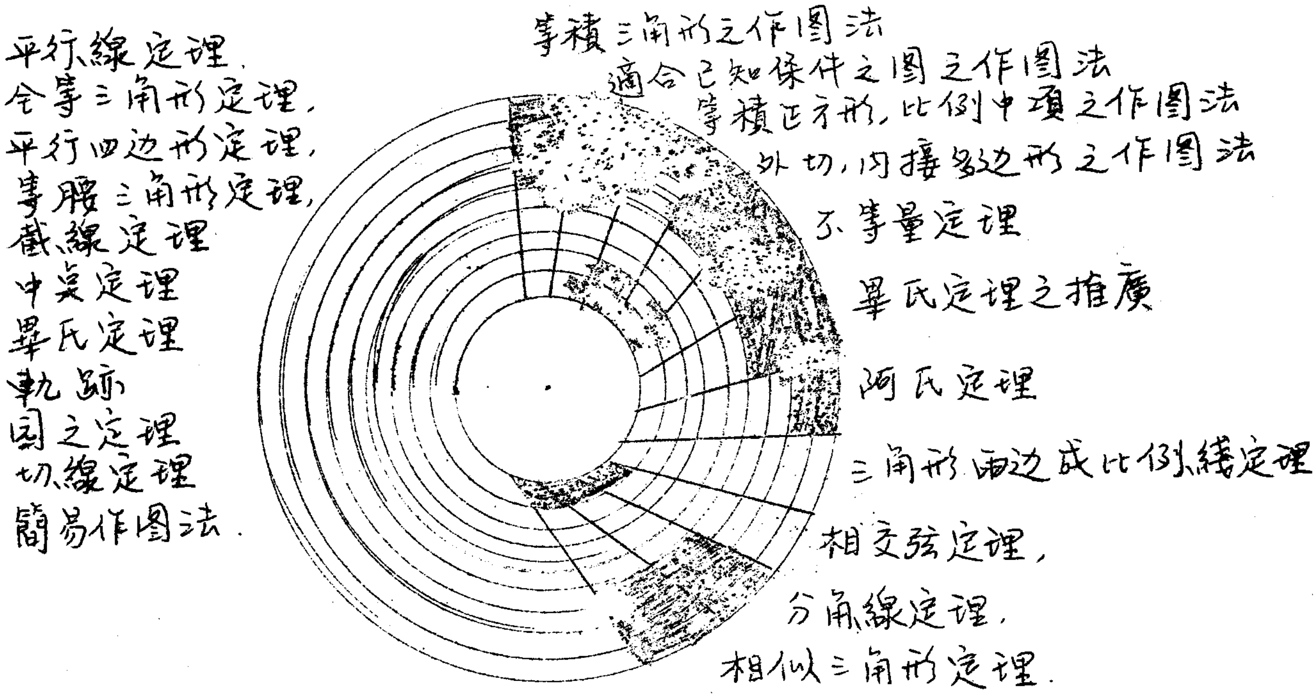


代數科課程之演變圖



不包括本課程範圍內的項目

幾何課程之演變圖



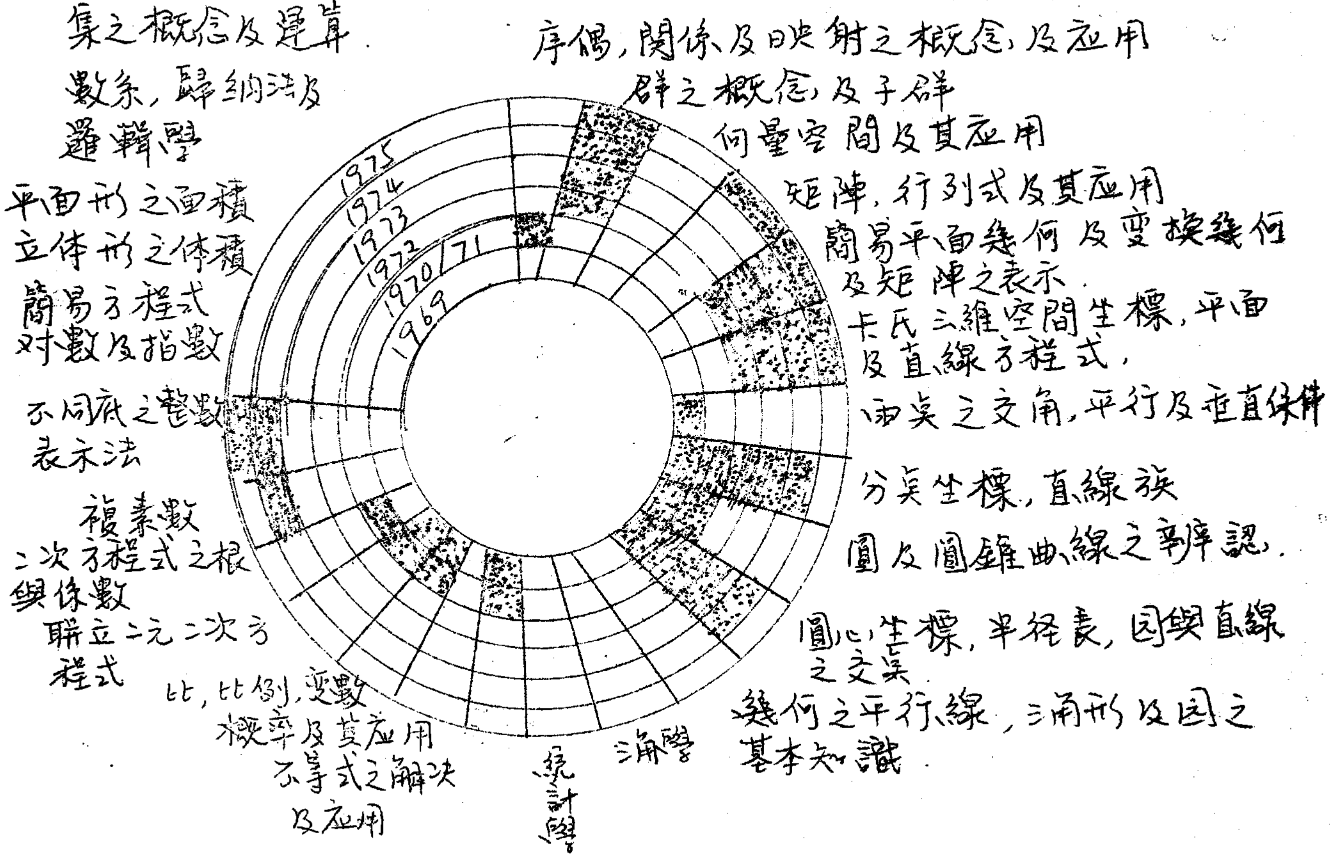
不包括在課程範圍內的項目

在算術課程方面，一九六七年删去基本股票和證券的問題，增添了速度，相對速度問題及各類圖解。又於一九七二年起，取消了分數、小數和基本四則運算原理。增加了圓錐體和球體的應用問題，質數，自然數之因子分解，及將度量衡改為公制等。至於比及比例則撥歸代數課程內。這都顯示社會和個人需求的改變。摒棄困難繁複之運算，注重概念之學習。在代數和幾何課程內，亦可見到相似的改變。在代數方面，增加了比，比例及變數法，一元二次方程式之根與係數之關係，無理數，根式方程式，指數定律。算術級數及幾何級數等。幾何課程方面，删去適合已知條件圓之作圖法，正多邊形之作圖法。定理學習方面，删去圓內相交弦之定理，相似三角形部份定理外，最意義重大的改變為免去學習各定理之証法，只注重其推理和應用。

由一九六九年起設立的「普通數學」課程乙，在內容方面，是參照在英國所推行的「新數學」實驗課程而修訂的。

由於選修的學生愈來愈多，漸漸的取代了課程甲的地位。在一九六九年至一九七五年間的一段時間內，課程內容亦有很大的改變。

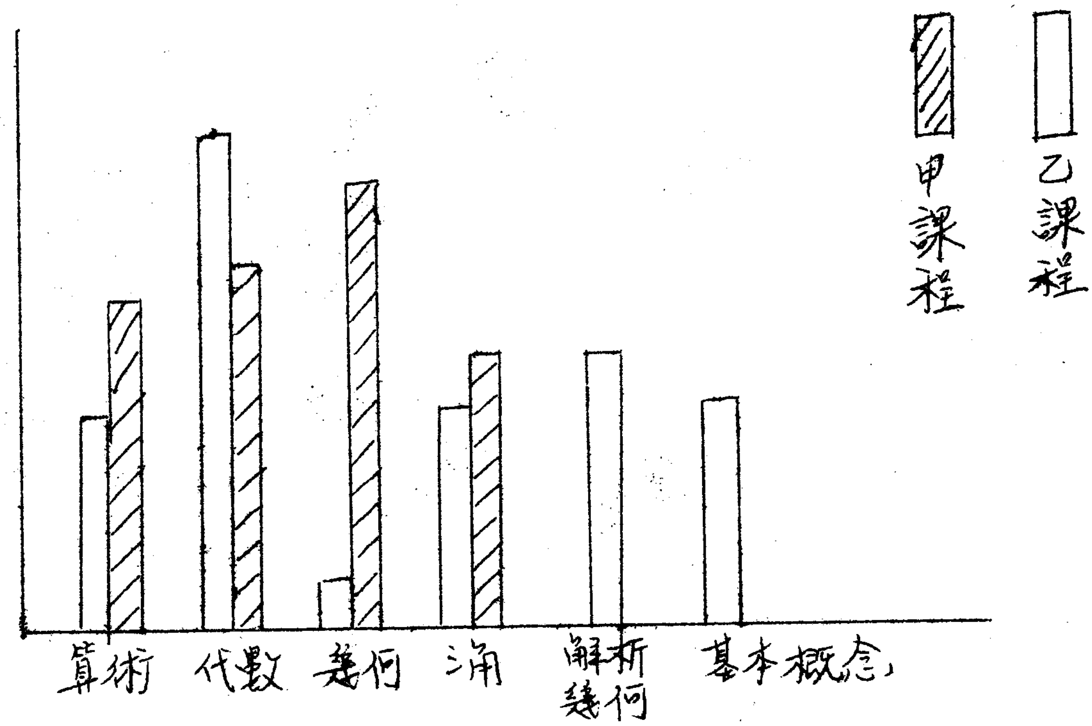
「普通數學」課程乙之演變圖



 不包括在課程範圍內的項目

從上面的課程圖表來看，「課程乙」在內容方面，包括有代數的基本運算，簡易方程式的解法和應用，簡易之因子分解等，再加入「數系」，「邏輯」，「集」，「群」，「向量」和「矩陣」注重辨証和推理，放棄繁複之運算工作。取締了幾何學內的假設，求証等抽象而困難的工作，只留下基本圖形的知識。「變換幾何」與「解析幾何」包括在課程內，用以訓練學生對形與數間之相互關係。三角學的內容，在「甲」、「乙」兩課程中，沒有太多的差異。數學歸納法，統計學和概率的應用，都被編入「課程乙」內。由一九六九年至一九七五年的一段期間內，「課程乙」的內容有很大的改變，「變換幾何」，「群」，「矩陣」和「行列式」都被删去，幾何學內的定理都被删去，只留下平行線，三角形和圓的基本知識。新增的

在「解析幾何」內有圓及圓錐曲線之辨認，直線族，圓與直線之交点。代數方面，主要為二次方程式之根與係數間之關係，不等式，級數等。若以一九七五年所公佈的「甲」「乙」課程作一比較，兩者間的差別，只在平面幾何，算術、統計學，向量，概率和數學歸納法等。



一九七五年「甲」「乙」課程內容之比較

又從一九六七年開始，中學會考數學課程內，增添了「附加數學」的新課程，內容包括以上所討論課程內的項目，再加上簡易的微積分學，及物理學內有關動力學的應用問題。這課程的目的是讓較優秀的學生有機會在數學上作更深一步的探討，來滿足他們的興趣，及幫助他們作升大學預科班內選修理科項目的準備。

第三章　結論

從中學會考數學課程分析的圖表資料中，「課程乙」比「課程甲」更適合教育原理，更能引起學者的興趣和更能滿足社會的需求，這可從參加中學會考數學科的人數百分

率得到証明。

	一九六九	一九七二	一九七五
課程甲	92.8%	61%	49%
課程乙	7.2%	39%	51%

在教育原理上，用「數論」的基礎和「邏輯」的運用，來訓練學生的歸納，推理，演繹和判斷的能力，較用抽象的平面幾何來得有效。但在實施新數學課程的時候，有很多理科的教師都感到學生們沒有學到足夠的數學知識和技能，去解答物理上的問題。為著配合各科的發展，「新數學」課程內便增添了不少应用上的知識和技能，如一元二次方程式和聯立二元二次方程式等。其他不合实際的或太抽象的項目，都一一予以刪去，如「變換幾何」，「矩陣」「行列式」等。在一九七五年以前，中學會考的「普通數學」課程因講授所用的語言有別，故分為中英文兩卷，同時課程的內容亦有很大的差別。由於英文中學會考的「甲」「乙」兩數學課程經不斷的改進，中、英文中學的數學試卷自一九七五年起，所考的題目是全部相同的。同時亦可預見不久的將來，「甲」「乙」兩課程亦會合而為一，這就是演變的結果。

基於數學一般應用，愈來愈廣，因為社會的進步使各種科技分工愈精細，對數學的需求更多和更精。這亦促使數學課程包括更豐富的內容。

最值得我們注意的是三年補助中學教育的施行。當局推行這政策的目的是使每一個適齡的兒童，都有接受九年教育的机會。就讀中學的學生人數將會增加一倍有多，個別差異會因此更加明顯，現在的數學課程必不能滿足學生的需求。另一目的是使大部份受到九年教育的學生，更能適應將來的社會——工業社會。更符合經濟和實用原則的數學課程，將會在短期內訂定和公佈，這又將是本港數學的另一次演進。

第四章　附錄

以下各附表皆以考試部所公佈之考試範圍手册為參考。

附表一　英文中學會考數學課程(甲)演進之比較

算術科	66	67	68	69	70/71	72	73	74	75
基本運算原理	✓	✓	✓	✓	✓				
質數及自然數之因子分解，含 2 3 4 5 8 9 11 之檢定法						✓	✓	✓	✓
最高公因數與最低公倍數，輾轉相除法	✓					✓	✓	✓	✓
英制及市制之度量衡	✓	✓	✓	✓	✓	✓	✓		
公制之度量衡	✓	✓	✓	✓	✓	✓	✓	✓	✓
分數與小數	✓	✓	✓	✓	✓	✓			
平方根計算法	✓	✓	✓	✓	✓	✓	✓	✓	✓
均數，比及比例	✓	✓	✓	✓	✓	✓			
百分率、賺賠率	✓	✓	✓	✓	✓	✓	✓	✓	✓
單利與複利	✓	✓	✓	✓	✓	✓	✓	✓	✓
普通平面形面積及立体形之体積包括									
1. 多边形	✓	✓	✓	✓	✓	✓	✓	✓	✓
2. 圓形	✓	✓	✓	✓	✓	✓	✓	✓	✓
3 多面体	✓	✓	✓	✓	✓	✓	✓	✓	✓
4. 直圓柱	✓	✓	✓	✓	✓	✓	✓	✓	✓
5. 圓錐体及球体				✓	✓	✓	✓	✓	✓
速度問題		✓	✓	✓	✓	✓	✓	✓	✓
對數之应用及有效數字	✓	✓	✓	✓	✓	✓	✓	✓	✓
準確之界限									
數據之搜集及分析各類圖解		✓	✓	✓	✓	✓		✓	
基本股票與証券問題	✓								

代數科	66	67	68	69	70/71	72	73	74	75
基本運算原理	✓	✓	✓	✓	✓	✓	✓	✓	✓
代數式與公式之運算及應用	✓	✓	✓	✓	✓	✓	✓	✓	✓
簡易代數式之因子分解(包括 $a^3 \pm b^3$)	✓	✓	✓	✓	✓	✓	✓	✓	✓
餘式定理及其應用	✓	✓	✓	✓	✓				✓
最高公因式,最低公倍式	✓	✓	✓	✓	✓	✓	✓	✓	✓
一元一次方程式之解法及應用	✓								
聯立二元一次方程式之解法及應用	✓	✓	✓	✓	✓	✓	✓	✓	✓
二次方程式之解法及應用	✓	✓	✓	✓	✓	✓	✓	✓	✓
二次方程式之根與係數間之關係						✓	✓	✓	✓
二元二次聯立方程之解法及應用(一為一次一為二次)	✓	✓	✓	✓	✓	✓	✓	✓	✓
指數定律,分指數零指數及負指數	✓	✓	✓	✓	✓	✓	✓	✓	✓
指數方程式									✓
根式,根式之應用,有理化因式,根式方程式									✓
比及比例,變數		✓	✓	✓	✓	✓	✓	✓	✓
級數,等差級數,等比級數		✓	✓	✓	✓	✓	✓	✓	✓
不等式之解法及其應用						✓	✓	✓	✓

三角學	66	67	68	69	70/71	72	73	74	75
角之量度，以度及弳為單位，弧長及扇形面積	✓	✓	✓	✓	✓	✓	✓	✓	✓
正弦，餘弦，正切函數及其圖像	✓	✓	✓	✓	✓	✓	✓	✓	✓
三角函數之基本關係，簡易恆等式	✓	✓	✓	✓	✓	✓	✓	✓	✓
直角三角形之解法及其應用	✓	✓	✓	✓	✓	✓	✓	✓	✓
二維及三維空間之簡易應用題	✓	✓	✓	✓	✓	✓	✓	✓	✓
簡易三角方程式 $(0^\circ - 360^\circ)$	✓	✓	✓	✓	✓	✓	✓	✓	✓
三角形正弦定律，餘弦定律.									
三角形之面積公式 $\frac{1}{2}ab\sin C$，$\sqrt{s(s-a)(s-b)(s-c)}$	✓	✓	✓	✓	✓	✓	✓	✓	✓

平面幾何	66	67	68	69	70/71	72	73	74	75
I 實用作圖法									
已知角之分角線及已知直線之中點及垂直平分線	✓	✓	✓	✓	✓	✓	✓	✓	✓
相等於已知角之作法	✓	✓	✓	✓	✓	✓	✓	✓	✓
60°，45°，30°角之作法	✓	✓	✓	✓	✓	✓	✓		
直線平行於一已知直線之作法	✓	✓	✓	✓	✓	✓	✓		
簡易三角形及多邊形之作法	✓	✓	✓	✓	✓	✓	✓	✓	✓
任意等分一已知直意或分一直線等於一已知比	✓	✓	✓	✓	✓	✓	✓	✓	✓
作等積之三角形	✓	✓	✓	✓	✓				

續平面幾何	66	67	68	69	70/71	72	73	74	75
圓之切線及兩圓之公切線之作法	✓	✓	✓	✓	✓	✓	✓	✓	✓
三角形之外接,內切及旁切圓之作法	✓	✓	✓	✓	✓	✓	✓	✓	✓
適合已知條件之圓之作法	✓	✓	✓	✓	✓				
等積正方形之作法		✓	✓	✓	✓				
比例中項及末項之作法		✓	✓	✓	✓	✓	✓		
外切或內接正多邊形(3,4,6或8)之作法									
II 理論部份									
凡直線角皆等	✓	✓	✓	✓	✓	✓	✓	✓	✓
對頂角相等	✓	✓	✓	✓	✓	✓	✓	✓	✓
相交二直線不能平行於同一直線	✓	✓	✓	✓	✓	✓	✓	✓	✓
二平行線為一截線所截則所得之內錯角相等,同旁內角相補及其逆定理	✓	✓	✓	✓	✓	✓	✓	✓	✓
三角形之內角和定理	✓	✓	✓	✓	✓	✓	✓	✓	✓
多邊形之內角和定理及外角和定理	✓	✓	✓	✓	✓	✓	✓	✓	✓
全等三角形定理	✓	✓	✓	✓	✓	✓	✓	✓	✓
等腰三角形定理	✓	✓	✓	✓	✓	✓	✓	✓	✓
不等量定理	✓	✓	✓	✓	✓				
平行四邊形之定理及檢驗	✓	✓	✓	✓	✓	✓	✓	✓	✓
截線定理	✓	✓	✓	✓	✓	✓	✓	✓	✓
中點定理	✓	✓	✓	✓	✓	✓	✓	✓	✓
等積形之定理	✓	✓	✓	✓	✓	✓	✓	✓	✓
畢氏定理	✓	✓	✓	✓	✓	✓	✓	✓	✓

