SCHOOL MATHEMATICS NEWSLETTER

November 1984

6

Mathematics Section, Advisory Inspectorate
Education Department, Hong Kong

Please ensure that every member of your mathematics staff has an opportunity to read this Newsletter.

The views expressed in the articles in this Newsletter are not necessarily those of the Education Department, Hong Kong.

Theme of cover design: The School Mathematics Newsletter will be blank without your contributions.

IN THIS ISSUE

FOREWORD

The contents of this sixth issue of the School Mathematics Newsletter (SMN) include articles on the teaching and learning of mathematics, problem solving, computer studies, and a report on the first Hong Kong Mathematics Olympiad. I hope that you will find this issue both informative and useful.

You may be aware that a new primary mathematics syllabus has been issued and will be implemented in 1985. You may also have heard about the amalgamation of the two existing secondary mathematics syllabuses (syllabus A and syllabus B). These are some of the notable features in the recent development in the teaching of school mathematics. Views and suggestions with reference to such development are welcome. Articles on other aspects of mathematics education are, of course, equally welcome. Please send your contributions directly to the Mathematics Section, Advisory Inspectorate, Education Department, Lee Gardens, Hysan Avenue, Hong Kong. The success of the SMN will not be achieved without your valuable support.

I would like to thank all those who have contributed to this issue of the SMN and to thank my colleagues who have helped in producing it.

C. P. Poon
Principal Inspector (Mathematics)

談談中學數學輔導教學

直心

輔導教學，從一九八二年九月開始試行，一般反應良好。中學方面，由一九八三年九月起，除中、英文兩科繼續施行外，更可推廣至其他科目。因此，許多數學科老師都積極研究是否有需要進行數學輔導教學及探討一些可行的辦法和應注意事項，部份學校更率先試辦，據說困難不大，但人手所限，只可有一兩班數學輔導教學班。筆者有機會跟一些熱衷輔導教學的老師接觸，現將他們的意見綜合，稍加整理，分述如下，供各位參考：

㈠ 目的：主要是爲數學科成績較同班同學落後的學生，提供額外的輔導，俾能追及（有人認爲太奢望）或追近一般水準，最低限度亦能增加學習數學的興趣，加強學習信心及改進自學能力。

㈡ 班級：教育署規定只限於中一至中三；中一優先，其次中二。

㈢ 人數：每組不超過20人，可按時（三個半月或半學年）更換部分或全部輔導生。

㈣ 對象：有些教師主張挑選成績最差的同學，但有更多教師認爲輔導下列三類學生，成效較大：

1. 資質不差，但學習態度不認眞的，
2. 資質較差，但認眞學習的，
3. 只有部份弱點或根基不穩，但肯學肯做的。

(五) 教師：最好有多年教數學經驗，具備三心 — 愛心、耐心和細心。在試辦期間，科主任若能親身試教，體驗其中苦樂，則對策劃將來的輔導教學，肯定有幫助。

(六) 場地：除普通課室外，更可利用下列特別室，但要注意安全措施：

美術、音樂、地理、生物、化學、物理、綜合科學、木工、金工、家政、縫紉、圖書館等。

此外，必要時，雨天操場、禮堂及禮堂入口附近空地，亦可利用。

(七) 模式：可能須要與中、英兩科的輔導教學配合。

(甲) 課內全時間整班輔導 — 先將輔導生編在同一班，然後再按程度或不同弱點，分成兩組，由兩位教師分別輔導。這模式的好處是編排時間、教材、進度等，都較容易，且不影響其他各班，困難在場地不足，因為每一堂數學都是輔導課的話，每星期便要佔用特別室最少五堂時間。同時，更換輔導生時，亦會帶來行政上的麻煩。

(乙) 課內全時間抽調學生輔導 — 全級各班在同一時間上數學課，（若班數太多，例如中一共有八班，可以分成兩批，A，B，C，D 四班同一時間上數學，E，F，G，H 四班則在另一時間上。這樣，在編時間表及調動教師方面，都比較容易。）從每班中抽出數個學生，加以輔導。好處在容易更換輔導生，不會集中輔導某一批學生，所以比較公平。壞處在輔導教師難與原任教師聯絡，因為可能多至四位或甚至五、六位，且各班課程編排進度不同，難與其他同學配合。

(丙)課內部分時間整班輔導 — 先將輔導生集中在同一班，每周抽出兩堂時間進行輔導教學，在這兩堂內，該班學生按程度或不同弱點，分成兩組，原任教師負責其中一組，另一位教同級數學的教師則負責另一組，兩人須緊密合作。這班的課程及教學法，不單在輔導堂，甚至在其餘各堂，都應該和其他各班不同。這模式實在是從模式甲演變出來，但減輕了場地不足和教師不足的壓力，因爲一九八三年九月所加的一位教師，可能只可以撥出一小部分時間來協助數學科的輔導教學。雖然這模式的成效，肯定不及模式甲，但總比較沒有輔導教學好，在有限的資源下，不失爲一折衷辦法。

(丁)課內部分時間抽調學生輔導 — 大致與模式乙相同，但每周只抽兩堂出來進行輔導教學，所以在編時間表及調動教師上，比較模式乙容易得多。

(戊)課餘加時輔導 — 每周兩次，每次約一小時。好處在靈活性大，解決許多行政上的困難，但必須獲得家長同意及分區教育主任批准。如果沒有好好地找出學生的弱點或沒有周詳的輔導計劃，便和由來已久的補課，沒有多大分別。

(八)方法：(甲)針對弱點輔導 — 輔導教學的英文名稱是'remedial teaching'，其中'remedial'一字，便有「對症下藥」的意思。較落後的學生，在弱點改善後，應該可以追近其他同學的水準。若採用這方法，就必須有完善的計劃及準備，甚至要自編教材。

(乙)加強個別指導 — 由於每組人數最多是二十，所以能夠個別照顧，再配合精簡教材，提供更多堂上練習機會，發覺錯誤，立即糾正。

(丙)複教 — 輔導教師將主要內容，提綱挈領地再講解一次，然後在學生做練習時，個別指導。此法只適用於模式戊。

(九)挑選學生：除根據考試或測驗成績外，更可以自編鑑辨測驗。教育署教育研究處所編寫的小五、小六數學科測驗(Attainment Test)，極具參考價值。此外，現任及上任教師對學生上課時／堂課／家課的觀察，以及與個別學生交談(interview)所得的印象，都有助於挑選。至於部分學校編某一班爲輔導班，該班學生同時接受中、英、數三科輔導，這樣編配是否適當，是否公平，有沒有這必要，便見仁見智了。雖然大多數情形是中、英落後的學生，數學成績也比較遜色，但是不患寡而患不均，同一批學生享受三重利益，是否過份呢？我們怎樣去照顧那些中、英不差，惟獨數學不濟的學生呢？值得我們三思。

(十)鑑辨測驗：初期不宜採用多項選擇題，只有對某一弱點再進行詳細分析時，多項選擇題才會有用。反之，做這類測驗時，宜要求學生將演算過程，連同算草，寫在卷內，從中找出其弱點，並加以登記。

進行鑑辨測驗或面談時，應注意下列各點：

1. 同一類型題目的題數要多一些，藉以保證測驗的可靠性。
2. 文字減至最少，除非故意測驗學生對文字題的了解。
3. 沒有時間限制，但應登記完卷時間。
4. 態度要和藹、友善，不要催促學生。
5. 當發覺學生做錯時，不要加以提示或中止他。

6. 鼓勵學生不用擦膠去改錯，應將錯處劃去，再加改正，從而研究他們的思路。

7. 面談時不要問一些帶引性或答案只是「對或不對」的問題。同時，若學生提出「我做得對不對？」一類問題，宜避免正面答覆。

8. 面談時嘗試叫學生一面做一面說出自己的做法或想法。

(十) 弱點登記表：（見附件一）

一般弱點應包括：

1. 根基不穩
2. 概念不清
3. 語文能力低落
4. 缺乏抽象思維能力
5. 計算法則模糊
6. 粗心大意

(十一) 精簡課程：（以中一為例）

1. 對日後學習影響不大的題材，可以刪去。例如：
 - (i) 二進制方面，僅保留認識、與十進制互化及加法。
 - (ii) 尤拉關係式（Euler's formula） $V+F-E=2$ 及利用釘板導出之多邊形面積公式 $(\frac{1}{2}m-1)+n$。
 - (iii) 嵌砌圖形（tesselation）及正多邊形的作圖。
 - (iv) 極座標及綫性方程 $ax+by=c$ 的圖像。

2. 較深入之題材，亦可刪去或押後。例如：
 - (i) 三角形內角和及外角的證明，可以略去。
 - (ii) 百分法的應用，只選擇最基本、最簡單的。
 - (iii)「負負得正」及「負正得負」，不嘗試去證明。

(iv)三角形全等及相似性質，押後至中二、中三的相應單元。

3. 較難或較繁的練習不做，例如：

(i)分數與百分數互化，只選擇那些能化成有限小數的。

(ii)括號的運用，極其量用至兩重括號。有關文字題亦只選擇較淺易的。

(三)對症下藥：例如想幫助學生善用以英文寫的課本及有效地用英文作答，編印一些類似附件二的詞彙對照表，會有幫助。有些老師甚至將它錄音，供學生課餘自修。又例如要針對根基不穩這弱點，就不妨選擇一些小六或甚至小五的練習給學生做。

(四)學生成績低落的其他原因：

(甲)個人因素：
1. 智商低下
2. 注意力不集中
3. 記憶力差
4. 情緒不安
5. 健康欠佳
6. 缺席過多

(乙)環境因素：
1. 家庭
2. 學校
(i)師生關係 — 不喜歡某老師
(ii)教材 — 乏味，過深，過繁，抽象
(iii)教學法 — 未能引導學生主動學習
(iv)學習風氣
3. 社會

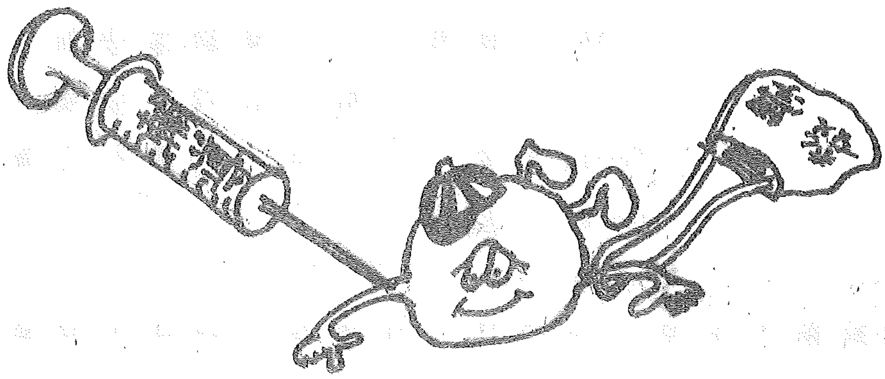
輔導教師若察覺上述原因，就要與其他老師或學生家長聯絡，尋求對策，配合輔導教學，雙管齊下，才會收效。

(十五)注意事項

1. 個別照顧
2. 教材富於伸縮性
3. 使學生有成功感，從而建立信心及培養興趣。
4. 要顧全學生的自尊心
5. 重視教師間的協調
6. 學習步驟：了解→練習→鞏固→記憶
7. 教材或作業目標要明確，範圍集中於某一細節。按部就班，循序漸進。文字簡易，多用插圖。
8. 作業時間要充足，盡快批改。
9. 根據學生的反應及進度，適當地調整輔導工作。

(十六)評鑑及紀錄

1. 教學紀錄
2. 弱點登記表
3. 再次測驗
4. 整體成果 — 根據測驗、觀察及家長報告等
5. 複查表
6. 個別紀錄 — 參閱附錄三



舉例：中一第一次鑑辨測驗（將數目填入格內）。

弱點 ＼ 課程／題數	小數	分數	百分法	比例	圖形	圖像	代數	幾何
概念								
計算								
解題								
步驟								
表達								
其他								

上表可用於個人，亦可用於全班統計，所列項目，視乎需要隨時更改。

其後的鑑辨測驗，範圍應較窄，例如要查出中三輔導生在幾何方面的弱點時，可以用下表：

弱點 ＼ 課程／題數	角	多邊形的角	全等	相似	平行	作圖	解析幾何
定義							
定理或公式							
解題							
計算							
證明							
其他							

有關四則運算及正負數的詞彙（舉例）　　　　附件二

number line	數綫		
sign	正負號		
positive/+ ve	正	+ 5	
negative/- ve	負	- 2	
add/plus	加	Add 5 to 3	3 + 5
	3 + 5 = 8	3 plus 5 is equal to 8	
subtract/minus	減	Subtract 4 from 3	3 - 4
	3 - 4 = -1	3 minus 4 equals negative one	
multiply/times	乘	3 times 4	3 x 4
divide/over	除	3 is divided by 4	3 ÷ 4
		3 over 4	$\frac{3}{4}$
larger than	大於	$4 > 3$	
smaller than	小於	$-4 < -3$	
in descending order	由大至小	$2 > 0.5 > 0 > -1\frac{2}{3} > -4$	
in ascending order	由小至大	-5, -4, 0.7, 1.2	

Arrange the following numbers in descending order.
將下列數目由大至小排列

Divide C by b and add the result to d.
是指　　　　d + c ÷ b

A, B and C have $32, $45 and $26 respectively. Who has the most money? How much money will C take from A so that A and C have the same amount of money? "respectively" 解作「分別是」，用以表示A有$32，B有$45，C有$26，試做這題。

輔導生個人紀錄表　　　　附件三

科目：________ 一九八　年　月　日至一九八　年　月　日

學生姓名：________ 班別：________ 組別：________

輔導教師：________ 原任教師：________ 班主任：________

(甲)學習情況		輔導前	輔導後
(1)平時上課：	正常	(　)	(　)
	普通	(　)	(√)
	精神恍惚	(√)	(　)
	談話玩耍	(　)	(　)
(2)堂上功課：	應付得來	(　)	(　)
	可應付部分	(　)	(√)
	只能抄襲	(√)	(　)
(3)家　課：	正常	(　)	(√)
	隨便應付	(　)	(　)
	抄襲	(　)	(　)
	常不交功課	(√)	(　)

