SCHOOL MATHEMATICS NEWSLETTER

學校數學通訊

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SMN

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<<學校數學通訊>>旨在為香港數學教育界提供一個溝通渠道，故此懇請各校長將本通訊交給貴校所有數科教師傳閱。

為使本通訊能成為教師的投稿公開園地，歡迎讀者提供任何與數學教育有關的文章。唯本通訊內所發表的意見，並不代表教育署的觀點。

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IN THIS ISSUE

affecting the mathematics education in Hong Kong in the near future? The introduction of the 'Targets & Target-Related Assessment' Project in the Primary One to Secondary Five mathematics curriculum and the implementation of the new Advanced Supplementary Level and the revised Advanced Level mathematics syllabuses in the Sixth Form mathematics curriculum are of course great events reshaping the mathematics education in Hong Kong in the 1990s. The 35th International Mathematics Olympiad to be held in Hong Kong in 1994 will also add much excitement to this new mathematics era. Therefore the Mathematics Section would like to appeal to all mathematics teachers/educators to make use of the channel provided by the SMN to express their views and suggestions on issues relating to the mathematics education in Hong Kong.

The Mathematics Section would like to thank once again all those who have sent in their articles and the fellow colleagues in the Section who have spent valuable time in producing this publication. Their contributions and efforts will be very much appreciated.

Mathematics Section,
Advisory Inspectorate.

SHARING OF TEACHING IDEAS :
An Extension of the Use of Geoboard in Form One

Ida Mok

When the Pick's theorem on the area of polygons is discussed in form one, the lesson usually proceeds as below :

1. Produce different polygons on the geoboard.
2. Find the area of the polygons and count the number of lattice points. Fill in the data in a table. (appendix)
3. Draw conclusion.
4. Apply the Pick's Theorem to find area of more polygons given in the exercise in the textbook.

However, enthusiastic students usually will not be happy with this alone. What is the reason supporting this Pick's theorem obtained by induction? Are there any other use of the theorem as most of the polygons we come across are not on the geoboard? These two questions naturally rise. Here, I try to suggest some possibility to answer these questions through student activity worksheets.

Worksheet I suggests a simple proof for the Pick's theorem by triangulation and comparing the angles of the triangles. It is designed such that students can work out a simplified

FOREWORD

This is the eleventh issue of the School Mathematics Newsletter (SMN). The Mathematics Section of the Advisory Inspectorate is very grateful to the mathematics teachers/educators who have contributed their articles and made this present issue come into existence.

The articles in this present issue cover a variety of topics, ranging from views on the mathematics curriculum and sharing of teaching experiences to interesting incidents/facts on the 32nd International Mathematics Olympiad (IMO) and recreational pastimes. There is also a new item "For Your Information" giving information of recent events/issues which might be of interest to readers.

It is always the wish of the Mathematics Section that the SMN can serve as an open, convenient and frequently used channel for people interested in mathematics and mathematics education to share ideas, experiences and suggestions so as to promote and improve the mathematics education in Hong Kong. The exchange of ideas and experiences is particularly important when there are new events happening. Will there be any new events

The School Mathematics Newsletter aims at serving as a channel of communication in the mathematics education in Hong Kong. School principals are therefore kindly requested to ensure that every member of their mathematics staff has an opportunity to read this Newsletter.

We welcome contributions in the form of articles on all aspects concerning mathematics eduction as the SMN is meant for a forum for articles written by teachers for teachers ; however, the views expressed in the articles in the SMN are not necessarily those of the Education Department , Hong Kong.

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proof with the knowledge of the sum of interior angles of polygon and angles at a point. Worksheet II suggests a chance to verify the extension of the Pythagoras theorem on the geoboard using the Pick's theorem. It provides an opportunity to use Pick's theorem and the geoboard to produce some other mathematical results. Worksheet III suggests a proof for the Euler's formula of polyhedrons from the Pick's theorem. This may give a surprise to the students as they realise that they can prove a relation in solid geometry by the geoboard. It also suggests a way how the Euler's formula can be proved to Form 1 pupils.

One reason for putting the Pick's theorem in the curriculum is that it provides a chance for exploration. The geoboard gives a good source for investigation in mathematics lessons. It is a pity that sometimes teachers and students cannot make good use of this opportunity. The activities suggested aim at making better use of this handy apparatus.

Reference

1. Detemple D. & Robertson J. M.; "The Equivalence of Euler's and Pick's Theorems", Mathematics Teacher, Mar. 74, 222-226
2. Ewbank W. A.: "If Pythagoras Had A Geoboard ...", Mathematics Teacher, Mar. 73, 215-221
3. Courant R. & Robbins H.: 1941, What is Mathematics?

Worksheet One

To Prove the Pick's Theorem

Part A

Fundamental triangles are triangles with no interior lattice points and no edge lattice points other than vertices.

(1) What different types of fundamental triangles can you make?

(2) What are the area of a fundamental triangles? Why?

Part B

Take any polygon with vertices on the lattice points. Triangulate the polygon with fundamental triangles.

(1) Count the number of fundamental triangles. What is the area of the polygon?

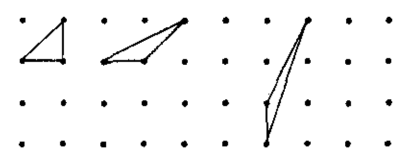
(2) What is the sum of all angles of the fundamental triangles?

(3) There are different ways of obtaining the sum in (2). Compare them and can you suggest a way to prove the Pick's theorem?

Teaching Notes :

Part A

1. Some examples of fundamental triangles are given below.

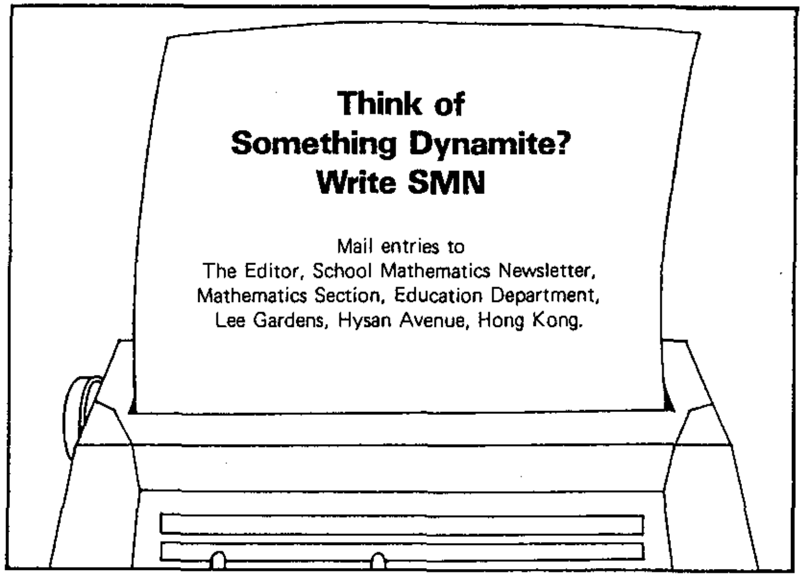


2. The area of each fundamental triangle is 1/2.

Part B

1. If the number of fundamental triangles is F, then the area of the polygon is F/2.

2. The sum of angles of all fundamental triangles is πF. Another expression for the sum is $\pi(B-2)+2\pi I$ which is the sum of the interior angles of the polygon and the angles at the interior lattice points.



Worksheet Two

The Extension of the Pythagoras Theorem

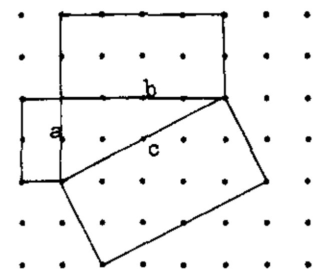
Make a right-angled triangle on a geoboard. Make different similar figures on the sides of the triangle. Find the area of the similar rectangles and fill in the table.

A = area of rectangle on side a

B = area of rectangle on side b

C = area of rectangle on side c

A	B	C



What conclusion can you make?

Teaching notes :

Proposition 31 of The Elements states that :

"In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle ."

- Euclid

Worksheet Three

The Euler's Formula from the Pick's Theorem

The Euler's Formula states that in a simple polyhedron let V denote the number of vertices, E the number of edges and F the number of faces, then always V-E+F = 2.

Now, we are trying to prove the Euler's formula from the Pick's theorem.

We remove a face from a polyhedron and stretch it to form a plane network. Figure 1 shows the network of a cube after this stretching. Since one face is removed, the value F of the network is less than that of the cube by one, but the value of E and V are the same. If we can show that V-E+F = 1 for the network then the Euler's formula is proved.

For simplification, we carry out the procedure on the cube. Place the network on the geoboard with vertices at lattice points (figure 2). Remember that stretching will not affect the result of V-E+F.

figure 1

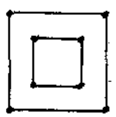
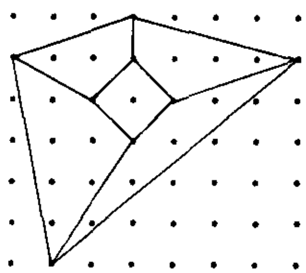


figure 2



Fill in the first line of the table.

V	E	F	V-E+F

Add lines by joining vertices together or joining vertices to interior lattice point to form fundamental triangles. The value of V, E and F may change after each addition. Fill in the rest of the table to record the changes.

What do you notice?

Triangulate the figure with fundamental triangles. Using this figure, write down the value of V, E and F in terms of A, B and I.

V = ________________

E = ________________

F = ________________

Using the Pick's Theorem, calculate the value of V-E+F.

Teaching notes :

Forming a fundamental triangle by joining two vertice will increase both the value of E and F by 1. Forming a fundamental triangle by joining two vertice to an interior lattice point will increase both the value of V and F by 1 and the value of E by 2. Hence, the value of V-E+F will not be changed during triangulation.

V = B + I

E = 3A + B/2

F = 2A

A = B/2 + I - 1 (Pick's theorem)

V-E+F = (B + I) - (3A + B/2) + 2A

= 1

Appendix

To Find the Area of a Polygon on a Geoboard

figure	B	I	A	B/2+I

B = the numebr of lattice points on the boundary

I = the numebr of lattice points in the interior

A = the area of the polygon

The Pick's theorem : $A = B/2 + I - 1$

習作在統計教學上的效能

黃毅英

一九八六年，筆者等(1)曾向全港三百三十八所中學，一千三百二十一位數學教師進行調查，研究統計在中學的教學情況，其報告於同年八月在加拿大舉行之第二屆國際統計教育會議中發表，調查中發現23.5%的教師認為中一至中五學生覺得統計比數學科較為無趣；反過來，只有12.8%的同學喜歡統計多於數學。在預科的情況，較喜歡數學的 6.2%而較喜歡統計的則為 3.9%。

至於如統計誤用等與考試範圍無直接關係的教學情況：4.5%的教師會對此等部份完全省去、55.2%不太講述，而只有 2.3%表示有足夠的闡釋。然而不少教師在調查中均回應現時教科書處理統計之內容不夠生活化和生動，不著重概念而只顧技巧。

隨着普及教育之推行，教學內容之生活化更形重要。若說普及教育之目的乃使未來公民獲得基本的知識、技能和素養，中學裏的數學內容中，大部份都不能在生活中直接得到應用。對於中學畢業後再不從事與數學有關工作的同學們，一大堆代數、三角、幾何的公式就只有一個去處：俏俏的從腦海中溜走，他們從數學學習中獲得益處之唯一可能，恐怕就是超越出公式的死板記誦和代入，而從中學習數學特有的解難策略及有條理的思考方法。

然而，統計與概率便稍有不同了。在資訊發達的社會裏，統計要算是一般市面最頻密接觸的數學內容。在傳媒中常見的財經走勢、產品銷量、玩具安全實驗報告等，要對統計有一定的認識，方可瞭解這些資訊的確切程度。

筆者等在上述的報告中續指出「習作之運用得不到重視，此能引起學生興趣的教學工具絕少得到運用」，更建議：「各團體，諸如香港統學會、大學之統計系及各學生統計學會應鼓勵舉辦中學生之統計習作比賽」。跟進了此項建議，香港統計學會即於該年(一九八六年)起舉辦全港中學生統計習作比賽；一九八八年開始分成初、高中兩組，而自一九八九年起則在分析數據之上容許參賽隊伍自行搜集數據、增加了比賽的趣味。

以下，筆者就數年帶隊參賽的經驗，與大家分享實際組織同學參加此類活動的詳情：

(一)　組隊與訓練

不少比賽活動都會被過份強調選拔精英與比賽成績所桎梏。故此，若組織者在同學之中精挑細選出數學成績較佳組隊、刻意訓練，不僅增加比賽壓力，且受益者就只局限於參賽的寥寥幾個。與上面引起學生學習統計興趣的原旨可謂大相逕庭。鼓勵同學自由組隊、選定題目並不限組數，此舉會令同學的學習更為主動，如此，導師反而不須擔心參賽的質素，因為如此的安排已拓造了積極的氣氛。

隊伍組成後，導師可稍為與同學們重溫一些統計的基本知識，既然有講述統計知識的安排，筆者建議乾脆在校內辦一些小型統計講座、讓其他同學亦有參加機會。

於是，每一參賽隊伍就可開始分工及訂定進程，工作步驟可能包括數據搜集、圖表陳列、分析、撰寫與圖表繪畫等。第一步自然就是先訂定題目了。

(二)　題目選定

題目的選定亦頗重要。好的題目未必一定能產生好的習作，但一些題目卻是很難作出數據搜集和分析的，導師應於早期提出。例如研究新市鎮的公共交通是否足夠，我們必須首先克服各類交通工具的量化問題，例如甲市鎮有五路巴士到市區；乙市鎮只有兩路，但有地鐵抵達。我們如何作出比較呢？接載人數？班次？收費？行走時間？輪候時間？這些恐怕均較難處理的。我們在附錄一中列出自第一屆獲獎作品之題目，以供參考。

此外，亦可同時考慮探討的範圍，常見的有：

1. 單項的探討。如：九一年港人消耗豬肉數量。
2. 多項探討。如：九一年港人各項肉類消耗量。
3. 單項或多項在總數中之比例：九一年港人各項肉類消耗量之比較。圓形圖是較常用的。
4. 單項在一段時間中的變化：近十年港人豬肉消耗量之增長。亦可對未來作簡單的預測。

5. 單項在總數中比例於一段時間中的變化：近十年港人豬肉消耗比重的增長。
6. 兩項相關性的比較：近十年肉食消耗與經濟增長的關係。

(三) 數據陳列與圖表

選定適當的統計圖表將數據陳列乃為至要。中學數學課程裏談到的統計誤用便舉出甚多此類的例子。事實上，透過統計習作的活動，同學們當更難掌握適當統計圖表的選。一九八六年香港中學生統計習作比賽的評審委員報告曾詳述了在習作中常見這方面的問題，今撮要譯出並放於附錄二中。

此外，隨着電腦的普及化，不少參賽者均以電腦軟件繪圖。例如 LOTUS 及 MATHCAD 等均是甚流行者，以電腦軟件協助繪圖當然較為方便，準確。有錯時修改亦易，然而歷屆得獎作品中觀察、不少手繪圖表也十分精確美觀；而恐怕評審的準則亦會着重圖表的適當運用及準確性多於視覺上的美觀吧！

(四) 撰寫與總結

由於要趕及參賽截止時間，在數據搜集之同時，亦可着手撰寫的工作。一般可分以下五部份：

1. 導言：闡述問題之出發。
2. 數據：收集過程與所得之數據。
3. 陳列：以適當圖表陳列。
4. 分析：按數據分析及總結。
5. 討論及附錄：可按現狀作一些討論或建議等。

報告中之分析中最大的毛病莫如下一些不從所得的數據或非用統計方法之結論。例如發覺近十年肉食消耗量與國民生產總值均有上升趨勢、我們難以斷定肉食消耗乃由於國民富裕而增加，也可能是因時代轉變，大家飲食習慣不同。又例如發覺近年離婚個案上升，就結論到一些離婚的原因；又或說比外國仍低等。因為外國數據已超越習作比賽規則的範圍。

(五) 活動的拓展

在現時，香港統計學會舉辦的中學生統計習作比賽仍是該類中唯一的。筆者認為其他團體實可多辦不同程度的統計習作比賽，促進學生學習統計學之興趣。而在個別學校中，透過數學學會、學生會、社組織等舉辦適合各校情況不同規模的比賽亦為甚值得考慮的。在校內舉辦，可有更大的彈性，又可探討與學校有關的話題，使習作內容更具趣味。

註 (1) Cheung P.H., Lam K., Siu M.K., & Wong N.Y. (1986). An Appraisal of the Teaching of Statistics in Secondary Schools of Hong Kong, Paper presented at the 2nd International Conference on Teaching Statistics, Canada.