續平面幾何	66	67	68	69	70/71	72	73	74	75
畢氏定理之推廣	✓	✓	✓	✓	✓				
阿氏定理	✓	✓	✓	✓	✓				
軌跡	✓	✓	✓	✓	✓	✓	✓	✓	✓
圓之對稱性質	✓	✓	✓	✓	✓	✓	✓	✓	✓
圓心角及圓周角	✓	✓	✓	✓	✓	✓	✓	✓	✓
共圓之檢驗	✓	✓	✓	✓	✓	✓	✓	✓	✓
等弧及等弦	✓	✓	✓	✓	✓	✓	✓	✓	✓
切線之性質,弦切角及圓之相切	✓	✓	✓	✓	✓	✓	✓	✓	✓
三角形兩边成比例線定理及逆定理		✓	✓	✓	✓	✓	✓	✓	✓
相交弦定理及逆定理		✓	✓	✓	✓				
分角線定理及逆定理		✓	✓	✓	✓				
相似三角形定理		✓	✓	✓	✓				

附表二　英文中學會考數學課程(乙)演進之比較

課程乙	69	70/71	72	73	74	75
質數,自然數之因子分解,含2,3,4,5,8,9,11之檢定法	✓	✓	✓			
最高公因數,最低公倍數,輾轉相除法	✓	✓	✓			
有效數字,準確之界限,近似值	✓	✓	✓	✓	✓	
整數不同底之表示法	✓	✓	✓	✓		
數系(自然數,整數,有理數,實數)	✓	✓	✓	✓	✓	✓
複素數	✓	✓	✓	✓		✓
數學歸納法及其簡易之應用	✓	✓	✓	✓	✓	✓
邏輯之學習	✓	✓	✓	✓	✓	✓
集之概念及子集	✓	✓	✓	✓	✓	✓
序偶,関係及映射之簡易概念,及应用関係及映射之圖像	✓		✓	✓	✓	✓

續課程乙	69	70/71	72	73	74	75
群之概念,結合律,分配及交換律 子群	✓	✓				
簡易矩陣(階數低於 4)及其应用 行列式	✓	✓	✓	✓	✓	
向量空間(維數不超過 3),純量與向量之積,向量之和,内積及簡易幾何应用	✓	✓	✓	✓	✓	✓
平行線,多边形内角和,相似三角形,畢氏定理	✓	✓	✓	✓	✓	
圓周角及弦切角定理	✓	✓	✓	✓	✓	
平面幾何之平行線,三角形及圓之基本知識						✓
平面形之面積及立体图形之体積	✓	✓				✓
因子分解 $ax+bx$, a^2-b^2, $a^2\pm 2ab+b^2$	✓	✓	✓	✓	✓	
簡易分式	✓	✓				
因子分解 $px^2+qx+r=(hx+k)(lx+m)$, p,q,r,h,k,l,m 皆為整數			✓	✓	✓	
最高公因式,最低公倍式				✓	✓	
公式之變換及應用			✓	✓	✓	
簡易一元一次方程式之解法及應用	✓	✓				
聯立二元一次方程式之解法						✓
一元二次方程式之解法及应用	✓	✓	✓	✓	✓	✓
一元二次方程式之根與係數間之關係			✓	✓	✓	✓
聯立二元二次方程式之解法及应用(一為一次,一為二次)			✓	✓	✓	✓
線式及二次函數之图解			✓	✓	✓	✓
不等式之解法及应用			✓	✓	✓	✓

續課程乙	69	70/71	72	73	74	75
指數定律、分指數、零指數、負指數及對數之應用	✓	✓	✓	✓	✓	✓
比及比例、級數			✓	✓	✓	✓
排列組合之意義						✓
計算尺之應用及簡易流圖	✓	✓				
簡易概率、和積定理及簡易應用	✓	✓	✓	✓	✓	✓
複利計算及應用			✓	✓	✓	
曲線之斜率、面積之估計、轉變之速率	✓	✓				
數據之搜集及分析、各類圖解、平均值、中位數及四分位數	✓	✓	✓	✓	✓	✓
正弦、餘弦及正切函數之圖解	✓	✓	✓	✓	✓	✓
角之量度、以度或弧為單位	✓	✓	✓	✓	✓	✓
三角函數之基本關係及簡易恆等式	✓	✓	✓	✓	✓	✓
直角三角形之解法及其應用	✓	✓	✓	✓	✓	✓
二維及三維空間之簡易應用題	✓	✓	✓	✓	✓	✓
簡易三角方程式（$0°—360°$）				✓	✓	✓
方位及坐標、經度、緯度	✓	✓				
簡易平面幾何變換	✓	✓				
立體之網形、尤拉氏定律	✓	✓				
卡氏正坐標（二維空間）	✓	✓	✓	✓	✓	✓
卡氏正坐標（三維空間）及平面之方程式、直線方程式	✓	✓				
兩點間之距離			✓	✓	✓	✓
分點坐標						✓
直線方程式、直線之特殊式、求滿足已知條件之直線方程式	✓	✓	✓	✓	✓	✓
兩線之交角、平行及垂直			✓	✓	✓	✓
簡易之軌跡	✓	✓	✓	✓	✓	✓

續課程乙	69	70/71	72	73	74	75
圓之方程式之辨認			✓	✓	✓	✓
圓心坐標,半徑,及圓與直線之交點						✓
圓椎曲線之辨認			✓	✓	✓	✓

附表三　一九七五年英文中學會考之普通數學與同等數學課程比較

課程(一九七五)	普通(新)	同等(舊)
算術		
質數,自然數之因子分解及因子檢定法	✓	✓
最高公因數,最低公倍數,輾轉相除法	✓	✓
分數及小數	✗	✓
百分率,賺賠率	✗	✓
單利與複利	✗	✓
平面形之面積及立体形之体積	✓	✓
速度問題	✗	✓
對數之應用	✓	✓
代數		
基本代數運算原理	✓	✓
簡易公式之應用與變換	✓	✓
餘式定理及其應用	✗	✓
最高公因式,最低公倍式	✓	✓
簡易之因子分解	✓	✓
簡易之分式運算	✓	✓
一元一次方程式之解法應用	✓	✓
一元二次方程式之解法及應用,根與係數間之關係	✓	✓
聯立二元二次方程式之解法及應用(一為一次,一為二次)	✓	✓

課程乙(一九七五)	普通(新)	同等(舊)
指數定律	✓	✓
指數方程式及根式方程式	×	✓
比及比例變數法	✓	✓
等差及等比級數	✓	✓
不等式之解法及應用	✓	✓
數學歸納法	✓	×
排列與組合之意義，簡易概率理論	✓	×
向量空間	✓	×
三角學		
三角函數及其圖像	✓	✓
直角三角形之解法及其應用	✓	✓
二維及三維空間之應用題	✓	✓
角之量度，以度或弳為單位	✓	✓
三角函數之基本關係及簡易恆等式	✓	✓
簡易三角方程式(0°—360°)	✓	✓
正弦定律，餘弦定律	×	✓
三角形面積之公式	×	✓
平面幾何	✓	×
統計學	✓	×

On the Method of False Position

Mr. K.Y. Li & L.S. Ko

A modified Method of False Position

To apply the method of False Position for finding a root α of $f(x) = 0$, it is common practice to look for an interval $[a, b]$, containing α such that $f(a)f(b) < 0$, say $f(a) < 0$ and $f(b) > 0$.

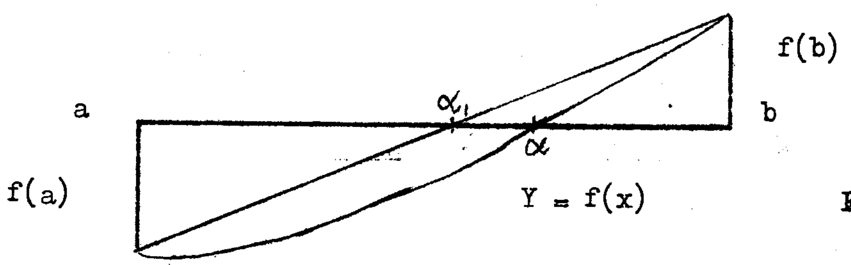


Figure 1

The first approximation α_1 of α is given by the formula

$$\alpha_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} \quad \dots\dots\dots\dots\dots\dots(1)$$

(See Figure 1.) In order to obtain a second approximation α_2 of α, we have to see whether $f(\alpha_1)$ is positive or negative. If $f(\alpha_1)$ is positive, then

$$\alpha_2 = \frac{af(\alpha_1) - \alpha_1 f(a)}{f(\alpha_1) - f(a)},$$

which is obtained by replacing b in (1) by α_1. If $f(\alpha_1)$ is negative, then we ~~replace a in (1)~~ by α_1 and obtain

$$\alpha_2 = \frac{\alpha_1 f(b) - bf(\alpha_1)}{f(b) - f(\alpha_1)}$$

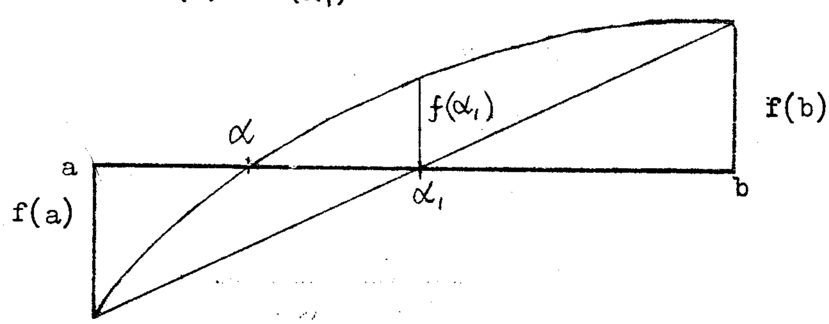


Figure 2(i) : $f(\alpha_1) > 0$; replace b by α_1

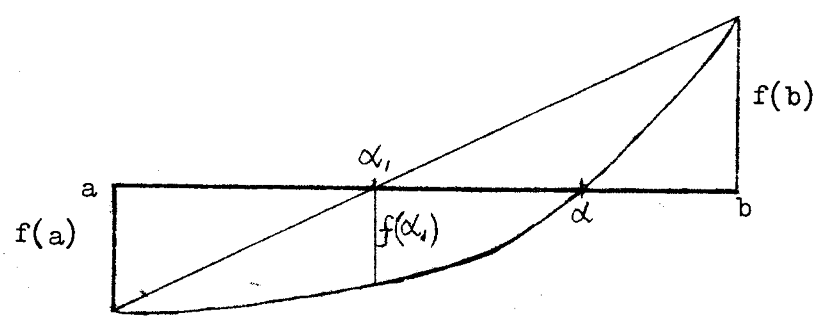


Figure 2(ii) : $f(\alpha_1) < 0$; replace a by α_1

(See Figures 2(i) and 2(ii).) In a similar manner, we obtain α_3, α_4......... which are found to converge to α.

One of the disadvantages of this method is the trouble of checking the sign of $f(\alpha_n)$ for every n = 1,2,3 .... in order to choose the appropriate formula for α_{n+1}. This disadvantage may be overcome if we modify the method slightly as follows :-

For the first approximation α_1 of α, we still use formula (1).

For all the other approximations α_{n+1} of α, we use the formula

$$\alpha_{n+1} = \frac{af(\alpha_n) - \alpha_n f(a)}{f(\alpha_n) - f(a)}$$

throughout. This means that, whatever the sign of $f(\alpha_n)$, we always replace α_n by α_{n+1} to obtain further approximation, keeping the point $(a, f(a))$ fixed all the time. (See Figures 3(i) and 3(ii)*. In this way, there is no need to check the sign of $f(\alpha_n)$. The convergence of this modified method is proved below.

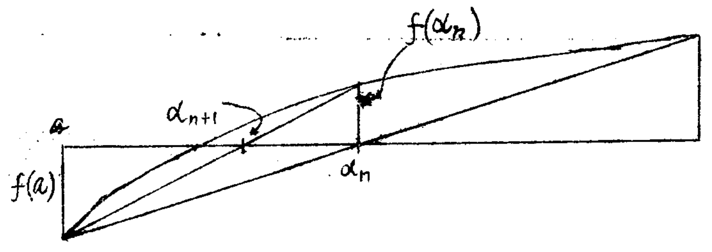


Figure 3(i): $f(\alpha_n) > 0$

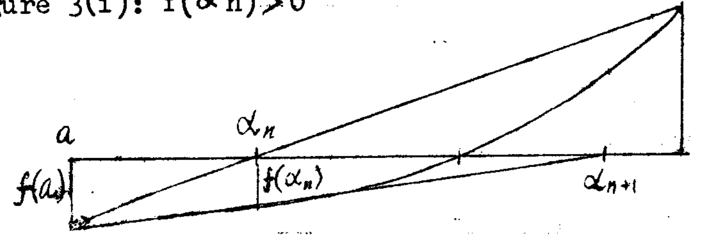


Figure 3(ii): $f(\alpha_n) < 0$

* It works equally well, of course, if we keep the point $(b, f(b))$ fixed and use the formula

$$\alpha_{n+1} = \frac{\alpha_n f(b) - bf(\alpha_n)}{f(b) - f(\alpha_n)}$$

throughout.

Convergence of the method

It is well-known that the iterative process

$$X_{n+1} = g(X_n)$$

converges if $|g'(\alpha)| < 1$, where α is the root of the equation $X = g(X)$ under consideration.

The iteration formula of our modified method is

$$\alpha_{n+1} = \frac{af(\alpha_n) - \alpha_n f(a)}{f(\alpha_n) - f(a)},$$

$$= a - \frac{(\alpha_n - a)\, f(a)}{f(\alpha_n) - f(a)}$$

To apply the convergence condition for the interative method, we put

$$g(X) = a - \frac{(X - a)f(a)}{f(X) - f(a)}$$

Then,

$$g'(X) = -\frac{[f(X) - f(a)]f(a) - (X - a)f(a)f'(X)}{[f(X) - f(a)]^2}$$

Since α is a root of $f(X) = 0$, we have $f(\alpha) = 0$, and

$$g'(\alpha) = -\frac{[-f(a)]f(a) - (\alpha - a)f(a)f'(\alpha)}{f(a)^2}$$

$$= 1 + \frac{(\alpha - a)f'(\alpha)}{f(a)} \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

Put $h = a - \alpha$ and expand $f(a)$ in Taylor's series :

$$f(a) = f(\alpha + h) = f(\alpha) + hf'(\alpha) + \frac{h^2}{2!} f''(\alpha) + \ldots\ldots\ldots\ldots$$

$$= 0 + hf'(\alpha) + \frac{h^2}{2!} f''(\alpha) + \ldots\ldots \qquad \ldots\ldots(3)$$

Dividing throughout by $f(a)$, we have

$$1 = \frac{hf'(\alpha)}{f(a)} + \frac{h^2}{2} \cdot \frac{f''(\alpha)}{f(a)} + \ldots\ldots \qquad \ldots\ldots(4)$$

From (2) and (4),

$$g'(\alpha) = 1 - \frac{hf'(\alpha)}{f(a)} = \frac{h^2}{2} \cdot \frac{f''(\alpha)}{f(a)} + \cdots$$

From (3),

$$\frac{f(a)}{h} = f'(\alpha) + \frac{h}{2} f''(\alpha) + \cdots$$

Hence if h is sufficiently small, we have

$$\frac{f(a)}{h} \doteq f'(\alpha)$$

$$\text{and } g'(\alpha) \doteq \frac{h^2}{2} \cdot \frac{f''(\alpha)}{f(a)} = \frac{h}{2} \cdot \frac{f''(\alpha)}{\left[\frac{f(a)}{h}\right]}$$

Hence,

$$g'(\alpha) \doteq \frac{h}{2} \cdot \frac{f''(\alpha)}{f'(\alpha)}$$

It is now clear that if α is a simple root of $f(X) = 0$ so that $f'(\alpha) \neq 0$ and if we choose a to be very close to α, then h is small and $|g'(\alpha)|$ can be made less than one. This means that the modified method is always convergent provided that the initial end value a is sufficiently near to the root α.

數學科設計學習

中華基督教會扶輪職業先修學校

1. 目的：藉此活動使學生能發揮思考及創作能力，並啓發學生對數學科學習興趣，從而獲得實習與理論之互相印證。

2. 項目：

名稱	輔導教師	有關班級
1. 座標的應用		商科班 商藝班
2. Surface of revolution		金工、電工 印刷班
3. 香港個別經濟資料系統報告		全級
4. Geometric exercise in paper folding		全級
5. The shapes of things		全級
6. Curve - stitching		全級
7. Ruled surfaces		全級
8. Geometric shapes		全級
9. 釘板研究各種常見圖形的性質		金工、電工 印刷班
10. Aids in the teaching of circular functions		全級
11. 放大尺		金工、電工 印刷班
12. 圓錐體的切割		金工、電工 印刷班

3. 參加資格：二年級學生

4. 報名辦法：1 由學生自由分組及選擇不同的項目。
   2 先報名者有優先選擇權。

5. 輔導教師工作：由一位教師負責一至兩項設計。先自行準備收集材料。然後約同各班選擇同一項目的同學，加以講解及指導。

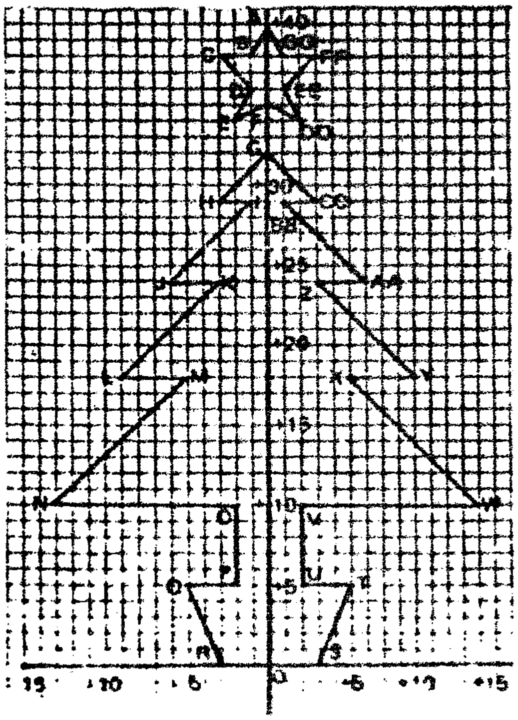
6. 評判：由輔導教師負責評分（20％）作爲平時習作分。
   a 初選：由輔導教師在六月九日前選出最佳作品（三至四種）。
   b 複選：由本校教師負責評判。

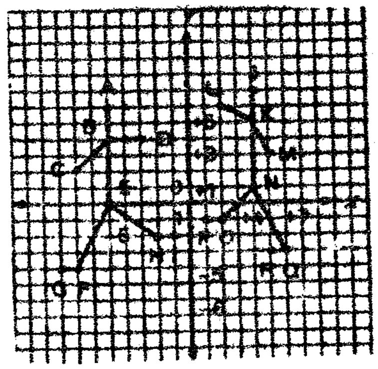
7. 獎品：分設冠、亞、季軍獎，優勝作品將作公開陳列。

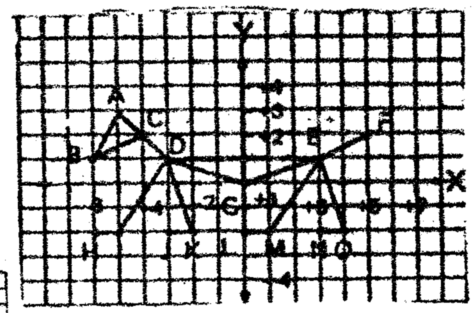
一、坐標的應用

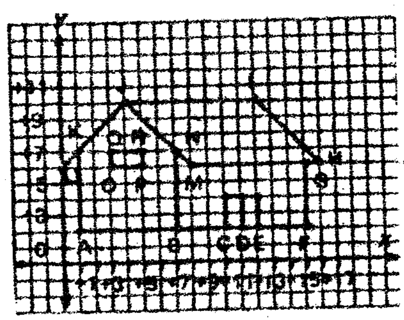
1. 材料：坐標紙.十字布,細鉄絲網.綉花線.膠繩.

2. 製法：先在坐標紙上設計圖案,再在十字布上用綉花線綉出或在鉄絲網上用膠繩穿出圖形,或用編織的方法織出圖形,亦可用打字机打出.