(乙)本科弱點：________

____

____

(丙)輔導項目：________

____

____

㈠輔導方法：______

㈡輔導效果：______

㈢建議事項：(1)可免輔導　（　）
(2)繼續輔導　（　）
(3)轉換輔導項目　（√）
(4)______

㈣備　　註：（舉例：該生曾接受分數四則輔導教學，頗有進步。）

㈤測驗及考試成績：

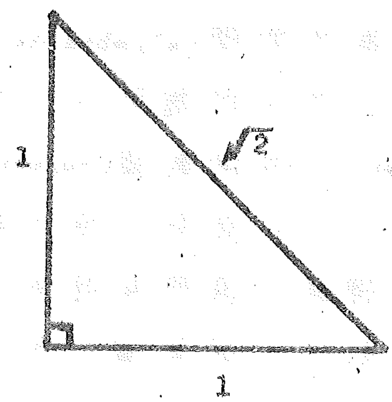
日　期	內　　容	成　績	備　　考
	（舉例：百分法的應用，期中考等）		

非標準微積分

黃毅英
黃棣珊紀念中學

對於古希臘人所謂「數」可能就只局限於「有理數」㈠。比如在畢達歌拉斯學派 (Pythagorean school) 裏便認爲物質是由整數個基本粒子所組成。印度的「勝論師」也認爲任何物質都可碎爲微塵或更細的單位，而物質間的差別只在於其組合方法的不同罷了。這種說法可能與近世物理學有相近之處，但由於這間接否定了精神因素，故在印度曾引起很大的爭論。姑勿論如何，若然物質只從整數個基本粒子組成，由一個物質到另一個物質，只須動用到p：q（p，q爲整數）這一類有理比數罷了。

但在另一方面，經由實數綫的概念和長度的測量使數字與幾何連上關係。我們可用直尺與圓規繪製出$\sqrt{2}$的長度。畢氏學派雖證明出$\sqrt{2}$不是有理數，但不承認它是一個數。而最初發現$\sqrt{2}$是無理數的希帕蘇斯 (Hippasus C5 B.C.) 還被指宣揚邪見。到希氏沉船海上也被稱因此而遭天譴㈡。這個便是後世稱作的數學第一個危機。（詳見〔1〕及〔2〕）



就這樣不了了之。到十九世紀對實數的理論仍十分含糊，當林德曼(Lindemann 1852-1939) 證明了π是超越數，導出第三

幾何問題（能否製一正方形其面積與一指定圓相等）不可解時（1882），克羅內克(Kronecker 1823-1891)仍寫信質問其為何研究此一問題，因為無理數（是故超越數）根本不存在㈢。其所謂不存在是指不能（有限地）從自然數構造出來。經過戴德金(Dedekind 1831-1916)（其「戴德金割 Dedekind cut）與康托爾(Cantor 1845-1918)（由集論建立區間套 nested interval）等的努力總算將實數系有效地建立起來而實數的性質則得以更深一步的認識。

可是康托爾的集合與超窮數(Transfinite numbers)產生出的詭論(paradox)竟觸起數學界的另一場極為劇烈的爭論，我們稱之為第三次數學危機㈣。

由自然數到有理數的擴展，主要是為了代數上四則的需要，但由有理數到實數，其擴充之目的並非為了使形如 $x^2-2=0$ 之方程有解。若於此方向發展，我們應轉到複數系(Complex number system)那邊。任何複系數方程均有複數解，這「代數基本定理」是高斯(Gauss 1777-1855)的不朽功績。我們叫這性質做代數性封閉(algebraically closed)。至於實數的作用，主要是為了分析上的需要，所得每個「本應收歛」(intrinsically convergent)㈤的數貫(sequence)確實收歛。例如3，3.1，3.14，3.141……就是一個本應收歛的有理數貫，但却不收歛於一個有理數。我們稱此為「完備性」(complete)（詳見〔3〕）。而收歛這問題是屬於數學上的一個無窮程序，我們可以見到 $\frac{1}{n}$ 漸近於0，但却永不等於零，這只是一種無窮趨近。

至於無窮概念，自古便十分模糊。自古希臘哲學家芝諾(Zeon 495 B.C.-435 B.C.)提出有關無窮的問題後㈥，一直沒有得過完滿的解決，直至十七世紀微積分的創立及日後的迅速發展，而導致所謂的第二次數學危機㈦。這危機的主要來源是

極限（以及微分、積分）中無窮少量的迷離意義。比方在

$$L = \lim_{\delta \to 0} \frac{(x+\delta)^2 - x^2}{\delta}$$

中的 δ，我們說由於它 $\neq 0$，所以式子成立且等於

$$\lim_{\delta \to 0} \frac{2x\delta - \delta^2}{\delta} = \lim_{\delta \to 0} (2x - \delta)$$

再旋即以 $\delta = 0$ 而得出 $L = 2x$。無窮大量的處境也是一樣。是故波爾察諾(Bolzano 1781-1848)便著有「無窮的詭論」(The Paradoxes of the Infinite)一書。這到了十九世紀科西(Cauchy 1789-1857)和外爾斯特拉斯(Weierstrass 1815-1897)利用 $\varepsilon - \delta$ 之定義把極限視作一個無窮迫近的進程，才把問題清晰起來。但無論如何，∞ 只代表一個趨向的無限(approaching infinity)而從來不是被納進數系的一個數。

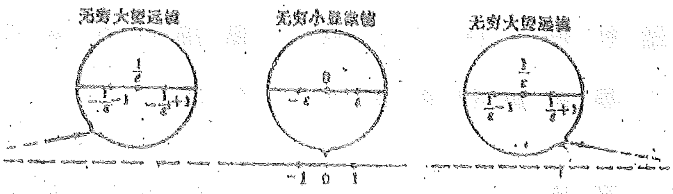
羅素(Russell 1872-1970)曾說「對於先前困擾數學界的無窮概念之解決大抵是我們這年代值得自豪的最偉大成就」(八)。但他們所指的是根據康托爾建立的超窮數理論，對「存在的無窮」(existential infinity；例如自然數的個數等）的解決，至於趨向的無窮，則依舊留作為一個程序。

直至本世紀六十年代，阿伯拉罕．羅便臣(Abraham Robinson 1918-1974)建立其「非亞基米德帶序域論(Non-Archemedian Ordered Field Theory)，才正式把無窮少和無窮大量引入數系當中。所謂帶序域，約言之，便是含有加、減、乘、除，有大小次序的系統，我們熟識的實數系 R 便是一個，但 R 適合如下的亞基米德原則(Archemede's Principle)：

若 $x\in R$，$x>0$，則有另一 $y\in R$ 使得 $0<y<x$。

在羅便臣建立的超實數系（hyperreal number system）R^* 內，雖然其中則與 R 類似，上面原則卻不成立。換言之，存在一大於零之數 ε，少於任何正實數。由於 R^* 是個域而 $\varepsilon\neq 0$，故 $\frac{1}{\varepsilon}$ 便是一無窮大量。同時，我們有 $R\subset R^*$。由這個引入了無窮小與無窮大量數系發展的數學分析，便成為一門嶄新的數學部門，稱之為非標準分析（Non-standard Analysis）（見〔4〕）。

你或會因 ε 之引入而感到不自然，因為它不存在於實數數綫上，但若看深一層，虛數 i 也不是非藏身於實數綫的嗎？〔5〕中介紹了一種觀察 ε 的方法，便是利用「無窮小顯微鏡」和「無窮大望遠鏡」（見圖）。



羅便臣除了能將整套舊有的微積分理論套入新的 R^* 又擺脫了無窮小量的含糊不清外，更利用新的工具解決了以前的一些懸案。不過，我們知道 R^* 除了是一個非亞氏帶序域外，它還得適合一些條件，如「任何於 R 的運算或函數都存在一於 R^* 上之延拓（Canonical extension）」，我們既不知 R^* 的實在結構，這些條件便要用高深的邏輯學去保證了。事實上，我們要用到所謂「高階邏輯」（High Order Logic）去保證 R^* 的存在，所以更遑論在中學裏應用和教授了。

在「非標準分析對中學微積分的冲擊」〔6〕一文中作者便指出如果我們不致要先教高階邏輯之時，非標準分析是很難於現時情況出現於中學數學的了。而且 R^* 既無簡單的模型亦非唯一，故產生種種問題，更使 R^* 含糊不清了。基仕那（Keisler）在〔7〕中試圖用加強 R^* 的條件去將之定形，他在〔3〕

中又作了點改進，但都未理想。〔6〕一文中便認爲後十年的發展都會朝這方向走，建立一個簡單清楚的 R^* 模型，使我們一開始學微積分便從 R^* 學起，放棄了以前 $\varepsilon-\delta$ 的煩瑣概念。

到「微積分即代數」〔8〕一文中，作者便逐步從 R 建立起 R^*。文中更聲稱此應爲日後教授微積分的正確方向。不過其中也少不免地利用了有限支集 (Finite support)，理想(Ideals) 及曹恩引理 (Zorn's lemma) 等工具。

不過，我們知道，以往我們會用兩種方法建立數系。在嚴謹地定奪其數學存在性時，我們可逐步從集合論（數學的基礎）建立上去。比如，0 可被定義作 $\varnothing$，1 爲 $\{\varnothing\}$，2 爲 $\{\varnothing, \{\varnothing\}\}$ 等，$Z = Z^+ \cup Z^- \cup \{0\}$，Q 是 Z 的商域(Field of quotient)，R 爲 Q 內本應收斂數貫的集的某一商集等等（詳見〔9〕）。另一方面，在研究其性質時，我們可把數系視作規限於某些公設(Axioms)的集合。例如，對於自然數，我們有皮亞諾(Peano 1858-1932)的五個公設，有理數則有帶序域的公設等。希爾伯特(Hilbert 1962-1943)便曾定出實數系的公設〔10〕，我們大可「依樣畫葫蘆」，只從 R^* 中抽出決定性的性質作爲定義，這樣 R^* 變得直觀親切，既避去了 $\varepsilon-\delta$ 之煩複，也免了一大堆邏輯的纏繞，這可能是非標準微積分打進中學數學教育的第一步。

在「非標準微積分」〔11〕一書中，超實數是適合以下公設的數系：

(1) 超實數是個域 (Field)，

(2) 它有大小關係 $<$，適合

(i) $a<b,\ b<c \Rightarrow a<c$

(ii) $a<b,\ a=b,\ a>b$ 三項只有且必有一項成立

(iii) $a > b,\ c > d \Rightarrow a+c > b+d$ 若加上 $\lambda > 0$，則 $\lambda a > \lambda b$，
若 $\lambda < 0$，則 $\lambda a < \lambda b$

(iv)若 $a > 0$，$n \in N$，則存在 $b > 0$，使 $b^n = a$。

(3) 存在一正無窮少量 ε，即 $\varepsilon > 0$，且對於任何 $\lambda \in R$, $\varepsilon < \lambda$
故無窮少量非只一個，2ε，3ε 等均是。如前所述，亦有負無窮小量及正負無窮大量。

$x - y$ 爲無窮小量記作 $x \approx y$，

(4) 對於有限的 $x \in R^*$，必有唯一之 $k \in R$ 使得 $k \approx x$，叫作 x 之標準部份 (standard part)，記作 st(x)。現在每個有限數都等於 $r + \varepsilon$，$r \in R$，ε 是無窮小量。

(5) 對每一個實函數 f，都有一個對應的超實函數，叫做 f 的自然延拓。

有了這些基本知識，我們再不需要 ε - δ。以前的

$\lim_{\delta \to 0} \frac{(x+\delta)^2 - x^2}{\delta} = 2x$ 以今日的語言，即 $st\left(\frac{(x+\delta)^2 - x^2}{\delta}\right) = 2x$

此處 δ 爲無窮小量，用關於st之代數運算即容易算妥。$f'(x_0)$ 亦變作

$$st\left(\frac{f(x_0+\delta) - f(x_0)}{\delta}\right)$$

而 $\int_a^b f(x)dx = st\left(\sum_a^b f(x)dx\right)$

其中 δ，dx 皆是無窮小量。

「非標準微積分」一書便這麼自然地將整個微積分用新的理論講述出來，不需要任何關於微積分的基礎知識；當然

囉，它是取代舊有 $\varepsilon-\delta$ 而講述微積分的教科書。講不定，幾年後，所有中學都轉用了非標準微積分哩！

不過，現時大部份的研究均集中於利用 R^* 研究 R 本身的函數，對於 R^* 本身的結構却未有太深入的認識，我們只知 R^* 既非完備(Complete)亦非亞氏(Archemedean)，但例如，如何使之變成拓撲空間呢？又可否拓展到 C^* $(=\{a+bi, a, b\in R^*\})$，而 C^* 是否代數性封閉呢？這些筆者都未及搜集，唯望資深者相告。

此文之成，實有賴漢君提供資料，於此致謝。

≪ 備 註 ≫

(一) 有認為 Rational 非指「有理」，是 Ratio - n - al 之誤傳。見項武義之「數學分析」。

(二) 有認為希氏是被畢派的人推下海的，見[2]。

(三) "Of what use is your beautiful investigations regarding π? Why study such problem? Since irrational numbers do not exist."

(四)「此句子是假的」便是一些類似的詭論。

(五) 即科西級列 Cauchy Sequence，若 $\forall \varepsilon>0$，$\exists N\in \mathbb{N}$，$(n, m>N \Longrightarrow |a_n-a_m|<\varepsilon)$，則 $\{a_n\}$ 為一科西級列。

(六) 其中一問題是。例如一箭射向一米之地，它需要一段時間到達半米之處，又要一段時間到距離¼米之處，如此共需無限段時間，故此箭永不能達至目的。一般地，根本沒有任何移動。

(七) 由於昧於無窮之意義，常引至詭論，例如：

$$0=(1-1)+(1-1)+\dots\dots$$
$$=1-(1-1)-(1-1)-\dots\dots$$
$$=1$$

(八) "The solution of the difficulties which formerly surrounded the mathematical infinite is probably the greatest achievement of which our age has to boast."