附錄一　　歷屆中學生習作比賽獲獎題目

1986-87年度

Hong Kong Populations (1976-1996)
Hong Kong Domestic Export of Textile Industry
從人口年齡分佈看香港人口老化問題
Tourism in Hong Kong
Workers in Hong Kong
香港人口老化現象
香港的人口變化
A Project on Textile Industry
近十年香港之房屋發展概況

1987-88年度

(初級組)
結婚在香江
The Rise of Productivity in the Manufacturing Industry
The Change in the Life Expectancy at Birth of the Hong Kong Population (76-86)
現代化與公共交通運輸工具
探討香港離婚日趨嚴重的影响及原因
Tourism in Hong Kong
香港航空業近年發展概要
Containerization of Hong Kong
Investigating the Top Three Killer Diseases in Hong Kong
Evaluation of the Contribution & Importance of Tertiary

Production in Hong Kong in Terms of the Employment Level and Wage Level
Domestic Export of Clothing in Hong Kong

(高級組)
Trends of Mortality and Causes of Death
Project on Employment, Wages and Household Expenditure
近十年本港之進口食品消費概況
Hotel Industry in Hong Kong
香港家庭結構的轉變與影响
Tourism in Hong Kong 1981-1991

1988-89年度

(初級組)
A Study of the Labour Market of Hong Kong
Occupational Accidents in Hong Kong
The Construction Industry in Hong Kong
香港人口統計－死亡
Tourism in Hong Kong
Public Transport
The Development of the Shipping and Port Industry in Hong Kong
A Study on School Medical Service Scheme (1978-1987)

(高級組)
香港陸上交通工具數量與人口及經濟之關係
Hong Kong's Population and Its Social and Economic Aspects
Have Hong Kong Got Richer ?

Housing in Hong Kong
Tourism
Mortality in Hong Kong
Time Series Analysis of Hong Kong's Imports and Exports

1989-90年度

(初級組)
Women Today: Employment, Marriage and Family Planning of Women in Hong Kong
香港醫療
Trends of the Characteristics of the Restaruant Sector
Tourism - Smokeless Industry
Life and Death - an Analysis
Statistics of Employment
Drug Abuse in Hong Kong
香港人口分析
環保在香港

(高級組)
香港近十年來老年人口增長與政府提供老人住院照顧服務情況
Urban Decay and Environmental Deterioration
Tourism in Hong Kong
Public Housing in Hong Kong (1979-1989)
Energy Consumption in Hong Kong
Municipal Wastes in Hong Kong
Women in Hong Kong

1990-91年度

(初級組)
Love Affairs in Hong Kong (Trends of Marriage in Hong Kong over the Past Decade)
Public Transport in Hong Kong
青少年濫用軟性藥物情況是否值得憂慮?
Crimes in Hong Kong
Energy
Research on Drug Addicts over the Past Decade
Out-Patient Attendances at Government and Government Assisted Hospitals

(高級組)
Review on the three Public Examinations and Tertiary Education in Hong Kong in 1980's
Juvenile Crime
Regional Migration in Hong Kong 1961-1986
中學生選購運動鞋的準則
Study Environment Vs Academic Performance
Foot-Health and Wealth

附錄二　　節譯1986年統計習作比賽評審委員報告

主題題目
以明確為佳。

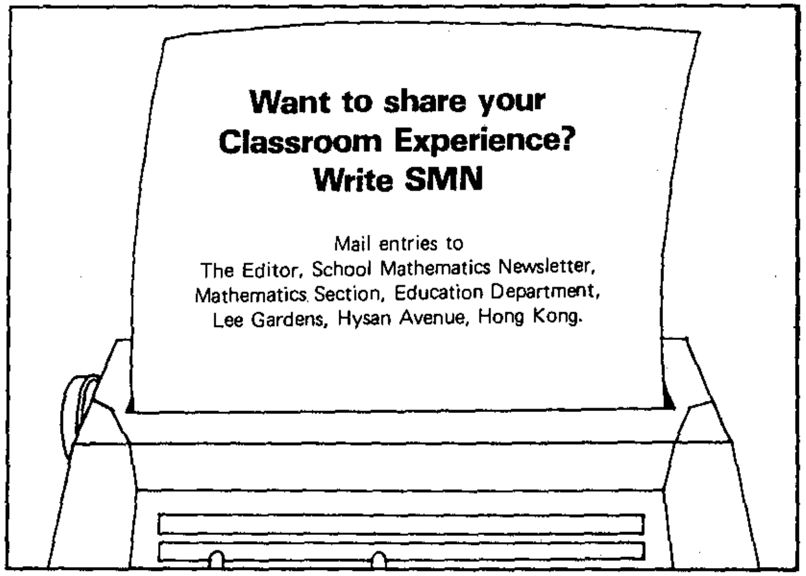
數據
說明其來源。

分析
一些習作缺乏分析，另一些誤用統計工具。統計工具不在乎高深，而在乎用之恰當。平均、百分比等亦能作出很多的分析來。

數據陳列
常見毛病
1. 頁數與圖表並無編號；
2. 圖表並無標題；
3. 圖表與標題不乎；
4. 內文並無引用圖表；
5. 兩軸並無標題；
6. 兩軸劃度不確；
7. 兩軸劃度不均勻；
8. 兩軸劃度不以「百萬計」，取代1,000,000,000；
9. 原點非為 0；
10. 矩形圖以 Y軸度數作代表面不用面積；
11. 圖表誤用，例如無關連之幾項而用綫圖等；
12. 在兩可比較的圖表間顏色、次序等不劃一。
其他問題如無引言與結論等。

解釋與結論
其中以誤解的問題最嚴重；又有結論非從數據分析所得的情況。

違反賽規
數據來源不依規定、又或反之照搬現成之數據。此外，亦有一些習作非為統計習作，亦即花絕大部份作與統計無關之論述等。



回應《略談運用統計圖表的一些問題》

老馮

閱罷黃毅英君在第九期通訊所發表的文章，深覺得益非淺。然而文中關於頻率多邊形的論點卻值得商榷。

首先，就算在理論中有連續的資料，但由於在量度的過程中必然出現誤差（比方說一把直尺就只有準確到毫米的刻度），所以現實中數據也只能達到某個精確度，也必然是離散的。這也是我們要學「有效數字」的原因。

筆者認為，使用頻率多邊形的目的在於反映資料的整體規律和趨勢（如股市走勢）。另外，當有多過一種數據存在的時候，比較兩個或以上的頻率多邊形就比使用條形圖或圓形圖等來得明朗。此外，除非使用電腦，否則在資料項數多的時候，繪畫條形圖或圓形圖等就會相當吃力。

附圖表達元素週期表中首十八種化學元素的原子數（atomic no.）和溶點（melting point）與及溶解潛熱（heat of fusion）之間的關係。圖中我們可以清楚見到溶點和溶解潛熱之間有極其相似，以原子數週期為8 的方式變化。如果以條形圖或圓形圖來表達，除了繪畫困難以外，上述所說的相似性和週期性就變得相當隱晦了。當然，嚴格來說附圖只是一幅折線圖而非一個頻率多邊形，不過同樣的論調還是可以說得通的。

以下幾點提議，或可作為參考：

一．只有在分析資料的整體規律或趨勢時才考慮用頻率多邊形。

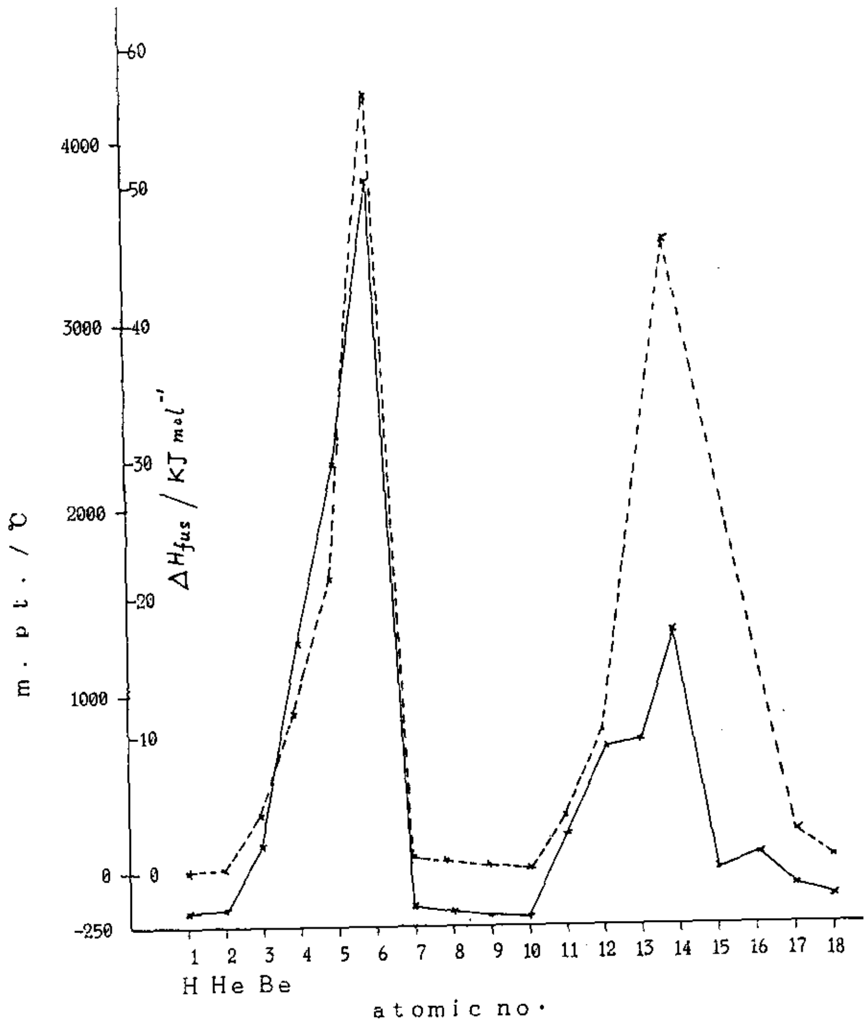
二．資料須量化。黃君文中圖一的例子已說明這點。如不量化，「走勢」便無從說起。

三．資料項數不多和只有一種數據的時候，不要使用頻率多邊形，因為在此情況下繪畫條形圖等並不困難，而分析走勢亦無意義。

四．提防錯誤的內插（interpolation）。譬如在附圖中，如果只標出原子數為單數的點而用折線聯起它們，就大大不妙了。對於只以單位遞增的獨立變數（如原子數、個數、負數等），最好可以收集到所有連續項（consecutive terms）的數據。

對於離散數據，頻率多邊形中的折線線段的確是沒有意義的，但卻未必須為此而放棄使用頻率多邊形。不過如黃君所說，要避免學生感到無所適從是必要的。

以上只是作者的一點意見，還望大家指正。現時中學生似乎都將統計理解為光背公式、只有計算的科目。黃君的文章就很有啟發性。未知他可會多寫幾篇文章以饗讀者？



Variation of melting point (solid line) and $\triangle H_{fus}$ (dotted line) for the elements H to Ar

A NEED FOR RESHAPING SECONDARY MATHEMATICS CURRICULUM

TSE Ping-nam

Introduction

Over the long term, changes in mathematics and mathematics curricula have been continuously taking place throughout the world. This is inevitable as mathematics is now growing more rapidly than ever before both in its basic and applied aspects. It is, however, unfortunate to see in Hong-kong that the secondary mathematics curriculum (especially Additional Mathematics) remained nearly fixed in basic contents for so many years. In addition, the implementation of compulsory education scheme is at present leading to serious cases of academic indigestion. Is there any justification for expecting all students to attain the same standards of performance? Is the present mathematics curriculum able to cope with the increasing need for mathematically trained people? Do the present syllabuses reflect the growing role that mathematics and its applications are playing in our society today? To what extent does the present curriculum promote or hinder mathematics teaching?

This article discusses a number of such issues and attempts to provide suggestions for the continual and healthy development of mathematics education.

Mixed Ability Schooling

As Hongkong moves towards universal secondary education, every teacher becomes more aware of the great differences in mathematical (as well as general academic) ability to be found in students. However, appropriate changes in the mathematics curriculum have not correspondingly taken place. The curriculum is still too academic and attainment-oriented, dominated by the goal of university entrance, and not able to cope with the wide range of student abilities. It thus forces students to attain a uniform standard rather than caters for their individual needs. To the contrary, educators now generally accept the proposition that there ought to be some differentiation in instruction in view of the recognized differences in ability, experience, and learning styles of students.

At present, about 90% of our Form 3 leavers can continue up to Form 4, whereas twenty years ago secondary education was offered only to the top 30% of the primary school leavers. So the fact that our school curriculum has remained basically unchanged in the past twenty years implies that our curriculum structure was originally designed only for what is now equivalent to our band one or at most band two students! A curriculum

which was planned for a selected 30% of the student population is certainly not appropriate for the 90%.

As a result of mixed ability schooling, there is a strong need for more flexibility in adapting the school curriculum, choosing suitable teaching materials, and injecting more local color and style. And unless students are allowed more freedom to choose what they wish to digest, the situation will get worse.

The Pedagogy of Applications

In recent years attention has shifted from rote learning and focused instead on so-called "problem-solving". This is a welcome development not only because mathematics is too complex and too vast to learn by rote, but also nearly every important use of mathematics does indeed involve problem-solving. However, the spoon-feeding method in Hongkong --- which some teachers find it necessary because of the unsuitability of our present syllabuses for the wide ability range in our schools --- has little in common with this focus of mathematics education. The heart of applied mathematics to problem-solving is : "Here is a situation; think about it." On the other hand, the heart of our usual mathematics teaching is : "Here is a problem; solve it." or "Here is a proposition; prove it." We have very rarely in mathematics allowed students to explore a situation for themselves and find out what the right problem to solve may be. It is this absence of individual exploration that actually makes for bad mathematics teaching. We are, in fact, giving a dishonest picture of mathematics if we do not allow students to participate in finding the right problem or theorem. The proverb that once a mathematician knows what he is trying to prove his job is half over, may be an over-simplification. Nevertheless the normal state of mathematical activity is one which involves the two aspects : a situation that is crying out for understanding, and a search for the right way to look at it. Unfortunately, we often exclude this intuitive discovery aspect of mathematics from our teaching.

The U.K. Cockcroft Report, entitled "Mathematics Counts" states that effective mathematics teaching at all levels should include the six approaches : (1) exposition by teachers; (2) discussion; (3) appropriate practical work; (4) consolidation and practice; (5) problem-solving; (6) investigational work. However, in Hongkong, the spoon-feeding method has little relation with most of these six approaches. Probably one of the major tasks of the curriculum builders is to keep these six requirements in mind while reshaping the curriculum. Also they should need to bear in mind the needs of our weaker students.

The Subject Content

Students in Forms 4 and 5 are now following syllabuses for Mathematics and/or Additional Mathematics --- the latter being offered primarily for gifted students in science and mathematics. The two syllabuses have remained basically unchanged in the past twenty years, though minor changes have taken palce such as the unification of two former mathematics

syllabuses into the existing one and the cancellation of Paper III (Mechanics) from Additional Mathematics syllabus. Students are still offered the same curriculum, the same depth of study and the same time allocation to the subjects. Still, there is no differentiation at all in the curriculum for different student abilities.

In addition, most students have a common belief that mathematics is to be pursued only in a clear-cut, logical fashion. Probably this belief is perpetuated by the way mathematics is presented in most textbooks --- often it is reduced to a series of definitions, theorems, and methods to solve various types of problems. And the theorems are usually justified by means of proofs and deductive reasoning. I do not mean to minimize the importance of proof in mathematics, for it is the very thing that gives mathematics its strength. But the power of imagination is as important as the power of deductive reasoning. As the mathematician Augustus De Morgan once said, "The moving power of mathematical invention is not reasoning but imagination." So why don't we start doing something to help our students?

The following should be considered for the future shape of school mathematics. Firstly certain primitive concepts and other key ideas which recur throughout the study of school mathematics should be added, for example, sets; logic; relations; functions; mappings; and so on. Many students are found in their working to wrongly use without understanding symbols like " => " and " <=> ". So an elementary treatment --- we should however avoid playing with students games of lengthy "truth table" problems --- would help them to make good use of those concepts in studying other topic areas as well as solving problems.

Secondly, a variety of adequate activities are needed which embrace imagination and reasoning, skills and techniques, and processes. Such aspect includes : (1) facts, definitions and assumptions; (2) language and symbolism; (3) deductive reasoning and proof; (4) informal, plausible, inductive reasoning; (5) graphical representation and interpreation; (6) algorithms, generalizations and other procedures including numerical procedures. That we often refer to as problem-solving necessitates the use of some or all of these things.