3. 參與者：二年級學生(商科.商藝班)每組三人

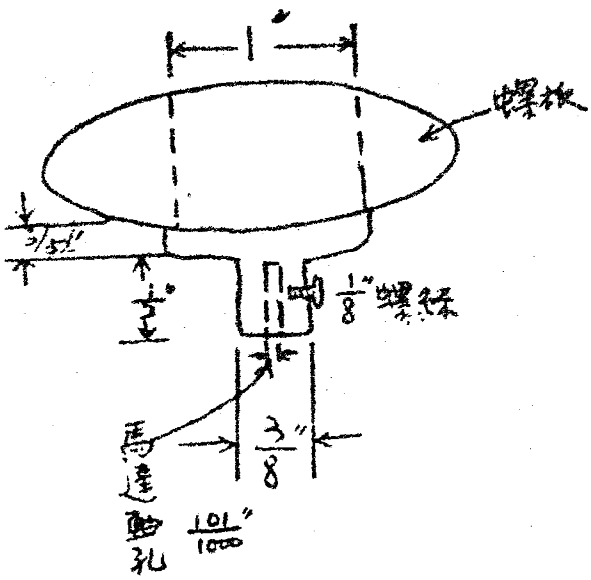
4. 内容：圖案,動物.人物或其他.

二、 Surface of revolution

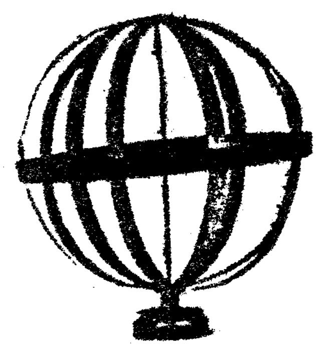
1. 材料：鐵柱或青銅柱．以青銅操作較佳

2. 製法：首先做好轉動部份，然後裝上馬達，馬達宜用直流低 mA 之馬達，現提意用 3V．30mA 左右之直流馬達，（Motor 3V 30mA）或欲可控制轉動速度快慢時，可另加一轉速控制器，而將 3V 提升為 6V 左右．
至於欲製造之平面圖形 可置於此圖的膠板上

轉動及支架部份



圓形部份



3. 參與者：二年級學生（工科生）每組 3 人

4. 內容：設計 8−10 個不同圖形 例

A 圓　B 長方形　C 三角形　D 拋物線形　E 橢圓形

三 香港個别經濟資料統計報告

1. 資料: Hong Kong Monthly Digest of Statistics 或其他官方公布資料.

2. 形式: 可用卷宗 (file) 之形式編製報告. 中英文均可表達方法宜整潔簡明.

3. 參與者: 二年級學生. 每組 人.

4. 內容: (中英文均可) 包括

a. 設計人及班別
b. 完成日期
c. 簡短說明
d. 目錄 (標明項目及頁數)
e. 原始資料表 (rawdata)
f. 頻率分佈表
g. 頻率線形圖 (histogram)
h. mean, mode, median
stand and deviation
i. 結論 將 h 項結果之意義加以簡單說明.

五. The shapes of things

1. 資料: 不同形狀的實物. 圖片. 模型. 剪貼簿 display-boards or tables

2. 形式: 學生將多方面搜集得的材料在剪貼簿或 display-boards 很整潔簡明的展示及說明.

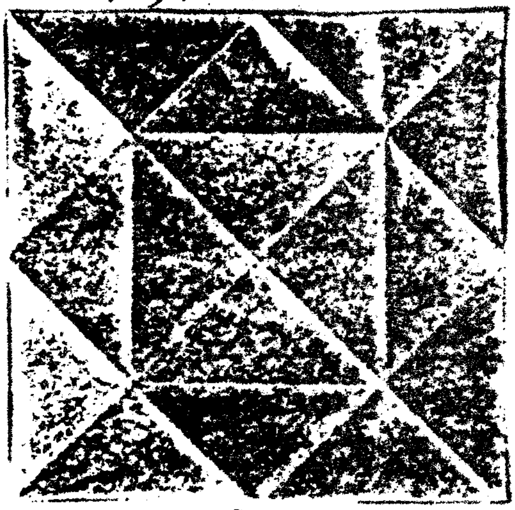
3. 參與者: 每組 8–10人

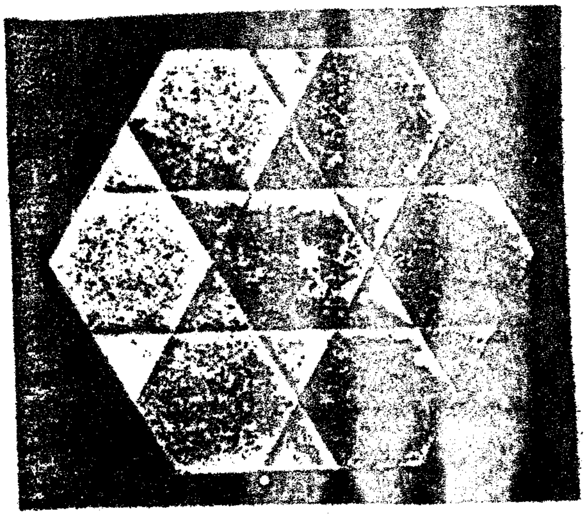
4. 內容

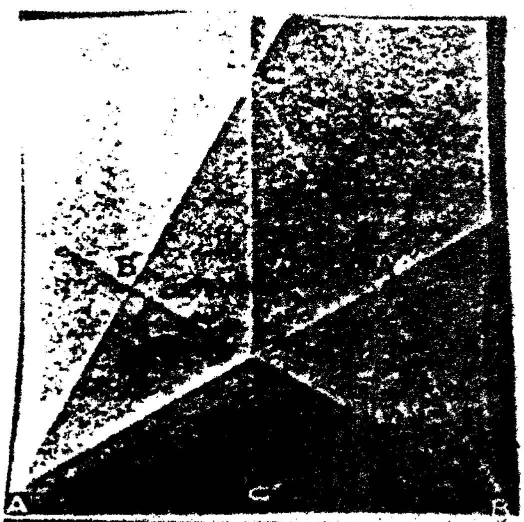
	Shapes	Examples
1.	Hexagon	snowflakes, bee-hives
2.	Octagon	sunflowers
3.	Pentagon	starfish
4.	Dodecagon	unusual snow-flakes
5.	Circular	wheels
6.	Parabola	missers of giant telescope, searchlight and radar beam antennas
7.	Ellipse	orbits of planets, satellites and carrets
8.	Spirals	shells, the distant galaxy of billions of stars whirling in space
9.	Spheres	earth, planets, spherical tanks (most efficient for storing gases or liquids under high pressure)
10.	Box shapes	buildings, rooms, walls, furniture
11.	Glindrical	pipes
12.	Cones	ice-criam, funnel, reverse funnel of a large blast fusnace

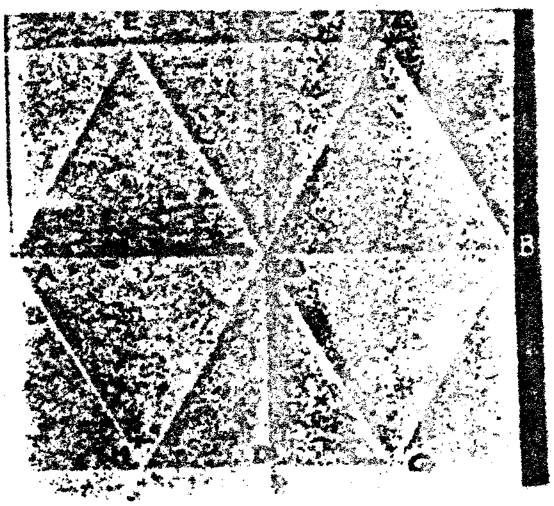
四. Geometric exercise in paper folding

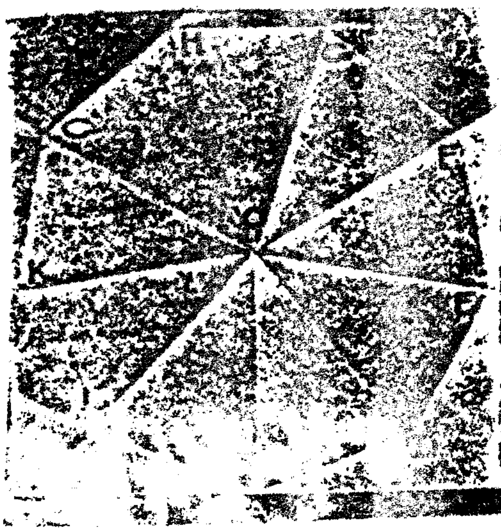
1. 材料：手工紙剪貼簿
2. 形式：用手工紙摺成不同的幾何圖形，附加簡明介紹各圖形的性質及對稱相似等特性。可摺得正方形、五邊形、六邊形、八邊形、十邊形、拋物線形、橢圓形等形狀，亦可證明畢氏定理。
3. 參與者：每組5人











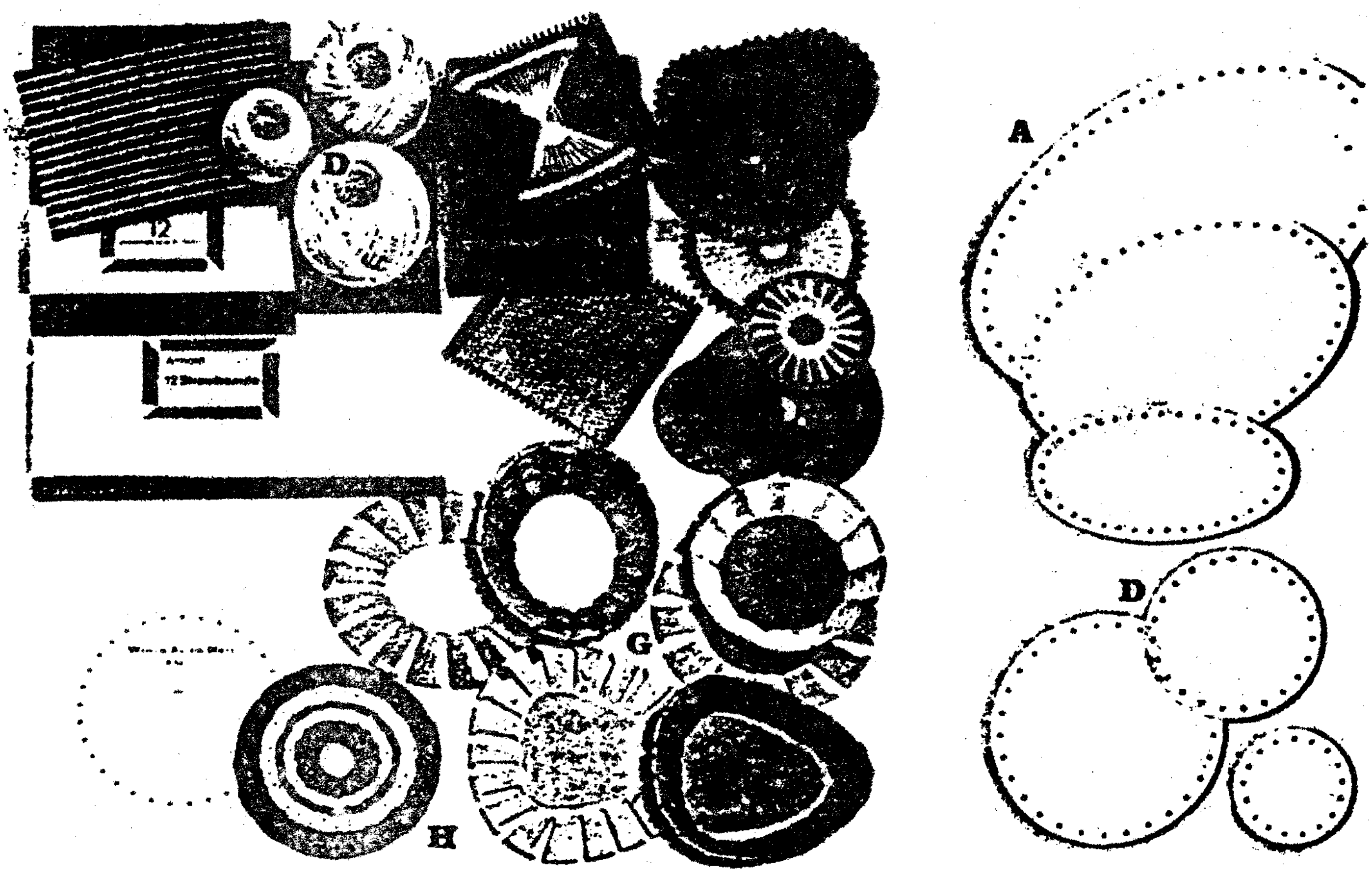
4. 參考書：Geometric exercise in paper folding (T Sundara Row)

六 Curue-stitching

1. 材料：膠片卡紙 發泡膠 罐蓋 木板等

2. 製法：在底板上先設計出圖案，利用直線等分或圓的等分，然後利用膠線毛線穿穿成不同圖案。

3. 參與者：每組5人



材料

設計

七、 Ruled Surfaces

1. 材料：膠片 鉄鏈 大紙盒 鉄架
2. 製法：先設計圖案、然後在立体架上利用膠條 鉄線(細) 綉花線等材料穿出
3 參與者：每組5人

b. Ruled surfaces ('Perspex')
1 and 3 Hyperbolic paraboloid; 2 and 4 hyperboloid; no. 4 adjustable

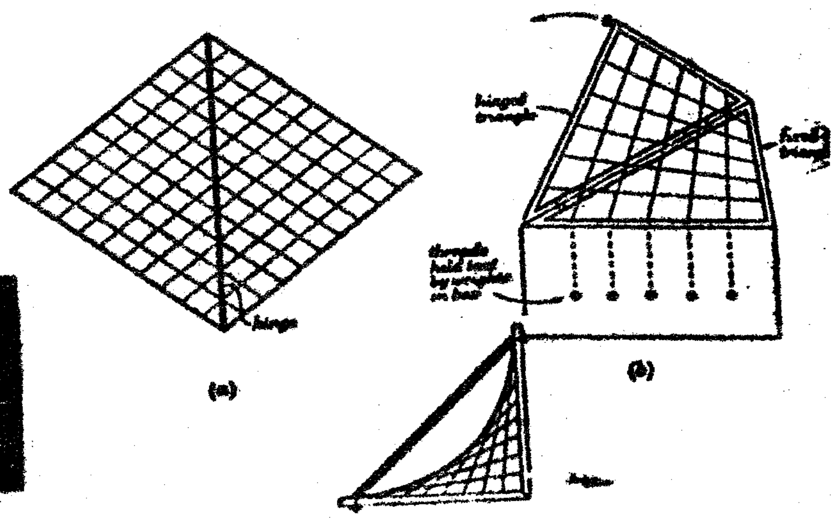
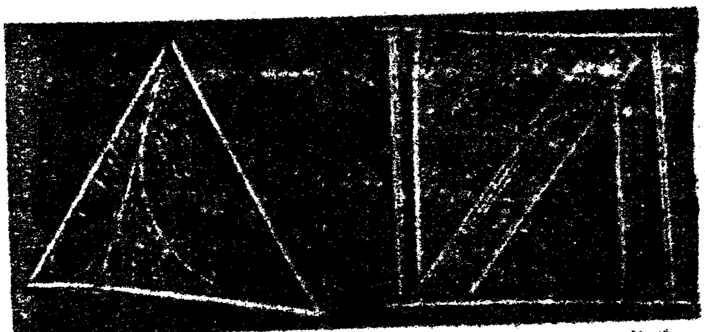


Fig. 232

c. Half-twist surface, quartic with double lines, icosahedron in octahedron



d. Six reguli in a tetrahedron, twisted cubic common to cylinder, hyperboloid, and cone

To locate the holes for threading, divide each diagonal AC, BD into an equal number of equal parts. Draw through each point of division parallels to the other diagonal, to meet the sides. At the points of intersection, erect ordinates to the

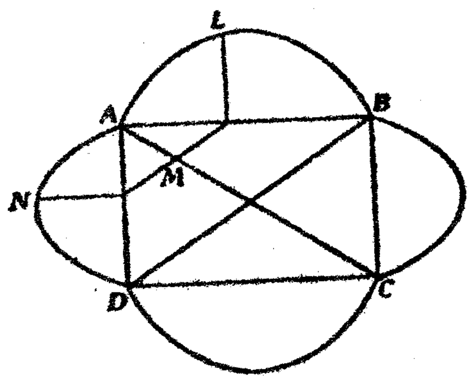


Fig. 233

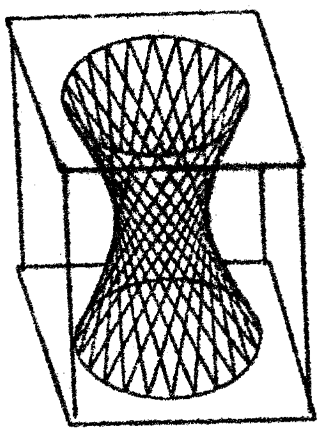


Fig. 223. Completed hyperboloid.

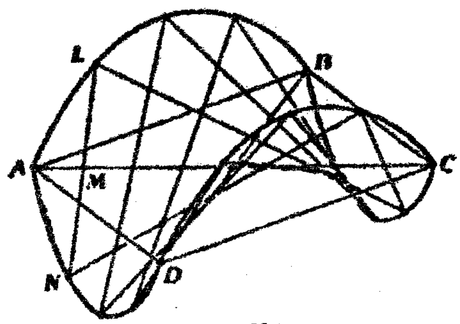


Fig. 234

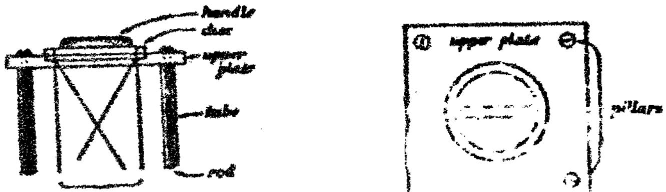
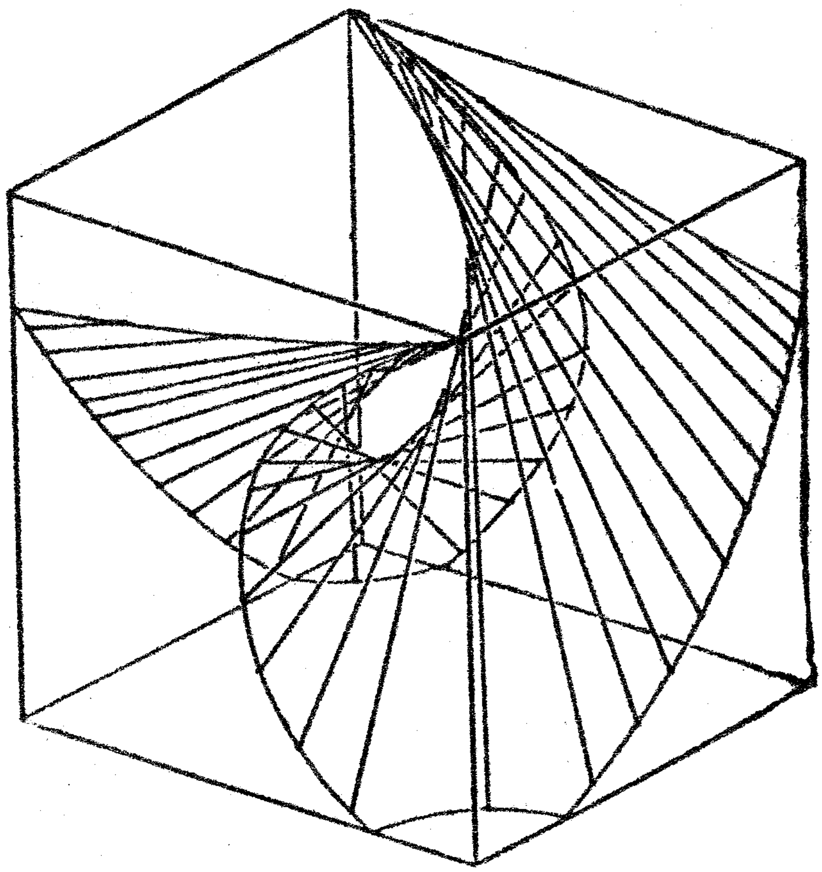
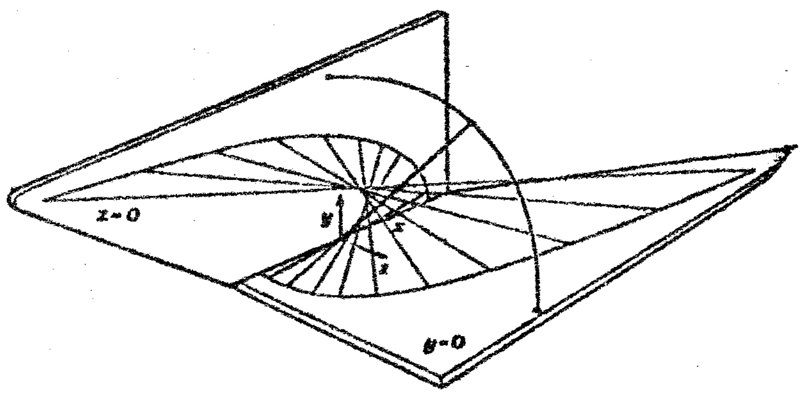


Fig. 229

parabolic arcs. These meet the arcs at the ends of a generator. For example in Fig. 233 the three points L, M, N will lie on a single generator. Fig. 234 shows the finished surface.