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數學・數學史・數學教師

蕭文强博士
香港大學

1 引　　言

本文題目出現的三項事物，都包含「數學」這個詞滙，它們之間顯然有極密切的關係。但似乎在很多人的心目中，這三項事物却沒有什麼關連。爲什麼我會這樣說呢？讓我先解釋一下。這裏的「數學」指對數學的探討，包括學習數學知識、瞭解數學新動態、討論數學問題、以至進行數學研究；「數學史」指對數學發展的認識和學習；「數學教師」自然指課堂上的數學教學了。有些人自己對數學興趣極濃，研究幹得出色，但對教學却不熱心，視作例行公事；對數學史更持輕蔑態度，認爲它與研究無干，只是供數學功力不足的人拿來擺弄的玩意吧。有些人對教學負責，把自己要教的材料準備充足，課堂上應付裕如，但對提高自己的數學修養却不重視，認爲既然自己只是教學而不是搞研究，何需提高；對數學史也不重視，認爲那是「花絮」而不是「正道」的數學，在教學上不能派用場的。所以，對一部份人來說，「數學」、「數學史」、「數學教師」可沒有什麼關連。

一篇短文決不能面面兼顧（本文原是給羅富國教育學院的演講的稿），爲了避免引起誤解，讓我首先聲明將不談什麼，但那不表示我認爲那些不重要。我將不談教學技巧，我只想提出一點：教師需要注意教學技巧，教學技巧是可以訓練的，所以教師需要自覺地訓練教學技巧；而且你一天任教師，你就得一天注意這回事。師範訓練提供這方面的基本知

識，大大地減輕了獨自摸索的苦況，但日後在課堂上的體會也是很重要。我也認爲，教學不單是技巧，更是藝術。要做一位好教師，除教學技巧外，還得注意兩方面，一是個人修養，二是本科學識。前者層次較高，我也不談了，只想引一段美國數學家MOISE的話：「教學這項活動，涉及一種意義十分不明確的人際關係。教師本人是一位表現者、講解員、監工、領頭人、裁判員、導師、權威人物、對話者和朋友。所有這些角色都不易擔當，其中有不少還是互不調和的。因此，要成爲一位老練成熟的教師，個人品格的細緻成長是不可或缺的。」至於本科學識，大抵沒有人懷疑其重要，我覺得奇怪的只是一點：沒有人相信只修畢小學數學課程便可以教小學數學，也沒有人相信只修畢大學數學課程便可以教大學數學，但爲什麼很多人却相信只修畢中學數學課程便可以教中學數學呢？我提出這疑問，並非提議所有中學數學教師必須修畢大學數學課程。反之，那未必是合適的做法。但顯然師範訓練中不能忽視本科的進修，而且那不能僅僅是把該科的大學課程「平移」過來就算了，更需要有的而發地選材。這是個重要的問題，必須集思廣益，全面探討，我也不敢談了。著名數學家教育家POLYA在一篇文章裏複述一位數學教師的妙語：「數學系給我們又厚又韌的牛排，嚼它不動；教育學院給我們淡而無味的清湯，裏面一丁點肉也沒有。」數學教師需要的是味道鮮美、營養豐富的濃郁肉湯！

我想在下文提出來跟大家討論的，中心思想大概是這樣子：我們已經肯定了教師學識的重要，但學識指什麼？我以爲那應包括三方面，即是「才」、「學」和「識」，三者互有關連，也互有區別，但相輔相成。清代文學家袁枚說過：「學如弓弩，才如箭鏃，識以領之，方能中鵠。」我們先討論「才」、「學」、「識」的關係，然後以此爲着眼點，看看「數學」、「數學史」、「數學教師」之間的密切關係。

2. 數學的「才」、「學」、「識」

數學在一般人的心目中佔什麼地位呢？大家都不會否認數學在科學研究、科技發展、社會科學、企業管理上的貢獻，矛盾却在於大家往往只見到這些成就而忘却了數學本身，難怪有人稱數學為「我們那看不見的文化」！而且大多數人或者不瞭解數學是什麼的一回事，或者只捕捉了片面零碎印象使以偏概全。受過普通教育的人，卽使不是藝術家也知道有雕刻、繪畫、……；卽使不是音樂家也知道有歌曲、旋律、……；卽使不是文學家也知道有詩、小說、……；卽使不是科學家也知道有核能、蛋白質、微生物、行星、……。但有多少人知道什麼是函數、公理系統、可換群、流形、……？再者，不少人雖然不高興別人指出他對藝術、音樂、文學、科學一無所知，却不介意別人說他對數學一竅不通，甚至認為不懂數學乃理所當然，說時縱非喜形於色亦必心安理得！你試向一位朋友說：「怎麼你的英文這麼差？」對方面紅耳熱地苦笑承認，而你大有可能從此少了一位朋友！但換了是說：「怎麼你的數學這麼差？」對方面有得色地呵呵笑，邊笑邊說：「是呀，在學校裏我一向最怕數學的，硬是弄它不通。」

為什麼會這樣子？我認為這是我們這群數學教師的「群恥」，「群恥」一天不除，我們的工作一天沒有做好。導致這現象的原因可能有好幾個，我只提我想到的一個吧。數學有它悠久的歷史，當近代物理、化學、生物猶處於發展的初期，數學已經背上了二千多年的輝煌成就，但中小學的數學課程却差不多只學到在這之前的數學！卽使在大學裏，當其他學科正從十九世紀以後的發展推向二十世紀的新發現，大部份學生的數學知識却終結於十九世紀初期！於是，數學漸漸形成它特有的一套語言，使非數學工作者感到難於親近。

同時，數學是一門累積的知識，它的過去將永遠融會於它的現在以至未來當中，加上它的確具有抽象思維的本質，要眞正瞭解它掌握它需要付出一定的時間和努力，並非所有人願意付出這樣的時間和努力（也沒有需要所有人成爲數學家）。由此衍生一個教學上的現象，就是側重了數學的技術性內容，把它作爲一門工具學科來講授。這樣做，教師可以在規定的時間內傳授一定份量的知識，也可以利用表面看來是淸晰條達的手法迅速地教懂學生這套特別的語言。然而，這樣做也掩蓋了數學作爲一門文化活動的面目，難怪很多認爲自己將來毋需使用數學的人覺得數學與已無干，也樂於表示自己跟枯燥的公式和刻板的計算打不上交道了。這使我想起劉徽＜九章算術註原序＞裏的一段話：「雖曰九數其能窮纖入微，探測無方。至於以法相傳，亦猶規矩度量，可得而共，非特難爲也。當今好之者寡，故世雖多通才達學，而未必能綜于此耳。」

所以，一個平衡健全的數學課程應該兼顧幾方面，粗略地可分爲三點：(1)思維訓練；(2)實用知識；(3)文化修養。打開任何一份數學課程綱要，都可以在「教學目的」這一項底下找到這三點。當然，表達方式各有不同，所用字眼亦各有異，但基本精神是一樣的。如果我們不拘小節，但求捕捉箇中精神，一個更抽象更籠統的說法便是在上一節結尾時提到的「才」、「學」、「識」。(1)相應於「才」；(2)相應於「學」；(3)相應於「識」。

「才」指才能，於數學而言，就是計算、推理、分析、綜合的能力，也是洞察力、直觀思維能力、獨立創作力。「學」指與專業有關的知識，都從前人繼承而來，例如勾股定理、二次方程解公式、極限理論、積分計算等等。「識」指對知識分析辨別、融會貫通、梳理出自已的觀點和見解這種能力。才而不學，是謂小慧；有學無識，只是「活動書櫥」；

不學則難以有識，即使有亦流於根底膚淺。所以，三者相輔相成。我們希望自己做到的，更希望我們的學生做到的，就是三者兼之。

三者之中，「才」是最不好討論，因爲雖然計算、推理、分析、綜合的能力還算可以訓練外（但也不易，而且效果難於測量），其餘像洞察力、直觀思維能力、獨立創作力是可培養而非可訓練的。不過，我願意介紹一些適合中學數學教師閱讀的參考資料：

G. POLYA, "How To Solve It", 2nd Edition, Princeton University Press, 1957

G. POLYA, On Learning, Teaching, and Learning Teaching, Amer. Math. Monthly, 70 (1963), 605-619

D. SOLOW, "How To Read And Do Proofs", Wiley, 1981

U. LERON, Structuring Mathematical Proofs, Amer. Math. Monthly, 90 (1983), 174-185

我曾經在兩次給中學生的講演上强調人類思維能力的可貴，奉勸各位同學不要只滿足於現成的解法，不要滿足於一定的程序而不加深究、不願自己動手幹動腦想，以致變得思考遲鈍、思路含糊。在這裏，我想再强調這一點，「才」是需磨練得來的。

其次，「學」的討論一定與教學法有關，我已經說過不談的。但「學」聯繫着「識」却是下面要討論的重點。爲方便起見，不如合起來稱爲「學識」。數學的「學識」可作縱橫看，縱方面就是追溯數學概念和理論的來龍去脈，橫方面就是探討數學的本質和意義。你或者問：「這麼大的題目，跟我的日常教學有關係嗎？數學的本質和意義是哲學上的問題，我只想教數學吧，管它什麼哲學觀點？我只想教懂學生現代數學吧，溯本尋源有何用哉？」

我不否認數學的本質和意義是哲學上的問題，而且每人對這問題有每人不同的見解和體會，我也不是要有一個人人一致的鐵定答案。但我不同意為了上述原因我們便迴避這個問題，我也不相信它對數學教師的工作沒有影響，讓我舉一個例子說明忽視這問題可能帶來的影響。各位對所謂「新數」、「舊數」之爭必已耳熟能詳（請參看以下一份很有份量的文章：

梁鑑添，評論近二十年來中學數學課程改革，

<抖擻>，38（1980），64—75）

很多時我們聽到類似這樣的一些批評，它說「新數」的教材不適合中小學生程度，因為中小學生的認知能力尚未發展到可以接受如此形式化處理的階段。雖然這是實情，但如果僅僅是這樣，反對便顯得乏力了，因為那就像說：「眞正搞數學的人是這樣搞的，可惜你們未達到那個程度，暫時只好改用別的你們可以接受的材料。」那豈非以「次貨」代「眞貨」嗎？而且，言下之意還有「對牛彈琴」之嘆！不是的，我認為數學家根本不是那樣搞數學，所以「新數」的設想從根本卽站不住腳。形成「新數」這種氣候和局面，是很多人（包括設計課程的人、編寫教材的人、教這些課程的人）對數學的本質和意義的信念的一種反映，這種信念是出於對「形式主義」的偏愛和誤解。德國大數學家HILBERT在本世紀初提出「形式主義」，視數學為沒有意義的符號進行沒有意義的紙上遊戲，那純粹是為了企圖解決數學基礎上的相容性難題。這個類似「釜底抽薪」的做法，是為了這個特定目標特意精心設計出來的，却不是說數學就是那樣子的活動。HILBERT本人的話，是最好的註解：「在我們的形式主義遊戲中所出現的公理和可證明的定理，乃是形成通常數學對象的那些概念的映像。」在他著名的<幾何基礎>卷首，他引用了十八世紀德國哲學家KANT的話作題詞：「人類的一切知識，皆始

於直覺，再發展爲概念，終於形成理念。」看看數學發展經過，當然更好明白這一點。我不打算作進一步的討論，但誰能再說數學本質的認識對數學教學沒有影響呢？

至於數學概念和理論的來龍去脈，是否陳年舊蹟？我看不是，因爲認識它的來龍去脈，有助於加深個人對數學的瞭解。通過歷史材料，我們也可以瞭解一個數學分支何時興旺、何時停滯、何時衰退，從中吸取成敗經驗，知道數學發展的規律，培養個人對數學的鑒識力，這些對教學是有幫助的。讓我就着個人經驗舉一個例子，今年我開了一門「代數數論」，即是討論有理數域的有限擴張，但我從數論的歷史談起，以 FERMAT'S LAST THEOREM 爲動機（那即是說 n 大於 2 時，$x^n+y^n=z^n$ 沒有非平凡整數解，問題至今猶懸而未決），引出以後的概念和定理，使學生明白那些抽象的理論紮根於實際問題。這樣做不只使課程較富趣味，更重要的是使它較富啓發。讓我再舉一個例子，就是很多學生視作畏途的 ε-δ 手法。一般書本上的定義使一些初學者看得頭昏腦脹，於是囫圇吞棗，終致消化不良！但如果我們試圖瞭解一下這種手法是怎樣演變來的，便會發覺就連 ε 這個符號也頗有點意思，它代表法文的「erreur」，是誤差的意思。十八世紀的數學家（如LAGRANGE）擅長以逼近法求近似值，譬如求 $f(x)=0$ 的根，自然地他們要估計誤差，譬如說，經過若干次逼近後所得的近似值與眞值相差多少？同樣的手法，到了十九世紀的數學家手中（如CAUCHY），却變成極限理論。他們反過來問，要逼近多少次才保證誤差不超過若干呢？這想法是近代數學分析嚴謹化的起步，也是 $\varepsilon-\delta$ 手法的基本思想。從這個角度看，$\varepsilon-\delta$ 手法只是具體的誤差估計吧，不是那麼高不可攀的。

從以上縱橫兩方面看，數學史明顯地能幫助我們增長學識。不只這樣，歷史還留給我們豐富的材料。如果我們從中

吸取營養，並和以今天的知識，以「事後諸葛亮」的眼光把古今結合起來，還可以在課堂上發揮具體的作用呢。兩年前我給了一個講演，便是專討論這件工作，故不再重複，有興趣的朋友可參看以下兩篇文章：

蕭文強，數學教學上如何古爲今用，<抖擻>，44（1981），70—73

蕭文強，活用數學系，<數學教學季刊>，2（1981），6—9

3. 實際的做法

以上我說明了數學上「才」、「學」、「識」的重要，也舉例說明了數學史對數學教師的用途。如果你接受上面的論點，剩下來應該討論的便是如何在師範訓練中增強對數學史的認識。容許我作一些建議，可行與否或應行與否，留待讀者爭辯討論吧。

我不認爲單單開設一門數學史課程可以達致上述目的，正如我不認爲在中小學獨立地講授數學史是合適的做法。我心目中的數學史，跟數學史家心目中的數學史有些不同，也跟一些人心目中的數學史不同。我心目中的數學史，並非單指數學個別課題之編年史，也並非單指數學家的生平軼事，而是既指數學知識的演變，也指創造這種知識的人、產生這些人和這種知識的客觀條件、還有這種知識的社會作用。我們要追求的是一種「歷史感」，這種「歷史感」不能單從一連串名字、一系列大事年表、一幀幀肖像、或者一頁頁小故事中得到。歷史是在長時間中由事件累積而成，「歷史感」也是在長時間中因學習歷史而由淡至濃，以至濃得與本科混爲一體而不可分。GOETHE曾經說過：「一門科學的歷史就是那門科學本身。」我的信念就是：數學史就是數學本身。所以，最理想的做法，是把師範訓練中的整體數學課程有機地

圍繞着數學史建立起來。至少，讓數學史的精神滲透到課程裏去。我可以提議幾本書適合教育學院的課本：

L.N.H.BUNT, P.S.JONES & J.D.BEDIENT, "The Historical Roots of Elementary Mathematics", Prentice Hall,1976.