Thirdly, the use of mathematics outside the discipline itself must be illustrated as a vital part of any school mathematics program. Indeed, at least one period in time-tables should be dedicated to such activity. It is common and sad to see that our students are difficult or unable to interpret a gas or electric bill, nor are they able to perform upon their mothers' requests simple estimation and calculation of foreign-exchange conversions. A suggestion of different topic areas for this aspect of mathematics is : (1) home --- home management, buying and selling; (2) leisure --- bookings, games, contests, watching slides; (3) travel --- public and personal transport, hotels and self-catering; (4) other subject matter fields --- science, commerce and industry.

Fourthly, any reform of the existing syllabuses should leave room for some consideration of the recent progress in mathematics. Such an inclusion will certainly help not only clarifying some or most students' misconception that mathematics

is a "dead" subject but also, serving the purpose of mathematics popularization. In fact, the twentieth century, especially the past forty years, has proved to be a "golden age" for new discoveries in mathematics. For example, with the help of computer facilities, a brief introduction of such topics as catastrophe theory, chaos and fractals will certainly make our curriculum more exciting and enjoyable. Also with respect to teaching modern statistics, many countries have revised their school curricula to include some new topics which are unfamiliar to most teachers except those who are the most recent graduates of statistics or those who update their knowledge in the field. Examples are two new graphical representations --- stem-and-leaf plots and box plots --- which are found in school statistics syllabuses in many countries but may not be known to the great majority of teachers in Hongkong. I do not suggest that they should replace the classical statistical graphs such as histograms and pie diagrams but I do recommend that they should be considered for inlcusion in a chapter of recent mathematical progress.

Fifthly, I do not completely agree to the belief that the history of mathematics must be included as an explicit means for broadening the curriuclum. This is not because the history is unimportant nor because it is lacking in potential interest, but because I do not believe that teachers and students can do adequate justice of it at this level. Instead, historical notes focusing mainly on the people may be provided, as I believe that a glimpse of the people of mathematics will provide a glimpse of the nature of mathematics.

Finally, there is currently much discussion over how the calculator can be used in classrooms and its implications for the curriculum. More emphasis should be placed on helping students to clarify mathematical concepts and to develop estimating skills, rather than on performing mathematical operations with large numbers. How and when to use the calculator effectively in the curriculum should now be considered.

Assessment and Examination

There is little point in discussing curriculum reform wihtout considering assessment and examination reform. At present, candidates taking Mathematics are assessed by one written paper and one objective test while those taking Additional Mathematics are wholly assessed by two written papers. The pass mark of most H.K.C.E.E. principal subjects is known to be around 30. So a student passed with Grade E in Mathematics may know a bit of algebra but nothing about trigonometry or geometry. This will not do any good to the student, schools, employers and the society. Therefore if there are totally ten basic knowledge items to be learnt by a student, then the criterion of an end-of-course examination should not aim at determining what the pass mark is but telling whether the student has equipped himself or herself with these ten basic items for further studies or work. Furthermore, those who mark the H.K.C.E.E. Mathematics examination are aware of the difficulty in discriminating the good candidates and the excellent ones --- probably the paper is too easy for them, especially for those also taking Additional Mathematics. The same difficulty can also be found in the poor performance of the weaker candidates. To them, the paper is obviously too

hard. Thus the present Mathematics examination is neither suitable for the top 40% of the ability range nor for the bottom 40%.

In the H.K.C.E.E. grades are essentially norm-referenced --- a roughly constant percentage of the candidates being allocated to each grade each year. In other words, our grades are indications of how well a candidate has performed in relation to others. Therefore in a norm-referenced examination which is attempted by candidates of a wide range of attainment, papers are bound to contain questions which are too difficult for those who attain low who attain low grades. We are, in fact, placing more emphasis on the efficient ordering of candidates and less on whether they are meeting the assessment objectives we have fixed. The most serious problem arising from such an examination is that it forces us into teaching approaches which we know are undesirable. If the concepts and processes one is trying to get across are too difficult or too abstract for the students one is teaching, then the tendency is to encourage repetitious problem-solving without understanding. This may lead to the precious Grade E, but to little else. Any interest in mathematics for its own sake, any appreciation of mathematical concepts and proofs, or any appreciation of mathematical application in everyday situations, is entirely lost.

In the U.K. the new General Certificate of Secondary Edcuation (G.C.S.E.) examination has moved from a basically norm-referenced standpoint towards a criterion-referenced one which is based on students' absolute performance. Criterionreferencing should in theory help candidates to demonstrate what they know, rather than what they do not know. If the G.C.S.E. examination turns out to be practicable, then it would, I believe, be worthwhile for Hongkong to consider something similar here, at least for a few foundation core subjects like languages and mathematics which are thought to be the basic competences a school leaver should possess. In fact, the TOEFL examination is a good, successful example of assessing candidates based on their absolute rather than relative performance.

Two New Mathematics Syllabuses in Forms 4 and 5

As said before, it is difficult to discriminate, based on the present Mathematics examination result, the performance of candidates of wide range of attainment. This single examination has certainly no discriminating power over such a wide range. So it would be better if different syllabuses and papers in a subject would be required for students of different levels of achievement. The grade descriptions of the Additional Mathematics examination, on the other hand, likely give a general indication of the standards of students' achievement --- since students in schools are usually selected to attend the subject. A Grade D, for instance, implies that a candidate will have shown a fair knwoledge of the subject content and is likely to be able to apply the knowledge, processes, and skills to structured situations, and to select a correct strategy to solve a multiple-concept problem.

It is therefore proposed that the two present mathematics syllabuses in Forms 4 and 5 (Mathematics and Additional Mathematics) should be amalgamated into two new syllabuses; let's call them Mathematics A and B. Candidates may only be allowed to take either of them in the same year. One syllabus, say Mathematics A, is primarily designed for the needs of students who want to have a well-informed mathematical literacy, more than just basic mathematical manipulative skills or the need of further study of mathematics. It is certain that the number of people who need mathematical training at this level is quite large --- probably everyone. Mathematics B, on the other hand, deals with a deeper level and is designed to broaden the mathematical experience of those candidates who could reasonably be expected to achieve Grade C or above in Mathematics A and intend to have a suitable basis for further study in mathematics and in other related disciplines. It does not mean, however, that a student taking Mathematics A may not enter for a science-related advanced course. Instead, both syllabuses should have the primitive aim of providing an acquisition of a foundation appropriate to the further study of mathematics and of other disciplines; but with Mathematics B designed for those who are thought to be particularly gifted in mathematics. So candidates passed with Grade B or above in Mathematics A are likely to have shown a good knowledge of the subject content. They are likely to be able to demonstrate powers of abstraction, generalization, and proof, and also their ability to continue their studies in mathematics to Advanced Level and beyond. Those who teach Advanced Level science subjects at present are well aware that their students need for their study a bit beyond Certificate Mathematics only.

The aims of both proposed syllabuses should also be broadened to cope with the necessity for employing the six approaches mentioned in "The Pedagogy of Applications". The set of aims for Mathematics paper of the G.C.S.E. examination in the U.K. is perhaps also appropriate for our purposes. I quote several aims which our present curriculum is unfortunately found missing :

1. development of oral, written and practical skills in a manner which enocurages confidence;
2. the ability to read mathematics and write and talk about the subject in a variety of ways;
3. development of an understanding of the part which mathematics plays in the world around them;
4. the ability of recognize when and how a situation may be represented mathematically, to identify and interpret relevant factors and, where necessary, to select an appropriate mathematical method to solve the problem;
5. the ability to produce and appreciate imaginative and creative work arising from mathematical ideas;
6. development of mathematical abilities by consideration of problems and the conducting of individual and cooperative enquiry and experiment inlcuding extended pieces of work of a practical and investigative kind;
7. appreciation of the interdependence of different branches of mathematics.

It must be noted, however, that the suggestion of combining the two present mathematics syllabuses into Mathematics A and B does not mean introducing a little bit of everything, nor does it mean diluting the existing syllabuses.

It should mean to introduce qualitatively as well as quantitatively different sorts of activity and experience so as to work towards the various aims set above. Such experiences could best be drawn from everyday situations so that students may respond to the demands of scoiety and the needs of the world at work.

Summary

This article broadly discusses some of the current issues in secondary mathematics curriculum, and preliminary suggestions for curriculum development and change are provided which represent some of my feelings and experience on mathematics curriculum as teacher. Of course, other problems as to what assessment pattern should be employed in the two proposed syllabuses require further investigation. My present major concerns include :

1. The curriculum must meet the varying needs and abilities of students in mathematics. It is presently being proposed to offer two different secondary mathematics courses after Form 3. But how to identify students for the courses and what specific topics to include is still a question.
2. Students must learn to be problem-solvers. In brief this means that students must have the capability and confidence to apply their mathematical skills towards finding a solution, especially when they are faced with a situation where the answer is not immediate.
3. There is a need to emphasize basic arithmetic skills. This arises from public concern that students are leaving schools without having the mathematical competency to

function in the society, and to pursue careers in which they are interested.

4. The role of calculators and computers in classrooms and their implications for curriculum design receive much attention. Because of their wide-spread availability, how to wisely incorporate these machines in the curriculum and the resulting changes are immediate issues.

I frequently encounter students who tell me about their unpleasant experiences with mathematics. I have true sympathy for them, and I recall one of my primary school teachers who assigned additional arithmetic problems as punishment. This can create only negative attitudes towards mathematics, which is, of course, unfortunate and undesirable. If curriculum reformers, teachers and parents have positive attitudes towards mathematics, their children cannot help but seeing some of the beauty of the subject.

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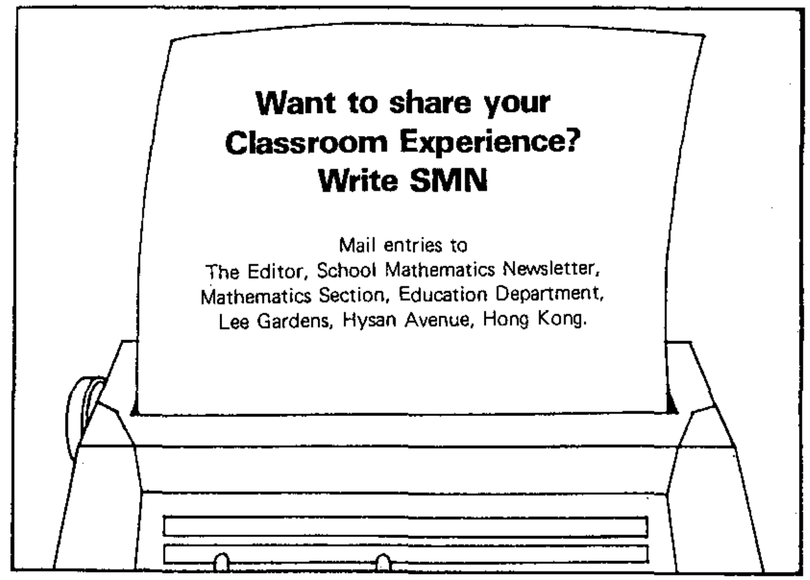


趣味速算

盧廣波

許多學生在沒有計算機在手時對數值計算難免面露難色。因此本人常常在授課時或課餘的數學活動中介紹一些心算和速算的方法，希望藉此給予他們在這方面的一點幫助。對於較高年級的同學來說，去証明這些速算法則的有效性亦是一種挑戰。以下是一些頗有趣味的速算法，原本不值一哂，但願能拋磚引玉。

(一) 14 × 19

處理方法如下：4 × 9 = 36
4 + 9 = 13
1 × 1 = 1
合加起來 = 266

14 × 19 = 266

15 × 17

處理方法如下：5 × 7 = 35
5 + 7 = 12
1 × 1 = 1
合加起來 = 255

15 × 17 = 255

16 × 12

處理方法如下：6 × 2 = 12
6 + 2 = 8
1 × 1 = 1
合加起來 = 192

16 × 12 = 92

透過以上的例題演繹，速算的方法大概已能清楚說明，但是這方法祇適用於兩個介乎十至二十內的兩位數相乘。

(二) 32 × 38

處理方法如下：2 × 8 = 16
(3 + 1) × 3 = 12
合加起來 = 1216

32 × 38 = 1216

93 × 97

處理方法如下：3 × 7 = 21
(9 + 1) × 9 = 90
合加起來 = 9021

93 × 97 = 9021

24 × 26

處理方法如下：4 × 6 = 24
(2 + 1) × 2 = 6
合加起來 = 624

24 × 26 = 624

以上的手法祇適用於兩個個位數相加等於十而十位數則相同的兩位數相乘的情形，例如 11 × 19、12 × 18、56 × 54、94 × 96 等。

國際數學奧林匹克競賽記趣

賴智強　香港隊副領隊

回想今年年初，香港數學學會邀請本人肩負起香港隊副領隊之職，負責帶領六名香港代表遠赴瑞典作賽以及閱卷評分之工作，當時著實有點猶豫，但又不想放棄參予此種國際盛事之機會，遂毅然答應，悉力以赴。

去年七月初香港區選拔賽入圍學生有六十多人，他們經過一年的集訓和測試後，其中六人被選為代表。由領隊梁達榮博士負責訓練及加強他們的解題技巧，而本人則負責照顧學生一切生活上的問題，以求適應當地的天氣及生活情況等，希望令他們除了能有一輕鬆愉快的旅程外，更能大開眼界。

領隊和兩位觀察員張百康先生和陳啟源博士比我們早兩天起程到 Uppsala，出席擬卷大會，在各地入圍之三十道擬題中，選出六題作為競賽題目，過程絕對保密。本人和六位代表則於七月十四日晚上啟程，途經泰國及哥本哈根，經廿小時航程後，終於到達瑞典 Sigtuna，隨即有一外籍接待員歡迎我們，他用很流利的普通話與我們交談，使我們驚訝不已。最慚愧的，莫如我們祗能用半鹹淡的普通話回應。

旅遊巴士瞬間便把我們送到一所環境幽美的中學－比賽的場地，也是我們的居處。這間中學附設有宿舍，而且整個舍堂均採用土產宜家傢俬，令人彷彿置身於宜家的陳列室中。

十五日大會安排我們外遊，可能是因為同聲同氣的緣故吧，無論早晚，香港隊總喜歡與澳門隊談天玩樂。十六日晚上我們到 Uppsala University 的禮堂參加盛大的開幕典禮，大會播出悠揚的樂韻使我們如置身於古典的歌劇院中。當天晚上我們還可以與領隊及觀察員遙遙相會。

十七日，比賽終於來臨了。各隊按號數被編到不同的試場應試。他們需於四個半小時內完成三條題目，每題佔七分。晚飯後我立即回到宿舍找領隊不獲，只能在書桌上找到當日的試題，我祇好獨自思考那些頗有趣味的試題。

比賽的翌日，我們目睹一些比賽場外的刺激情形。原因是賽會准許學生在開考後卅分鐘內以書面向大會發問。他們的問題會由一些年青力壯的「跑腿」踏單車由試場送到領隊們手中，然後又把領隊們的答案送回試場。當日車來車往，「跑腿」們非常落力，汗流夾背，我不禁默默的多謝他們。

此後，我們四人便日以繼夜的閱卷和評分。在一天之內要同時參考大會提供的題解，並將香港隊的三十六條題解給予評分，因為翌日便要與協調小組（Coordination Team）「交手」，為港代表爭取理想之評分，這是全旅程中最疲勞的一夜。

面對協調員，我和領隊需先替各隊員的題解擬分，並提出理由去說服協調員接受我們的意見，通過我們所擬的分數。這是一個極具挑戰性及重要性的關鍵，我們必須揣測評判的標準而給分，太少分當然對自己組員不利，太高的分數亦很容易被否決，如何才能

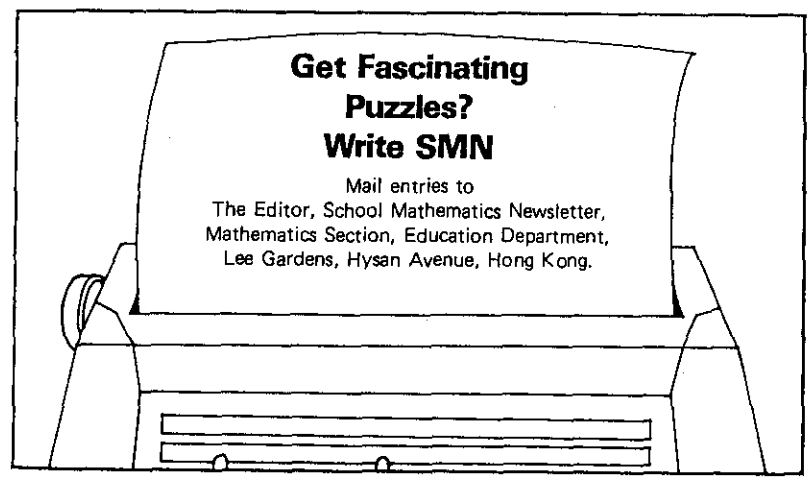
恰到好處，實在費煞思量。再者，學生在應考時，思路極為混亂，我們要從他們寫下的零碎答案中，東拼西湊，找出一條足可舖陳的脈胳來說服協調員。最困難的是要在很短促的時間內去完成這迫切的任務真使人透不過氣，但看到大部份的題目都獲得我們理想的分數，我們的努力並沒有白費，內心頓然充滿莫名的喜悅。