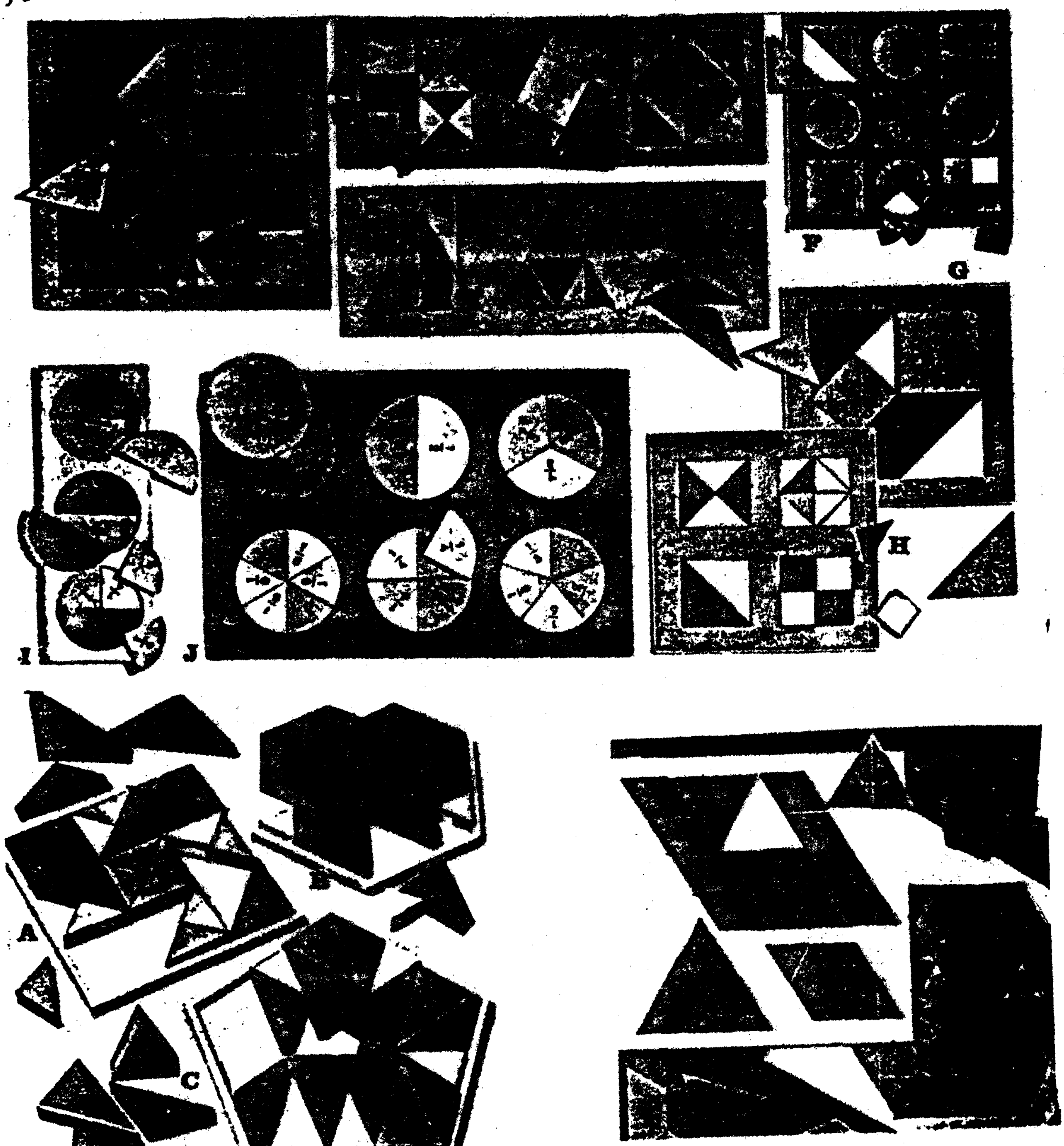




Reference : Mathematical models
H. M. Cundy and A.P. Rollett

八 Geometric shapes

1. 材料：膠片.卡紙.地膠夾板
2. 製法：先設計一完整的幾何形狀.或一圖案為底.然後利用不同形狀或相同形狀的圖形去砌拼.可作教育用具.
3. 參與者：每組5人

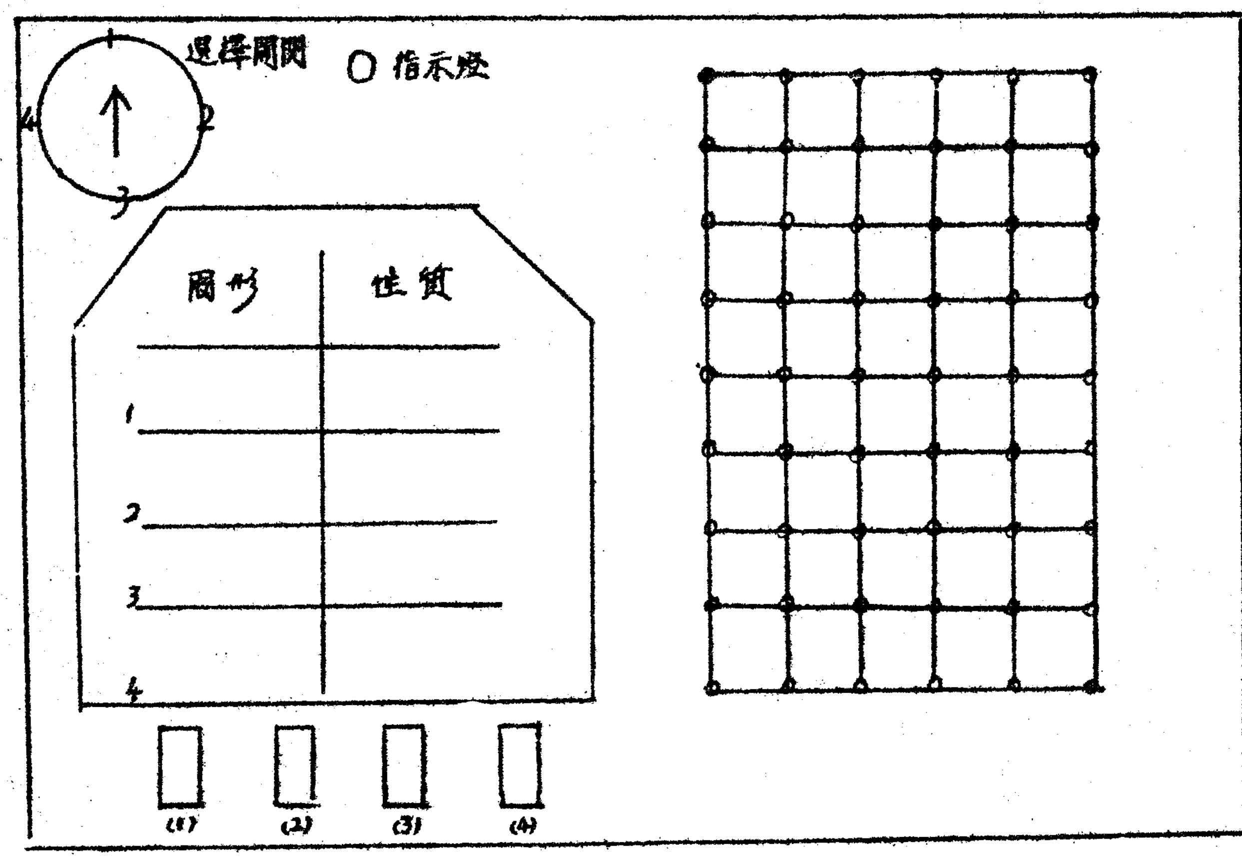


4. 內容：平行四邊形.五邊形.六邊形.圓形 人物.動物或 pentominoes

九. 利用釘板研究各種常見圖形的性質及利用電按鈕選擇適當的答案

材料：夾板，顏色圖釘．電擊按鈕．電池盒．燈泡．四辦選擇旋鈕．電線．

用途：從理論或以死記去灌輸學生各種幾何圖形之性質．如其對角線是否互相垂直是否相等．是否平分等．學生每每因所記憶者太多．而未能完全認識．若以釘板上繞着不同的幾何圖形時，則學生可自行學習．並可加深學生的認識．和學習的興趣．並加鼓勵其自動自發的精神．



AIDS IN THE TEACHING OF CIRCULAR FUNCTION

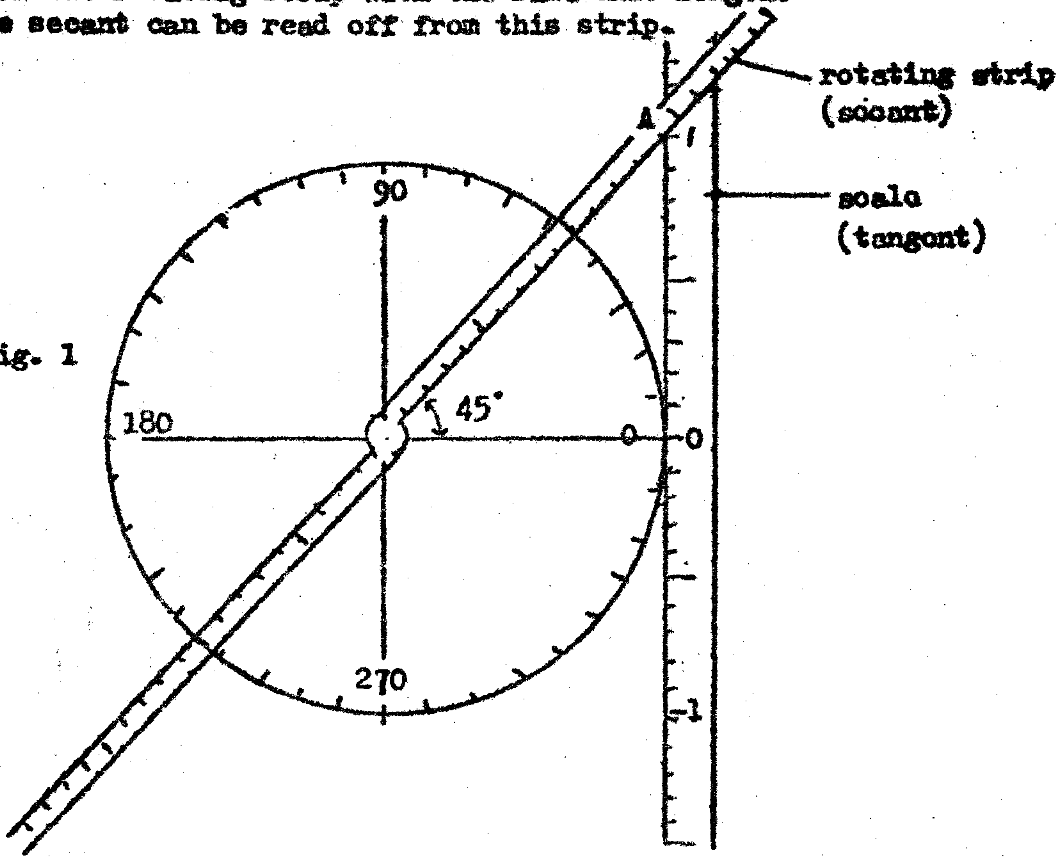
I. Sine, cosine and tangent are the three simple trigonometrical functions. The easiest one is probably the tangent which comes first in many courses of study. A simple device can be made to tell the tangent and secant of all angles.

How to construct :

i) Divide a circle into degrees and hinge a long strip at the centre
ii) With the radius as one unit length, mark off a uniform scale, positively and negatively with zero at the middle
iii) Fix the scale to the marked circle, tangent at the zero mark (fig.1)
iv) Mark the rotating strip with the same unit length.

The secant can be read off from this strip.

fig. 1



How to read the scale : The intersecting point A (fig. 1) tells us 2 values —— the value of the tangent is marked on the scale and the value of the secant on the rotating strip.

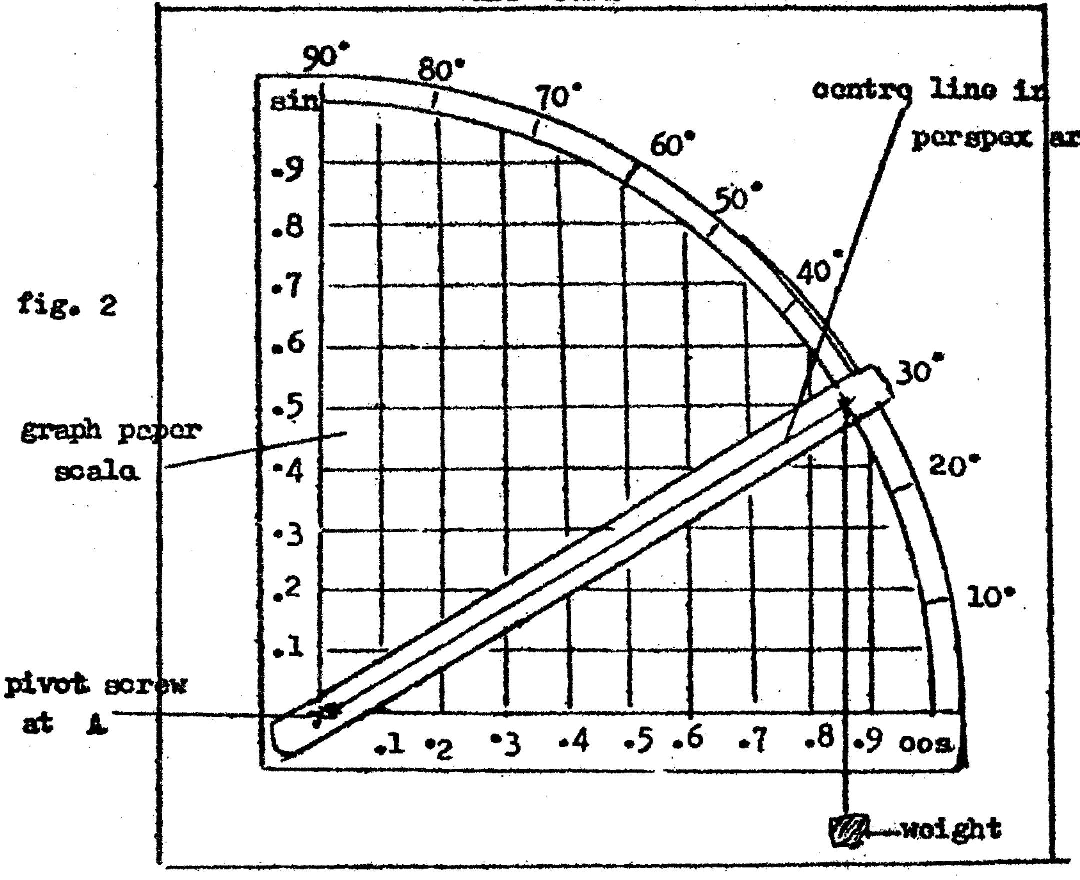
As an example, refer to the fig. : tan 45 = 1
sec 45 = 1.4

II. The second one deals with angles in the first quadrant (0°– 90°) and lacks the versatility of more complicated models but it performs its limited functions ewll.

CONSTRUCTION

i) A quadrant of radius 10 in. is drawn on a graph paper and marked out as shown in fig. 2
ii) This marked sheet is stuck to a 12 in. square of cardboard
iii) A strip of Perspex, 11 in by $\frac{1}{2}$ in with a line 10 in. long scored down its centre, is pivoted about the zero mark (A).
(The strip is fitted from behind with a countersunk screw, which projects out $\frac{1}{2}$" in front, exactly 10" from the pivot screw so that it is immediately above the circumference of the quadrant.)
iv) A bright thread, fitted with a bob weight, is attached to this screw.

fig. 2



This model, when used, is attached to the wall. The length of the Perspex arm (the hypotenuse) is considered to be one unit in length; when this arm is set to any angle, the plumb-line cuts along the base-line a distance equal to the adjacent side. This, as the length of the hypotenuse is unity, gives a direct reading of the cosine of the angle.
By reading horizontally across ,(from the end of the arm to the vertical scale) a direct reading of the sine of the angle is obtained. The tangent is obtained by taking the ratio of these two distances cut off.

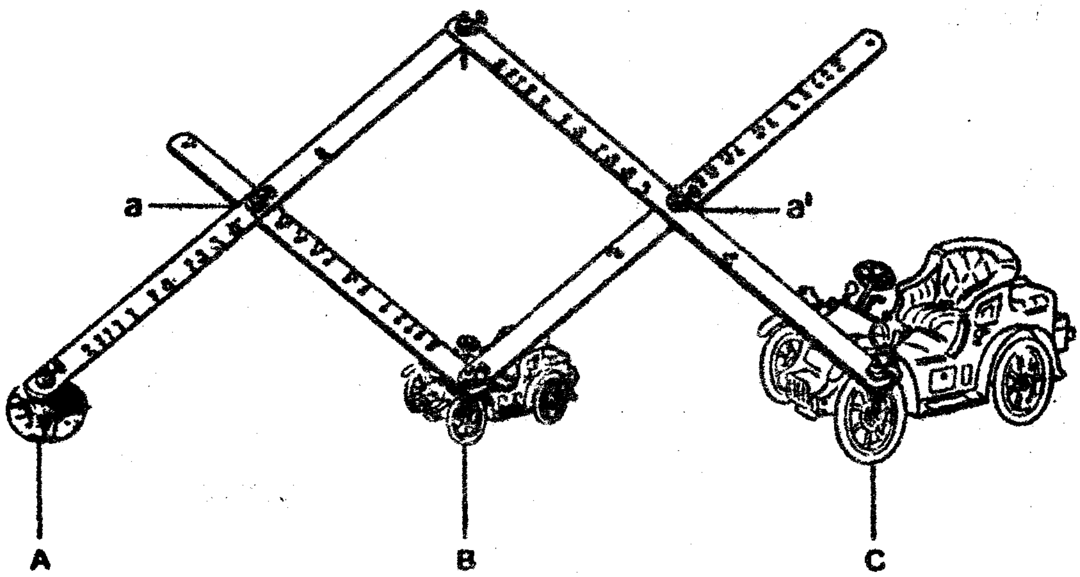
Thus $$\tan = \frac{\sin}{\cos} = \frac{\text{opp. side}}{\text{hypotenuse}} \div \frac{\text{adj. side}}{\text{hypotenuse}} = \frac{\text{opposite side}}{\text{adjacent side}}$$

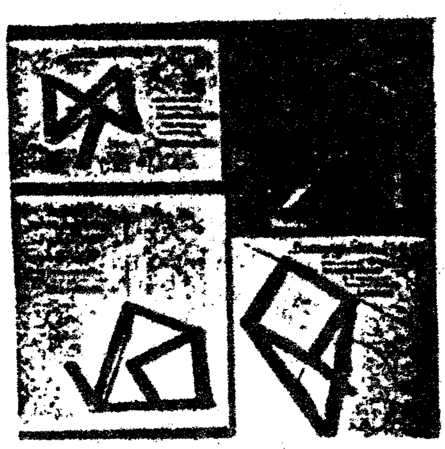
十一. 放大尺

材料: 鋁

製作: 先將鋁片依已定尺寸切好. 銼平. 然後劃線. 鑽孔. 車螺絲. 再裝配. 造成後. 繪出三個例圖.