H.EVES, "An Introduction To The History Of Mathematics",4th Edition, Holt, Rinehart & Winston, 1976

E.SONDHEIMER & A.ROGERSON,"Numbers and Infinity -- A Historical Account Of Mathematical Concepts", Cambridge University Press, 1981

H.EVES & C.V. NEWSOM, " An Introduction To The Foundations and Foundamental Concepts Of Mathematics", Holt, Rinehart & Winston, 1965

M.KLINE, "Mathematics in Western Culture", Oxford University Press,1953.

李儼，杜石然，＜中國古代數學簡史＞，港版，商務印書館，1976

我也可以提出一些供數學教師參考的「資料的資料」：

Mathematics Appreciation Courses: The Report Of A CUPM Panel (Bibliography & Reference List), Amer. Math. Monthly, 90(1983), C11 - C20

L.Leake, What Every Mathematics Teacher Ought To Read (Seventeen Opinions), Math. Teacher, 65(1972), 637-641.

L.LEAKE, What Every Secondary School Mathematics Teacher Should read -- Twenty-four Opinions, Math. Teacher, 76(1983), 128-133

合起來它們列舉了三百種以上的（英文）書籍文章，使人目不暇給。當然，還有很多資料沒給包括在內，其中一本不在上述名單却十分值得教師閱讀的書就是：

M.KLINE, "Why Johnny Can't Add", Vintage Books, 1973.

至於合適的中文參考資料也有不少，我個人最熟悉的自然是下面兩種：

蕭文强，＜爲什麽要學數學？ — 數學發展史給我們的啓發＞，學生時代出版社，1978

（抖擻文選：數學教學論叢），商務印書館，1981。

我們不可能在這裏詳細討論課程內容，但舉一個例子或者可以把我的設想表達得較爲清楚。幾何向來是課程設計上

的「疙瘩」，不論「新數」、「舊數」都未曾很好地解決這個問題。在六十年代（或之前）唸中學的朋友，一定還記得當時的綜合幾何是多麼困難的一部份，但起初幾課却又似多麼無聊做作。先來一大堆說了有如不說的定義（例如直綫是有長度沒有濶度的東西），然後證明一些看來明顯不過的定理（例如兩直綫相交，對頂角相等）。當學生正困惑於什麼需證明、什麼不需證明之際，形勢却急轉直下，接踵而來的是大批使人不知從何入手的習題，猶其作圖問題與軌跡問題，往往連班上「高手」也給難倒了！六十年代後期「新數」入侵課堂後，處理幾何的方法走向兩個極端，一是加入更多公理使它嚴謹化，一是完全拋却證明而單從直觀角度學習幾何知識。甚至有人認爲綜合幾何根本不應在中學課程佔一席位，不如以解析幾何代替了它。我不知道八十年代中學幾何是什麼樣子，但我從學生的反映，它一定不再是二十多年前我學的一樣了。曾經有位學生告訴我：「看了古希臘數學後，我才知道反證法對幾何也有用，以前在中學我只在代數用它。」另一次我測驗時擬了一道題，問能否用一條不經切割的鐵綫屈成一個正八面體的骨架，用意原在考察學生對圖論的認識，誰料全班四十五人中只有七八個答對，其餘的學生人人皆知運用什麼定理，可惜他們弄不清正八面體是什麼樣子。有人以爲那是正八邊形，也有人以爲那是正八邊形爲底的角錐！我不是要「復古」，但我覺得綜合幾何在教育上仍有其優點，若經適當編排，它是訓練抽象思維和邏輯思維、培養空間想像力的好工具，而且不少愛好數學的朋友一定還記得當年如何對綜合幾何「一見鍾情」！不過，要領略綜合幾何之美，單是「學」恐怕不足，猶需有「識」。特別是教師本人應該知道一點幾何的歷史，才曉得怎樣佈置教材。適合中學的綜合幾何材料，差不多全部在公元前四世紀希臘數學家EUCLID的「ELEMENTS」前六卷找得到。明代徐光啓與意大利傳教士利瑪竇合譯「ELEMENT」亦只譯了前六卷（稱爲<幾何原本>），使後世不少人誤以爲「ELEMENTS」就是幾何課

本。事實上，「ELEMENTS」十三卷包羅不少題材，不單是幾何知識，而其編排處理的手法，更奠下後世數學公理化的基石，開了數學演繹精神的先河。這本書對後世的數學發展影響至大，也是人類思想史上的一個里程碑，難怪徐光啓評曰：「由顯入微，從疑得信。蓋不用爲用，衆用所基。眞可謂萬象之形囿，百家之學海。」關於這本書的性質及編寫目的，衆說紛紜，近年來更出現一些新的觀點和論證，使「定案」變「懸案」，這本珍貴文獻更值得研究了。雖然我們不是要做數學史家，但這樣重要的一本著述顯然是應該認識的。通過這本書的學習，突出重點（卷一不妨細讀），回顧古希臘數學的發展，探討歐氏幾何與非歐幾何的演變，對於增進我們的幾何「學識」是十分有幫助的。

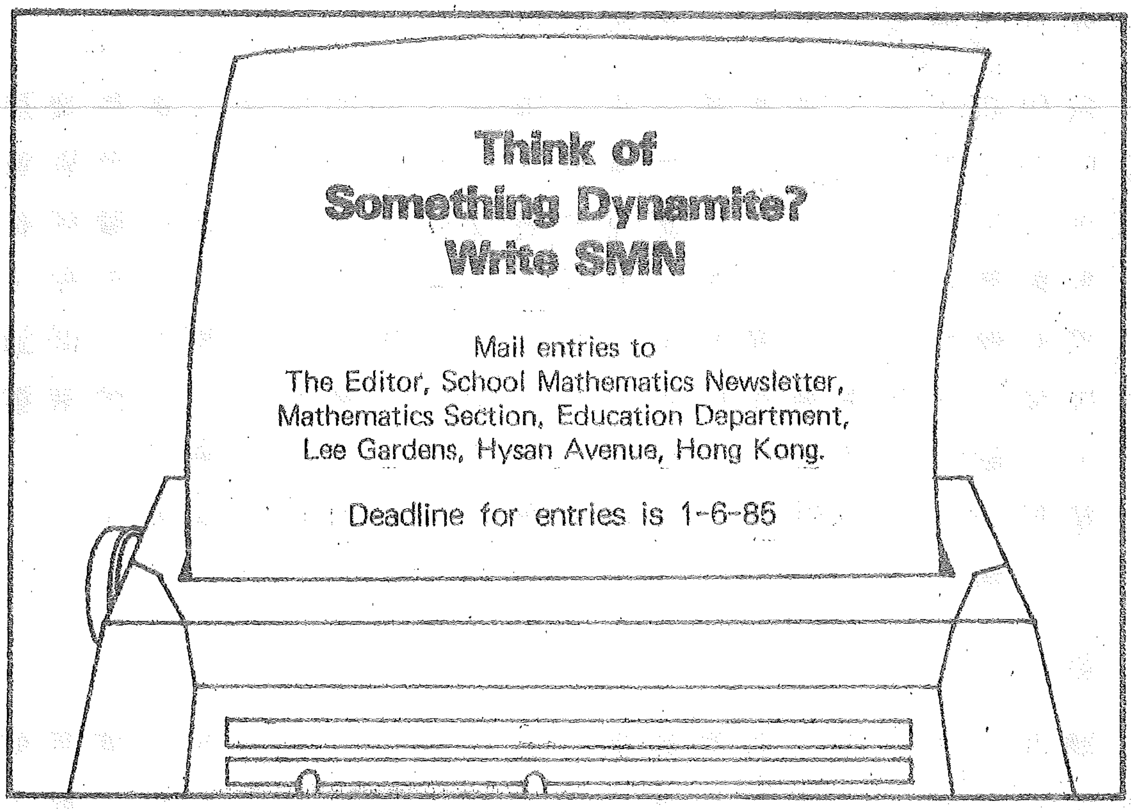
我們在香港大學數學系裏也開設了一門叫做「數學發展史」的課，課的宗旨可分爲兩點：(1)除了使學生明白個別選講課題的發展經過以外，更希望通過這些選講課題的闡述使學生對數學有個整體認識，把它看成是人類文化的一部份，是人類集體智慧的累積結晶，是一門生機蓬勃的學科。(2)通過專題探討，培養學生的獨立探討能力、書寫和口述的表達能力。有關這門課的詳細情形，可參看以下的文章：

梁鑑添，蕭文强，一門與數學發展史有關的課程，
<抖擻>，41（1980），38—44。

4. 結　　語

讓我重複一遍，數學教學的目標是(1)思維訓練(2)實用知識(3)文化修養，三者應有適當的平衡。要同時達到這三點，一定有客觀條件上的困難，但作爲數學教師，我們必須肩負這項責任。經常接觸數學，可保持本身的活力和熱情；想想我們學科的歷史、本質和意義，或者是一種激勵和鼓舞。近代著名數學家WEYL說過：「我們並非宣稱數學應享有科學之皇后的權利，有其他科目與數學是有同等甚至更大的教育價

值。但數學立下所有心智活動所追求的客觀眞理的標準，科學及科技是它的實用價值之見證。如同語言及音樂，數學也是人類思維的自由創作力的主要表現形式，同時它又時通過建立理論來認識客觀世界的一般工具。所以數學必須繼續成爲我們要教授給下一代的知識和技能中的基本成份，也是我們要留傳給下一代的文化中的基本成份。」這就是我們的事業。



小學數學科主任的工作

教育署輔導視學處數學組

在小學裏各學科均設有科主任，負責策劃、統籌及監察該科的有關教學事宜。在很多學校裏，科主任都盡了很大的努力，把工作做得有聲有色，使科務得以順利推行。但是却有部分校方沒有將科主任的職權作明確界定，也沒有將權力下放，只讓教師輪流擔當。於是居其位的科主任認爲這個職位徒然加重工作負荷，便採取避之則吉的態度，其實教學工作能否有效地推行，科主任的投入與否，有着一定的影響力，所以校方必須「選賢任能」，而獲委爲科主任的則應認清自已的職責，並切實履行。

小學數學科科主任的工作，大要可分爲諮詢及統籌兩方面：

㈠ 諮詢方面

1. 介紹「小學數學課程」的精神與數學教學的新趨勢。
2. 推薦適當的參考書。
3. 介紹數學科用具及其使用法。
4. 在選用新課本過程中，作初步的建議，供有關教師參考。
5. 建議提高教學質素的方法。
6. 向教師介紹各種進修途徑，並鼓勵他們參加。
7. 向沒有教學經驗或新到任的教師提供指導與協助。

㈡ 統籌方面：

1. 與教師共同商討一個明確的教學目標。
2. 與各級共同擬定一個完整而有系統的教學內容。
3. 因應課程需要及學生程度，與教師共同訂定各班級的教學進度（進度表上所列的不應只是課本目錄的翻版，而應有較詳盡的內容，以及列出對課本所作之增删）。

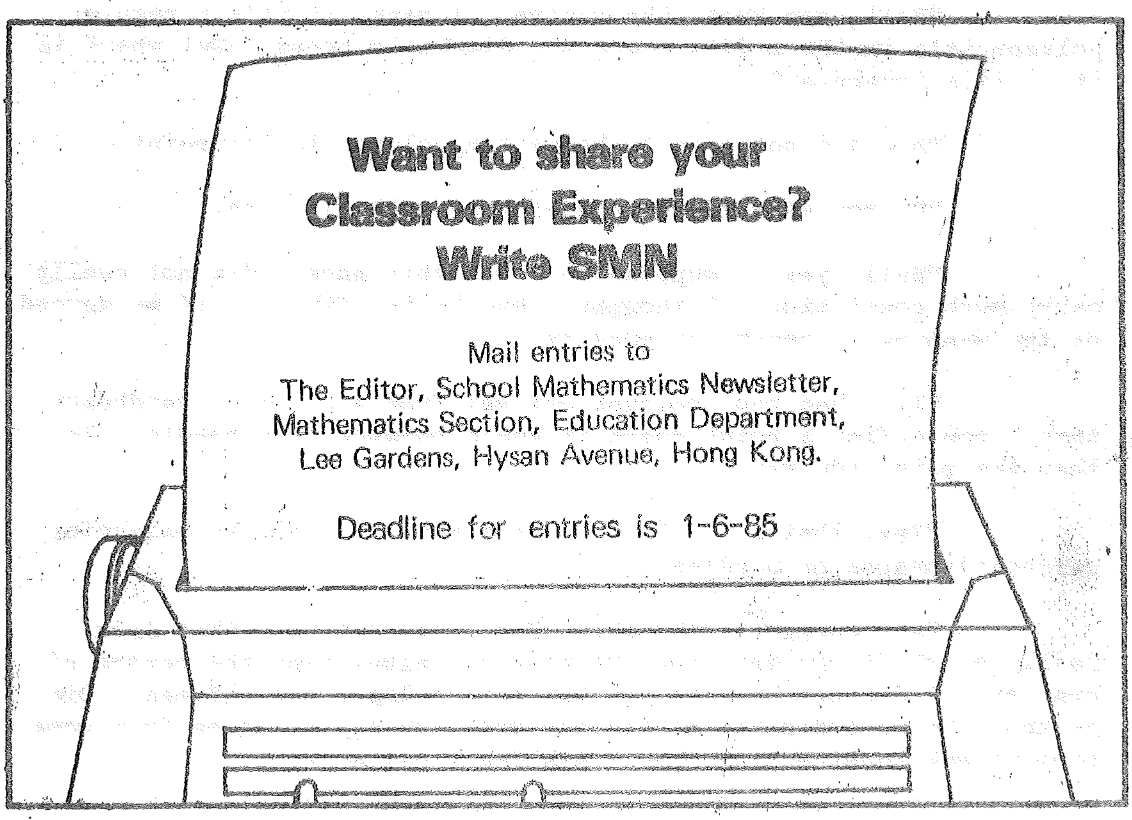
4. 配合學校情況，與教師擬定一個可行的輔導教學計劃。
5. 與教師商討如何採用適合學生程度的教學法教導學生。
6. 與教師共同選擇一套適合學生使用的課本。
7. 與教師共同商議及選購適當的參考書籍及教學用具。
8. 經常留意學生的學習情況，進行教學上的檢討。
9. 經常檢查教學用具是否充足，操作性能是否良好，及是否被教師充分使用。
10. 籌劃課外活動（如遊戲、展覽、比賽及設計等）以配合實際教學，藉以提高學生的學習興趣。
11. 審查各級的測驗或考試題目，務求能切合課程需要，以及能衡量與判別學生的眞正能力。
12. 主持科務會議。科務統籌雖然是科主任的職責，但若要科務工作順利推展，各有關老師的支持是很重要的。一個理想的科主任必須懂得好好的利用科務會議與教師交換意見，鼓動教師積極參與科務工作。因此科務會議不應拘泥於每學期舉行一次，而應鼓勵教師隨時提出問題共同探討，找尋答案，倘能與有關教師達成一相互間認爲可行之協定時，應再在科務會議上提出追認，並加以記錄，以供其他教師參考，及作爲日後檢討的根據。而科務會議的內容，亦不應只限於一般事務的討論，例如試題的數量，學生家課的形式，工作的分配等，而應對下列一些問題作深入的討論，例如課程的內涵及其精神，交換教學心得，解決教師所遭遇的困難，試題內容的分佈與編排，家課的份量，班級間教學工作的協調，全年數學活動的策劃以及對考試結果、教學進度等作出檢討，以爲下學年擬定教學計劃之參考。

科主任的職責除了上列的諮詢及統籌科務兩大範疇外，同時擔負橋樑的作用，一方面溝通校方與教師間的意見，使校方的既定方針能順利推行，而老師的困難，亦能向校方反

映。另一方面作爲教師間的橋樑，協調各班級間的差異和不同年級與不同年度間教學的連貫性。

要達到以上的種種目的，科主任必須對教學有濃厚的興趣，同時更要留意課程的發展，參閱有關的數學書刊，多參加研討會或研習班，並多方面與其他數學教師交流經驗，才可以提高本身的專業水平和明白現代的教學趨向。此外，若科主任能擔任不同級別的數學課，便更能全面而確切地瞭解校方的眞實情況，對科務推行有一定的幫助。

從以上所見，一位理想的科主任所擔任的工作，其實是相當繁重的。希望他們能斟酌學校情況，考慮任務的緩急輕重，抱着「出一分力，發一分光」的精神，盡力而爲。而校方及有關教師更應衷力合作，全力支持，才能發揮科主任的積極性，從而能順利推行數學科的教學工作。



So You Think You Have a Problem?

