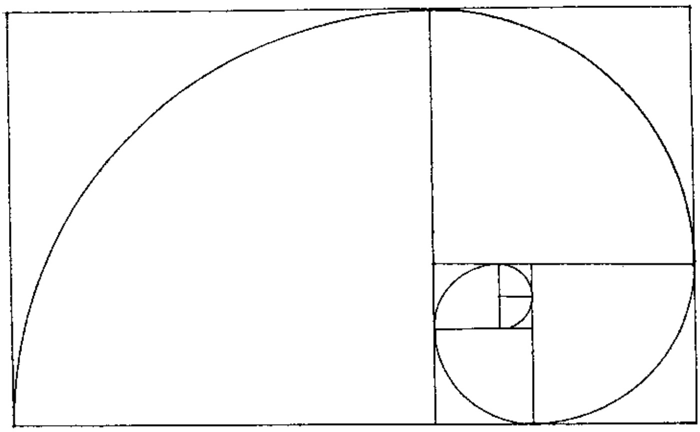
SMN 學校數學通訊 School Mathematics Newsletter

第十二期 Issue 12 1993



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The School Mathematics Newsletter aims at serving as a channel of communication in the mathematics education of Hong Kong. School principals are therefore kindly requested to ensure that every member of their mathematics staff has an opportunity to read this Newsletter.

We welcome contributions in the form of articles on all aspects of mathematics education as the SMN is meant for an open forum for teachers of mathematics, however, the views expressed in the articles in the SMN are not necessarily those of the Education Department, Hong Kong.

Please address all correspondence to :

The Editor,
School Mathematics Newsletter,
Mathematics Section,
Advisory Inspectorate Division,
Education Department,
12/F., Wu Chung House,
197-221, Queen's Road East,
Wanchai, Hong Kong.

Telephone : 8926553

〈〈學校數學通訊〉〉旨在為香港數學教育界提供一個溝通渠道，故此懇請各校長將本通訊交給貴校所有數學科教師傳閱。

為使本通訊能成為教師的投稿公開園地，歡迎讀者提供任何與數學教育有關的文章。唯本通訊內所發表的意見，並不代表教育署的觀點。

來稿請投寄：

香港灣仔
皇后大道東197-221號
胡忠大廈十二樓
教育署輔導視學處數學組
學校數學通訊編輯收

電話：8926553

IN THIS ISSUE

ISSUE 12 1993

FOREWORD

Welcome to the twelfth issue of the School Mathematics Newsletter (SMN).

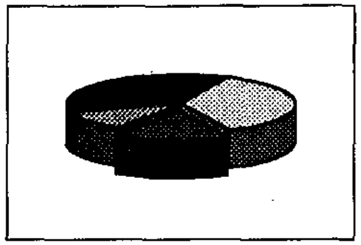
As usual, the articles in this present issue come from different individuals interested in the field of mathematics and mathematics education, like classroom teachers and teacher educators. The Mathematics Section of the Advisory Inspectorate Division would like to thank them sincerely for their contributions, without which this issue of the SMN will not come into existence.

Under the compulsory education system, mathematics teachers are faced with the tremendous challenge of teaching students of very different abilities, motivations and aspirations. To meet this challenge, mathematics teachers need to equip themselves with a repertoire of mathematical skills and teaching strategies to cope with different teaching situations. To this end, the articles in this publication cover a variety of relevant topics, ranging from teaching methodologies (e.g., experiments on Mastery Learning and Cooperative Learning) to contemporary mathematical concepts (e.g. Fuzzy Logic). There are also some interesting puzzles to tap readers' mind. We do hope all readers will find the content of this issue informative and stimulating.

Once again the Mathematics Section wishes to express its gratitude to all contributors, and also to our fellow colleagues in the Section who have made good efforts in producing this issue of the SMN.