資格: 二年級金工科男生 三人一組.





十二　圓錐体的切割

材料：模型部份：水晶膠，不銹鋼，蠟。

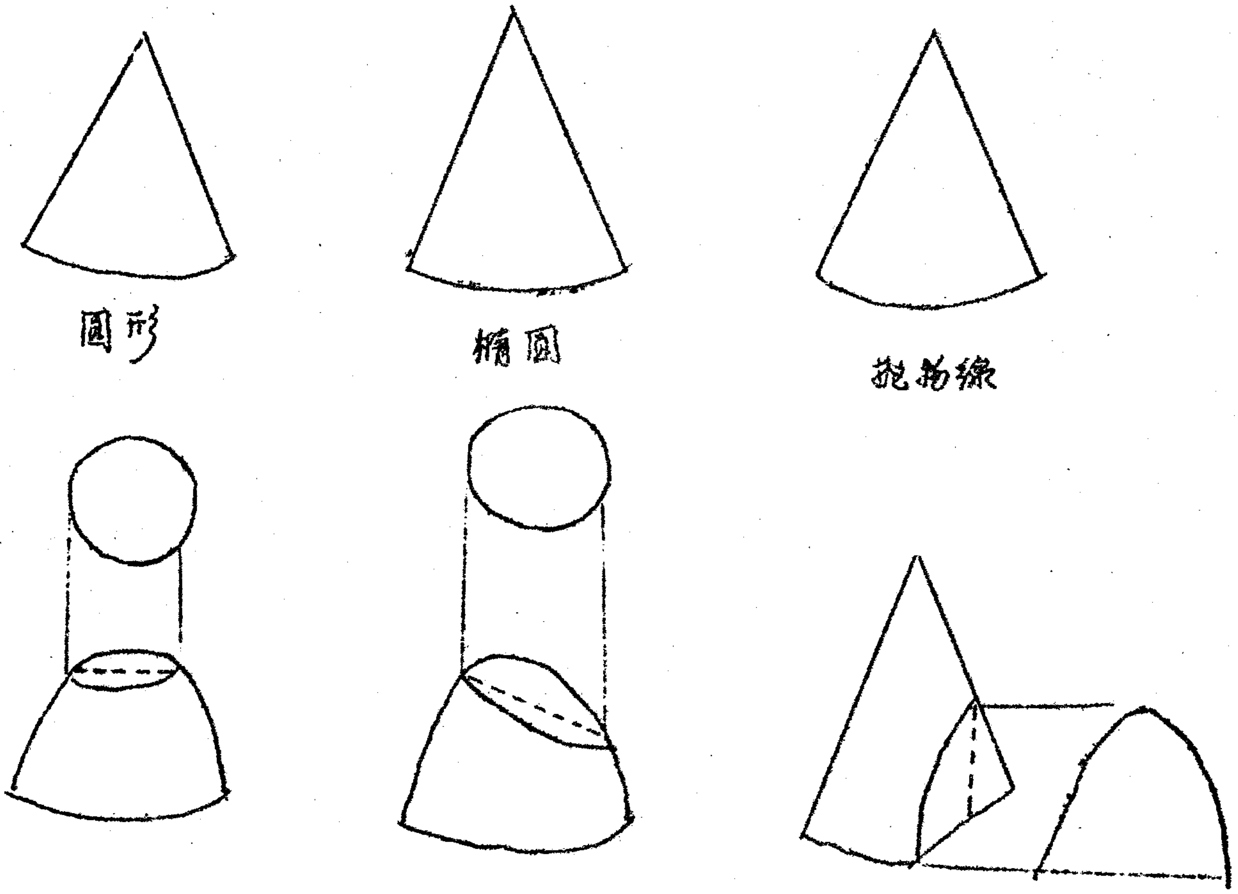
　　　繪圖部份：圖畫紙

製作：模型部份：a. 用不銹鋼車成圓錐形.(依已定的尺寸及大小)

　　　　　　　　b. 用卡紙先作一圓錐體模型，再倒入水晶膠或蠟，凝固而成錐體，再分割將之切成所需之形狀

　　　繪圖部份：先將鉛筆及繪圖儀器依已定的尺寸大小畫於畫紙上.如可能時加上墨.

資格：二年級男生三人一組.



Rounding error could be astonishing

S.B. TENG

Mathematics Section, Advisory Inspectorate, E.D.

In the newly issued Form 4 & 5 CDC Mathematics Syllabus (the sequel to the Provisional CDC F.1 - III Mathematics Syllabus), the topic on gradient of a tangent to a curve at a point is included. It is suggested in the Teaching Notes that teachers may begin with successive chords and when one end point of the chord is made to approach the other end point then in the 'limiting' position it is hoped that students can see the gradients approaching a definite number. Particular cases where no tangent exists are, of course, not included at this stage. The use of pocket calculator in this process is considered essential.

This is all very well if teachers as well as students possess sophisticated calculators which are accurate up to, say, 10 digits or more. Unfortunately this is not very likely. With the ordinary calculators, the accumulated rounding errors could be very astonishing.

Teachers therefore should be careful when this topics is taught. Otherwise, students may observe the gradient, instead of approaching a limit, behaving rather strangely.

To illustrate the above point, here are some examples. In these examples, the Casio fx-201 P and the HP 25 programmable calculators are used for fast comparison. Teachers and students, of course, need not buy programmable calculators since these are still quite expensive.

Example 1 $\quad y = x^3 - 2x^2 - x - 5$

We know at $x = 2$, the gradient is 3. Now we try the successive chords method. Since when $x = 2$, $y = -7$ hence

$$\text{gradient at } x = 2 \text{ is equal to } \frac{-7-(x_o^3 - 2x_o^2 - x_o - 5)}{2 - x_o}$$

$$\text{i.e. grad}_{x=2} = \frac{x_o\left[x_o(2-x_o) + 1\right] - 2}{2 - x_o} \quad \text{after simplification}$$

and where x_o should be taken as close to 2 as possible. Here are the results from the two calculators.

The HP - 25 Programme

```
ENT
ENT
ENT
 2
 -
STOO
 X
 1
 -
 X
 2
 +
RCLO
 ÷
f   FIX 9
```

The Casio 201 P Programme

```
ENT  1 :
2 = K2 - 1 :
3 = 1 x 2 + K 1 x 1 - K2:
4 = 3 ÷ 2 :
ANS  4 :
```

The gradient

Xo	HP-25 result	Casio 201 P result
1.9	2.61	2.61
1.99	2.9601	2.9601
1.999	2.996001	2.996001
1.9999	2.99960	2.9996
1.99999	3.	2.99996
1.999999	3.	2.9997
1.9999999	3.	2.997
1.99999999	3.	2.97
1.999999999	3.	2.7

Therefore it would be difficult to convince the students to see that the gradient approaches 3 if we happen to use the fx-201P calculator.

Example 2 : $y = x - \sin x$

At x = 2, gradient is 1.41614684
Using the successive chords method

$$\underset{x=2}{\text{Grad}} = \frac{1.09070257 + (\sin x_o - x_o)}{2 - x_o}$$

The fx - 201 P programme

ENT 1 :
2 = K2 - 1 :
3 = 1 sin - 1 + K 1.09070257 :
4 = 3 ÷ 2
ANS 4 :

The HP-25 programme

g Rad
ENT
ENT
Sin
-
1.09070257
-
X ⇄ Y
2
-
÷
f Fix 9

The Gradient

X_o	HP-25 result	fx-201 P result
1.9	1.370026577	1.3700266
1.99	1.4115931	1.411593
1.999	1.415689	1.41569
1.9999	1.41607	1.4161
1.99999	1.4158	1.416
1.999999	1.413	1.41
1.9999999	1.38	1.4
1.99999999	1.1	1
1.999999999	-2	1

This shows even a sophisticated machine like HP 25 finds it difficult to handle simple trig. function.

Example 3 : $y = 2x^{\frac{1}{2}} + 7$

We are to find the gradient at x = 4. We know it is 0.5. Below are the results (we omit the equation and the programmes)

X	HP-25 result	fx-201 P result
3.9	0.503164680	0.5031647
3.99	0.5003128	0.500313
3.999	0.500032	0.500032
3.9999	0.5	0.50002
3.99999	0.5	0.5002
3.999999	0.5	0.502
3.9999999	0.5	0.52
3.99999999	0.6	0.6
3.999999999	0	2

From these examples, it is no wonder teachers should feel wary about this topic. Nevertheless if handled with care and ample preparation, this topic can still be fun, interesting and worthwhile doing at this level.

Here are some practical hints when this topic is taught

1. Always try the working beforehand

2. Stay away from trig. or log function. Keep to polynomials (In fact this is what is suggested in the teaching notes)

3. Avoid using very extreme values.

4. Once you have tried a few examples and succeeded in convincing your students that this method works, you may introduce the derivative method.

Here are some examples you can try with your class :

1. $y = 2x^2 - 3x + 5$

2. $y = 3x^3 - 4x^2 - x + 5$

3. $y = 11 - 2x + 3x^2$

4. $y = x^3 - 2x^2 - x - 5$

Master Mind with the Programmable Calculator

FUNG

Mathematics Section, E.D.

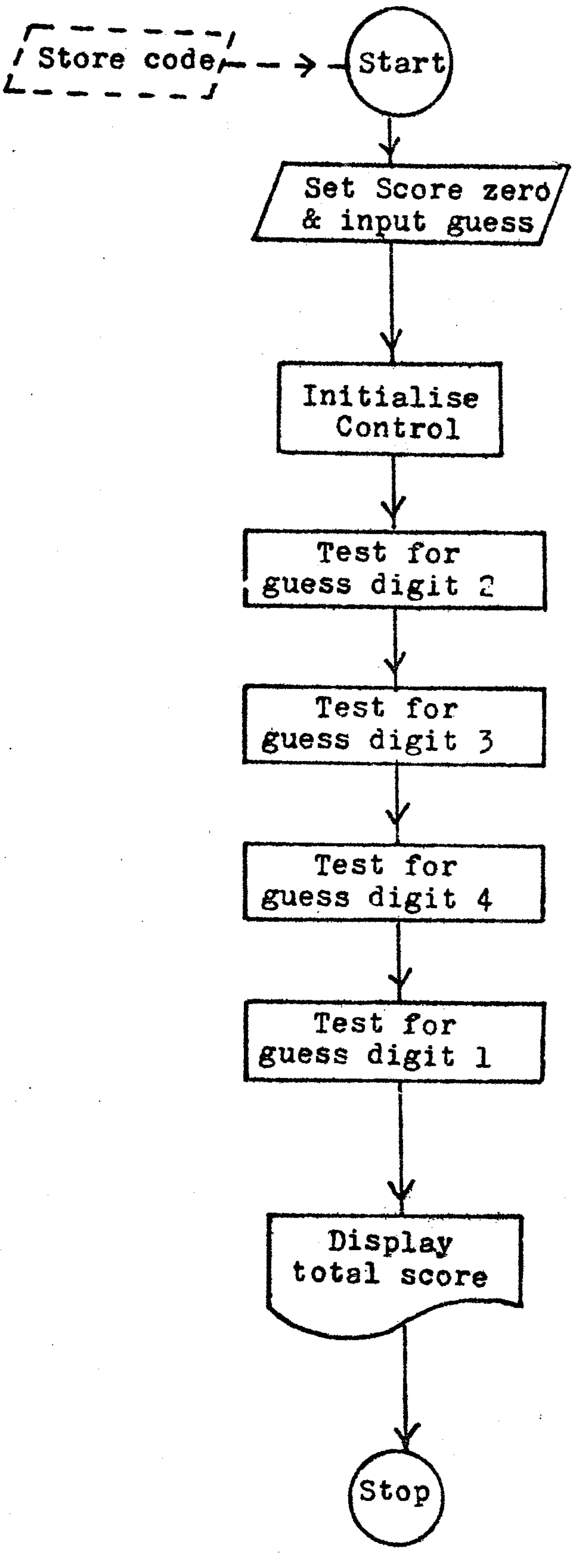
Master Mind is unquestionably an excellent Mathematical Game. But you must have a very co-operative opponent— one who never makes mistakes in placing the key pegs, and, at the same time always remains patient during the long idling periods waiting for you to solve the secret code. Such an opponent is difficult to find. Happily I found it in the programmable calculator.

I have tried my hand on two different models, the Casio fx-201 P and the Hewlett - Packard HP 25. The game programmed is the standard 4-digit version (no doubles). Programmes for the two machines are given below :

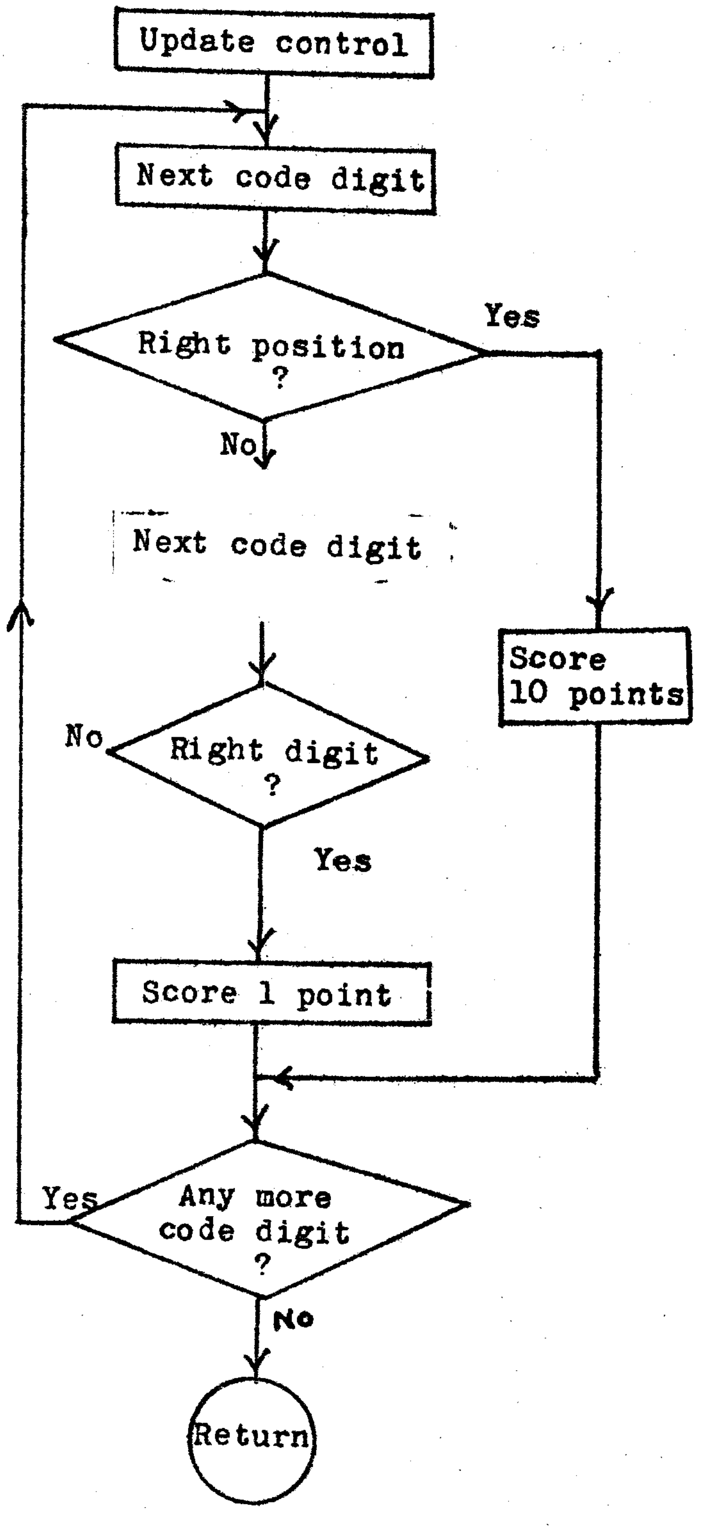
fx-201 P

Programme

Step	Key Entry
001 - 011	ENT 6 : 0 : 3 : 5 : 7:
012 - 019	I = K1 : GOTO 1 :
020 - 026	3 = 5 : GOTO 1 :
027 - 033	3 = 7 : GOTO 1 :
034 - 040	3 = 0 : GOTO 1 :
041 - 043	ANS 6 :
044 - 053	SUB1 : I = I x K2 :
054 - 057	9 = IM :
058 - 068	IF 3= IM : 2 : 5 : 2 :
069 - 078	ST#2 : I = I x K2 :
079 - 089	IF3 = IM : 4 : 3 : 4 :
090 - 099	ST#3 : 6 = 6 + K1 :
100 - 102	GTO 4 :
103 - 116	ST#4 : IF9 = IM : 2 : 6 : 2 :
117 - 127	ST#5 : 6 = 6 + K1 :
	(ST#6 :)



Subroutine for Test



Memories

0	D(1) -- Guess digit 1
1	C(1) -- Code digit 1
2	C(2)
3	D(2)
4	C(3)
5	D(3)
6	Score
7	D(4)
8	C(4)
9	Control 2
I	Control 1

U ser Instructions

Step	Action	Machine Mode	Input/keys	Output
1	Key in programme.	WRITE		
2	Store Code.	MANUAL	C(1) [ENT] [1]	[1 C(1)]
			C(2) [ENT] [2]	[2 C(2)]
			C(3) [ENT] [4]	[4 C(3)]
			C(4) [ENT] [8]	[8 C(4)]
3	Start playing.	COMP	[STA]	[6 0]
	Set Score zero.		[ENT]	[0 0]
	Input guess.		D(1) [ENT]	[3 0]
			D(2) [ENT]	[5 0]
			D(3) [ENT]	[7 0]
			D(4) [ENT]	[6 Score]
4	To carry on game with new guess, go to step 3.			
5.	To start new game, go to step 2.			

Scoring

For each right digit in the right position: 10 points

For each right digit in the wrong position: 1 point

Thus a score of 21 indicates 2 digits in the right position and 1 digit in the wrong position. A score of 40 means the secret code is broken.

Programming Notes

The programme is not a complicated one, but to squeeze it in a calculator with only 127 key steps (not programming steps) is quite a problem.

The most interesting part of the programme is the use of the Indirect Memory I which stores only the first digit of a number. By updating the control with "I=IxK2", a cycle of four numbers 2, 4, 8, 1 (16) will be stored in I in turn, to be used as contorl for both entry and exit to the Subroutine.