Matthew D. Linton
School of Education, University of Hong Kong

"You're a mathematician. You'll be able to help me."

That sounded an ominous opening. It was Jack who had approached me in the common room. Jack was in the English department, with a special interest in drama - and if I had been quick I would have remembered that he had recently started dabbling in home computing. I shrugged my shoulders non-committally, and waited for the worst.

"How do you find the centre of a polygon?"

Now what sort of a question was that? What did he want? I thought I had better probe a little.

"How do you mean 'centre' of a polygon?"

"Well, you know, the centre. I mean, if it's a regular polygon it's in the middle where the diagonals cross. But where is it if it's irregular?"

This did not seem to be getting closer to the point.

"Do you mean the centre of gravity?" I tried.

"Well, yes, I suppose so," but this answer did not really carry much conviction. I thought I had better find out if we agreed on the meaning of centre of gravity.

"If I had your polygon cut out from a piece of cardboard then I could find a point where it would balance on a needle. Is that the point you want?"

"Yes, that's OK. But I don't want to do it by balancing cardboard shapes on needles."

Fair enough, I thought. But I was not sure that Jack really wanted to go into the business of calculating the centre of gravity. I did not know as yet how this polygon was defined. Did he know the co-ordinates of its vertices, or did it arise from some geometrical construction. So I stalled a bit more.

"This is a practical problem, is it?"

"Oh yes, it's practical."

"Well then, instead of balancing your cardboard shape on a needle, you could hang the shape up from one corner and use a cotton plumb line to mark on the card a vertical line. Then repeat this using a different corner, and where the two lines cross that's the centre of gravity."

"It's not as practical as that. No, the polygon is drawn on a computer screen."

Now we were getting somewhere. He really did want to <u>calculate</u> the centre of gravity.

"Do you know the co-ordinates of all the corners of the polygon?" I checked.

"Yes."

"And you want the computer to calculate the co-ordinates of the centre of gravity?"

"Yes."

At last I had a clear statement of the problem.

"Oh that'll be no problem. I'll work you out an algorithm overnight."

"That's great," said Jack, please with the thought that he had left his problem in what he imagined were capable hands.

The following day I greeted Jack with the news that, whilst there was no difficulty in the calculation of the centre of gravity for a general polygon, it was however a bit of a grind. I had devised a scheme in which the polygon was divided into triangles with point masses proportional to their areas placed at their centres of gravity, and then found the centre of gravity of the point masses. I leave it to you, gentle reader, to reconstruct my scheme — or to devise a better one. The calculation of each triangle area was going to involve the evaluation of a 3 x 3 determinant, and I was afraid that the whole thing was going to take up more lines of coding and a longer execution time than Jack had bargained for. So I pointed out that if he used

$$\underline{\bar{r}} = \frac{1}{N}\sum_{i=1}^{N} \underline{r}_i$$

where the $\underline{r}_i$ are the position vectors of the N vertices, he would be finding the centre of gravity of equal point masses placed at the vertices of the polygon. Whilst this would not in general be the same as the centre of gravity of the polygon (except in the case of a triangle), I ventured to suggest that it would not be very far away and might suit his purposes, and furthermore would need only a few lines of coding.

Jack was grateful for my efforts, and said that he would try my suggestions to see if they worked.

"Worked for what?" I wondered. I was not convinced that I had solved the right problem. It seemed high time to dig a bit deeper.

"What do you want this for, any way?" I tried as an opening.

"I want to rotate the polygon on the computer screen, and I want the right centre for the rotation."

Oh dear! Why had I not asked earlier what the problem was all about? A rotation problem. So it probably had nothing to do with centre of gravity at all. And what did Jack mean by the 'right centre'? Surely you could rotate a polygon about any point?

"Well Jack, I don't think you need bother with centre of gravity. In fact you can rotate it about any point you want."

"Oh, but I want it to look right," he replied.

What did he mean by 'look right'? I began to feel that at last we were approaching the real problem. I picked up a cigarette packet handily lying on the table in front of us, held one corner between thumb and forefinger and twirled it round.

"Look, you can rotate this packet about a corner if you want," I gave as explanation for my demonstration.

"But that doesn't go round the middle. It wouldn't look right on the screen — in fact it might go off the screen."

This reply still seemed to be stating the problem in the original vague terms like 'round the middle' and 'look right'. I decided to take the last remark about going off the screen as the clue to this search for the real problem.

"Do you mean you want to rotate about the point which makes the polygon sweep out as small an area as possible? As the polygon is rotated, the corner furthest from the centre of rotation will trace out a circle, and all other corners and edges of the polygon will remain inside this circle. Are you trying to find the point of rotation that makes this circle as small as possible?"

Jack seemed to think that that was exactly what he wanted. So here, at last, was the problem. Find a point $\underline{r}$ so that $\max\{|\underline{r} - \underline{r}_i|\}$ is a minimum. I showed Jack that for a triangle the point he wanted was the circumcentre, and that it certainly was not the centre of gravity. But I could not see where the point would be in the case of a general quadrilateral.

"You'd better let me think about this overnight," I suggested.

Overnight thoughts quickly revealed that my assertion that the circumcentre was the required point for a triangle was only true for an acute-angled triangle, and that in the case of an obtuse-angled triangle the point was the mid-point of the longest side. I decided to call the wanted point the 'centre of rotation' and the corresponding circle swept out by the polygon the 'circle of rotation'. Some moments thought convinced me, as I am sure they will convince yourselves, that the circle of rotation must pass through at least two vertices of the polygon. If it passes through only two vertices then these will form the end points of the longest diagonal. If it passes through three vertices then they will make an acute-angled triangle, the circumcentre of which will be the centre of rotation. For four or more vertices on the circle of rotation then the centre of rotation will be the circumcentre of any triangle formed by three of the vertices, and there will be at least one of these triangles that is acute-angled.

As a first step in locating the centre of rotation, I thought it a good idea to find the longest diagonal, and then, using the mid-point of this diagonal as centre, construct a circle through the end points of the diagonal. If this circle lies outside (or on) all the other vertices of the polygon then we have constructed the circle of rotation. If, however, there are some vertices beyond this circle then my next thought was that the circumcentre of the triangle formed by the most distant vertex and the end points of the longest diagonal would be the centre of rotation. Some playing around with ruler and compasses showed that this hope was false, but that the point so found could not be very far from the point I was looking for.

I thus returned next day with the recommendation for Jack that he use the mid-point of the longest diagonal, and although that may not be the centre of rotation it would probably do quite well. The only exact solution I could devise was to find the circumradius of all possible acute-angled triangles formed by any three vertices of the polygon, and then select the largest of these. This did not seem like a worthwhile chore. If any reader can invent a quick algorithm for locating the centre of rotation I would be pleased to hear about it.

Next day I found Jack at his micro, and there on the screen was the very polygon he wanted to rotate. It was the outline plan of a stage set. Of course, Jack was a drama man. And then I saw what all this had been about, and what the trouble was. Jack had a program that could take a stage plan and redraw it tilted so as to look as though it were viewed from the middle of the front row of the circle. It then constructed vertical flats where required to give quite a realistic impression of a stage set. Jack wanted to find out how the set looked from a seat at the side of the theatre, so he first wanted to rotate the stage plan through the appropriate angle. The tilting and drawing of the flats would then follow to give the view from the side seat.

"Look at this rotation," said Jack as he demonstrated this part of the program. The stage plan was redrawn at an angle, but was distinctly distorted. It was then that I realised that Jack had had at the back of his mind the mistaken notion that his rotation was not working properly because he had chosen the 'wrong' point to rotate about, and to correct matters he needed to find the 'right' one.

"Oh, it doesn't matter where you chose to rotate about," I said, "provided the plan stays on the screen. What's wrong is your rotation program — that's all."

Some uninviting lines of unstructured BASIC were produced for me, so I pleaded, "No time at the moment, but I'll write you some coding by tomorrow. No trouble," to avoid having to debug Jack's program.

By the following day Jack had rewritten his program himself. It involved calculating the distances from all the vertices to the arbitary point chosen as centre of rotation, which I pointed out was unnecessary — but it worked : the stage plan rotated, it then tilted and the flats were erected and all the sight lines for our theatre goer in one of the less expensive seats were clearly displayed. I gave Jack a few lines of somewhat more efficient coding, but I do not think he ever used them. After all his problem was now solved — he had done it himself, and it had nothing to do with the centre of a polygon (whatever that is).

A Suggestion to Prove $m_1 m_2 = -1$ iff $\ell_1 \perp \ell_2$ for Form Two Students

Leung Nim Sang
Grantham College of Education

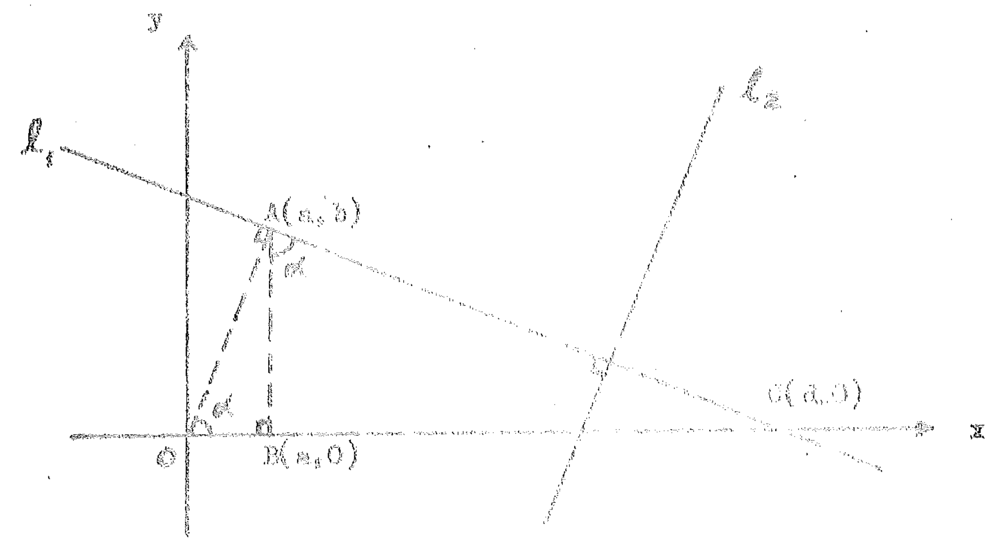
Although proof of the captioned property is not required at the Form 2 level according to the CDC Mathematics Syllabuses, teachers and more capable students may find this article informative. — Editor

The above property is proved by the inductive method in many textbooks. However, the more capable students may not be satisfied with this method. The following suggestion is one way of proving it by the deductive method. (Let's consider the case that neither ℓ_1 nor ℓ_2 are perpendicular to the x-axis.)

(a) To prove:

If ℓ_1 and ℓ_2 are perpendicular then $m_1 m_2 = -1$.

Proof:



In the diagram, ℓ_1 and ℓ_2 are perpendicular and the slopes are m_1 and m_2 respectively. A line OA passing through the origin is drawn parallel to ℓ_2 cutting ℓ_1 at A. From A a line AB is drawn perpendicular to x-axis cutting it at B.

Let $\angle AOB = \alpha$, A be (a, b), C be $(d, 0)$.

Obviously, $\angle BAC = \alpha$

In $\triangle ABC$,

$$\tan \alpha = \frac{BC}{AB}$$

$$= \frac{d - a}{b}$$

As $OA // \ell_2$,

$$\therefore m_2 = \text{slope of } OA$$

$$= \tan \alpha$$

$$= \frac{d - a}{b}$$

By definition,

$$m_1 = \frac{b - 0}{a - d}$$

$$= \frac{b}{a - d}$$

Hence, $m_1 m_2 = -1$

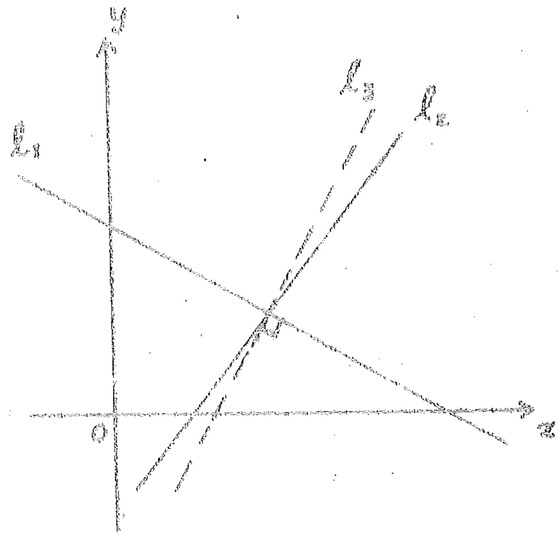
(b) To prove :

If $m_1 m_2 = -1$, then ℓ_1 and ℓ_2 are perpendicular.