在協調進行的同時，各國得分亦即時張貼於總部，各代表不時到場觀看，緊張之情，溢於言表。有些更不停抄錄，分析形勢，好不熱鬧。見到自己拿得滿分，雀躍不已；拿到零分的雖然有點不快，最後亦泰然處之，彰顯體育精神。

總括來說，香港隊在是次比賽中，於組合、圖論及數論的題目上均表現不錯，但在幾何基礎太弱，失分太多，其他國家則輕取滿分，距離太遠，使香港隊排名二十九。獎牌方面，有兩位隊員榮獲銅獎及一名獲得優異獎，對他們來說，是一次難忘的經歷。

比賽過後，各人都獲安排遊覽瑞典名勝，大家都拋開連日來的作戰心情，輕鬆下來，彼此的友誼更躍進一步。二十三日晚我們一行十人參加閉幕典禮，在華麗寬敞的禮堂裡，五十多個國家的參賽者聚首一堂。得獎者魚貫上台接受他們的獎狀。當香港隊員上台領獎時，我們也自覺沾上一份光榮。閉幕禮在音樂，雜技，歌聲，笑聲及掌聲中結束，給我們留下深刻的回憶。儘管大家的語言及文化有異，但在歡樂的氣氛中，我們都感受到人類和諧共處之可愛。

這次瑞典之遊對本人來說實是人生大事，但對香港來說，能成為九四的主辦國可算是九七前一大盛事，估計將會有超過六十個國家參加。但慶幸之餘，我們先要面對一些難題。主辦國必須提供約六百人之起居飲食，保守估計，費用高達六百萬港元；其次是大會需要兼精數學及外語的人才參予籌劃及推行工作，這也是一個很大的困難。雖然如此，香港既是一個國際城市，能被選中主辦一個集世界數學精英於一堂的盛會，實在是香港的光榮。身為香港數學教學的一份子，當然希望能參予其中，對是次比賽寄予厚望。因此，我在此謹以至誠，希望香港政府及各界人仕，傾力以赴借出宿舍及比賽場地，有志於數學教育的同仁投身籌劃工作，積極推動，共襄此盛事。



STORIES OF THE 32ND IMO(1991)
SIGTUNA, SWEDEN

by

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- By accident, I was the leader of the Hong Kong International Mathematical Olympiad (IMO) team during the year 1991. Perhaps I should briefly describe what happened during the past few months.

- During the summer of 1990, around 300 secondary school students sat for a selection contest, subsequently around 60 students were chosen for further trainings. From the month July up to December of 1990, the students were taught topics in number theory, combinatoric and geometry. After December all training activities ceased because the students had to prepare for public or internal examinations. The students then sat for the Asian Pacific Mathematical Olympiad at March and the final selection contest at May of 1991. Subsequently 6 students were chosen as the team members of the HK IMO team. The 6 students were then given a bit more of training during the month of June. Prepared or not prepared, they were going to participate in the IMO to be held in Sweden.

- The 32rd IMO was held in Sweden from July 12 to July 23, 1991. We (two observers and I) arrived there several days before the team did. As the team leader I was a member of the Jury, composed of the leaders of all participating countries, a chairman and a deputy chairman from the host country. The first duty of the Jury was to choose 6 questions out of 30 questions. The 30 questions were chosen by the host country from the questions submitted earlier by the team leaders.

- About 10 of the 30 questions were rejected by the Jury because roughly of the following reasons: (i) a question was posted before, (ii) the solution of a question followed easily from a well known theorem, (iii) a question was not aesthetically pleasing, or (iv) a question might cause trouble during coordination (to be further explained later). After that the Jury concentrated to choose 6 questions by applying the following criteria: (i) questions of various degrees of difficulty should be included, (ii) questions of various topics should be included. Eventually 6 questions were chosen, they were used to examine the students. As it turned out, the choice was not necessarily perfect.

- Following the tradition, the team members took 2 $4\frac{1}{2}$-hours examinations during the mornings of 17th and 18th of July. 3 questions were asked in the 1st exam and 3 in the 2nd. After that the exam scripts of a team were handed back to the team leader. The leader and the deputy leader then brought the scripts to the coordinators during allocated times and they discussed (or argued) on how many marks should be awarded to a solution. The process was called coordination. Needless to say, we were not completely happy during the two days of coordination. However bear in mind that 2 coordinators might have to read about 100 questions during the two days and to award marks consistently, the process might be more painful for them.

- Our team won 2 bronze medals and a honorary mention during this contest. Although I did not expect our team performed as good as the strong teams I hoped that our team members could have done better. In particular if they handled the geometry problems more competently then their results would be more solid.

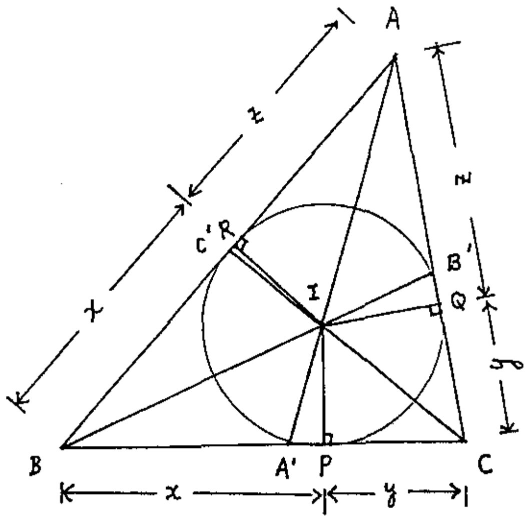
- I shall briefly describe the six questions.

- Question 1 : Given a triangle ABC, let I be the centre of its inscribed circle. The internal bisectors of the angles A, B, C meet the opposite sides in A′, B′, C′ respectively. Prove that

$$\frac{1}{4} < \frac{AI \cdot BI \cdot CI}{AA' \cdot BB' \cdot CC'} \le \frac{8}{27}.$$

- Our team did poorly in this question. This probably indicated that their training in geometry is not sufficient. More importantly I believed that this reflected a profound neglect of geometric reasonings in our secondary schools. One curious thing was, the secondary school students in Hong Kong did not have any proper training in combinatoric nor number theory, but our team did better in questions relating to these topics.

- The crucial point of this question is to express the ratios in terms of the sides of the triangle.



- Observe $\frac{AI}{AA'} = \frac{AA' - IA'}{AA'} = 1 - \frac{IA'}{AA'}$.

And $\frac{IA'}{AA'} = \frac{\Delta BIC}{\Delta BAC} = \frac{x + y}{2x + 2y + 2z} = \frac{a}{a + b + c}$, (see diagram).

Hence $\frac{AI}{AA'} = \frac{b + c}{a + b + c}$. Similarly $\frac{BI}{BB'} = \frac{a + c}{a + b + c}$, and

$\frac{CI}{CC'} = \frac{a + b}{a + b + c}$. (According to the proposer, this fact is well known).

Using the arithmetic-geometric inequality, the right hand inequality immediately falls off, with equality holds if and only if a = b = c. Concerning the left hand (strict) inequality, the solution of the proposer is quite unnatural. I cannot resist the temptation of presenting a nice proof by a Turkey contestant. Let $p = a + b + c$, $\ell = b + c - a$, $m = a + c - b$, and $n = a + b - c$. Then $\ell + m + n = p$. And

$$\frac{b + c}{a + b + c} = \frac{1}{2}\,\frac{(a + b + c) + (-a + b + c)}{a + b + c} = \frac{1}{2}\,\frac{p + \ell}{p}.$$

Similarly $\frac{a + c}{a + b + c} = \frac{1}{2}\,\frac{p + m}{p}$ and $\frac{a + b}{a + b + c} = \frac{1}{2}\,\frac{p + n}{p}$.

Hence $\frac{AI}{AA'} \cdot \frac{BI}{BB'} \cdot \frac{CI}{CC'} = \frac{1}{8}\,\frac{(p + \ell)(p + m)(p + n)}{p^3}$

$$= \frac{1}{8}\,\frac{p^3 + (\ell + m + n)p^2 + (\ell m + \ell n + mn) + \ell mn}{p^3}$$

$$= \frac{1}{4} + \frac{(\ell m + \ell n + mn) + \ell mn}{p^3} > \frac{1}{4},$$

Using the facts $\ell > 0$, $m > 0$ and $n > 0$, we obtain the strict inequality.

- It is certainly worthwhile to see if similar estimates may be obtained if I is replaced by the orthocentre, or the circumcentre, etc.

- Question 2 : Let $n > 6$ be an integer and $a_1, a_2, \cdots, a_k$ be all the natural numbers less than n and relatively prime to n. If

$$a_2 - a_1 = a_3 - a_2 = \cdots = a_k - a_{k-1} > 0,$$

prove that n must be either a prime number of a power of 2.

- First we denote by T(n) the set of natural numbers less than n that are relatively prime to n. Essentially the question asks when does T(n) forms an arithmetic progression (A.P.). It was worried that the students did not know what an A.P. means, and hence eventually the question was formulated in such a curious manner. Now clearly if n is a prime number, then $T(n) = \{1, 2, \cdots, n - 1\}$, hence T(n) forms an A.P.. Also if n is a power of 2, say $n = 2^m$, then $T(n) = \{1, 3, \cdots, 2^m - 1\}$, hence T(n) again forms an A.P. with the common difference d = 2. Now it is necessary to prove that if n is not a prime number nor a power of 2, then T(n) does not form an A.P. Roughly speaking, the solution of the proposer goes as follows.

- (i) Let n be an odd composite (not prime) number, let p be a prime number that divides n. Suppose T(n) forms an A.P., then since $1, 2 \in T(n)$, the common difference d must equals 1. This implies $T(n) = \{1, 2, \cdots, n - 1\}$. However $p \notin T(n)$, and this implies a contradiction.

- (ii) Let n be an even composite number and such that 3 does not divide n. Suppose T(n) forms an A.P., then since 1, 3 $\in$ T(n), the common difference d must equals 2. However since n is composite and is not a power of 2, there exists a prime number p (p > 3) that divides n, and p $\notin$ T(n), hence again we obtain a contradiction.

- (iii) Let n be an even composite number, n > 6, and such that 3 divides n. Again assume that T(n) forms an A.P.. Now suppose p is the smallest prime number that does not divide n, then clearly the common difference d must equals p - 1, hence p - 1 must divide (n - 1) - 1 = n - 2. Suppose now q is any prime number that divides p - 1, then q must divide n - 2. However by the minimality of p, q also divides n, hence q divides 2 and q = 2. This implies $p - 1 = 2^s$, or $p = 2^s + 1$. It is not hard to show that if p is a prime number p must be of the form $2^{2^t} + 1$ (Fermat prime). This implies the first 3 terms of T(n) are 1, $2^{2^t} + 1$, and $2^{2^t} + 1 + (2^{2^t} + 1 - 1) = 2\cdot 2^{2^t} + 1$ respectively. However since $p \neq 3$, we must have $t \geq 1$, and therefore 3 must divides $2\cdot 2^{2^t} + 1$. And this again leads to a contradiction.

- Needless to say, the third case is the hard case. Nevertheless one might feel uneasy that a Fermat prime pops up in the proof. Afterall there are not too many known Fermat primes around. In fact, by exhibiting two numbers relatively prime to n that differs by 2, one of our students showed that the common difference must equals 2 and thus leads to a contradiction. Unfortunately his proof was not complete and he could only scored 6 marks (out of 7).

- After slight modifications, the new proof of the third case might go as follows.

- Let $n = 2^{s_1} 3^{s_2} p_1^{t_1} \cdots p_\ell^{t_\ell}$, where $p_1, p_2, \cdots, p_\ell$ are prime numbers satisfying $3 < p_1 < \cdots < p_\ell$. Then $2\cdot 3\cdot p_1 \cdots p_\ell \pm 1$ are 2 numbers relatively prime to n that differs by 2. Hence if T(n) forms an A.P. then the common difference must equals 2. However since $3 \notin T(n)$ we see that this leads to a contradiction. Note that the argument is also valid if $n = 2^{s_1} 3^{s_2}$. However there is still a catch in the argument, namely if $n = 2\cdot 3\cdot p_1 \cdots p_\ell$, then $2\cdot 3\cdot p_1 \cdots p_\ell + 1 = n + 1$, and is too big. Hence we assume $n = 2\cdot 3\cdot p_1 \cdots p_\ell$. Now since n > 6, we see that $p_1 p_2 \cdots p_\ell > 1$. Hence $p_1 p_2 \cdots p_\ell \equiv 1$ or 2 (mod 3). In case $p_1 p_2 \cdots p_\ell \equiv 1$ (mod 3), (example $n = 2\cdot 3\cdot 7$), then $2\cdot p_1 p_2 \cdots p_\ell - 1 \equiv 1$ (mod 3), and $2\cdot p_1 \cdots p_\ell - 3 \equiv 2$ (mod 3). We then observe that $2p_1 \cdots p_\ell - 1$ and $2p_1 \cdots p_\ell - 3$ are two numbers relatively prime to n that differs by 2. In case $p_1 p_2 \cdots p_\ell \equiv 2$ (mod 3), (example $n = 2\cdot 3\cdot 5$), then $2p_1 \cdots p_\ell + 1 \equiv 2$ (mod 3), and $2p_1 \cdots p_\ell + 3 \equiv 1$ (mod 3).

We then observe that $2\cdot p_1 \cdots p_\ell + 1$ and $2p_1 \cdots p_\ell + 3$ are two natural numbers relatively prime to n that differs by 2. Both cases will lead to contradictions if T(n) forms an A.P.

- Question 3 : Let S = {1, 2, 3, ···, 280}. Find the smallest integer n such that each n-element subset of S contains five numbers which are pairwise relatively prime.

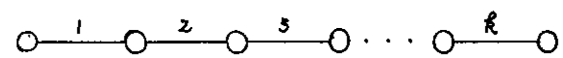
- Let A_1, A_2, A_3 and A_4 be subsets of S consisting of multiples of 2, 3, 5 and 7 respectively. By the inclusion-exclusion principle we calculate $|A_1 \cup A_2 \cup A_3 \cup A_4| = 216$. By the pigeon hole principle it is easy to see that $A_1 \cup A_2 \cup A_3 \cup A_4$ does not contain five numbers which are pairwise relatively prime. Hence 216 < n. It is natural to guess that n = 217. Of course the hard part of this question is to show this fact. Now the proof of the proposer is quite ad-hoc. It would be nice to see if there is any general principle that covers this type of questions. What happens if 280 is replaced by an arbitrary number m?

- Question 4 : Suppose G is a connected graph with k edges. Prove that it is possible to label the edges 1, 2, 3, ···, k in such a way that at each vertex which belongs to two or more edges the greatest common divisor of the integers labelling those edges is equal to 1.

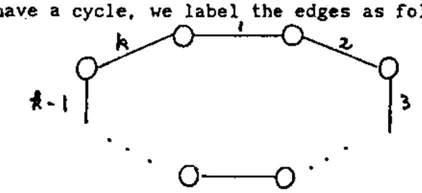
- (The definitions of a graph, a vertex, an edge, and connectedness were also given.)

- From the very beginning this question caused trouble. First there was the problem of defining the terms vigorously, whether the graph was a simple graph or a multigraph, whether the intersection of two edges formed a vertex, etc. Then there were worries of coordination, whether the students could properly express their ideas, whether the leaders were honest enough to present the ideas of the students to the coordinators, etc. For me the essential shortcoming of the question was that it depended basically on one idea. When one got hold of that idea the question was almost done. Then one encountered the problem of expressing oneself properly. In fact I believed that two of our students scored low marks in this question, not because they did not know how to solve the question, but because they did not know how to express themselves properly.

- Imagine how we are going to label a linear chain. It is quite natural to label the edges of the chain from one end to the other end consecutively as follows.



Or if we have a cycle, we label the edges as follows.



Now the two labellings satisfy the requirement of the question since consecutive integers are relatively prime (have greatest common divisor equals 1).