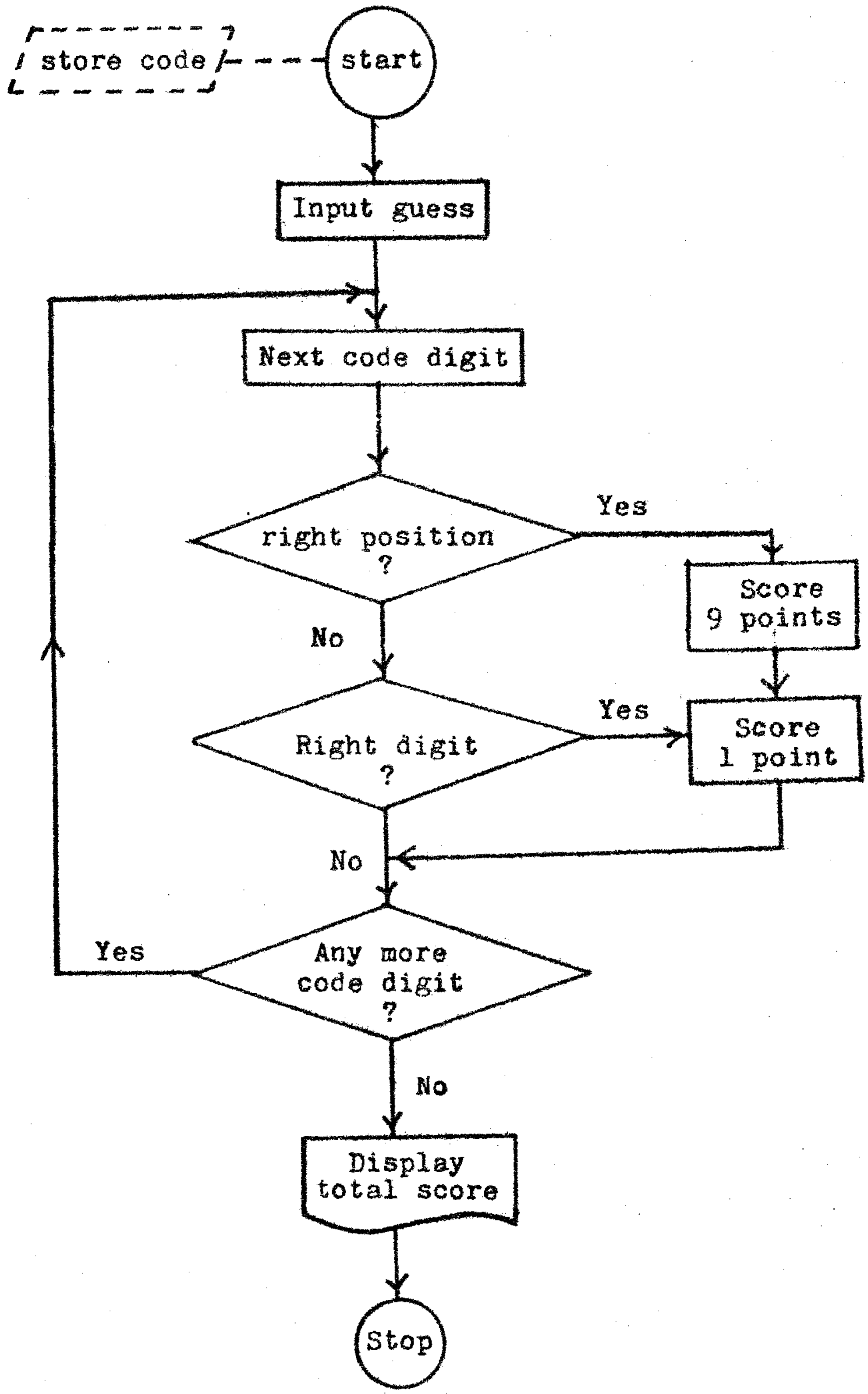
The programme is the shortest one I can make out. However, it still totals 130 steps, 3 steps too many for the fx-201P. Accidentally, I found out that if the last line of a Subroutine is a dummy command, it can be left out. This unexpected allowance helps to shorten the programme and makes the whole thing possible. I wonder if the designer of the calculator himself notices this.

There is a flaw in this programme for the fx-201P. The Score Memory has to be set zero manually (the first [ENT] in step 3 of the User Instructions) before inputing a new guess. In the following programme for the HP25, this is not necessary.

HP 25

ogramme

Line	Key Entry
01	STO - 4
02	RCL 0
03	f x≠y
04	GTO 07
05	9
06	STO + 4
07	↓
08	RCL 4
09	f x=y
10	GTO 23
11	↓
12	RCL 7
13	f x=y
14	GTO 23
15	↓
16	RCL 6
17	f x=y
18	GTO 23
19	↓
20	RCL 5
21	f x≠y
22	GTO 25
23	1
24	STO + 4
25	RCL 1
26	RCL 3
27	f x=y
28	GTO 44
29	g x<0
30	GTO 40
31	g x=0
32	GTO 36
33	STO - 3
34	RCL 7
35	GTO 03
36	RCL 2
37	STO - 3
38	RCL 6
39	GTO 03
40	RCL 1
41	STO 3
42	RCL 5
43	GTO 03
44	RCL 4



Memories

1	D(1) -- Guess digit 1
2	D(2)
3	D(3)/Control
4	D(4)/Score
5	C(1) — Code digit 1
6	C(2)
7	C(3)
0	C(4)

User Instructions

Step	Action	Machine Mode	Input/keys	Output
1	Key in programme.	PRGM		
2	Integer display.	RUN	[f] [FIX] [0]	
3	Store code.	RUN	C(1) [STO] [5]	
			C(2) [STO] [6]	
			C(3) [STO] [7]	
			C(4) [STO] [0]	
4	Input guess.	RUN	D(1) [STO] [1]	
			D(2) [STO] [2]	
			D(3) [STO] [3]	
			D(4) [STO] [4]	
			[R/S]	Score
5	To carry on game with new guess, go to step 4.			
6	To start new game, go to step 3.			

Programming Notes

As there are only 8 data memories in the HP25, two of the memories initially used to store the guess digits have to play a dual roles. After the first loop of the test, Memory 4 is changed to be the scores accumulator and Memory 3 is used as the control.

The programme contains only 44 steps and the calculator has not been used to its full capacity. A more ambitious programmer may extend it for the 5-digit game with doubles or even insert a random number generator to supply new code.

- (End) -

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University lecturers, college of education lecturers and mathematics teachers who wish to contribute articles for publication are more than welcome. Contributions need not be typed. For further information, please contact the Editor, School Mathematics Newsletter at 5-774001 ext. 36

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Presentation of Geometry Proof

K.F. HO, S.W. PUN and C.S. POON

"Geometry is out-of-date. It requires too much time in teaching and learning". "Geometry is no longer a subject that pupils have to study. It can be replaced by Analytic Geometry". "Geometry becomes so unfavourable that even in the recent Certificate of Education Examination Syllabus topics on Geometry were reduced to a minimum and formal prooofs will not be asked as well". However, we do not agree with these point of views.

Geometry is still a branch of mathematics that is worth teaching and learning in any secondary school. One of the objectives in Mathematics education is to develop the pupil an ability to reason logically. The most vital element in geometry is its deductive reasoning no matter whether the subject is developed rigorously or not. In fact, mathematics consists of plenty of if-then statements. The deductive reasoning appeals to these if-then statements. The pupil would learn how to reason logically by proving a geometrical fact or theorem, step by step, from a hypothesis to a conclusion.

Analytic geometry, however, is not an adequate substitute for geometry. The reasoning used in analytic geometry is primarily computational. Computation is a form of reasoning, but it is only one of the many forms mathematical reasoning may take. The fact that computation is a form of reasoning is usually obscured by the prominence in it of mechanical calculations. Therefore, analytic geometry is not a subject in which the pupil can perfectly learn to reason.

Why is geometry so unfavourable among teachers as well as pupils ? Teachers feel that too much time and effort and required to teach the proofs in geometry and that it is very tedious to correct a pupil's proof. On the other hand, pupils have to memorize lots of postulates, definitions and theorems in order to restate them in proving a geometric fact. They always confuse a theorem with its converse, ignore some important working steps and do not clearly show the thought process involved in a proof. They are forced to memorize the proofs of some geometric facts, and consequently, they may lack understanding.

The traditional method of displaying a proof in two column-statement reason" form has more or less the responsibility for the above weak points in teaching and learning geometry. Many teachers and texts have presented a geometry proof with a heading of the "Given - ; To prove - " format. We contend that the use of this format is not only misleading, but logically incorrect. Very often, theorems are stated using an if-then statements; for instance, "If, in $\triangle$ ABC, AB = BC, then $\angle A = \angle C$". When we prove this theorem we prove the whole implication. We certainly cannot prove $\angle A = \angle C$. However, in the format stated above pupils would be required to write :

Given : $\triangle$ ABC with AB = BC

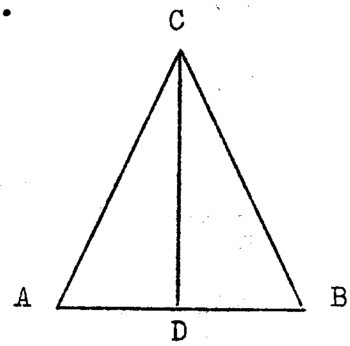
To prove : $\angle A = \angle C$

The nature of the hypothesis of a theorem is concealed. The pupils hardly notice that the so called "Given" part is an assumption. So they are likely to confuse the theorem with its converse or to use a statement which is required to prove. We have other discontentments to the two-column form.

This form does not allow pupils to show clearly their thought process; that is, it does not show conveniently which statements imply a given statement. It is not easy to point out the source of error and determine how much a pupil's proof is correct.

Here we would like to introduce another form of proof which we believe, would lessen the inconvenience of the two-column form. Notice that we are not going to suggest it as a replacement but as an alternative to the two-column form. An example of proving a geometric theorem by the two-column form and the new form is given as the figure below.

<u>Theorem</u> : Prove that if the median to a side of a triangle is perpendicular to that side, then the triangle is isosceles.



A. <u>Two-column Form</u>

Given : The figure as represented at the right with CD ⊥ AB and CD as median on AB

To prove : AC = BC

Proof :

<u>Statements</u>	Reasons
1. CD is the median on AB	1. Given
2. AD = DB	2. Definition of median
3. CD ⊥ AB	3. Given
4. ∠ADC = ∠BDC	4. Right ∠ s
5. CD = CD	5. Common
6. △ADC ≅ △BDC	6. S.A.S.
7. AC = BC	7. Corr. parts of congruent triangles

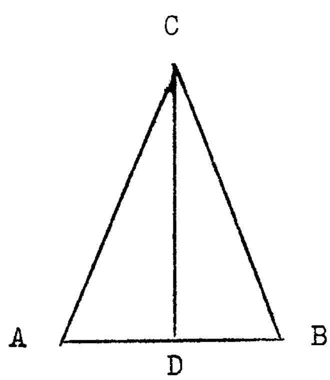
Q.E.D.

B. <u>Flow-chart Form</u>

To prove :-

If : In the figure at the right, CD ⊥ AB and CD is the median on AB

then : AC = BC



Proof :

CD is the median in AB ⟶ AD = DB

CD ⊥ AB ⟶ ∠ADC = ∠BDC

CD = CD

} ⟶ △ADC ≅ △BDC ⟶ AC = BC

The new form is just as a flow-chart arranging the implications, horizontally in the logical sequence of the proof and we replace the "Given : -; To Prove : - " format by " To Prove : If : - ; then : - " format which is equally effective and more accurate to demonstrate the nature of the proof. The advantages to this flow-chart form are many:

(a) The true nature of proof is emphasized

(b) The relationship between the statements is graphically demonstrated.

(c) It shows much clearly the thought process involved

(d) It is easier to pointout the source of pupils' trouble

(e) Teachers are able to determine how much the proof the pupils understands

(f) Teachers can write the chart in reverse order to demonstrate the analytic thinking of the proof

One might notice that this method writes no reason for each implication and thus discourages the memorization of postulates and theorems. As a matter of fact, as the pupils writes a conclusion he has to check whether it is valid under the former assumptions. This form puts emphasis not only on correct thought process but also on understanding. One obvious disadvantage is that the form is not suitable for theorems or exercises which involve many steps of working since a lengthy proof of this form is apt to cause confusion. We recommend that the teachers use this method of presentation after several lessons of two-column form of proofs and make a comparative evaluation between this and the two-column form.

趣味數學的妙用

—關麗娟—

數學這門學問，對於很多中學生，都是既懼怕而又覺得難明的；尤其是女同學，大部份對數學毫無興趣，視它如鬼魅一樣；每逢上數學課，饒你多麼淺白，饒你老師說得多麼簡易，他們仍是搔首難明，魂遊天外。

作爲數學教師的我們，面對著一群害怕數學的學生，應該怎樣去施教才好呢？讓我告訴你一個妙方吧！就是告訴學生一些趣味數學，即如稱銀問題、黃金分割等；下次上課的時候，不要再討論網絡的奇點、偶點，而是告訴學生有名的七橋問題，由學生自己去思想，學生會驚訝於你的改變，同時沒有了密密麻麻的數目字，沒有了 x，y，z，他們一定會有興趣去思考，最後你才引出網絡的問題，到此他們才發現網絡的妙用，在他們的心目中，這不是數學，而是一些有趣的問題，而你亦已完成了你要教授的責任，學生亦已在不知不覺中學習到數學，那不是很好嗎？

或者你仍在懷疑趣味數學的實用性，那就讓我來告訴你一些關於我的體驗吧！當我正要向一群中一學生講解公式設立的問題的時候，而他們卻感到極爲乏味且不知從何設立，尤其是 $1+2+3+\cdots\cdots+n=\frac{(1+n)n}{2}$ 這一個公式的設立，我面對著這一個難題，正感到不知如何是好，忽然想起趣味數學，於是便向學生講述 Guass 的軼事，先將 $1+2+3+\cdots\cdots+n$ 列出，然後將其首尾調轉排列，變成 $n+(n-1)+\cdots\cdots+3+2+1$，再將兩式相加便得出 $(n+1)n$，因此 $1+2+3+\cdots\cdots+n=\frac{(n+1)n}{2}$，學生經我如此講解後，都對公式設立的問題發生濃厚的興趣，而興緻勃勃的參與討論。從以上的一個例證，雖然仍不能看出趣味數學的妙用，但也可以體味到其功效了。

或者你還沒有發現趣味數學的妙用，又或者你所認識趣味數學的例子不多，那也不打緊，就立刻到圖書館翻翻吧！

這不特對你的學生有所裨益，而你也可能有意想不到的收穫。從下堂開始，試試把趣味數學加進你的教學過程中，看你的學生有甚麼反應？你一定會深切體會到趣味數學的妙用，而你亦不用再爲學生的搔首蹝躕，魂遊天外而煩惱了。

'The Mark Six' Problem

S.B.T.

The 'Mark Six' Lotteries have become way of life with many oy of us. Indeed, who could withstand the temptation of putting down a few dollars when the prize money goes up to a staggering 7-digit figure. Lady Luck may just smile at you once and 'once' is more than enough.

To bet on the 'Mark Six' we may use the single selection ticket or the multiple selection ticket. The latter allows you to choose up to 15 numbers. If one day when the prize money "astronomical you or your syndicate decide to bet on 16 numbers, have you ever wondered how many selection tickets you need to use? Our problem then is: to decide the minimum number of tickets (multiple and/or single) that need to be used for betting 16 (or 17, 18 etc.) numbers such that each selection is made only once.

One obvious solution to the problem is to open a telephone betting account and then you can bet on any total of numbers (provided you have enough money in your account) But other than that, do you have an answer to the above problem?

MARK SIX LOTTERY (76-77)

L.L. Li

There are two plausible reasons for studying the Mark Six. First, there is the hope of making some easy money if one can "sort of predict" the outcome of draws. Second, there is the academic question of whether one could "sort of predict" the outcome of draws at all.

Those who do not believe that each draw is a random selection of seven balls out of thirty six usually hold the opinion that truely random macroscopic systems simply do not exist. Some newspaper columnists go as far as giving predictions of probable numbers and number doublets in upcoming draws.

In this note we study, phenomenologically, the numbers drawn in the years 1976 and 1977 without attempting to draw any conclusion as to whether the lottery machine is "fair". Here, we take the view that the "number" associated with any ball is but a label for that particular ball and that it does not carry any mathematical significance. There were 152 draws, giving 152x7=1064 numbers. If 152 is considered "sufficiently large", then one would expect the frequency for drawing any particular ball to be

$$\frac{1064}{36} = 29.56.$$

Fig. 1 shows the actual frequencies for the thirty six balls. 16 balls were drawn with frequencies less than the average expected frequency and 20 balls came out with higher frequencies. The actual frequencies range from 14 to 41. We also note, in passing, that balls labelled by multiples of six are "favorites" (by chance ?).

In fig. 2, we show a histogram of the number of balls n occurring with frequencies f in the ranges indicated on the horizontal axis. One cannot say much about the distribution.

For better presentation of the distribution, let us define relative frequency $p \equiv \frac{\text{actual frequency f}}{\text{expected frequency}}$. The meaning of p is clear : p=1.0 means the ball was drawn as often as expected, and p=1.5, for example, means it came out 50% more often than expected. Fig. 3 shows a histogram of n against p, which now resembles the binomial and/or the Poisson distribution and which is suggestive of the existence of some "popular" balls.

Finally, we test the "fairness" of the distribution at a significance level of 0.05 by use of the Chi-square test. From the data of fig. 1, we arrive at $\chi^2 \simeq 42$. For $\nu = 35$, $\chi^2(0.95) \simeq 50$. It looks like the test is affirmative despite the "implication" of Fig. 3.

1/ 23	2/ 17	3/ 21	4/ 28	5/ 24	6/ 39
7/ 30	8/ 37	9/ 27	10/ 29	11/ 28	12/ 31
13/ 34	14/ 38	15/ 32	16/ 38	17/ 24	18/ 33
19/ 32	20/ 31	21/ 32	22/ 31	23/ 33	24/ 41
25/ 33	26/ 24	27/ 17	28/ 26	29/ 29	30/ 35
31/ 22	32/ 23	33/ 22	34/ 34	35/ 31	36/ 35

Fig. 1

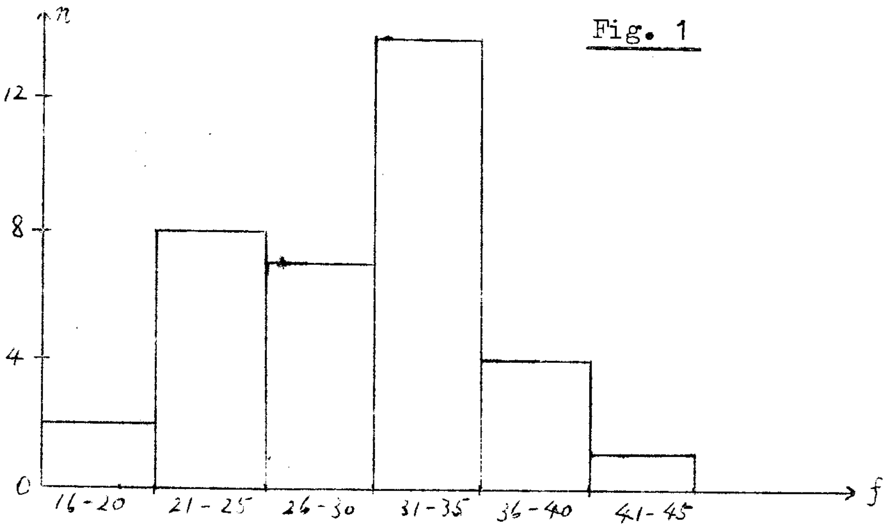


Fig. 2

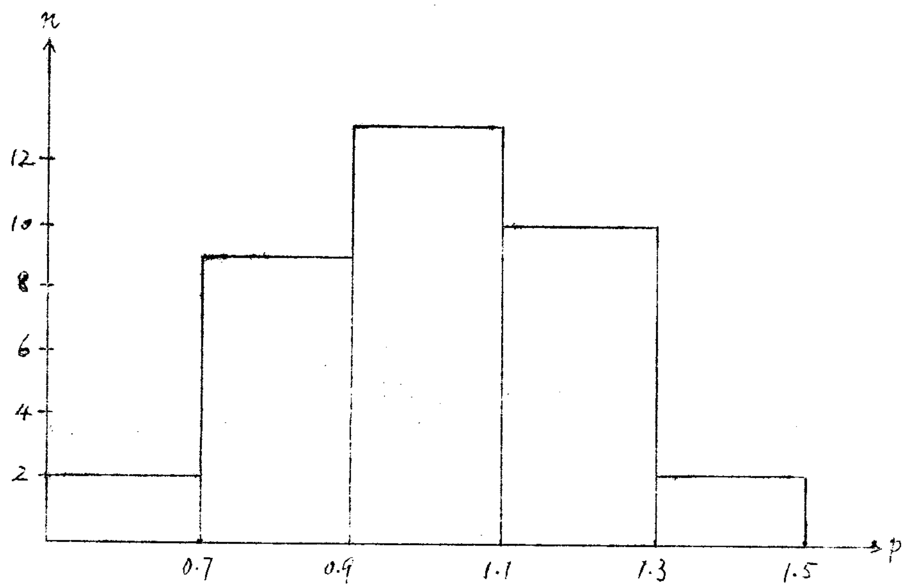


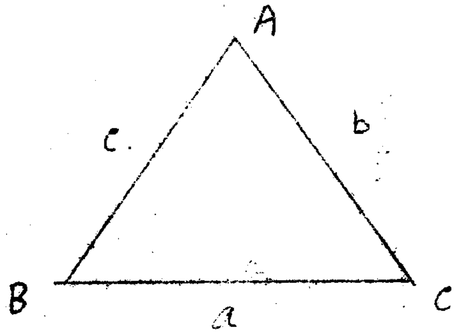
Fig. 3

$\Delta = \frac{1}{2} a\ b \sin C$

陳瑞蓮

三角學中很多公式都和三角形面積有很密切的關係。茲就我所能想出的，分列如下：

1. $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ $\quad (s = \frac{1}{2}(a+b+c))$