Proof :

In the diagram,

through the intersecting point of ℓ_1 and ℓ_2 a line ℓ_3 perpendicular to ℓ_1 is drawn.



Let the slope of $\ell_3 = m_3$.

From (a), $m_1 m_3 = -1$

But given $m_1 m_2 = -1$

$\therefore m_2 = m_3$

$\therefore \ell_2$ and ℓ_3 coincide ,

i.e. ℓ_1 is perpendicular to ℓ_2.

Why Doesn't $(a+b)^2$ equal a^2+b^2 ?

K. Robin McLean
School of Education, Liverpool University

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It was the second session of a PGCE Mathematics for non-specialists option course. In the group of twenty there were chemistry graduates, a number of geographers fairly well-versed in statistics and some students who had done no mathematics since they took 'O'-level. I was attempting to show how algebra could be developed from number work in secondary schools when Margaret, a mature biology student, said "I know that $(10 + 9)^2$ isn't the same as $10^2 + 9^2$, but why isn't it the same?"

What an opportunity for the rest of the group to try their hand! Everyone rose to the challenge, all eager to help. Soon the board was covered with explanations of all kinds. Brackets were multiplied out, other numbers were substituted in place of 10 and 9, someone based an argument on analogies with third and fourth powers, pictures of squares were drawn and areas examined. Each fresh explanation was greeted with approval by the majority of students, but Margaret's response was always the same : "Yes, I can see that. That's very nice, but I still don't understand why $(10 + 9)^2$ isn't the same as $10^2 + 9^2$."

We broke up for coffee and, on resuming the class, Sue, a PE student, wrote on the board :

	19	10	
	19	10	
	19	10	
	19	10	10^2
	19	10	
	19	10	
	19	10	
	19	10	
	19	10	
19^2	19	10	
	19	9	
	19	9	
	19	9	
	19	9	
	19	9	9^2
	19	9	
	19	9	
	19	9	
	19	9	

"Obviously", said Sue, "If you add up the left-hand column you'll get more than if you add up the right-hand column for, in each row, the number on the left is bigger."

Silence fell. That said it all. Margaret was satisfied.

The Effectiveness of Definition and Exemplification on Learners with Different Cognitive Styles

Tsang Yok Sing
Pui Kiu Middle School

INTRODUCTION

In the secondary school classroom most of the teaching of concepts in mathematics (and in other subjects as well) is done verbally. Definition and exemplification are two strategies most commonly employed by teachers. How do these two strategies compare with one another in effectiveness? Do students with different characteristics respond to these strategies in different ways? These questions are worth investigating, for while most mathematics teachers use both definitions and examples in the teaching of concepts, opinions differ on the relative effectiveness of the two, and this often affects the teacher's decision on the priority of each teaching strategy, and the allocation of teaching time.

The present study examined the effect of these two instructional approaches on field-dependent and independent learners. Field dependence/independence was first studied by Witkin and his associates as a consistent manner of perceiving the upright by an individual (Witkin, 1949). Witkin used the rod-and-frame test, in which the subject was presented with a luminous square frame enclosing a luminous rod in a dark room. Both rod and frame were tilted, at different angles, and the subject's task was to adjust the rod to a position where he perceived it as upright. It was found that some individuals referred primarily to the felt position of the body for perception of the upright, while others relied mainly on the position of the frame. The terms field independent and field dependent were used to characterise these different styles of perception.

Later research reported that field independent correlated strongly with disembedding abilities. People who were good at perceiving the upright independent of external field were found to perform better in an embedded-figures test, which required the subject to locate simple shapes embedded in more complex figures (Witkin et al., 1954). Subsequently, convenient paper-and-pencil tests claimed to measure the same thing as the rod-and-frame test were developed to replace the complex gadgets used in earlier laboratory experiments (Witkin, Oltman, Raskin and Karp, 1971).

The convenience of the embedded-figures test has made it very popular in research on individual differences in learning. Positive results have been reported in a number of studies on the relationship between cognitive style (as measured by the embedded-figures test and designated by field dependence-independence) and various learning tasks.

The learner variable used in the present study was measured by an embedded-figures test similar to that used by Witkin. Subjects scoring high on the test were said to be analytic, and those with low scores were called global.

METHOD

The experiment was conducted in a Chinese secondary school. The concept chosen for the experiment was the mathematical concept of function, and subjects were students in the third year of their secondary education. Teaching was presented in the form of instruction booklets, written in Chinese, distributed to each individual student.

Three different booklets were prepared. The first booklet, designated Book D (Definition), contained an explicit definition of a function as a special kind of relation. The second booklet, called Book X (Examples), consisted of examples and nonexamples of the concept of function. The explicit formulation of the definition was not given, but attention was drawn to the defining attributes as they were present in an example or absent from a nonexample. A third booklet, Book C (Combined) was a combination of parts of the first two booklets. The lengths of all three booklets were approximately the same.

Concept attainment was assessed by the following abilities: (1) to distinguish between examples and nonexamples of the concept; (2) to account for the identification of nonexamples in terms of defining attributes of the concept; (3) to determine whether an attribute is a necessary consequence of the defining attributes. The test consisted of three sections. In Section A a list of relations were given, and the students were asked to classify them as examples or nonexamples. Students were also required to give a reason whenever they thought that a relation was not a function. The answer would be considered correct if it pointed out one or more of the criterial attributes that were not exhibited by the relation. The total number of correct answers in this part constituted the score for Section B. Section C contained statements some of which were necessary consequences of criterial attributes of functions while others were not. Students were asked to decide whether each statement was a necessary consequence.

The following hypotheses were formulated for testing:

(1) Analytic students will perform better than global students in the learning of mathematical concepts under similar conditions.

(2) Students presented with the definition and examples of a mathematical concept will perform better than students presented with either definition or examples alone.

(3) The difference in performance between analytic and global students will be greater when no examples are presented than when examples are given.

(4) The difference in performance between the example and no example conditions will be greater for global than for analytic students.

A hundred and fifty-seven Middle 3 students (80 boys and 77 girls) took the embedded-figures test. Among them 154 (78 boys, 76 girls) went through the experiment subsequently.

Administration of the embedded-figures test took one 40-minute period for each class. The subjects were divided into three approximately equal groups according to their embedded-figures test score. Those scoring high were labelled Analytics, those with low scores called Globals, while those in between made up a Middles group.

The second part of the experiment followed about a week after the embedded-figures test. A double period was used for each class. Book D was given to a third of the analytics, Book X to another third, and Book C to the rest. The middles and the globals were treated similarly. A random number table was used to determine which subject should read which book and the name of each subject was written on the cover of the book assigned to him or her before the teaching session.

The booklets were given out and the subjects were told that they could use up to forty minutes to read through the contents. However, if anyone finished early and thought he had learned all he could from the booklet, he could raise his hand for the test, but at the same time the booklet would be taken away from him.

RESULTS

The results of each section of the post-test were analysed separately.

(1) Analytics versus Globals. To test the hypothesis that analytics perform better than globals on the whole, the one-tailed t-test for independent groups was used to compare the mean scores of the two groups for each section. The results show that the analytics did significantly better than the globals as hypothesised..

(2) Difference in Instructional Method. To test the hypothesis that simultaneous presentation of both definition and examples facilitates learning better than presenting either alone, the mean score of subjects learning from Book C was compared with the mean scores of those learning from the other books. Again the one-tailed t-test was used. The results do not support the hypothesis that the combined method is more effective than the other two. In fact, in nearly all cases the mean score for the Combined Group is lower than the mean score for either of the two other groups.

(3) Analytics versus Globals for Different Instructional Methods. The third hypothesis differs from the first in that it takes instructional method into consideration. To test this hypothesis, the performance of the Analytic-Definition Group was first compared with that of the Global-Definition Group. Then another comparison was made between the Analytic-Examples Group and the Global-Examples Group. The results support Hypothesis Three. With a definition approach, the analytics excelled significantly. But with an examples approach, no significant difference was found between the analytics and the globals.

(4) Definition versus Examples for Analytics and Globals. The last hypothesis compares the effects of definition and of examples on learners with different cognitive styles. To test this hypothesis, two-tailed t-test were used. Only results in Section B support the hypothesis: the globals benefited more from examples than from the definition, at the .10 significance level. In all other cases no significant difference was found between the two instructional approaches. However, the analytics who learned from the definition had a higher mean score than analytics learning from examples, whereas among globals it was those learning from examples that did better.

Mean scores in Section A and B for each cognitive style x method group are as displayed in Tables 1 and 2. The figures in the two tables exhibit a similar pattern. For both the analytics and the globals, scores resulting from the combined method fall between scores obtained when using the other two methods. However, going from D to C and then to X, the direction of change of the mean score for the analytics is exactly opposite to that for the globals. For the former it is D > C > X, whereas for the latter X > C > D.

Table 1

Mean Scores in Section A
(Expressed as Percentages of the Total Score)

		Method of Instruction			
		Definition	Combined	Examples	All
Cognitive Style	Analytic	70.1	67.8	62.3	66.7
	Middle	62.2	59.2	68.8	63.0
	Global	58.3	60.4	62.3	60.4
	All	63.6	62.2	64.4	63.4

Table 2

Mean Scores in Section B
(Expressed as Percentages of the Total Score)

		Method of Instruction			
		Definition	Combined	Examples	All
Cognitive Style	Analytic	51.0	42.7	39.2	44.3
	Middle	33.3	31.7	31.4	32.1
	Global	18.8	25.0	31.4	25.2
	All	34.6	33.0	34.0	33.9

DISCUSSION

A possible explanation for the fact that globals did not learn most efficiently from a combined method, is that the initial presentation of the definition might force the globals out of their preferred cognitive mode into an analytic mode, which is not as efficient for them as a whole-form comparison method. If this explanation is correct, then reversing the order of presentation of definition and examples would produce a different effect on global learners. Set in their preferred global mode by an initial presentation of example, they may benefit more from the definition that follows.

The findings of the experiment accentuates the role played by examples in concept learning by global students. In the teaching of mathematical concepts, standard definitions are followed by most teachers and textbooks. Greater variations exist in the selection and organization of examples. The results of the study suggest that examples should play a much more important role in the teaching of concepts than simply serving illustrative purposes. Also, the order of presentation may affect the efficiency of definition and examples. Further research along this line may reveal methods for improving the learning of students who find mathematics difficult.

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The First Hong Kong Mathematics Olympiad

Mathematics Section
Advisory Inspectorate, E. D.

The Mathematics Olympiad was first introduced in 1972 by the Mathematics Club of the Northcote College of Education as an internal activity for the N.C.E. students. The competition was open to secondary schools in 1974 and was called the "Inter-school Mathematics Olympiad". This competition was held annually until 1983 and was restricted to a maximum of fifty invited schools because of limited resources.

In 1983 the Mathematics Section of the Advisory Inspectorate of the Education Department and the Mathematics Department of the Northcote College of Education decided to expand the competition so as to include more participating schools. The competition was re-named as the "Hong Kong Mathematics Olympiad".

All secondary schools were invited to participate in the First Hong Kong Mathematics Olympiad. 122 schools took part in the Heat Events held in December 1983. The 40 schools with the highest scores became finalists. The Final Event took place in February 1984. Results were as follows : -

Champion : Wong Tai Shan Memorial College

1st Runner-up : S.K.H. Lui Ming Choi Secondary School

2nd Runner-up : C.N.E.C. Christian College

The First Hong Kong Mathematics Olympiad was considered a success. It was very well received by the participating students. Thus, the Second Hong Kong Mathematics Olympiad will continue to be launched in the coming school year.

Bar Codes

Poon Hoi Cheung

As you push a shopping cart up and down the aisles at a supermarket, you will never fail to notice a sequence of dark and white pattern of bars printed clearly on the labels of cereal packets, marmalade jars, canned goods etc. In fact, more and more grocery products and merchandise now bear a symbol of the electronic age — the bar code. Although it has been used for more than 10 years, few people seem to know much about it. This article, therefore, is written as a layman's guide to the code.

The bar code system is part of a scheme covering most of the western world to give retailed goods unique identifying numbers. The scheme will give better and faster communication between the various parts of the manufacturing and retain chain. At present, there is a local department store using bar-coded labels to automate its daily sales operations. If you shop at its supermarkets, you will experience this at first hand. With the new system (known as point-of-sale or POS system), the bar code is scanned by an optical reader/scanner (including handheld devices) or a low-power laser beam. The unit is linked to a computer, which holds the prices of the various goods. The computer displays the name and price of the product and this information is also printed on the bill receipt. This not only improves services to the customers, but also allows the company to up-date inventories and sales statistics on its products, assuring a more efficient and more profitable operation.

Although many attempts have been made to Standardize the code for a wide range of products, there is still no unique system of code. Two important bar codes are the Universal Product Code (UPC), widely adopted in the United States and the European Article Numbering (EAN), a popular code for European countries. In Hong Kong, a new bar code system called the Local Article Numbering (LAN) — a modified version of the UPC (Version E) is used on local and Chinese products. As a bar code reader usually can be used to read more than one system, automation is therefore achieved.

Let us now investigate closely the structure and characteristics of the Universal Product Code Regular Version : UPC-A (most commonly used in grocery applications) and see how it is encoded. UPC-A consists of 10 numeric characters and the standard symbol (a machine-readable form) is in the form of a series of parallel light and dark bars of different widths. The basic characteristics of the symbol are as follows (see figures 1 and 2) :

a) Series of light and dark parallel bars (30 dark and 29 light for any 10-character code) with a light margin on each side.

b) Each character is always represented by 2 dark bars and 2 light spaces.