- When one gets hold of this idea the question becomes quite easy. In case if the graph contains a leaf (a vertex of degree 1) we start to label consecutively from this leaf along a chain as long as possible, from 1 up to m, say. If $m < k$, there remain edges still unlabelled. Now since the graph is connected, there are edges linked to the labelled chain which are unlabelled. We may start from an arbitrary unlabelled edge linked to the chain and start to label a new longest possible chain $m + 1$, $m + 2$, $\cdots$, etc. Continuing in this manner we eventually label all the edges of the graph. And it is not hard to show that this method of labelling satisfies the requirement of the question. In case if the graph does not contain any leaf we may start from an arbitrary edge and perform similar operations. And hence the question is done.

- Finally we observe that the statement is still valid if the graph contains loops or multi-edges, for we may simply label a simple subgraph first. Also the condition of connectedness is needed, just try to label the edges of 2 triangles from 1 to 6.

- <u>Question 5</u> : Let ABC be a triangle and P an interior point in ABC. Show that at least one of the angles $\angle PAB$, $\angle PBC$, $\angle PCA$ is less than or equal to 30°.

- The solution of this question by the proposer is long and difficult. Actually if one is familiar with geometric inequalities, the question becomes quite easy.

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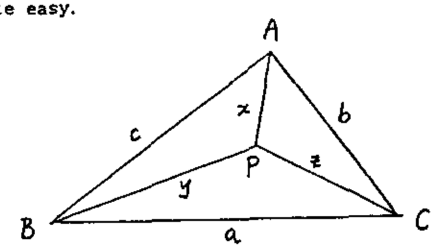


Assume by contradiction $\angle PAB > 30^\circ$, $\angle PBC > 30^\circ$, and $\angle PCA > 30^\circ$. Clearly we may also assume $\angle PAB < 120^\circ$, $\angle PBC < 120^\circ$ and $\angle PCA < 120^\circ$. Now the area of ΔABC equals $\Delta = \frac{1}{2}$ cx sin PAB + $\frac{1}{2}$ ay sin PBC + $\frac{1}{2}$ bz sin PCA

$$> \frac{1}{4}cx + \frac{1}{4}ay + \frac{1}{4}bz, \text{ (see the diagram).}$$

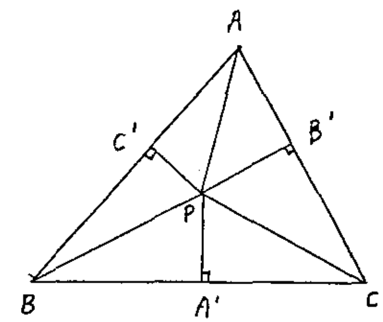
On the other hand, by the cosine law we have

$$x^2 = b^2 + z^2 - 2bz \cos PCA > b^2 + z^2 - \sqrt{3}\, bz,$$

$$y^2 = c^2 + x^2 - 2cx \cos PAB > c^2 + x^2 - \sqrt{3}\, cx, \text{ and}$$

$$z^2 = a^2 + y^2 - 2ay \cos PBC > a^2 + y^2 - \sqrt{3}\, ay.$$

Summing up we have $bz + cx + ay > \frac{1}{\sqrt{3}}(a^2 + b^2 + c^2)$. Combining with the first inequality we get $4\sqrt{3}\,\Delta > a^2 + b^2 + c^2$. And this precisely contradicts the so-called <u>Weitzenbeck inequality</u> : $4\sqrt{3}\,\Delta \le a^2 + b^2 + c^2$.



On the other hand if we drop perpendiculars from P to the sides of the triangle as indicated. Then we have

$$PC' = PA \sin PAB > \frac{1}{2} PA,$$

$$PB' = PC \sin PCA > \frac{1}{2} PC, \text{ and}$$

$$PA' = PB \sin PBC > \frac{1}{2} PB.$$

Summing up, we have $PA' + PB' + PC' > \frac{1}{2}(PA + PB + PC)$. And this precisely contradicts the so-called <u>Erdös-Mordell inequality</u> : $PA' + PB' + PC' \le \frac{1}{2}(PA + PB + PC)$.

- It may be debatable if the inequalities are well known, but certainly they are known. Using the criterion that the solution of a question should not follow easily from a theorem, perhaps this question should not be included at the very first place.

- <u>Question 6</u> : An infinite sequence x_0, x_1, x_2, $\cdots$ of real numbers is said to be bounded if there is a constant C such that $|x_i| \le C$ for every $i \ge 0$. Given any real number $a > 1$, construct a bounded infinite sequence x_0, x_1, x_2, $\cdots$ such that

$$|x_i - x_j|\,|i - j|^a \ge 1$$

for every pair of distinct non-negative integers i, j.

- Since I cannot say anything more than the proposed solution I should refrain from giving any comments. Clearly if such a sequence exists it cannot be convergent and hence not monotonic. Perhaps our readers would like to try on this question.

- Accordingly 51 students obtained full marks in this question. Hence there must be many different sequences satisfying the required property. Unfortunately the many different solutions were not recorded.

- When the result of the North Korean team on this question was posted on the Notice board it stirred up a lot of speculations, since all of the six team members scored full marks. During the eighth (last) Jury meeting it was disclosed that all of them presented the same solution, although it differed from the proposed solution. The North Korean leader defended this by claiming the students bad seen this question before. Even worse, all of the six team members gave the same solution in question 3, their solution and the proposed solution looked just alike. Eventually the team was disqualified by an (almost) unanimous decision of the Jury.

- Finally I would like to make a few remarks.

- (i) The original purpose of the IMO was to stimulate the innovative spirit of the student. When a student first encounters a question he trys to investigate various subcases, consider related issues, etc, and eventually solves the question. However it is well known that many strong teams are trained very extensively. It is quite possible that when they go to a math contest, they have seen somethings similar to the questions. There is a possibility that this kind of "over-training" in fact upset their interest in mathematics. Moreover since the proposers have to cook up somethings that the students have not seen before, the questions become more and more unnatural. That explains partly why there is a term "competition mathematics".

- (ii) Back to our team the situation is quite different. Because of various reasons our team was not trained enough. Many of the basic techniques were not handled competently by some of our team members. Depend on how the public, the government and the mathematical community at large view this event, perhaps we should try harder to prepare our team.

- (iii) The host country (Sweden) put up a great effort in organising this event. I look forward to the 35th IMO (1994) to be held in Hong Kong.

MY EXPERIENCE IN TEACHING MATHEMATICS IN EVENING SCHOOL

K i K i

I have been teaching Mathematics in an English evening school for six years and find the job challenging as well as rewarding. In my school, most of the pupils are working in the nearby factories and a large percentage of them are new immigrants from China. The spread of pupils' ages is wide, for example, the age of S1 pupils in the academic year 1990/91 ranged from fourteen to forty-six years old. As far as I know, they get little time for study and some of them even need to work on Sunday! It was also observed during the lessons that some of them were tired after working hard for a whole day and fell asleep occasionally. In general, they are weak in English and in basic Mathematics skills such as addition and subtraction of fractions. However, regarding learning attitude, most of the pupils are mature, attentive eager to learn and with self-initiative. The school operates five evenings per week and there are four lessons (each lasts for forty minutes) per evening. Furthermore, there are three mathematics lessons in a week. The time allocated to the subject is extremely inadequate.

Having taken into consideration the background of pupils and the particular features of the school, I realize that the teaching strategies adopted should be practical. In fact, they are found to be quite effective. For the purpose of experience sharing, these strategies are given in the paragraphs that follow.

In view of the inadequate time allocation, some topics/subtopics in the "Syllabus for Mathematics (Form I - V) 1985" may be deleted for the sake of saving time. However, the following two points should be kept in mind.

(i) The deletion should not affect the continuity of the syllabus. For instance, "Binary Number", which is only taught in S1 and has little connection with other topics in the syllabus, may be deleted. On the other hand, it is unwise to delete the topic "Introduction to Co-ordinates" in S1 as pupils will not get adequate background knowledge to study other related topics such as "More about Co-ordinates" in S2 and "Co-ordinate Geometry of Straight Lines" in S3.

(ii) All essential parts of Mathematics required for HKCEE should not be deleted but the depth of treatment can be adjusted to suit the level and ability of pupils.

To help pupils deal with the lengthy and complex calculation, the proper use of calculators should be taught so that the time spent on computation can be reduced. In this way, use of four figure tables become unnecessary.

If pupils are interested in Mathematics, they will surely learn it better. Therefore, simple home-made teaching aids, if possible, should be used to facilitate teaching. For example, we can make use of a paper triangle to elucidate the theorem "sum of angles of a triangle is 180°". By tearing the triangle into 3 pieces (so that each piece contains one angle of triangle) and fitting the three angles together, we can provide pupils with a simple visual verification of the theorem! Colour chalk, which is a handy and effective teaching aid, should be used whenever necessary. Questioning technique should also be fully utilized to conduct our lessons. However, as most of the pupils are adults and they are lacking in confidence in answering questions, it is desirable not to address questions directly to a pupil to avoid possible embarrassment. Hence, choral answers are usually expected. Moreover, to sustain pupils' interest in the subject, we should choose examples and problems which are highly related to daily life situations. For instance, when we come to the topic "Statistical Data" in S1, we can quote real data (such as distribution of age of Hong Kong people) from "Hong Kong Annual Report".

As we all know that pupils who constantly meet with failure will be discouraged. Therefore, to help pupils develop confidence, the questions assigned should be interesting and commensurate with pupils' ability. Moreover, homework should be given to consolidate the subject contents which have been learnt during the lessons and pupils should be encouraged to work on their own. It should also be marked conscientiously with special attention to pupils' presentation and returned promptly to ensure immediate feedback.

Holding short tests (as short as ten minutes) after finishing one topic can effectively help pupils revise as well as consolidate what they have learnt. One point which should be noted in test construction is that the level of difficulty as well as the quantity of the test questions should match with pupils' ability. Otherwise, the assessments will discourage rather than encourage pupils to learn. Also, our attitudes play an important part in building up pupils' confidence. Therefore, we should be more patient with pupils particularly those in evening schools. It is because their standard, in genral, is very low.

Teaching Mathematics in evening school is not an easy task, but it is also a challenge and a pleasure to teach mature pupils. My experience given in the context are by no means exhaustive. Your opinions and experience are also helpful to me. If you have any suggestions or comments, please let me have them via the Editor of the School Mathematics Newsletter.

數學教學對話錄——
一個與乘法有關的遊戲

許國輝

有老師問我：「應該要求學生背乘數表嗎？」

「不應該！」我衝口而答，但接着又說：「應該！」

那位老師被我的出爾反爾、前後矛盾的答案弄得目瞪口呆，我連忙解釋說：「在引入乘法概念的階段，我們應提供足夠的活動，讓學生明白乘法的原理，並通過數數活動的記錄引導學生編出乘數表，再安排多種不同形式的練習，如十行表上的活動或附有答案的自習算卡及數學遊戲，使學生通過多次運用來熟習乘數表。而不應該獨沽一味背乘數表。」

「那麼，為什麼又說應該？」她奇怪地問。

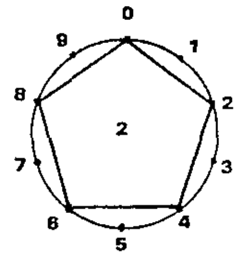
「如果一名學生在三年級學兩位乘一位數時，仍未熟習乘數表。教師在診斷其問題後，應該要他背乘數表，好使他亡羊補牢，趕上進度。」

「然則，有那些遊戲可以令學生熟習乘數表的？」

「遊戲方式很多。以下一個遊戲對一些『唸口杦』的學生或許有幫助，特別是經常記錯乘積的個位數字的學生，如「八七六十三」。通過發現乘法個位數字規律的遊戲，他會明白8的

倍數中個位數字是：0,2,4,6,8，而不會有3字尾。」

「這遊戲是怎樣玩的？」



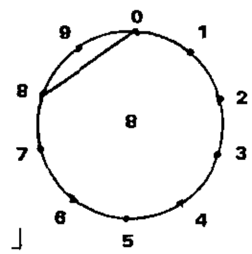
「請你先唸出2的倍數。」

「2，4，6，8，10，12，14，……」

「你看！我將你唸2的倍數的個位數字，在右面有10點的圓內，連結對應的點畫了一個圖形。」

「眞有趣！2的倍數的個位數字竟組成了一個五邊形。」

「請你唸出8的倍數，這次一面唸一面畫吧！」



「啊！我又畫出一個……」

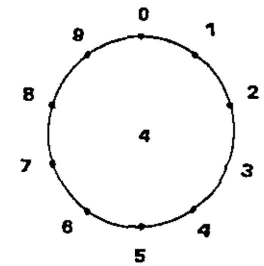
「巧妙嗎？兩個圖形爲什麼會相同呢？」

「2和8的圖形相同，是因爲其個位數字一樣，而且數字排列前者爲2，4，6，8的順序，後者則是倒序。」

「你的分析正確。你試畫出『4的倍數的圖形』來。」

「4的倍數的個位數字雖然也是0，2，4，6，8，但次序是：4，8，2，6，0。我猜畫出來的圖形會有所不同，讓我試試！」

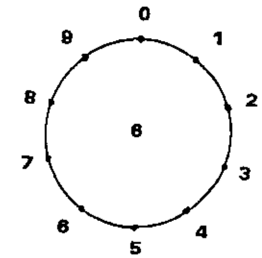
「你找到了！你猜『6的倍數的圖形』又如何？」



「咦！6的圖形和4的圖形竟然……」

「你可知道這是什麼道理？」

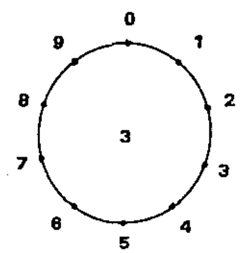
「當然不是巧合。第1組相同的圖形是2和8，第2組是4和6，我猜想：第3組相同的圖形會是3和7。」

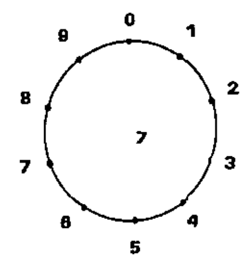


「爲什麼你有這樣的假設？」

「哦！我觀察到2與8的和是10，4與6的和也是10。因此，我的推論是：既然3與7的和也是10，它們的圖形應該是一樣。」

「好！你自己求證吧！」



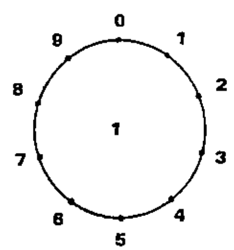


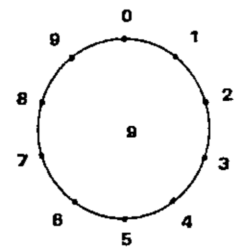
「哈哈！我對了！」她高興得嘴也合不攏。

「恭喜你！你證明了你的發現果然是對的。那麼，1和9的和也是10，它們的圖形又會相同？究竟圖樣會是怎樣的呢？」

我窮追不捨，繼續向她的智慧挑戰。

她滿懷信心地說：「我肯定圖樣也相同，我畫給你看！」





「果然不出你所料，你可以概括你的發現嗎？」

「我發現了：兩個數的和是10，它們的倍數的個位數字在一個10點的圖內，所組成的圖形是相同的。」

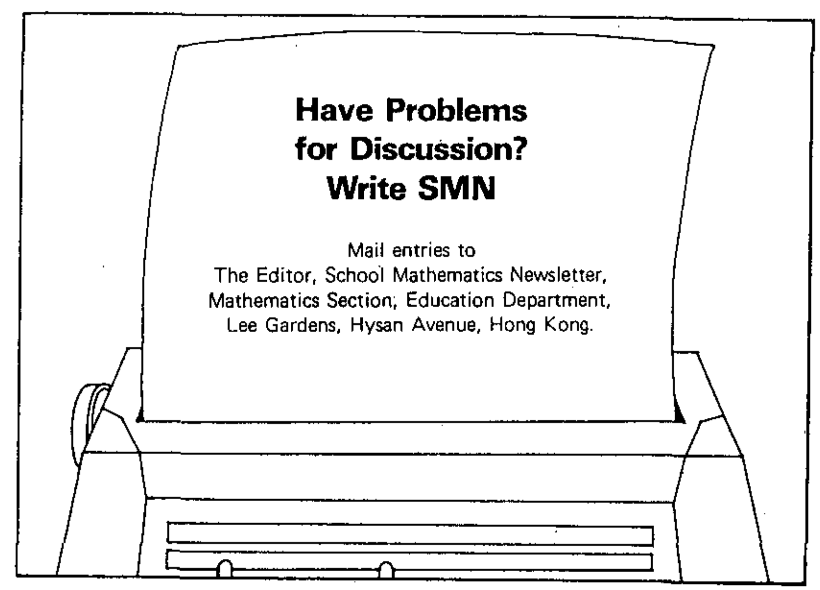
「你剛才體驗了數學思考方法的歷程。這個遊戲好玩嗎？有啓發性嗎？如果我說：『數學只是一種符號的遊戲。』你同意嗎？」