[証] $\Delta = \frac{1}{2} a\ b \sin C$

$= \frac{1}{2} ab \cdot 2\sin\frac{C}{2}\cos\frac{C}{2}$

$\overset{(*)}{=} a\ b\sqrt{\frac{(s-b)(s-a)}{ab}} \cdot \sqrt{\frac{s(s-c)}{ab}}$

$= \sqrt{s(s-a)(s-b)(s-c)}$

(*) A 為 Δ 一內角；a, b, c 為三邊，$s = \frac{1}{2}(a+b+c)$

則 $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$

$\cos\frac{A}{2} = \sqrt{\frac{s(s-c)}{bc}}$

[証]：$\because \cos 2A = 1 - 2\sin^2 A$

$\cos A = 1 - 2\sin^2\frac{A}{2}$

$\therefore \sin\frac{A}{2} = \sqrt{\frac{1}{2}(1-\cos A)}$

$= \sqrt{\frac{1}{2}\left(1 - \frac{b^2+c^2-a^2}{bc}\right)}$

$= \sqrt{\frac{1}{2} \cdot \frac{(a+b-c)(a-b+c)}{bc}}$

$= \sqrt{\frac{(2s-2c)(2s-2b)}{4bc}}$

$= \sqrt{\frac{(s-b)(s-c)}{bc}}$

$\therefore \cos\frac{A}{2} = \sqrt{\frac{1}{2}(1+\cos A)}$

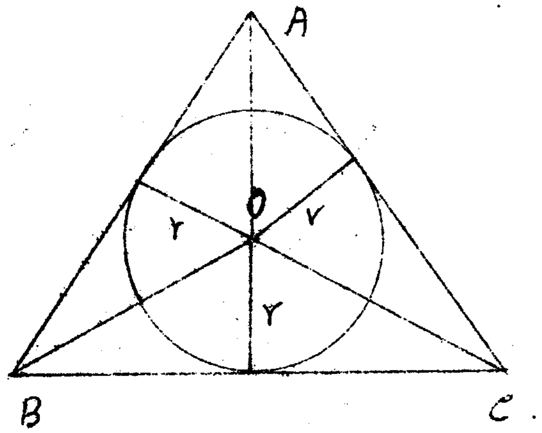
$= \sqrt{\frac{1}{2}\left(1 + \frac{b^2+c^2-a^2}{2bc}\right)}$

$= \sqrt{\frac{1}{2} \cdot \frac{(b+c+a)(b+c-a)}{2bc}}$

$= \sqrt{\frac{2s(2s-2a)}{4bc}}$

$= \sqrt{\frac{s(s-a)}{bc}}$

2. 三角形內切圓半徑　$r = \frac{\Delta}{s}$



〔證〕：設 $\Delta ABC = \Delta$

$a + b + c = 2s$

則 $\Delta ABC = \Delta ABO + \Delta BCO + \Delta CAO$

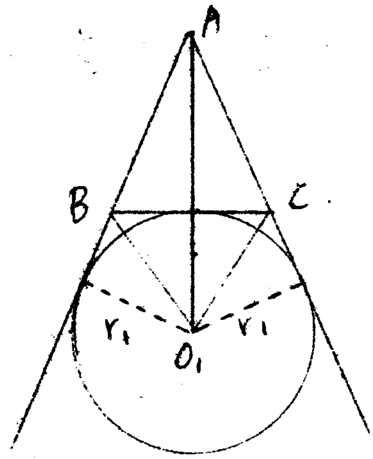
$\therefore \Delta = \frac{1}{2}cr + \frac{1}{2}ar + \frac{1}{2}br$

$= \frac{1}{2}r(a+b+c)$

$= sr$

$r = \frac{\Delta}{s}$

3. 三角形傍切圓半徑 $r_1 = \frac{\Delta}{s-a}$，$r_2 = \frac{\Delta}{s-b}$，$r_3 = \frac{\Delta}{s-c}$



〔證〕$\Delta ABC = \Delta O_1AB + \Delta O_1CA - \Delta O_1BC$

$\therefore \Delta = \frac{1}{2}cr_1 + \frac{1}{2}br_1 - \frac{1}{2}ar_1$

$= \frac{1}{2}r_1(b+c-a)$

$= \frac{1}{2}r_1(2s-2a)$

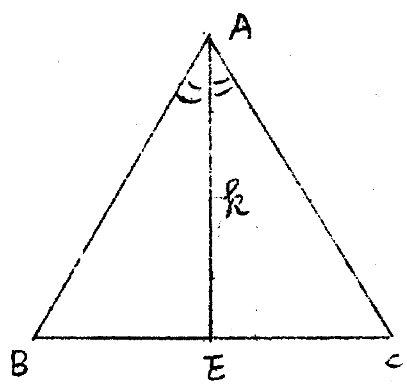
$= r_1(s-a)$

$\therefore r_1 = \frac{\Delta}{s-a}$

同理亦得 $r_2 = \frac{\Delta}{s-b}$

$r_3 = \frac{\Delta}{s-c}$

4. 在 ΔABC 中，AE 為 $\angle A$ 的分角線，則 $AE = \frac{2bc}{b+c}\cos\frac{A}{2}$



〔證〕：設 $AE = k$

$\Delta ABC = \Delta ABE + \Delta AEC$

$\frac{1}{2}ab\sin C = \frac{1}{2}ck\sin\frac{A}{2} + \frac{1}{2}bk\sin\frac{A}{2}$

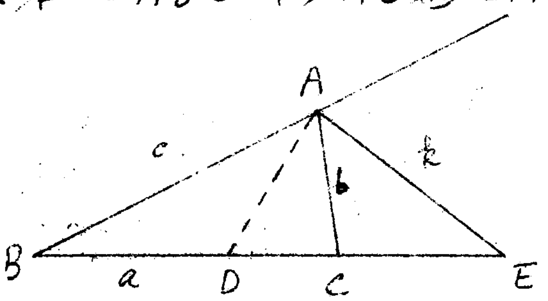
$\frac{1}{2}ab\sin C = \frac{1}{2}k(c+b)\sin\frac{A}{2}$

$bc\sin\frac{A}{2}\cos\frac{A}{2} = \frac{1}{2}k(b+c)\sin\frac{A}{2}$

$2bc\cos\frac{A}{2} = k(b+c)$

$k = \frac{2bc}{b+c}\cos\frac{A}{2}$

5. 在 $\triangle ABC$ 中，AE 為 $\angle A$ 的外角分角線，則 $AE = \frac{2bc}{c-b}\sin\frac{A}{2}$



$(c > b)$

[証]：作內角分角線 AD 及外角分角線 AE，則 $\angle DAE = 90^\circ$

設 $AE = k$

由 $\triangle ABC = \triangle ABE - \triangle ACE$

$$\frac{1}{2}bc\sin A = \frac{1}{2}ck\sin(90^\circ + \frac{A}{2}) - \frac{1}{2}bk\sin(90^\circ - \frac{A}{2})$$

$$bc\sin A = ck\sin(90^\circ + \frac{A}{2}) - bk\sin(90^\circ - \frac{A}{2})$$

$$2bc\sin\frac{A}{2}\cos\frac{A}{2} = k(c-b)\cos\frac{A}{2}$$

$$\therefore k = \frac{2bc}{c-b}\sin\frac{A}{2}$$

[註]：若 $b > c$，則 $k = \frac{2bc}{b-c}\sin\frac{A}{2}$

6. 正弦定律

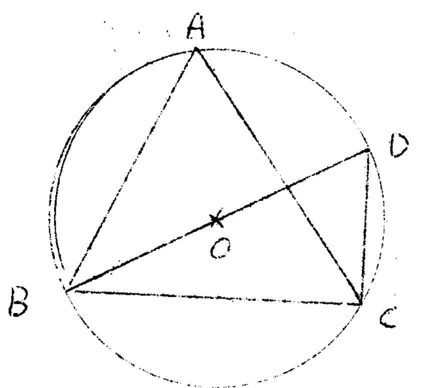
[証]：$\triangle = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$

$$\therefore ab\sin C = bc\sin A = ac\sin B$$

$$\text{i.e. } \frac{1}{ab\sin C} = \frac{1}{bc\sin A} = \frac{1}{ac\sin B}$$

$$\frac{c}{\sin C} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

7. 三角形的外接圓半徑 $R = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C}$



[証]：令 BD 為 $\triangle ABC$ 外接圓直徑，則

$$\angle BDC = \angle BAC = \angle A$$

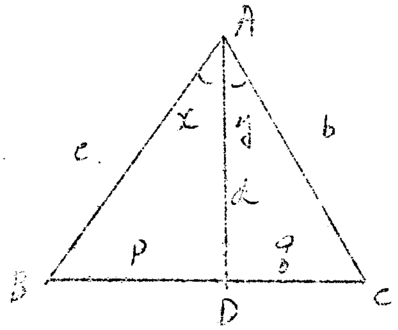
$$\angle BCD = 90^\circ$$

$$\therefore \frac{BC}{BD} = \sin\angle BDC$$

$$\therefore \frac{a}{2R} = \sin A$$

同理亦得 $R = \frac{b}{2\sin B} = \frac{c}{2\sin C}$

8. 分角線定理推廣 $\frac{p}{q}=\frac{c\sin x}{b\sin y}$

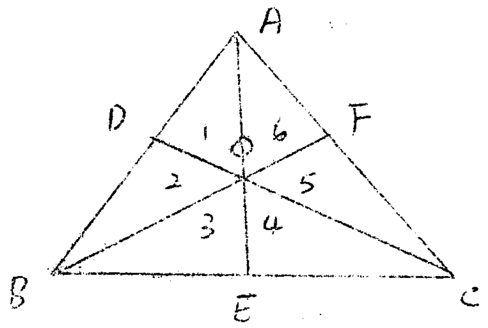


[証] ∵ △ABC與△ADC等高，故

$$\frac{p}{q}=\frac{\triangle ABD}{\triangle ADC}=\frac{\frac{1}{2}cd\sin x}{\frac{1}{2}bd\sin y}=\frac{c\sin x}{b\sin y}.$$

9. 若 AD為∠A之分角線，則 $x=y$，故 $\frac{p}{q}=\frac{c}{b}$.

10. 西氏定理 $\frac{AD}{BD}\cdot\frac{BE}{EC}\cdot\frac{CF}{FA}=1$.



[証] ∠1 = ∠4，∠2 = ∠5，∠3 = ∠6

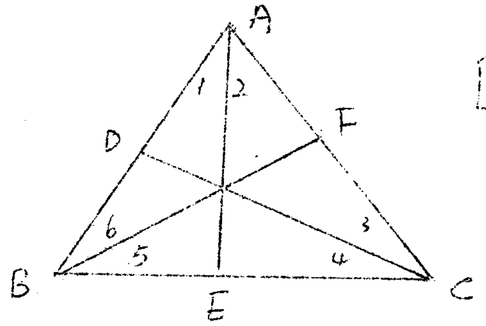
由分角線定理推廣得

$$\frac{AD}{DB}\cdot\frac{BE}{EC}\cdot\frac{CF}{FA}$$

$$=\frac{AO\sin\angle 1}{BO\sin\angle 2}\cdot\frac{BO\sin\angle 3}{CO\sin\angle 4}\cdot\frac{CO\sin\angle 5}{AO\sin\angle 6}$$

$$=1.$$

11. 西氏定理三角表示式 $\frac{\sin\angle 1}{\sin\angle 2}\cdot\frac{\sin\angle 3}{\sin\angle 4}\cdot\frac{\sin\angle 5}{\sin\angle 6}=1.$

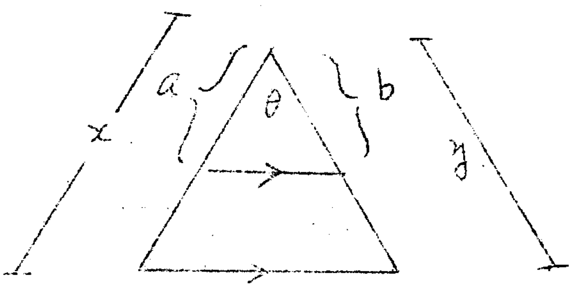


[証] $1=\frac{BE}{EC}\cdot\frac{CF}{FA}\cdot\frac{AD}{DB}$ （西氏定理）

$$=\frac{AB\sin\angle 1}{AC\sin\angle 2}\cdot\frac{BC\sin\angle 5}{AB\sin\angle 6}\cdot\frac{AC\sin\angle 3}{BC\sin\angle 4}$$

$$=\frac{\sin\angle 1}{\sin\angle 2}\cdot\frac{\sin\angle 3}{\sin\angle 4}\cdot\frac{\sin\angle 5}{\sin\angle 6}.$$

12. 兩相似△面積之比等於其對應边平方之比



[証] $\frac{\frac{1}{2}ab\sin\theta}{\frac{1}{2}xy\sin\theta}$

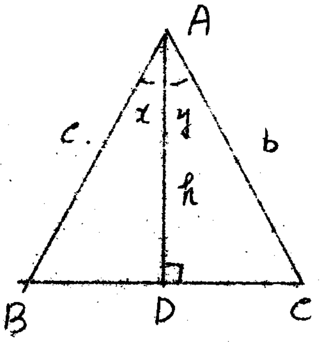
$$=\frac{a}{x}\cdot\frac{b}{y}$$

$$=\frac{a}{x}\cdot\frac{a}{x}\quad\left(\because\ \frac{a}{x}=\frac{b}{y}\right).$$

$$=\frac{a^2}{x^2}$$

13－16 為複角函數公式

13. $\sin(x+y)=\sin x\cos y+\cos x\sin y$



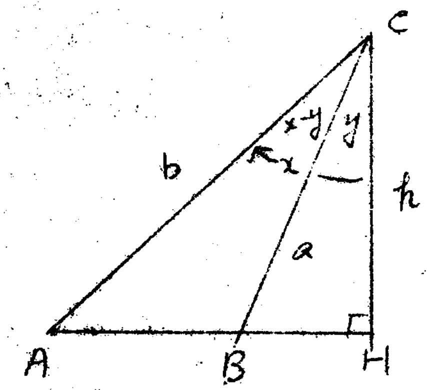
[證]： $\triangle ABC=\triangle ABD+\triangle ADC$.

$$\frac{1}{2}bc\sin(x+y)=\frac{1}{2}ch\sin x+\frac{1}{2}bh\sin y$$

$$\sin(x+y)=\frac{h}{b}\sin x+\frac{h}{c}\sin y$$

$$=\cos y\sin x+\sin y\cos x.$$

14. $\sin(x-y)=\sin x\cos y-\cos x\sin y$



[證] $\triangle ABC=\triangle ACH-\triangle CBH$

$$\triangle ACB=\frac{1}{2}ab\sin(x-y).$$

$$\triangle ACH=\frac{1}{2}bh\sin x$$

$$\triangle BCH=\frac{1}{2}ah\sin y$$

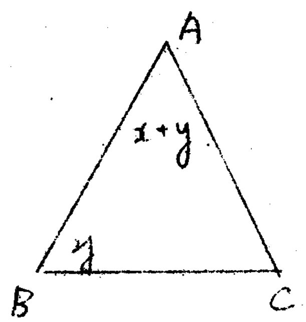
$$\frac{1}{2}ab\sin(x-y)=\frac{1}{2}bh\sin x-\frac{1}{2}ah\sin y$$

$$\sin(x-y)=\frac{h}{a}\sin x-\frac{h}{b}\sin y.$$

$$\sin(x-y)=\cos y\sin x-\cos x\sin y.$$

【編輯按：由13式亦可推出14式.】

15. $\cos(x+y)=\cos x\cos y-\sin x\sin y$.



[證] $\because \sin(A-B)=\sin A\cos B-\cos A\sin B$.

設 $A=x+y$， $B=y$.

則 $\sin[(x+y)-y]$

$$=\sin(x+y)\cos y-\cos(x+y)\sin y.$$

$$\cos(x+y)=\frac{1}{\sin y}[\sin(x+y)\cos y-\sin x]$$

$$=\frac{1}{\sin y}[(\sin x\cos y+\cos x\sin y)\cos y-\sin x]$$

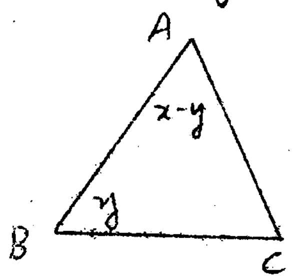
$$=\frac{1}{\sin y}[\sin x\cos^2 y+\cos x\sin y\cos y-\sin x]$$

$$=\frac{1}{\sin y}[\sin x-\sin x\sin^2 y+\cos x\sin y\cos y-\sin x]$$

$$=\frac{1}{\sin y}(\cos x\sin y\cos y-\sin x\sin^2 y)$$

$$=\cos x\cos y-\sin x\sin y$$

16. $\cos(x-y) = \cos x\cos y + \sin x \sin y$



〔証〕 $\because \sin(A+B) = \sin A\cos B + \cos A \sin B$

設 $A = x-y$ ， $B = y$

則 $\sin[(x-y)+y]$

$= \sin(x-y)\cos y + \cos(x-y)\sin y$

$\cos(x-y) = \frac{1}{\sin y}[\sin x - (\sin x\cos y - \cos x \sin y)\cos y]$

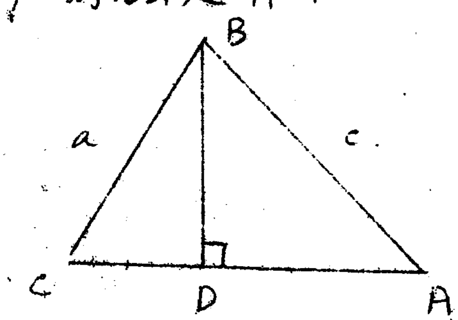
$= \frac{1}{\sin y}[\sin x - \sin x\cos^2 y + \cos x \sin y \cos y]$

$= \frac{1}{\sin y}[\sin x + \cos x\cos y \sin y - \sin x + \sin x \sin^2 y]$

$= \cos x\cos y + \sin x \sin y$

〔編輯按：由13及14兩式亦可推出15及16.〕

17. 餘弦定律



〔証〕 $DA = c\cos A$

$CD = a\cos C$

$\therefore b = a\cos C + c\cos A$

同理亦得： $a = c\cos B + b\cos C$

$c = a\cos B + b\cos A$

由 $a = b\cos C + c\cos B$ 得

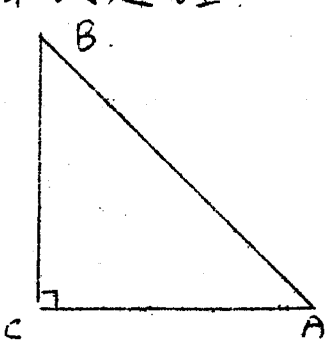
$a^2 = ab\cos C + ac\cos B$ ------(1)

同理 $b^2 = ab\cos C + bc\cos A$ ------(2)

$c^2 = ac\cos B + bc\cos A$ ------(3)

(1)−(2)−(3) 得 $a^2 = b^2 + c^2 - 2bc\cos A$

18. 畢氏定理



〔証〕：由餘弦定律

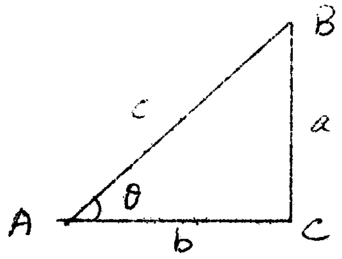
$c^2 = a^2 + b^2 - 2ab\cos C$

當 $\angle C = 90^\circ$

$c^2 = a^2 + b^2 - 2ab\cos 90^\circ$

$c^2 = a^2 + b^2$

19. $\sin^2\theta + \cos^2\theta = 1$



[証]: $\sin^2\theta + \cos^2\theta$

$$= \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$$

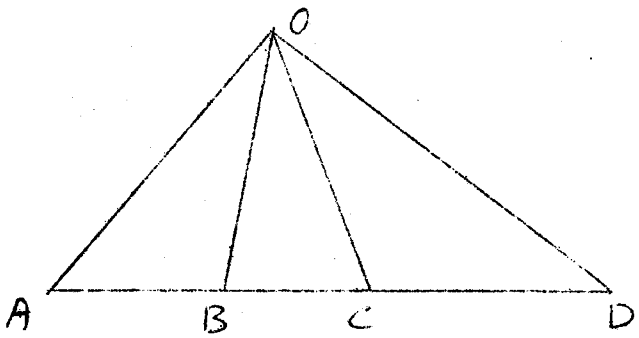
$$= \frac{a^2+b^2}{c^2}$$

$$= \frac{c^2}{c^2}$$

$$= 1$$

20. 自線外一点 O 引四直線至線上 A, B, C, D 四点，則

$$\frac{AC \cdot BD}{AD \cdot BC} = \frac{\sin AOC \cdot \sin BOD}{\sin AOD \cdot \sin BOC}.$$



[証]：因 △OAC、△OAD，△OBD，△OBC 等高

$$\frac{AC \cdot BD}{AD \cdot BC} = \frac{AC}{AD} \cdot \frac{BD}{BC}$$

$$= \frac{\triangle OAC}{\triangle OAD} \cdot \frac{\triangle OBD}{\triangle OBC}$$

$$= \frac{\frac{1}{2} OA \cdot OC \sin\angle AOC}{\frac{1}{2} OA \cdot OD \sin\angle AOD} \cdot \frac{\frac{1}{2} OB \cdot OD \sin\angle BOD}{\frac{1}{2} OB \cdot OC \sin BOC}.$$

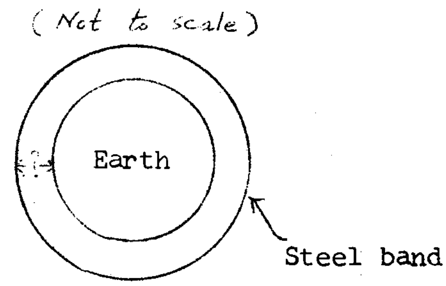
An Imaginary Activity Lesson in Lower Forms

K.T. Wong

Mathematics Section, E.D.