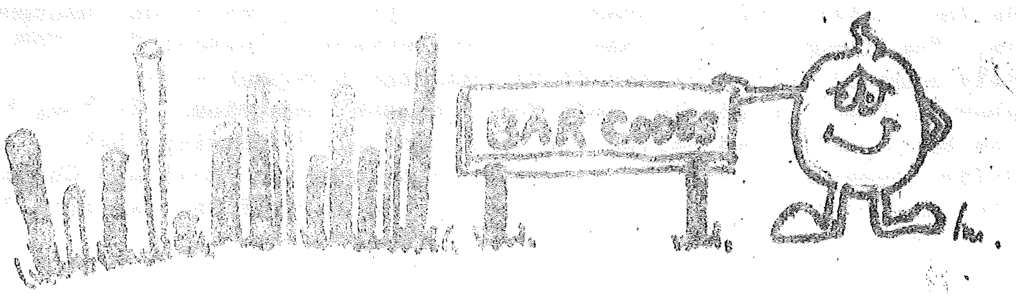
c) Each character is made up of 7 data element called a 'module' which may be dark or light.

d) A bar may be made up of 1, 2, 3 or 4 dark modules.

e) The symbol also includes the following two characters :-

- one modulo 10 check character embedded in the right-most position of the symbol to ensure a high level of reading reliability.
- one character, embedded in the left-most position of the symbol, shows which number system a particular symbol encodes.

f) The symbol size may be larger or smaller than norminal (3.76 cm x 2.61 cm. including the light margin).



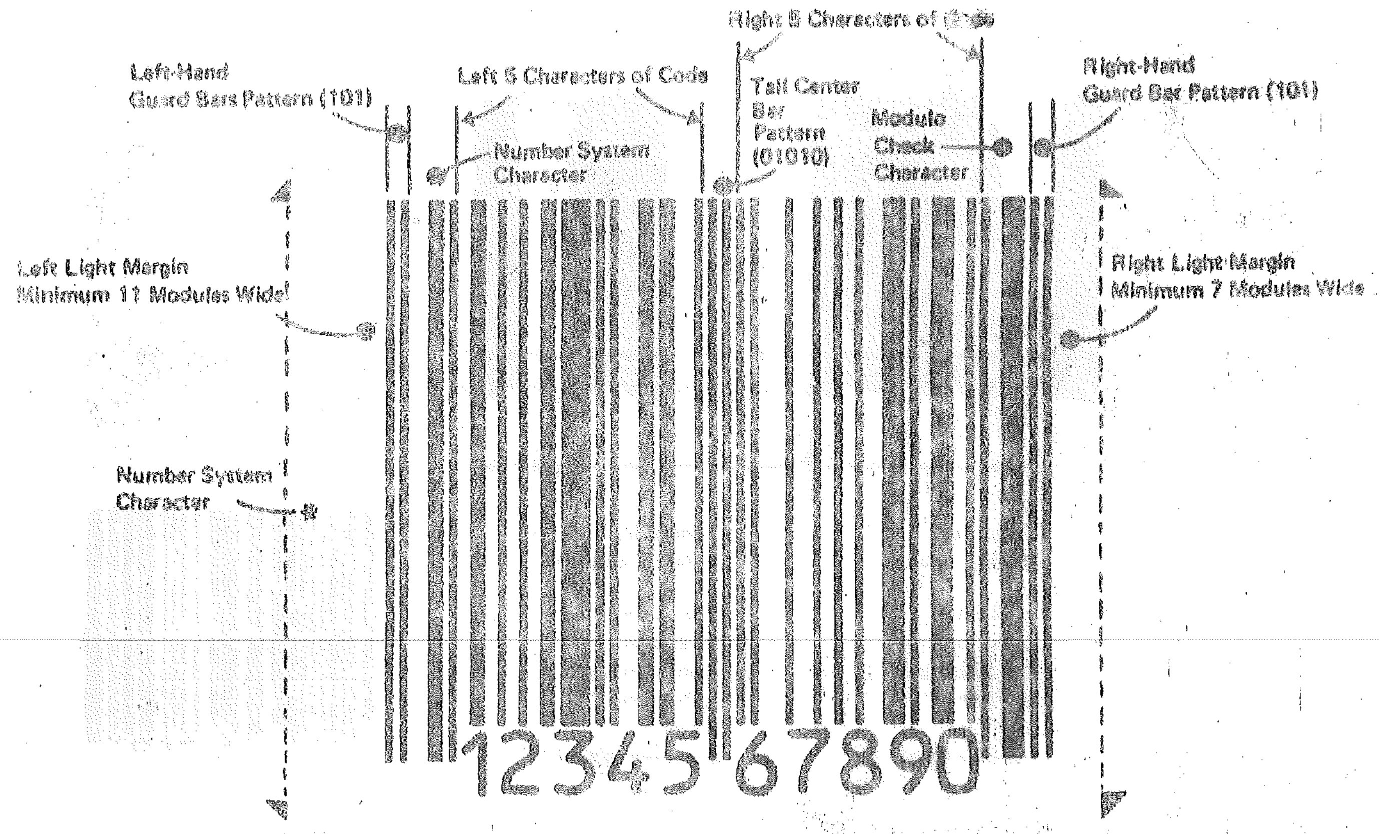
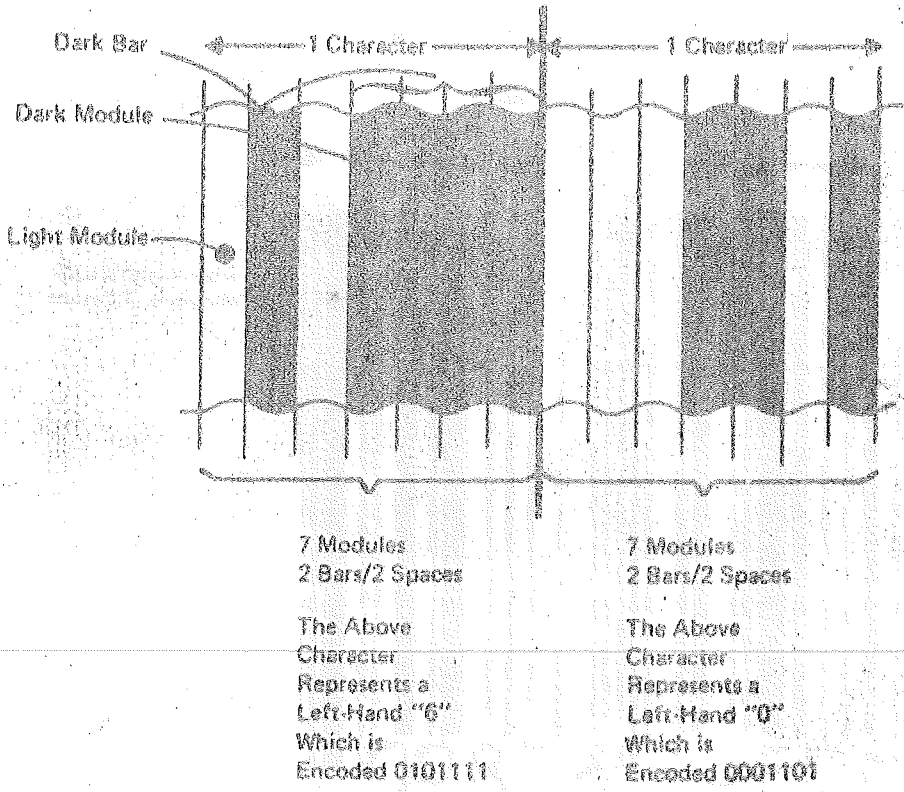


FIGURE 1 UPC STANDARD SYMBOL
(not to scale)

An example of a regular symbol is shown in Figure 3. The left side of the symbol begins with a left margin and the guard bars (enclosed 101). The left guard bars are followed by the encoded number system character. This character is usually identified with a human readable character immediately preceding the left margin. The number system character is followed by 5 characters (manufacturer I.D.), centreband (encoded 01010), 5 more characters (item I.D.), the modulo 10 check character (implicitly encoded), the right guard bars (encoded 101) and right margin.



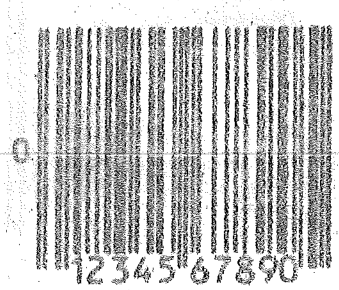


FIGURE 2 CHARACTER STRUCTURE

FIGURE 3

Dark modules represent 1's while light modules represent 0's. The number of dark modules per character on the left side is always 3 or 5 and the number is always 2 or 4 right-hand characters. The following table is the encoding system for the right and left halves of a regular symbol.

Decimal value	Left characters (Odd parity)	Right characters (Even parity)
0	0001101	1110010
1	0011001	1100110
2	0010011	1101100
3	0111101	1000010
4	0100011	1011100
5	0110001	1001110
6	0101111	1010000
7	0111011	1000100
8	0110111	1001000
9	0001011	1110100

The module 10 check character is calculated by the following procedure :-

Step 1 : Starting with the number system character, add up all the characters in the <u>odd</u> positions.

<u>Example</u> : $0 + 2 + 4 + 6 + 8 + 0 = 20$

Step 2 : Multiply the sum obtained in step 1 by 3.

<u>Example</u> : $20 \times 3 = 60$

Step 3 : Starting again at the left, sum all the characters in the <u>even</u> positions.

<u>Example</u> : $1 + 3 + 5 + 7 + 9 = 25$

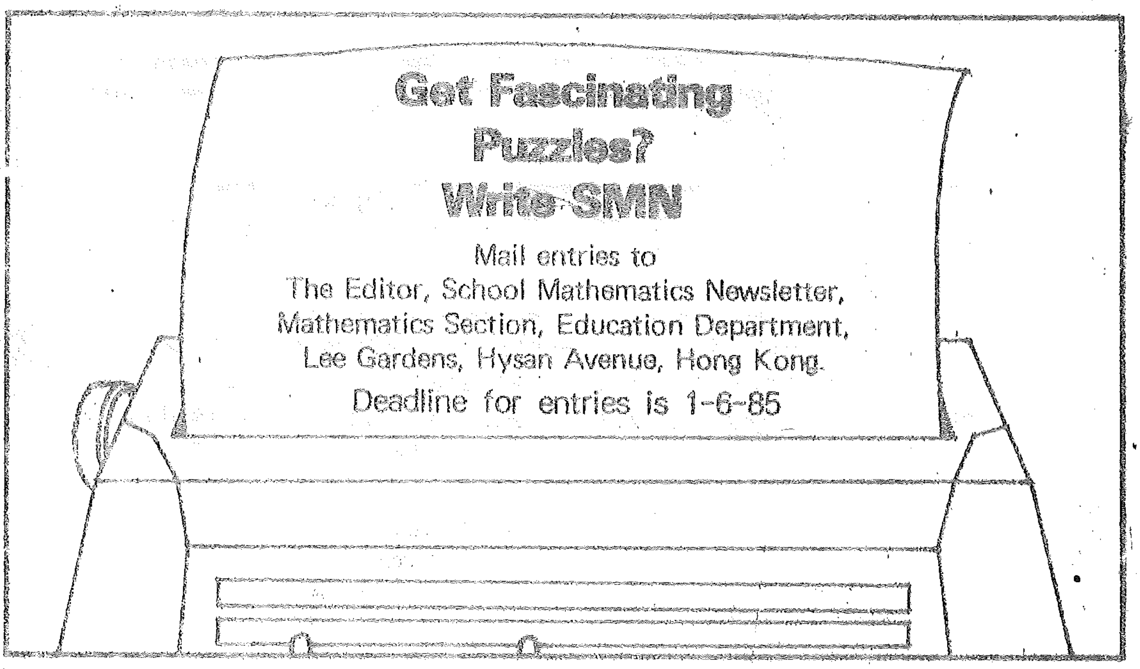
Step 4 : Add the product of step 2 to the sum of step 3.

<u>Example</u> : $60 + 25 = 85$

Step 5 : The module 10 check character value is the smallest number which when added to the sum of step 4 produces a multiple of 10.

<u>Example</u> : $85 + 5 = 90$; 5 is therefore the check character.

As an exercise, readers may examine and decode some sample bar codes to ensure that it is correct. [redacted]



A Plan to Provide Computer Literacy for Secondary School Teachers

Anthony Pau
IEEE (H.K.) Computer Chapter

Why Computer In Education

By 1990 at least half of our total workforce will hold computer-related jobs. It can be office automation in corporations, management information systems in banking institutions or it could be automation systems in assembly lines. These future projections dictate that today's teachers be computer literate if they expect to prepare their students successfully for the technological era in which they will live. Defining "computer literacy", however, is not easy because there are so many varied attitudes as to what it actually constitutes.

Computer Literacy for In-service Teachers

The following outline represents the areas used to define computer literacy:

1. Communication. Teachers must be familiar with computer terms and jargon so that they can effectively communicate with their students.

2. Understanding. Teachers must generally know how computers work and be able to identify their various parts.

3. Survival. Teachers must have some knowledge of the process of entering and retrieving information from a computer. Too often we hear about a computer's error or a computer's problem that caused something to go wrong. The truth of the matter is that often it was really a human error that initially caused the failure. Therefore, teachers must be able to intelligently discuss how the information was input into the computer and how the results were obtained.

4. Applications. I think teachers need to know how computers are used in the home, in business as well as in education. In other words, they must understand the general uses that presently exist so that they can discuss them with their students. This would also allow them to carry on intelligent conversations with parents, educating them as to what a computer can and cannot do.

5. Instruction. Students expect to use the computer fully. For us to be in control of the computer, we need to be able to program. If we are able to program, then the computer can be used as a problem-solving device. Learning a programming language is essential if the computer is to be used in a form other than merely switching it on and off and responding to it. Primary and intermediate teachers should know LOGO and BASIC. Intermediate and secondary teachers should know BASIC, especially now that it is included on most microcomputers. For senior high teachers, PASCAL is most appropriate. Advanced Placement exams use PASCAL in testing students for computer science advanced placement. Of course, there are other languages such as PILOT and FORTH. If teachers know at least one language appropriate to their grade level, then they can fully utilize the computer in their classroom.

6. Futuristics. Teachers must be able to discuss artificial intelligence, robotics and the general direction that computers are heading so that they can make students aware of what lies ahead for them in terms of future opportunities.

There are also other general topics included under computer literacy. One of these includes:

7. Societal Implications. Teachers must understand the abuse and misuse of a computer, and what can be done about these problems so that students realize that such issues exist. In addition, they must realize that information in a computer can be incorrect, and how an individual goes about making a change or a correction. In other words, teachers should not accept everything the computer prints out as gospel.

Training Workshop

The IEEE (Institute of Electrical and Electronics Engineers) Hong Kong Computer Chapter is a gathering of computer professionals. This Institute is organizing computer workshops for in-service teachers. These training workshops will be provided free of charge to secondary school computer education teachers, especially those who are excluded from the Hong Kong Government subsidized computer studies pilot scheme. Planning, organizing, delegation and following through ..... these are tedious tasks, to be sure, but we are making progress.