「這個說法似乎『兒戲』一點，嚴嚴正正的科學，怎麼好說它是『遊戲』！」

「2＋8＝10這條算式中，『2』『8』『10』『＋』『＝』都是符號。所謂『2＋8』是表示『2』和『8』這兩個符號在這裡的關係是相合；所謂『＝』表示在它前後的兩件東西在量上相同。所以歸根到柢『2＋8＝10』只是三個符號和兩個關係的聯繫。因此，數學可以定義爲『使用符號來研究關係的科學』。」

「這個定義，夠學術性，不過又好像抽象一點。我倒喜歡『數學是符號的遊戲』這個講法。」

「不過，數學的定義是怎樣也是人爲的而已，它並不重要。數學教師重要的任務反而是：引導學生接受數學、明白數學、運用數學，進一步喜愛數學。要達到這些目標，教師須設計出一個情境，讓學生用遊戲的心情學習數學。」



小學數學科輔導教學要點

數學組

為了改善小學教育的質素，教育署在1981年發表了「小學教育及學前服務白皮書」，提出了多項改善小學教育質素的建議，期望學校在獲得額外的教師和其他資源後，可積極參與提高教育質素的工作，施行輔導教學是其中一項。

輔導教學的目的在於為那些中、英、數三科基本科目較同班同學落後的學生，提供額外的輔導，以幫助他們解決學習上的困難。

教育署在一九八二年二月發出了教育署資助學校通告八二年第二十六號，列出小學輔導教學的行政指引和中、英、數三科的教學指引，以供學校參考。同時教育署有關單位也配合該通告舉辦數個小學輔導教學研討會，對與會校長和教師介紹輔導教學推行的概念和方法。其後，輔導視學處數學組也舉辦了一系列的小學數學科輔導教學研討會和教師進修班，讓有關教師了解輔導教學的理論和實踐。

以下是根據小學數學科輔導教學教師進修班的內容，簡撮推行輔導教學的一些要點，並在附件內包含一些有關的資料和講義，以供教師參考之用。

(一)找出個別學生的弱點

(A)通過日常觀察。
(B)利用交談來進一步了解學生。
(C)編製適當的診斷測驗，找出學生在數學科某方面的弱點。
(D)調查學生的家庭背景，包括在家中做功課之環境等。

(二)針對弱點加以輔導

(A)編排合適的活動，堂課及家課，題材必須與學生日常生活有關，所涉及的數字不宜過大，務使學生恢復學習信心。
(B)教師應有愛心及耐性，增強學生的自信心及培養學生對數學科的興趣。
(C)必要時邀請家長交換意見，要求家長與校方配合，俾收得更佳的輔導效果。

(三)安排單元式的短測驗

(A)在完成每個單元時，編製對該單元的測驗，算題以測驗基本知識及概念為主。如有需要，可擬較多的同類題目，以確定學生是否真正掌握應有的計算技巧或數學概念。
(B)將測驗成績記錄，以備將來參考及比較。

(四)輔導教學的教材

(A)教師必須不斷檢討及修改教材以配合學生的能力。
(B)教材除輔助學生了解以往之課程外，還須顧及配合正常班之進度。

(五) 教師須作教學紀錄

(A) 學生出席紀錄。
(B) 教學日誌。
(C) 教學進度。
(D) 成績紀錄。

(六) 學生個人紀錄及複查表
(A) 學生個人紀錄
學生測驗及考試成績。
教師對學生弱點之紀錄及評語。
教師對學生進步情形之紀錄。
(B) 學生個人學習複查表
本年度課程摘要。
以往各年課程摘要。

附件：
(一) 輔導教學指引一數學科（節錄自教育署資助學校通告八二年第二十六號有關輔導教學指引事。）
(二) 小學數學科輔導教學
(三) 輔導教學循環
(四) 小學數學科學生常見的錯誤
(五) 記錄及複查表
(六) 小學數學科輔導教學模式的比較

輔導教學指引 —— 數學科

節錄自　教育署資助學校通告八二年第二十六號

有關小學輔導教學指引事

目　的

輔導教學的主要目的，是爲成績較同班同學落後的學生，提供額外的輔導。在數學科方面，教師應着重基本概念的領會，雖然這樣做法或許會引致學生未能學習比較精深的數學。

組　織

(甲) 時間表的編排

數學科的輔導教學最好能在分開兩日的兩個數學教節進行，如其中一節能編爲每隔一個星期六上兩個相連教節則更佳。

(乙) 學生的挑選

挑選學生接受輔導教學，除了總指引第七段內建議的辦法（根據上一學期或學年的成績）外，還可根據學生在某方面的數學概念和技巧的弱點而挑選。最好的方法是利用有關科目的鑑辨測驗。如果沒有適用的鑑辨測驗，教師亦可以普通堂上測驗／考試的成績，對學生的活動／堂課／家課的觀察，或與個別學生的討論作爲準則。

(丙) 教師的編排

輔導教師應該熟識所教的科目內容和學生的背景。因此，除總指引中第九段所建議的安排外，另外一個方法是由對學生有最深切認識的班主任擔任輔導教學，而由另外一位教師負責原班的課節。

(丁)協調工作

輔導教師和原班教師之間，在挑選學生、選擇教材、教學方法及教學進度各方面均應有緊密聯繫，因此，學生由原班調往輔導小組或由輔導小組調回原班時，其學習均不會脫節。

(戊)輔導教師的工作

輔導教學要做得成功，必須要有週詳的準備功夫。學生的習作不應只是重覆堂上所做的，應針對輔導小組學生的弱點和需要來設計習作。教師可以將各種數學概念和技巧，分成多個簡易的小部份，使學生做習作時不致有太大困難。教師應記着，成就感可使學生對自己恢復信心。

教師應使數學的內容較有意義，輔導小組學生所做的習題應與他們切身環境有關，而不應太注意將來的用途。習題亦應較富趣味性，使學生對數學科發生興趣。

為克服部份學生在語文上的困難，習題的字句應盡量簡短。如果可能的話，應以圖形或數學符號代替文字。此外，習作最好是一些作業紙，學生可以直接在紙上演算，節省時間和勞力。

教師應盡快批改學生的堂課／家課。迅速知道自己的成績對學生十分重要，尤以成績較差的學生為然。

評核學生的進度，是輔導教學中重要的一部份。教師應該將學生的習作、測驗或其他評核方法的成績保存詳細的紀錄。

附件二

小學數學科輔導教學

主要目的：輔導教學指引一為成績較同班同學落後的學生，提供額外的輔導。

希望達到效果：學生一 a.學習態度轉變。
b.提高自信心及興趣。
c.與同班同學的程度拉近距離。
教師一發現及指正學生的錯處。

輔導前的診斷

診斷方式：1. 診斷測驗。
2. 交談。
3. 教師觀察。
4. 活動。

診斷時注意要點

1. 教師態度要和藹友善。
2. 不應催促學生。
3. 當學生計算錯誤時，不要中止他及加以提示。
4. 發問要適應學生反應。
5. 別發問帶引性問題。

6. 嘗試叫學生一面做，一面說出他的做法或想法。
7. 鼓勵學生把錯處劃去，再將改正寫出。
8. 當學生顯示焦慮、厭倦或抗拒時，應即中止診斷。
9. 保持診斷過程的記錄。

輔導前的準備

1. 充份了解個別學生的弱點／困難。
2. 對有關教材涉及的概念／技巧作細緻的分析、研究。
3. 準備教材／作業。
4. 周詳計劃。

預備教材應注意的事項

1. 目標明確，範圍集中於某一細節。
2. 按部就班，循序漸進。
3. 眞實感。
4. 趣味化。
5. 文字簡易，多用插圖。
6. 作業紙（WORKSHEET）勝於工作卡（WORK-CARD）或板書。
7. 若有需要，可用精簡教材，將艱深的部份刪去。
8. 富於伸縮性。

輔導教學的進行

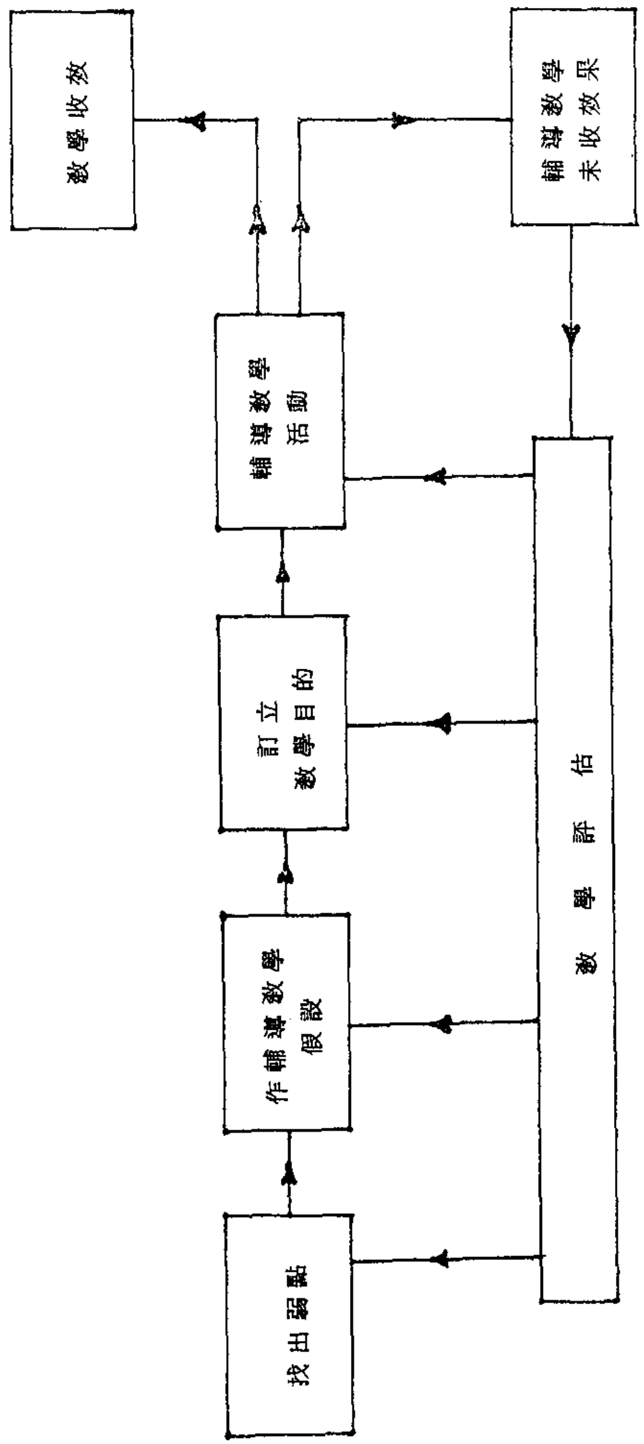
1. 每節有具體的教學目的。
2. 注意時間的分配。
3. 培養良好的學習氣氛：引起學習的動機，積極參與及熱烈的討論。
4. 用啓發式教學。
5. 注意發問的技巧，引起學生的積極思維。
6. 安排適當的活動→了解→練習→鞏固→記憶。
7. 多用教具／圖解分析問題。
8. 培養成功感，從而建立學生對數學學習的興趣與信心。
9. 多給鼓勵，要顧全學生的自尊心。
10. 盡快批改學生堂課／家課並作記錄。
11. 根據學生的反應，適當地調整輔導工作。

總結

數學科輔導教學原則

1. 個別照顧。
2. 透徹、細緻。
3. 富於伸縮性。
4. 培養成功感，從而建立學生對數學學習的興趣與信心。
5. 活動→了解→練習→鞏固→記憶。
6. 要顧全學生的自尊心。
7. 教師間的協調。

附件三



輔導教學循環 （譯自 Fredricka K. Reisman, Diagnostic Teaching of Elementary School Mathematics : Methods and Content. 1977)

附件四

小學數學科學生常見的錯誤

㈠ 概念上的錯誤

(i) 概念或法則未有存在

(ii) 概念或法則有所混淆

(iii) 概念之學習不全

(iv) 概念應用錯誤

(v) 概念建於不正確的資料上

□ 與運算有關的錯誤

□ 與數位有關的錯誤

□ 與進位／退位有關的錯誤

㈡ 計算程序上的錯誤

□ 算法正確，但錯用他法之運算口訣

□ 算法及口訣均錯

□ 口訣正確，但錯用算法

□ 不正確運用算法

㈢ 運算的錯誤

□ 運算次序錯誤

□ 不完整的運算

㈣ 粗心大意做成的錯誤

㈤ 不作答

附件五

記　錄

為什麼要做記錄

1. 了解學生的發展與成長
2. 方便教師檢討自己的工作

教師工作的記錄

1. 輔導小組登記冊
2. 輔導工作計劃／進度的記錄
3. 測驗／考試成績的記錄

個別學生的記錄

1. 學生的背景
2. 複查表
3. 成績紀錄
4. 對學生的長處／弱點／困難的評語
5. 學生習作的剪貼簿
6. 學生自己做記錄

給家長的報告

應該採用那些記錄

1. 因應數學上的要求
2. 衡量在記錄上所花的時間與所收的效果

四年級上學期數學科輔導教學複查表

______午部______組　　學生姓名______________　輔導教師______________

項目	複習／輔導日期	理解程度					備註
		優	良	常	差	劣	
＜因數和倍數＞							
倍數及公倍數的認識							
最小公倍數的認識							
因數及公因數的認識							
最大公因數的認識							
＜乘法＞							
乘數為一位數複習							
兩位數乘兩位數							
兩位數乘三位數							
乘法的綜合性質							
＜除法＞							
一位數乘三位數複習							
兩位數除兩位數							
兩位數除三位數（不退位）							
兩位數除三位數（退位）							
兩位數除四位數							
除法應用題							
＜正比例＞							
簡易歸一法							
＜分數＞							
簡單分數的認識							
分數的種類							
假分數、整數、帶分數的互化							
分數的擴分和約分							
分數化簡							
同分母分數的加法							
同分母分數的減法							
同分母分數的加減混合計算							
＜小數＞							
小數的意義、讀和寫、位值、大小比較							
小數的加法和減法							
小數的加、減混合計算							
＜容量＞							
毫升的認識							
＜立體圖形＞							
錐體、柱體的一些性質							
＜統計圖表＞							
象形統計圖的認識							

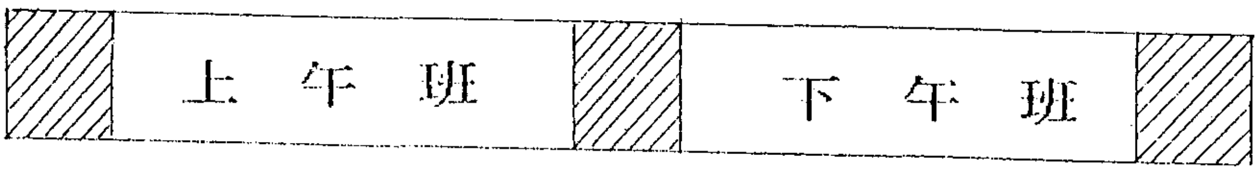
附件六

小學數學科輔導教學模式的比較

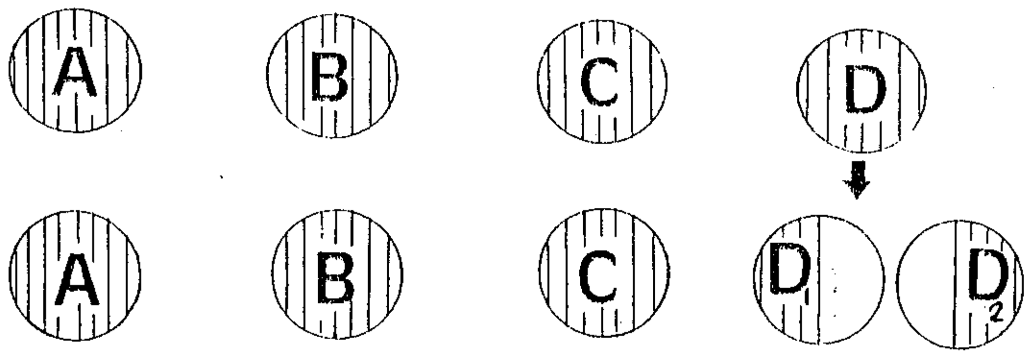
○ 優點
△ 缺點

模　式	學生方面	教師方面	學校方面
抽離輔導	○一星期各天上學及下課時間一致 ○接受個別照顧機會較多 △要接兩位不同老師的教導 △由於轉場地影響，上課時間少了 △上輔導課時要追舊單元，同時學新單元	○一星期各天上學及下課時間一致 △不同學生有不同困難，在短時間內不易一一解決 △原班教師與輔導教師要經常作協調工夫 △由於轉場地影響，授課時間少了 △輔導教師要替學生追舊單元，同時教新單元	○一星期各天上學及下課時間一致 △由於輔導小組相當多，安排場地有困難
大班輔導	○只接受一位教師的教導 △接受老師的個別照顧較少	○一位教師任教全星期各節 ○學生程度劃一，設計教學較易 △對學生的個別照顧較少	△學生要接受多於一科的輔導機會不大 △編訂時間表較難 △安排場地較困難（需要大課室）
課外輔導	○額外上課時間多了，學習可較透徹 ○只接受一位老師的教導 ○接受老師的個別照顧較多 △一星期各天上學及下課時間未必一致	○額外授課時間多了，輔導可較完整 ○可安排由一位教師兼任原班教師及輔導教師 ○在適當的安排下，輔導小組可與原班的課程配合進行 △在正常上課時間以外，仍要授課	○安排場地較易 ○編訂時間表較易 △一星期各天上學及下課時間未必一致