"Class, I am going to ask you an interesting question. Could you choose the correct answer and give your reasons to support your choice?

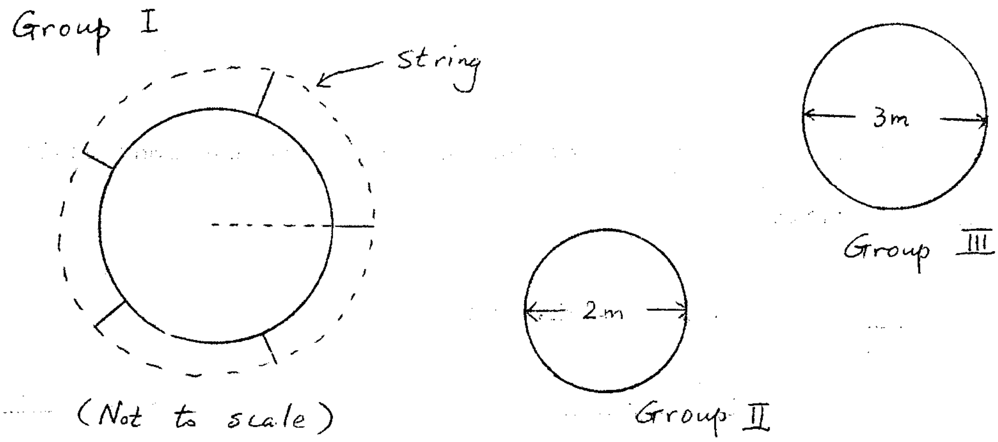
"Suppose the Earth were a perfect sphere of radius 6376 km and that it were possible to put a steel band around the equator; but somebody made a mistake, and the band was fabricated 2 m too long. If the excess were equally distributed as shown in the diagram, how high above the surface of the Earth would the band have to be suspended ? (A) Could you crawl under it? (B) Could a snake wriggle under it ? (C) Could you pass a sheet of paper under it ?"



Most pupils answered "No" to all these questions, some chose (C) but nobody chose (A), thinking that 2 m distributed over 40,000 km ($2 \times \pi \times 6376$) would have a negligible effect on the position of the band, which would still lie practically flat against the surface.

"Now let's stop arguing and do a simple experiment and see whether we can find the correct answer from the results obtained."

The class were then divided into groups equipped with metre rules and strings, and went to the playground. On the playground there were some pre-drawn circles of different diameters, say from 1 m to 5 m. Each group was asked to use a string of length 2 m longer than the circumference of the circle allocated to them to make a larger concentric circle. (not



exactly a circle). The gap between these two concentric circles was measured along the radius in five different positions and average of the readings was taken. When the class returned to their classroom and reported their findings, surprisingly the answers given by the groups were almost the same, i.e. about 32 cm, in spite of the different sizes of the circles they had dealt with.

At this stage, some pupils drew this conclusion : "No matter how large or how small the circle is, the gap will be the same. As the steel band around the equator of the Earth was fabricated 2 m too long, the case is similar to the experiment we have just done. Therefore the gap should also be about 32 cm and the correct answer to your question should be (A), i.e. I can crawl under it."

"We'd better prove it" said the teacher.

"Let r be the radius of the Earth, R be the radius of the circular steel band. From $C = 2\pi r$ where C is the circumference

we have $r = \dfrac{C}{2\pi}$

$$\therefore \quad R = \frac{C+2}{2\pi} = \frac{C}{2\pi} + \frac{2}{2\pi} = r + \frac{1}{\pi}$$

∴ The height above the surface of the Earth the band would have to be suspended should be

$$\frac{1}{\pi} \text{ m} = 0.318 \text{ m} = 31.8 \text{ cm}$$

Can you notice anything interesting from the answer $\frac{1}{\pi}$?"

"The length of the equator C has disappeared."

"Yes, it means the radius is immaterial. The excess length 2 m is the only factor which affects the height. This also explains the fact that you got similar results from different circles."

As an extension to this lesson, the teacher can introduce some knowledge about π such as

(1) Early Chinese achievement in the study of π,

e.g. Chou - pei' Suan-King (周髀算經) regarded π as 3 (" 方徑一周四，圓徑一周三 ")

Tsu Chúng-chih (祖冲之), 429 - 500 A.D., is the first man in this world to obtain such a precise value of π by pointing out that

$$3.1415926 < \pi < 3.1415927$$

(2) Some curious approximations of the value of π,

e.g. $\frac{22}{7}$, $\frac{355}{113}$, $\sqrt{10}$ etc.

All these can be easily memorized. (From 1, 1, 3, 3, 5, 5 to $\frac{355}{113}$)

(3) Mnemonics to assist the recall of a good approximation to π,

e.g. " See I have a rhyme assisting

3 1 4 1 5 9

My feeble brain its tasks oft-times resisting."

2 6 5 3 5 8 9

(4) Latest development in the task of finding the value of π by using an advanced computer.

A Fortran Program for Primes Less than 36100

JOSEPH SHIN

Mathematics Section, E.D.

This program constructs a table of primes, up to 36100, by a procedure similiar to the 'sieve of Eratosthenes'. It is known that if $n \leqslant N$, and n is not prime, then n must be divisible by a prime not greater than $\sqrt{N}$. We now write down the numbers

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ..., 36100

and strike out successively:

(i) 1

(ii) 4, 6, 8, 10, ... (i.e. all even numbers starting from 4)

(iii) 6,9, 12, 15, ... (i.e. all multiples of 3 staring from 6)

(iv) 10, 15, 20, 25, ..(i.e. all multiples of 5 starting from 10)

⋮

We continue the process until all multiples of 189 starting from 378 are struck out. The numbers which remain are primes.

```
      PROGRAM FOR PRIMES LESS THAN 36100
      DIMENSION IPRIME (36100)
      DO 5 J = 1, 36100
 5    IPRIME (J) = J
      DO 35 J = 4, 36100, 2
 35   IPRIME (J) = 0
      DO 25 K = 3, 190, 2
      L = 2 * K
      DO 15 J = L, 36100, K
 15   IPRIME (J) = 0
 25   CONTINUE
      N = 0
      DO 45 I = 1,36100
      IF (IPRIME (I). LE.1) GO TO 45
      N = N + 1
      IPRIME (N) = IPRIME (I)
 45   CONTINUE
      WRITE (6,70) (IPRIME(I), I = 1, N)
 70   FORMAT (1H  , 20I6)
      STOP
      END
```

Remark

(1) COMPILE TIME = 2 SEC
(2) CPU TIME = 52.16 SEC
(3) ARRAY AREA = 144400 BYTES

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Official journal of the School Science & Maths. Assn. Inc., published monthly, October through May at

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Letters to the Editor

Dear Sir,

It was pleasure in reading your recent School Mathematics Newsletter. As a mathematics teacher, I found the articles useful and inspiring (especially those in the Classroom Notes). It would be convenient to keep each Newsletter to oneself for frequent review. Some of my colleagues share the same desire with me. I wonder, therefore, if we can obtain Newsletters of our own. It would be just fair, of course, that we should pay for the paper and printing.

I hope that I have not cause you too much trouble.

Yours faithfully,

(Tsang Mak Yuet Kwai)
Queen's College

Dear Sir,

Your Newsletter contains some interesting articles besides news. The present practice of sending one copy of it to a school means that most people will not have enough time to go over it carefully, and much of the effort that go into its production will be wasted. Is it possible for individual mathematicians to subscribe to the magazine ?

Yours faithfully,

(Robert Shin)
Kwun Tong Govt. Sec. Tech. School

17. Some summation formulae for binomial coefficients

a) $$\sum_{s=0} \left(-\tfrac{1}{4}\right)^s \cdot \binom{n-s}{s} = (n+1)/2^n$$

b) $$\sum_{s=0} t^s \cdot \binom{n-s}{s} = \frac{1}{x-y}\left[x^{n+1} - y^{n+1}\right]$$

Where $x = \frac{1}{2}\left[1 + (1+4t)^{1/2}\right]$ and $y = \frac{1}{2}\left[1 - (1+4t)^{\frac{1}{2}}\right]$

(Vajda, Mathmeatical Gazette)

18. A graphical representation of quadratic equations

Let the equation be

$$x^2 + a x + b = 0$$

Take rectangular cartesian co-ordinates plotting a against b so that every point in the plane represents an equation with real coefficients. The locus of all equations having equal roots is the parabola

$$a^2 = 4b \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; (*)$$

All points inside the parabola (*) represent equations with complex roots and all points outside (*) represent equations with real roots.

(Anthony Bayes, Mathematical Gazette)

19. On $\sum_1^n r^3 = \left(\sum_1^n r\right)^2$

Let $S(0) = 0$
$S(1) = 0$
$S(r)$ = the sum of all possible products formed two at a time from the first r integers $(r \geq 2)$

Then $$\sum_1^n r^2 + 2\,S(n) = \left(\sum_1^n r\right)^2$$

and
$$S(1) - S(0) = \tfrac{1}{2}(1^3 - 1^2)$$
$$S(2) - S(1) = \tfrac{1}{2}(2^3 - 2^2)$$
$$S(3) - S(2) = \tfrac{1}{2}(3^3 - 3^2)$$
+) $$S(n) - S(n-1) = \tfrac{1}{2}(n^3 - n^2)$$

$$S(n) = \tfrac{1}{2}\left(\sum_1^n r^3 - \sum_1^n r^2\right)$$

Thus $$\sum_1^n r^3 = \sum_1^n r^2 + 2S(n)$$

Hence $$\sum_1^n r^3 = \left(\sum_1^n r\right)^2$$

(Roger F. Wheeler, Mathematics Gazette)

20. A triangle inequality

ABC is a triangle. If O is a point on the triangle such that the distances of O from A,B,C are X, Y, Z whilst the perpendicular distances of O from the sides BC, CA, AB are x, y, z respectively, then

$$X + Y + Z \geqslant 2(x+y+z)$$

(P. Erdös, Aufgabe 7 Mathematikai Lapok)

21. A formula for π

$$\pi = 32 \tan^{-1}\left(\frac{1}{10}\right) - 16 \tan^{-1}\left(\frac{1}{515}\right) - 4 \tan^{-1}\left(\frac{1}{239}\right)$$

(G.F. Freeman, Mathematical Gazette)

22. A relation between progressions

If an arithmetical progression a (1), a(2), a (3), ... and a geometrical progression b(1), b(2), b(3), ... satisfy the conditions

$$a(1) = b(1),\ a(2) = b(2),\ a(1) \neq a(2) \text{ and } a(1)\cdot a(2) > 0$$

then for n = 3, 4, 5, ...

$$a(n) < b(n) \qquad \text{if } a(1) > 0$$
$$a(n) > b(n) \qquad \text{if } a(1) < 0$$

(D. Djokovic, Mathematical Gazette)

23. Magic squares In any 4 x 4 magic square, if T is the total of the numbers in each row, column and diagonal, then the numbers in the four corner cells always give a total T.

(D.B. Eperson, Mathematical Gazette)

24. Two trigonometrical inequalities

For $0 \leqslant x \leqslant \pi$

$$\frac{2}{\pi^2} \leqslant \frac{1 - \cos x}{x^2} \leqslant \frac{1}{2} \quad \text{and}$$

$$\frac{1}{\pi^2} \leqslant \frac{x - \sin x}{x^3} \leqslant \frac{1}{6}$$

25. "Proof" of the remainder theorem !!

Divide f (x) by x - a :

$$\begin{array}{r|l} & f \\ \hline x - a & f\,(x) \\ & f(x) - f(a) \\ \hline & \qquad f(a) \end{array}$$

(T.M. MacRobert)

26. Applications of the inequality of the means to prove $(1 + 1/n)^n$ is an increasing function of n

$$\left\{\left(1+\frac{1}{n}\right)^n\right\}^{1/(n+1)} < \left\{1 + \left(1+\frac{1}{n}\right) + \left(1+\frac{1}{n}\right) + \ldots + \left(1+\frac{1}{n}\right)\right\} \Big/ (n+1) = 1 + \frac{1}{n+1}$$

$$\therefore \left(1+\frac{1}{n}\right)^n \qquad \left(1+\frac{1}{n+1}\right)^{n+1}$$

(J.St. - C.L. Sinnadura, Mathematical Gazette)

27. Runs of squares

$$3^2 + 4^2 = 5^2$$

$$10^2 + 11^2 + 12^2 = 13^2 + 14^2 \quad \ldots\ldots$$

$$21^2 + 22^2 + 23^2 + 24^2 = 25^2 + 26^2 + 27^2$$

$$36^2 + 37^2 + 38^2 + 39^2 + 40^2 = 41^2 + 42^2 + 43^2 + 44^2$$

(T.H. Beldon, Mathematics Gazette)

28. Solution of $\sqrt{x+7} + \sqrt{x-1} = 2$ ---------- (1)

Since $(x + 7) - (x - 1) = 8$, then

$$\frac{x+7 - (x-1)}{\sqrt{x+7} + \sqrt{x-1}} = \frac{8}{2}$$

giving

$$\sqrt{x+7} - \sqrt{x-1} = 4 \text{ ---------- } (2)$$

$$(1)+(2) \qquad 2\sqrt{x+7} = 6$$

$$\therefore x = 2.$$

By substitution, 2 is not a root of (1). Hence equation (1) has no solution.

(Richard Beetham, Mathematical Gazette)

29. <u>The majic of squares</u>

$$5^2 + 15^2 + 25^2 + 35^2 + 45^2 + 66^2 + 76^2 + 86^2 + 96^2 + 106^2 + 116^2$$

$$= 6^2 + 16^2 + 26^2 + 36^2 + 46^2 + 56^2 + 77^2 + 87^2 + 97^2 + 107^2 + 117^2$$

30. <u>$\text{Sin } x \geqslant x - x^3/3!$ for $x \geqslant o$</u>

We restrict our consideration for $x \geqslant o$ only

Let $f(x) = \text{Sin } x - x + x^3/3!$ (so that $f(o) = 0$)

Then $f'(x) = \cos x - 1 + x^2/2$, $f'(o) = 0$, and

$$f''(x) = -\text{ Sin } x + x,$$

Since $-\sin x + x \geqslant 0$, so that $f''(x) \geqslant o$ which menas that $f'(x)$ is an increasing function. Thus

$f'(x) \geqslant f'(o) = 0$ which means that $f(x)$ is an increasing function.

$\therefore \text{ Sin } x - x + x^3/3! \geqslant 0$

i.e. $\text{Sin } x \geqslant x - x^3/3!$

31. <u>$\text{Log } x \leqslant x-1$ for $x > o$</u>

May be proved by differential calculus.

32. <u>A.M. $\geqslant$ G.M.</u>

Let $x_1, x_2 \ldots\ldots, x_n$ be n positive numbers and let

$$A = (x_1 + x_2 + \ldots\ldots x_n) / n$$

Now

$$\log \frac{x_1}{A} \leqslant \frac{x_1}{A} - 1 \qquad \text{(by classroom note 31)}$$

$$\log \frac{x_2}{A} \leqslant \frac{x_2}{A} - 1$$

$$\vdots$$

$$+) \quad \log \frac{x_n}{A} \leqslant \frac{x_n}{A} - 1$$

$$\log \frac{x_1 \; x_2 \; \ldots\ldots \; x_n}{A^n} \leqslant 0$$

Thus $A^n \geqslant x_1 \; x_2 \ldots\ldots \; x_n$

i.e. $A \geqslant \sqrt[n]{x_1 \; x_2 \ldots\ldots \; x_n}$

13. C is a point on a quadrant arc AB, centre O, and E is a point on the arc AC. The perpendicular from E to OB meets OC at F; D is the foot of the perpendicular from C to OB and H is the foot of the perpendicular from E to OA. If HF bisects OD at G, show that E trisects AC.

(Archibald J. Finlay, Mathematical Gazette)

14. Arrange the digits from 1 to 9, to form a 3 x 3 determinant giving the greatest possible value.

(Sinclair Grant, Mathematical Gazette)

(ANS : 412)

15. If a graph of a cubic function $y=f(x)$ meets the x-axis in the points (a,o), (b,o), (c,o), then the tangent at the point

$$\left(\frac{a+b}{2},\ f\left(\frac{a+b}{2}\right)\right) \quad \text{passes through } (c,o)$$

(T. Nakazawa, Mathematical Gazette)

16. The formula $\frac{1}{2}n(n-3)$ for the number of diagonals of a convex polygon with n sides is well known. Assume that all the intersections are distinct, at how many points inside the polygon do these diagonals intersect?

(ANS : $n(n-1)(n-2)(n-3)/24$)

(W.R.S. North, Mathematical Gazette)

17. If $a(1)$, $a(2)$, $a(3)$, ... and $b(1)$, $b(2)$, $b(3)$, ... are sequences of positive numbers, we write

$$\{a(n)\} < \{b(n)\}$$

to mean

there exists a number N such that $a(n) < b(n)$ whenever $n > N$.

Which of the following are true?

(i) If $\{a(n)\} < \{b(n)\}$ and $\{b(n)\} < \{c(n)\}$ then $\{a(n)\} < \{c(n)\}$

(ii) If it is not the case that $\{a(n)\} < \{b(n)\}$ then $\{b(n)\} < \{a(n)\}$

Justify your assertions with proofs or counter-examples.

18. For $x > 0$, prove that

$$x/(1+x) < \ln(1+x) < x$$

19. $f(x)$ is a real function that satisfies, for all x, y

$$f(x+y) + f(x-y) = 2f(x)f(y).$$

Prove that either $f(0)=0$, or $f(0)=1$ and $f'(0)=0$.

20. If n is a positive integer, prove that the final digit of its square cannot be 2,3,7 or 8. Prove further that the final digit of the sum of the squares of the first n integers cannot be 2, 3, 7 or 8.

21. Let

$$a(n) = \frac{1}{2\sqrt{2}}\left\{(1+\sqrt{2})^n - (1-\sqrt{2})^n\right\}$$

Establish a linear relationship between $a(n)$, $a(n+1)$ and $a(n+2)$, and deduce that $a(n)$ is an integer for all positive integers n.

22. Suppose f is a twice differentiable function with $f''(x) < 0$ for all $x > 0$. Show that

(i) $f\left(\frac{x_1+x_2}{2}\right) > (f(x_1)+f(x_2))\cdot\frac{1}{2} \qquad (x_1, x_2 > 0)$

(ii) If $0 < a < b$ then

$f(ta+(1-t)b) \geq tf(a)+(1-t)f(b)$ for all $0 \leq t \leq 1$.

23. The number 1649 of four digits has the property that the numbers

16, 64, 49

formed by pairs of successive digits from 1649 are all squares. Determine all numbers (of 3, 4, 5, ... digits) with this property.

24. There are 10 stacks of coins, each consisting of 10 half-dollars. One entire stack is counterfeit. Assuming that you do know the weight of a genuine half-dollar and that each counterfeit coin weighs one gramme more than it should. It is known that one can identify the counterfeit stack by a single weighing of coins on a pointer scale. Do you know how?

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