Without well-defined affective and cognitive goals, a teacher training course in computer education could wander in many different directions. The objectives of the training workshop are:

1. To give teachers a broad base of knowledge in computer education rather than to provide mastery in any specific area of computer science including programming.

2. To motivate teachers to initiate a computer education program in their schools.

3. To provide teachers with activities that can be used in their classrooms.

These goals derive from a global concern of computer educators - to give students experience with computers to increase their options for future study and work. It is not necessary for teachers to be expert programmers to provide students with valuable computer experience. Therefore, to meet the above goals, a course content is being developed that blends programming, graphics, off-line activities, as well as reviews of software and courseware. The diversity of topics and of uses of the computer is designed to communicate to teachers the versatility of the computer as an educational tool. The content of the course also provides a framework within which teachers can develop a computer education curriculum specific to the need of his or her own students.

Teachers, school-masters, principals and administrators who are interested to participate or enroll in this training workshop program for in-service teachers, please feel free to contact the author.

Where To Find Updating Correspondence

Finally, for those who would like to keep abreast, they can write to obtain correspondent from the IEE Computer Society TECHNICAL COMMITTEE ON COMPUTERS IN EDUCATION (TCCE). The Scope and Goals of the TCCE are listed below.

The Technical Committee on Computers in Education will:

1. Aid in the technical aspects of specifying, designing, implementing and evaluating the hardware/software/systems that form an educational computing system.

2. Provide a data base of Society papers, books and articles that are relevant to educators.

3. Develop both hardware and software standards that directly relate to the educational uses of computers.

4. Analyze and develop basic requirements for authoring languages.

5. Provide local tutorials.

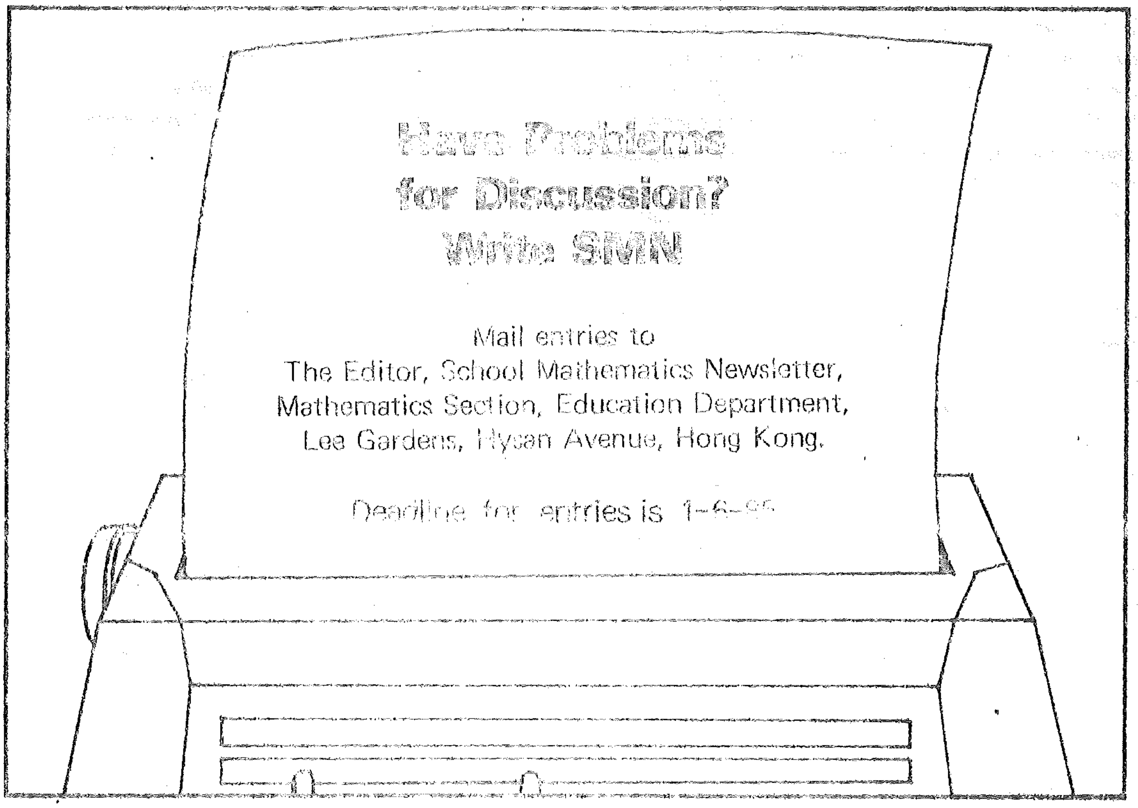
6. Cooperate with the Educational Activities Board to develop a range of educational programs.

7. Sponsor conferences related to computers in education.

8. Promote and provide recognition for research and significant accomplishments in the field of educational computing.

9. Investigate the application of the latest computer research and developments to help in the solution of important educational problems.

TCCE has established a precollege committee to study curriculum needs as well. The committee also sponsors the development of standards in this area. THERE ARE NO MEMBERSHIP FEES. TCCE only requires that you be a qualified, experienced, interested and motivated person who wants to improve the status of computers in education. Simply fill out a copy of the membership form on the following page. The TCCE will select its membership from industry and education. You need not be a member of the Computer Society to join this technical committee. This committee will allow educators and computer society members to work together to effect change on a natural level.



Tripping The Light Fantastic
Optical Computers Will Illuminate the Future

Michael Kieran

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It is 1987. A six-year-old sits on the living-room floor, mesmerized by the ever-changing images which dance before him on the screen. Intent on exploring Western Canda, he taps out a series of commands on the small keyboard in his lap, and up on the screen pops a three-minute color video-tape of aerial photography of the Rockies. A few more commands, and the screen displays superb nature photography of the region's birds and animals. He goes on to maps and still photos of sites of interest along the way, then branches off to the geography pages, where he finds a colorful computer-generated simulation of the violent naturel forces that were responsible for the creation of the mountain chain. A reference to fossils leads him to select related pages on evolution, then continental drift and petrochemicals. Left alone with this new 'toy', another child might begin with the Rocky Mountains, but end up on one of the moons of Jupiter.

This scenario - in which a personal computer is hooked up to an optical disk recorder containing more than 50,000 color images - is technically possible today and will become common as costs drop during the next five years. In fact, optical disks, which resemble ordinary phonograph records, but can hold 10 thousand times as much information as magnetic 'floppy disks' of equivalent size, are the forerunners of a new generation of information tools based on tiny laser chips smaller than a single grain of salt.

The semi-conductor lasers used to read and write information on optical disks are microscopic versions of the conventional lasers used in industry, commerce and medicine. These miniscule devices are actually perfect crystals that are ideal for use in computers because they can be modulated, or turned on and off, billions of times per second. "The importance of the semi-conductor laser is that it opens up for humanity a technology which, over the next two decades, will revolutionize the way we process, store and communicate information," says Michael H. Coden, president of Codenoll Technology Corporation, a Yonkers, New York manufacturer of fibre-optic components and systems.

"The semi-conductor laser is, in principle, identical to every other laser : It is a light amplifier, and it works by stimulated emission of radiation. The amazing things about the semi-conductor laser are its size, its extremely low power consumption and its ability to be modulated many times in a tiny fraction of a second."

The new technology, photonics, derived from photos, the Greek word for light, could have fully as important an effect on how people use information as the invention of electronics.

Long-range research is directed toward the design and construction of optical computers - ultra high-speed machines with vast computing power and memory resources operating on tiny pluses of light, rather than electricity. Computers based on light will not only be a thousand times faster than the current generation of electronic digital computers, they will also be able to view and process entire frames of information instantaneously, rather than having to figure them out one bit at a time. To gauge the impact of this advance, imagine how a telegraph operator from the 1880s would react if shown a color TV set.

Photonics isn't just the far-fetched dream of some light-headed engineers. Optical disks that can be connected to personal computers and business microcomputers are just arriving on the market and optical storage of information is already well advanced. Some optical disks can pack more than two billion bytes, or characters of information, per side - enough to store the entire Encyclopedia Britannica! The demand for storage media with extremely high capacities is expected to accelerate because of the growth of interative computer programs and the demand for archival storage of business and government documents.

Jim St. Lawrence, Director of the Interative Technologies Laboratory at the New York Institute of Technology, says the most exciting aspect of connecting optical disks to personal computers is interactivity - the ability to have the computer present images and sounds appropriate to the users' responses. Individual pictures, video sequences and sounds can be selected from the optical disk and displayed virtually instantly.

One example he cites is an electronic mannequin developed in 1981 by the American Heart Association in Dallas, Texas to teach people how to use cardio-pulmonary resuscitation (CPR) in order to save the life of a heart-attack victim. Sensors in the mannequin connect to a microcomputer, which is programmed to retrieve appropriate video and audio sequences from an optical disk. As a student practices CPR on the dummy, a doctor on the television screen provides individualized coaching. "A little more to the left, and don't press quite so hard. Not that far. Good, now just a little slower. Perfect."

St. Lawrence says studies have shown that instruction through such an interactive optical disk program is more than twice as effective, in half the time, as when a real doctor teaches people CPR.

The current systems have limitations. Although optical disks can store computer programs, they work best when used just to store and retrieve images, with magnetic floppy disks used for the actual programs. This is because minor errors on a video picture may be undetectable to the eye, but even a tiny error in a computer program can cause it to crash.

Also, with current optical disks information can be written onto the disk, but not erased. This is an advantage in banking and other financial uses where a permanent audit trail is essential, but can be a problem in applications such as data management in which information is frequently updated.

To be an ideal storage medium for use with computers, optical disks should be erasable, so that unnecessary information can be written over as new information is added. An optical disk recorder with read, write and erase capabilities, called the Video High Density (VHD) system, was demonstrated earlier this year by Matsushita Electric Industrial Co. Ltd. of Japan. It will be launched commerically next year.

According to Jim St. Lawrence, the widespread use of optical disks with small business computers and home computers won't occur until costs drop and standard interfaces are established, probably within four or five years. "But once that occurs," he says, "the possibilities are endless."

The amount of information that can be stored using optics is nothing short of astonishing. One disk can already contain words or pictures equivalent to 50,000 or more color printed pages, and machines now on the drawing board have densities 10 times higher. When connected to a microcomputer, a disk player can retrieve and display on a video terminal, in a thousandth of a second, any one of the images.

For uses in education and entertainment, computer program designers are now working on systems that would make it easy for a user to jump between many different images and texts.

Don Sawyer, a research scientist at Bell Northern Research Ltd. in Nepean, just outside Ottawa, says optical disks linked to computers will have a major impact on culture and recreation. "We're talking about a whole new breed of artists. You can have a book, and add your own pictures and music. There will be stories where you can stop and, if you like, trace the life history of one character separately. And stories will be written with optional parts and the reader can choose which way it evolves."

A major advantage of optical storage systems is that they are ideally suited for use with optical-fibre transmissions. Optical-fibres are hair-thin strands of glass that are smaller, lighter and require less maintenance than conventional copper cables, but can carry far more information each second. Even though optical-fibre transmission systems consume far less power than copper cables, they can carry voice, data and video signals simultaneously, and can transmit them far greater distances before a repeater is needed to amplify the signals.

In June, Bell Canada announced that by the end of this year it will use fibre-optics exclusively for new cables between telephone switching offices.

Experts in optical technology agree we've hardly begun to tap the potential of computers that use light not only for storage and transmission of information, but also for processing. Some predict the optical computer will be a revolutionary tool that within 10 years will make today's most powerful computers look like dullards.

The fundamental components of any digital computer are switches capable of two different positions - zero and one, negative and positive, up and down. In an electronic computer, the switches are transistors, and even the fastest transistors in the most powerful microchips cannot change states in less than about a nanosecond, or a billionth of a second. However, an optical device could theoretically switch from one state to the other in about a picosecond, or a thousandth of a billion of a second.

Although there have been significant discoveries in the field of machine intelligence, progress has been hampered for lack of an optical device that could accomplish the switching function performed by transistors in electronic machines.

A solution may have been found. Earlier this year, Eitan Abraham and his colleagues at Heriot-Watt University in Edinburgh announced they had developed an experimental version of an optical transistor, capable of switching times of a few picoseconds. The device, called a transphasor, would initially be used to build optical computers with a logical organization much like today's conventional machines.

However, according to Prof. Abraham, "Optical switching systems need not be thought of merely as rapid substitutes for electronic devices. On the contrary, the greatest benefits of optical switches could come from applications that cannot be duplicated by other means." Such applications would be extensions of the work currently being done in 'expert systems' - computer programs that use complex rules and relationships to teach a machine how to make judgments that normally would be made by a human mind.

Although optical computers will perform tasks impossible with electronic machines, the microelectronics industry is in no danger of fading away. Optical technology will be gradually integrated into electronic systems where performance and cost-effectiveness justify the expense, but electronics will continue to be cruical to virtually every aspect of information technology.

Optical computers with artificial intelligence are still confined to the laboratory, but when they emerge during the next decade their capacity to store and rapidly interact with visual information, and their ability to operate at the speed of light could dramatically reshape our attitudes toward education, business and culture.

DO YOU KNOW ?

Mathematics Teaching Centre (Secondary)

The Mathematics Teaching Centre (Secondary) has recently moved to the Advisory Inspectorate Teaching Centres at 4 Pak Fuk Road, North Point, Hong Kong. Visits to the centre may be arranged by appointment with the Senior Inspector (Mathematics Teaching Centre) on 5-614364.

Second Hong Kong Mathematics Olympiad

The Heat Events of the Second Hong Kong Mathematics Olympiad will be held in December 1984 while the Final Event will take place in February 1985.

Computer Education Section, Advisory Inspectorate

The Computer Education Section with an establishment of four inspectors was set up in June 1984 to take care of the subject Computer Studies and matters relevant to computer education.

FROM THE EDITOR

I wish to express my sincere thanks to those who have contributed articles and also to those who have helped in the preparation of this issue of SMN.

Grateful acknowledgement is made to the editors of Mathematics in School and enRoute Magazine and also to Mr. K. Robin McLean, and Mr. Michael Kieran for their kind permission to reprint the articles.

Readers are cordially invited to send in articles, puzzles, games, cartoons, etc. for the next issue. Contributions need not be typed. Write : The Editor, School Mathematics Newsletter, Mathematics Section, Education Department, 4th floor, Lee Gardens, Hysan Avenue, Hong Kong.

For information or verbal comments and suggestions, please contact the editor on 5-8392488.