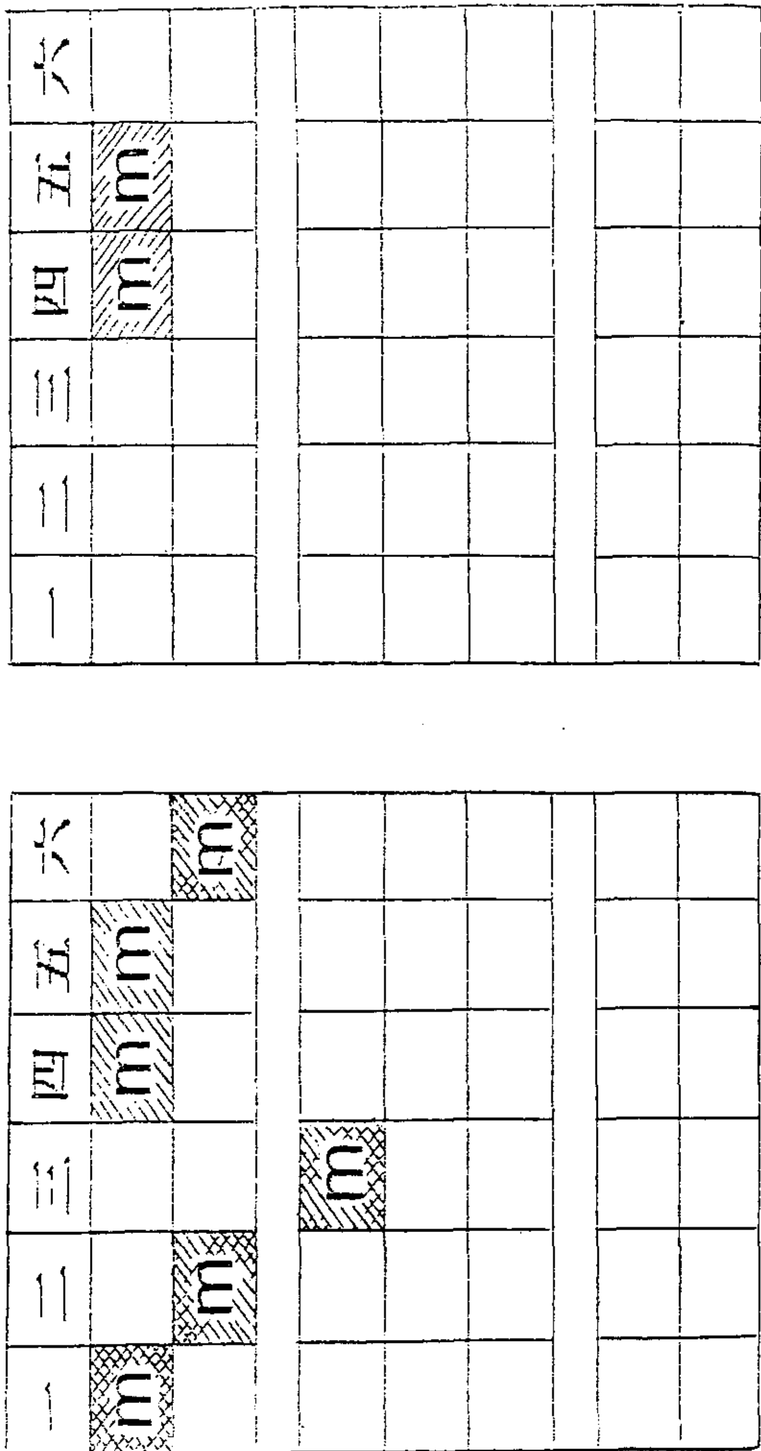
課外輔導



大班輔導



抽離輔導



概算

逸

在我們計算過程中所運用的估計技巧，稱爲「概算」。在數和計算這個範疇裏，估計技巧最能發揮它的用途。一些教育家均認爲數學課程發展的當前急務，是在數學上加强概算的訓練。主要原因是概算的概念重要，而且很多學生對概算存有恐懼感，他們對概算的意義模糊不淸，也不曉得爲什麼需要概算。當學生需要做概算時，他們唯一的方法是用紙和筆找出準確的答案，再用四捨五入法「找出」它的估計量。更嚴重的是在他們的觀念中，數學上的問題只能接受精確的答案，概算根本不足以登大雅之堂。他們會理直氣壯地說：「爲什麼不直截了當地把準確答案算出來呢？」

故此，爲了要提高數學效果，首先要讓學生認識及熟習多種找尋估計量的方法，其次是培養學生有一個接受概算的心態。這包括認識概算是什麼，什麼時候去應用概算和在不同的情況下所需要概算的準確程度，而且更要懂得使用適當的概算方法，此外更能夠分辨一個筆算得出來的答案是否合理，從而承認概算在數學上的地位。

與解答應用題相似，學習概算包括要學習多種的技巧，並且需要一段較長的時間來練習，但概算不應視作爲一個獨立的課題來教授，而應該融會於現存課程中的不同的範疇裏。概算技巧大概可分爲下列幾種：

(1) 把注意力集中在數的最左邊的數字上：

這種方法最適宜在加數時運用，因爲最左邊的數字所表示的值最大，因此對答案的影響爲最大。這種方法可分爲兩個步驟進行，例如：

$2.99 + 4.27 + 0.56 + 3.78 + 1.38 = ?$

2.99	第一步	把最左的數字加起來：
4.27	↓	$2 + 4 + 3 + 1 = 10$
0.56	第二步	把答案作合理的調整：
3.78		$0.99 \approx 1$
+ 1.38	↓	$0.27 + 0.78 \approx 1$
		$0.56 + 0.38 \approx 1$

$10 + 3 \approx 13$

最初估計量　需調節的數字　最後估計量

在分數加法中，這種方法也同樣有效：

$\frac{6}{7} + 2\frac{4}{5} + 1\frac{1}{8} + 4\frac{1}{2} = ?$

$2 + 1 + 4 = 7$

$\frac{6}{7} + \frac{1}{8} + \frac{4}{5} + \frac{1}{2}$ ……………… 答案大於 2

∴ 估計量：9

把最初的估計量 (estimate) 作合理的調整，是十分重要的，所用的方法則視乎問題而改變，這種概算方法的好處是無論把 3 個數目連加或 6 個數目連加均適用，甚至當二年級的學生計算 214 + 316 + 125 時，教師不妨也提示學生說：「2 + 3 + 1 是 6，答案是否比 600 多些呢？」

至於減法方面，上述的概算方法雖然也可以應用（例如：682 − 498 …… 6 − 4 是 2，故初步估計是 200，把答案作合理的調整……。估計結果是少於 200），但是由於減法的運算每次只局限於某 2 個的數目，故上述的方法作用不大，而用四捨五入法已足夠了。

在乘法方面，這種概算方法，加上心算技巧，也能發揮很大的效果呢！

例如：627 × 4 = ？

627 × 4 = ？

第一部：把最左的數字乘起來

$600 \times 4 = 2400$

第二部：調整答案

27 × 4 大於 100

故 $627 \times 4 \approx 2500$

其實，傳統除法的試商方法已採用了上述的概算技巧，但同學們常常在決定位值時犯上錯誤，故教師在授課時，應特別留心。例如：

找出 3684 ÷ 7 的近似值

第一部：首先找出商的第一個數字

```
    5
7)3684
```

第二部：決定商中第一個數字的位值

```
  百十個
    5
7)3684      答案約是 500
```

第三部：調整答案

```
    5
7)3684
  35
   184      故答案是 500 多些
```

由於上述的步驟集中處理第一個數的數字及其位值上，因此估計量往往比實際算出的數值小，故調整的步驟是必須的。

(2) 找出一個較集中的數值：在日常生活的問題上，我們常常需要計算一些數目，而這些數目的數值是十分接近的，例如：

試估計商店一星期內紙包飲品的銷售量：

星期日	392 包	第一步：估計平均數
星期一	396 包	每日約 400 包
星期二	403 包	第二步：把平均數乘以頻數
星期三	405 包	400 包 × 7 ＝ 2800 包
星期四	400 包	
星期五	398 包	答案：每星期約售 2800 包
星期六	401 包	

上述的方法可應用於整數、分數及小數的計算，雖然它只能應用於某一類的問題上，但不失爲一個簡單、快捷的方法呢！

(3) 在多位數的乘法中，我們常用四捨五入法來找尋估計量，第一步是把數字加大或縮少來方便計算，第二步是計算答案。第三步是當需要時把答案調整；而調整方法有下列三種：

例(一) 28 × 56 （加大）（加大） 30 × 60 ↑ ↑

28 × 56 一定少於 1800，故答案一定少於 1800

例(二) 62 × 23 （減少）（減少） 60 × 20 ↓ ↓

62 × 23 一定比 1200 大，故答案一定比 1200 大

例(三) 62 × 79 （減少）（加大） 60 × 80 ↓ ↑

兩個數目已互相調節，故可取 4800 爲估計答案

上述的各種方法均能提供一個合理的答案，至於如何選擇，則視乎情況，運算方法及所要計算的數字而定了。

(4) 把數目中容易組合成相容數字 (compatible numbers) 的抽出來運算，在必要時，甚至把數目略爲加大或縮少以方便計算，例如：

27 ＋ 49 ＋ 38 ＋ 65 ＋ 56 ＋ 81 ＝ ？

17, 39, 38, 65, 66, ＋ 81

接近 100（17 ＋ 81）；38、65 接近 100；39、66 接近 100

答案約是 300

在連加法中，我們把容易配對的數字連結起來，以方便心算。這方法運用於除數中，也甚爲有效，例如：3388 ÷ 7 大約是多少，我們如果把算題變爲 3500 ÷ 7 來心算，便容易得多了。

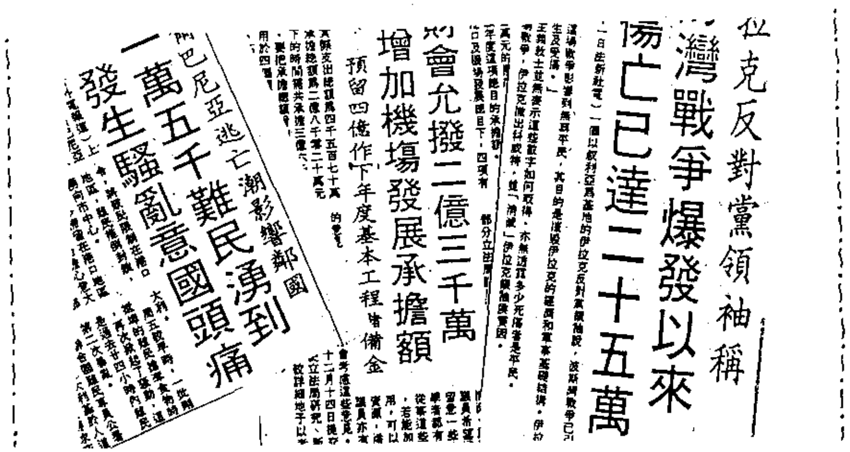
(5) 與上述方法相似的是在數目中找尋〝特別數目〞(special numbers) 以方便心算，例如：

題目	思考	估計
$\frac{7}{8}+\frac{12}{13}$	每個數字均接近 1	1 ＋ 1 ＝ 2
$720 \times \frac{23}{45}$	$\frac{23}{45}$ 接近 $\frac{1}{2}$	$720 \times \frac{1}{2} = 360$
816 × 9.84 %	9.84 % 接近 10 %	816 × 10 % ＝ 81.6
436.2 ÷ 0.98	0.98 接近 1	436 ÷ 1 ＝ 436
103.96 × 14.8	103.96 接近 100 而 14.8 接近 15	100 × 15 ＝ 1500

當教授分數、小數和百分數的運算時，教師應該多鼓勵同學們運用相容數字和特別數字來找出估計量，更可藉此幫助判斷筆算得出來的答案是否合理。

在日常的授課中，教師可以因應不同的課程內容而教導學生運用各種不同的估計技巧，假若能配合清楚的講解，恒久的練習及靈活的應用，必定能使同學們受益良多。不過，從另一個角度來說，假若同學們在任何情況下，只接受準確的答案，而無法接受一個合理的估計量，他們便很難有學習和應用估計量的自發性了。至於教師應怎樣幫助學生們對估計技巧建立自信心，明白它的性質和用途，以下的方法頗爲有效：

1. 利用實際的例子，讓同學們明白到日常生活中所提及的數據，很多只是估計量，這些資料是很容易在報章中找到的：



此外，多以實際生活的例子作堂上的練習，使估計技巧成爲他們生活經驗的一部份，例如：

1. 弟弟有 5 元，他最多可買蘋果幾個？
2. 買水果各 2 個，約付多少元？
3. 買 3 個梨或 6 個橙，那一種要多付些錢？

2. 强調在很多情況下，精確的計算並非必要的，例如：「一個顧客在一張 $ 262 元的賬單上加上 10 % 多些的小賬，他共需付多少？」或是「把一件原價 98 元的衣服減爲 79.5 元出售，約打了多少折？」等問題，合理的估計量已足夠了。此外，同學們更應該明白，在很多的情況下，可以接受的估計量是多於一個的，而估計的方法也不是一成不變或需要依據很多的法則，不同的方法也能找出合理的估計量呢！

3. 在利用概算時，盡量避免繁複的程序，讓學生相信估計技巧是容易學習的，並且樂於使用。

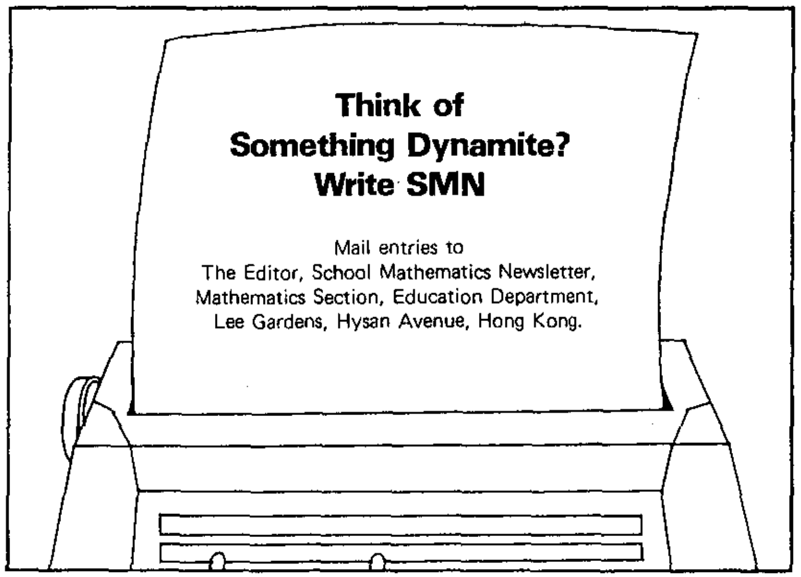
4. 讓學生熟習概算時所運用的數學語言，以便更明白概算的意義，例如：
   - 大約 $12\frac{1}{2}$
   - 接近 9
   - 15 多些
   - 比 7 少些
   - 6 和 7 之間，但較接近 6

5. 多與學生討論，鼓勵他們說出思考的過程以彼此交流經驗及互相學習，讓他們能習慣利用快捷的心算，解決問題。

6. 概算與慣用的計算有很大的分別。假使我們希望同學們能有信心地接納概算爲數學的一部份，我們一定要把概算應用於不同年級的課程裏，而事實上在小學的數學課本中，每一個年級均有不少的計算題和文字題，這些都是練習和應用概算的好題材。假如教師常常鼓勵同學們先估計後計算，便不會因計算錯誤而接受荒謬的答案了。舉例來說，貨車車輪的直徑是 0.85 米，圓周是多少？只要他們能概算出圓周大約是直徑的 3 倍，便很容易知道圓周大約是 3 米了。

*　　*　　*　　*

總括來說，在現代人的生活中，我們常常需要運用概算來解決問題，並且計算機現已非常普及，學生們也絕少用紙筆作繁複精深的計算，所以他們應該懂得用概算來判別所得的答案是否合理，但要使同學們更自信、更有效地運用概算，前面的道路仍是十分漫長呢！況且，概算的運用包括了認知和感性的因素，倘若這方面能與估計技巧一起發展，便更能收到事半功倍的效果。



PASTIMES

(1) Can you suggest a set of numbers a, b, c such that

(i) exactly one of the six permutations of the three numbers is consecutive;

(ii) the quadratic expression $a^2x^2+b^2x+c^2$ is factorizable in the set of integers?

(2)

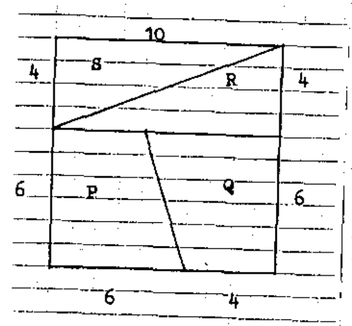


Figure 1

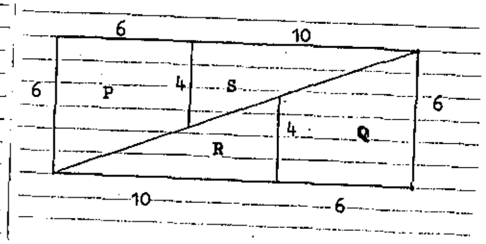


Figure 2

In figure 1, a 10x10 square has been divided into four portions P, Q, R and S. It seemed that these portions can be fitted together to form a 6 x 16 rectangle as in figure 2. Clearly the area of the rectangle is 4 sq. units less than that of the square. What has happened to the missing part (i.e. the 4 sq. units)?

(3) It is asserted that triangles with sides 2a-1, 3a-2, 3a-1, for all natural number a, can never be isosceles. Do you agree? Please state the supporting reasons.

(4) In what number scales are 111 a multiple of (i) 7 (scale 10) and (ii) 12 (scale 3)?

(5) A Major Square of the third order is a square of side three units in which the sum of the numebrs in each row, column and diagonal are the same.

(a) Complete the two Magic Squares below.

	1	
	5	
		2

		35
		38
	44	

(b) Rearrange the nine numbers in each of the squares in (a) in ascending order. What can you say about the two squares formed?

(6) A palindromic number is a number which is the same as the number formed by reading backward (e.g. 272). With a pair of different 1-digit numbers, two palindromic 3-digit numbers can be formed. For example, with the pair (1, 2), 121 and 212 are formed.

Find the pairs of different 1-digit numbers which can generate two 3-digit palindromic numbers such that their sum is also a palindromic number with

(a) 3 digits,

(b) 4 digits.

(7)

Column A	Column B	Column C
RENE	TAYLOR	1596-1650
BLAISE	DESCARTES	1623-1662
GOTTFRIED	LAGRANGE	1647-1716
WELHELM		
LEONARD	PASCAL	1701-1783
CARL	LEIBNITZ	1777-1855
FRIEDRICH		
AUGUSTIN	GAUSS	1789-1857
LOUIS		
ISAAC	EULER	1643-1727
BERNHARD	CAUCHY	1826-1866
BROOK	RIEMANN	1685-1731
JOSEPH	NEWTON	1736-1813
LOUIS		

By matching Columns A, B and C, you can obtain the names and the life-spans of ten prominent mathematicians. What are they?

(8) Are the 3-digit numbers formed by x, y and 9 (where x, y are positive integers less than ten and their sum is nine) divisible by 9?

Someone has asserted that those 3-digit numbers formed by x, y and 9 are divisible by 99 if the tenth digit of the numbers is 9. Do you agree with the assertion? If so, please prove it.

Solution to PASTIMES

(1) Suggested solution : a=3, b=5, c=4 .

(2) In figure 1, the gradients of the sloping edge of Q and R are 2/5 and 1/3 respectively. However, in figure 2, the gradient of the diagonal of the rectangle is 3/8. As 2/5 > 3/8 > 1/3, this indicates that Q and R together fill more than half the rectangle, and this also happens with the pieces P and S. Therefore, there must be an overlapping part along the diagonal of the rectangle (which is shaded as in figure 3) of which the area is 4 sq. units.

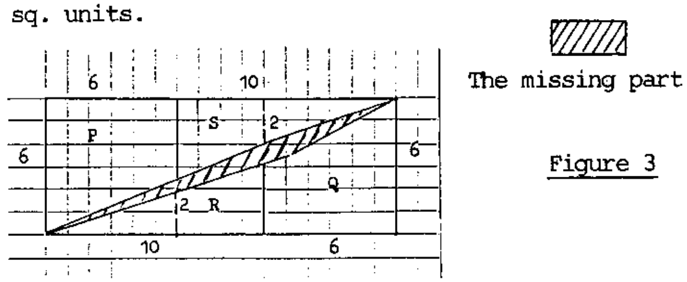


Figure 3

(3) Yes, the assertion is correct.
Suppose that there exists isosceles triangle with the said three sides, the one of the following must be true

(i) 2a-1 = 3a-2

(ii) 2a-1 = 3a-1

(iii) 3a-1 = 3a-2

However, in case (i), we get a=1, but 1, 1, 2 can not be the sides of a triangle. Considering case (ii), we get a=0. Clearly -1, -1, -2 cannot be sides of a triangle. At last, there is no solution for a in case (iii). This contradicts the assumption. Hence, triangles with sides 2a-1, 3a-2 and 3a-1 can never be isosceles.

(4) (i) In the number scale n, 111 represents n^2+n+1. Table 1 shows the value of n^2+n+1 for $7k \leqslant n \leqslant 7k+6$ and k is a natural number.

n	$7k$	$7k+1$	$7k+2$	$7k+3$	$7k+4$	$7k+5$	$7k+6$
n^2+n+1	$7M^{(i)}+1$	$7M^{(ii)}+3$	$7M^{(iii)}$	$7M^{(iv)}+6$	$7M^{(v)}$	$7M^{(vi)}+3$	$7M^{(vii)}+1$

$M^{(i)}$ to $M^{(vii)}$ are constants

Table 1

It is easily seen that 111 is a multiple of 7 in scale n when n is equal to 7k+2 or 7k+4.

(ii) 12 (scale 3) = 5 (scale 10)
Table 2 shows the value of n^2+n+1 for $5h \leqslant n \leqslant 5h+4$ and h is a natural number.

n	$5h$	$5h+1$	$5h+2$	$5h+3$	$5h+4$
n^2+n+1	$5N^{(i)}+1$	$5N^{(ii)}+3$	$5N^{(iii)}+2$	$5N^{(iv)}+3$	$5N^{(v)}+1$

$N^{(i)}$ to $N^{(iv)}$ are constants

Table 2

From table 2, it can be seen that 111 cannot be a multiple of 5 in any number scale.

(5) (a)

8	1	6
3	5	7
4	9	2

41	20	35
26	32	38
29	44	23

(b) The two sequences

1, 2, 3, 4, 5, 6, 7, 8, 9, and

20, 23, 26, 29, 32, 35, 38, 41, 44

are arithmetic progressions.

(6) (a) There are 16 paris, namely, (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (2, 3), (2, 4), (2, 5), (2, 6), (2, 7) (3, 4), (3, 5), (3, 6) and (4, 5)

(b) There are 4 pairs, namely, (2, 9), (3, 8), (4, 7) and (5, 6).

(7) Names and life-spans of ten prominent mathematicians are :

RENE DESCARTES (1596-1650)
BLAISE PASCAL (1623-1662)
ISAAC NEWTON (1643-1727)
GOTTFRIED WELHELM LEIBNITZ (1647-1716)
BROOK TAYLOR (1685-1731)
LEONARD EULER (1707-1783)
JOSEPH LOUIS LAGRANGE (1736-1813)
CARL FRIEDRICH GAUSS (1777-1855)
AUGUSTIN LOUIS CAUCHY (1789-1857)
BERNHARD RIEMANN (1826-1866)

(8) Yes. For example, if x=3, y=6, the six numbers formed by 3, 6, 9 (i.e. 369, 396, 639, 693, 936, 963) are divisible by 9. The assertion is correct and the proof is shown as follows :

Without lose of generality, let the number formed by x, y, 9 be x9y

$$\begin{aligned} \therefore \text{The number} &= 100x+90+y \\ &= 100(9-y)+90+y \\ &= 99(10-y) \text{ which is divisible by 99 as } 10-y \text{ is an integer.} \end{aligned}$$

FOR YOUR INFORMATION

An English-Chinese Glossary of Terms Commonly Used in the Teaching of Mathematics in Secondary Schools (amalgamated version)

This Glossary, prepared by the Curriculum Development Council, has been sent to Printing Department and would be issued to schools in early 1992. It is a revision and amalgamation of the ones previously issued to schools in 1988 and 1989 for use in Secondary 1 to 5 and Secondary 6 to 7 respectively.

As its name implies, the Glossary provides Chinese translation of those English terms commonly used in the teaching of Mathematics at secondary and sixth form levels in secondary schools. It is intended to facilitate the wider use of Chinese as the medium of instruction in Mathematics.

Teaching Syllabi for Two Advanced Supplementary Level Mathematics Subjects

The draft teaching syllabi for Advanced Supplementary Level Mathematics and Statistics, and Advanced Supplementary Level Applied Mathematics have been endorsed by the Curriculum Development Council Co-ordinating Committee (Sixth Form). Printed copies of the said teaching syllabi would be issued to schools in due course.

In the preparatory stage of the two teaching syllabi as well as their corresponding HKEA examination syllabi for the two said AS Level subjects, a joint nuclear working committee with members from the relevant Subject Committees of the Curriculum Development Council and the Hong Kong Examinations Authority was formed to work out the main frame for the syllabi according to the suggestion made by the Working Group on Sixth Form Education. The teaching syllabus does not pretend to provide a great new insight into what mathematics teachers should be trying to achieve. Rather it seeks to provide an indication on how and to what depth a certain topic should be treated through its suggested notes on teaching and time ratio.

1990 - 91 School-based Curriculum Projects

In the context of this scheme, school-based curriculum projects refer to projects undertaken by schools to adapt centrally designed curriculum to match the needs and abilities of their pupils.

In 1990 - 91, three projects (two at primary level and one at secondary level) related to mathematics learning had been developed, tried out and chosen for exhibition. The projects at primary level were respectively "Fraction for fun" for P3 and P4 pupils and "Line Symmetry and Rotational Symmetry" for P6 pupils. The former aims at providing materials to help pupils learn and applying fractions in daily use. It was designed by a group of teachers of True Light Middle School of H.K. Primary Section. The latter aims at (1) teaching basic mathematical concepts of line and rotational symmetry; and (2) stimulating the interest and creativity of pupils in the learning of mathematics through the discovery and appreciation of symmetrical shapes from the environment. The project designers were a group of teachers of Sau Mau Ping Catholic Primary School (AM). The one for secondary level was titled "Beyond + - x ÷". The project aims at

(1) enhancing pupils' interest towards mathematics;
(2) helping pupils apply their mathematical knowledge in problem solving; and
(3) introducing a new branch of mathematics, namely network and graph theory, which is not included in the present syllabus. The project designers were a group of teachers of St. Joan of Arc Secondary School.

The 32nd International Mathematical Olympiad 1991

The 32nd IMO was held from 12 to 23 July 1991 in Sigtuna, Sweden. The actual contest took place on 17 & 18 July in National Boarding School (Sigtunaskolan Humanistiska Laroverket, abbreviated as SSHL). It was the fourth time for the Hong Kong Team to participate in this international competition. Among the 55 participating countries or territories, Hong Kong ranked 29 with a total score of 91 and obtained two bronze medals and one honourable mention.

The results of the 32nd IMO are tabulated as follows:

Position	Team	Score
1	Union of Soviet Socialist Republics	241
2	Peoples Republic of China	231
3	Romania	225
4	Federal Republic of Germany	222
5	United States of America	212
6	Hungary	209
7	Bulgara	192
8	Iran	191
	Vietnam	191
10	India	187
11	Czechoslovakia	186
12	Japan	180
13	France	175
14	Canada	164
15	Poland	161
16	Yugoslavia	160
17	Republic of Korea	151

Position	Team	Score
18	Austria	142
	United Kingdom	142
20	Australia	129
21	Sweden	125
22	Belgium	121
23	Israel	115
24	Turkey	111
25	Thailand	103
26	Colombia	96
27	Argentina	94
	Singapore	94
29	Hong Kong	91
	New Zealand	91
31	Morocco	85
	Norway	85
33	Greece	81
34	Cuba	80
35	Mexico	76
36	Italy	74
37	Brazil	73
	Netherlands	73
39	Tunisia	69
40	Finland	66
	Spain	66
42	Philippines	64
43	Denmark	49
44	Ireland	47
45	Republic of Trinidad	46
46	Portugal	42

Position	Team	Score
47	Mongolia	33
48	Luxembourg	30
	Indonesia	30
50	Switzerland	29
	Iceland	29
52	Cyprus	25
53	Algeria	20
54	Macau	18
55	Bahrain	4

There were altogether 312 participants.

The question papers of the 32nd IMO are attached in next two pages.

32nd International Mathematical Olympiad

Sigtuna, Sweden

Version: English

FIRST DAY

17 July

1. Given a triangle ABC, let I be the centre of its inscribed circle. The internal bisectors of the angles A, B, C meet the opposite sides in A', B', C' respectively. Prove that

$$\frac{1}{4} < \frac{AI \cdot BI \cdot CI}{AA' \cdot BB' \cdot CC'} \leq \frac{8}{27}.$$

2. Let $n > 6$ be an integer and $a_1, a_2, \ldots, a_k$ be all the natural numbers less than n and relatively prime to n. If

$$a_2 - a_1 = a_3 - a_2 = \cdots = a_k - a_{k-1} > 0,$$

prove that n must be either a prime number or a power of 2.

3. Let $S = \{1, 2, 3, \ldots, 280\}$. Find the smallest integer n such that each n-element subset of S contains five numbers which are pairwise relatively prime.

Time: $4\frac{1}{2}$ hours.
Each problem is worth 7 points.

32nd International Mathematical Olympiad
Sigtuna, Sweden

Version: English

SECOND DAY
18 July

4. Suppose G is a connected graph with k edges. Prove that it is possible to label the edges $1, 2, 3, \ldots, k$ in such a way that at each vertex which belongs to two or more edges the greatest common divisor of the integers labelling those edges is equal to 1.

   [A *graph* G consists of a set of points, called *vertices*, together with a set of *edges* joining certain pairs of distinct vertices. Each pair of vertices u, v belongs to at most one edge. The graph G is *connected* if for each pair of distinct vertices x, y there is some sequence of vertices $x = v_0, v_1, v_2, \ldots, v_m = y$ such that each pair v_i, v_{i+1} $(0 \leq i < m)$ is joined by an edge of G.]

5. Let ABC be a triangle and P an interior point in ABC. Show that at least one of the angles $\angle PAB$, $\angle PBC$, $\angle PCA$ is less than or equal to $30°$.

6. An infinite sequence $x_0, x_1, x_2, \ldots$ of real numbers is said to be *bounded* if there is a constant C such that $|x_i| \leq C$ for every $i \geq 0$.

   Given any real number $a > 1$, construct a bounded infinite sequence $x_0, x_1, x_2, \ldots$ such that

   $$|x_i - x_j||i - j|^a \geq 1$$

   for every pair of distinct non-negative integers i, j.

The allotted time is $4\frac{1}{2}$ hours.
Each problem is worth 7 points.

FROM THE EDITOR

I would like to express my gratitude to those who have contributed articles and also those who have given comments and suggestions.

The SMN cannot survive without your contribution. You are, therefore, cordially invited to send in articles, puzzles, games, cartoons etc for the next issue. Anything related to mathematics education will be welcomed. We particularly need articles on experience sharing, fresh classroom ideas and teaching methodology. Please write as soon as possible to

The Editor,
School Mathematics Newsletter,
Mathematics Section,
Advisory Inspectorate,
Education Department,
Room 528, Lee Gardens,
Hysan Avenue, Hong Kong.

For information or verbal comments and suggestions, please contact the editor on 839 2574 or 561 4364 or any member of the Mathematics Section on 839 2488.