Mathematics Section
Advisory Inspectorate Division

統計圖表誤用



黃毅英
中文大學教育學院
課程與教學系

一名中學生在完成基本普及教育後，若不再升學或從事有關數理科的工作，則統計學可能是這麼的一個普通市民在日常生活中最常遇到的數學內容。而圖表具有視覺的效果，最為各行業所樂於使用，因此也最常被誤用的。首屆香港統計習作比賽評審員報告便列舉了以下十二項參賽者最常見的毛病。

一．圖表沒有編號；

二．圖表欠缺標題；

三．圖表標題不當；

四．圖表未被內文引用；

五．坐標軸無標記；

六．坐標度數不確；

七．坐標度數不平均；

八．坐標單位太長，如用 1,000,000 而不用「以百萬計」；

九．不以 (0,0)作原點；

十．矩形圖不以面積而以高度表頻數；

十一．圖表選擇錯誤，如：
(a) 以折線圖表離散數據，
(b) 圓形圖不以百分比作顯示；

十二．相約圖表之顏色、度數、次序等不統一。

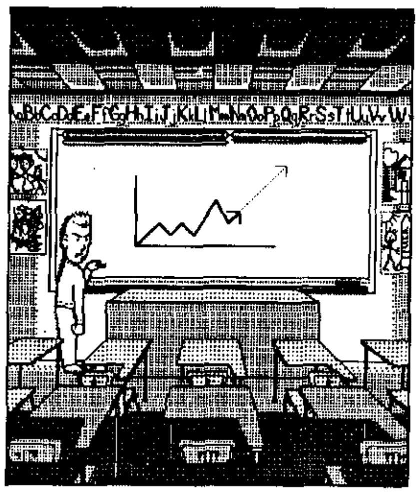
不過筆者以為圖表上的數字「錯誤」往往不一定是誤用圖表，圖表不以(0,0)作原點便是一例。有時為了放大看其趨勢或有此需要，而且每一點的 x 值都與適當的 y 值對應，所以並不存在「錯」的問題。所謂「錯」其實是圖像中帶來的錯覺吧。

圖表上引起的錯覺其實是與閱讀者的成熟程度有關的。例如將某棒形圖的棒用鮮艷的顏色來顯示，亦算是一種誤用，但若閱讀者「不為所動」，著色的棒形圖亦無不可。故此，讓未來的公民(學生)獲得

此種閱讀統計圖的成熟程度可能比使其曉得計算平均值、標準差更為重要，且亦為數學科推行公民教育的一種體現。

要達到這些目的，最好能從常見的報刊雜誌抽出不同的例子、而非光光自行製作統計圖誤用之例子。這不只令教學變得生動，且可讓學生對於這些圖表有更深的感受。（以下抽出一些例子 ＊），若教師直接指出各圖表之毛病，所達到的效果可能不大。反之，教師可先讓學生指出各圖表之特式、相同與不同，這樣便更能提高學生閱圖之成熟程度 。

＊ 作者提供的統計圖表，因版權關係而未能刊登，編輯部謹此致歉！



在小六數學科試行合作學習的經驗

胡少偉

(I)合作學習與數學教學

(一)甚麼是合作學習

根據 The International Encyclopedia of Education : Cooperative learning (合作學習) is an instructional approach in which learners attain their goals and rewards through interdependence and cooperation with one another.

Jerry Rottier and Beverly J.Ogan 在 Cooperative learning in Middle-level schools 一書中提出合作學習對剛踏入青春期的少年學習者 (young adolescents) 有以下的優點：

(1) Cooperative learning tends to promote higher achievement.

(2) Cooperative learning promotes the greater use of reasoning.
(3) Cooperative learning promotes a postitive relationship.
(4) Cooperative learning promotes more positive attitudes toward subject matter.
(5) Cooperative learning promotes higher self-esteem.

(二)在數學教學施行合作學習的好處

根據 David W.Johnson 及 Roger T. Johnson,在數學教學中施行合作學習的好處有:—

(1) Mathematical concepts and skills are best learned as a dynamic process with the active engagement of students.
(2) Mathematical problem solving is an interpersonal enterprise.
(3) Mathematical learning groups have to be structured cooperatively to communicate effectively.

(4) Cooperation promotes higher achievement in Mathematics than competitive and individualistic efforts.

(5) By working cooperatively, students gain confidence in their individual mathematical abilities.
(6) Choices of which mathematics courses to take and what careers to consider are heavily influenced by peers.

(三)適用於數學科的合作學習模式

適用於數學科的合作學習模式有三個:

(1)小組成就分部法 (Student Teams and Achievement Divisions)

(2)小組遊戲比賽法 (Teams - Games - Tournaments)

(3)組員互助法 (Team Assisted Individualization)

小組成就分部法是將全班分成若干組，每組四至五人，當中有不同能力程度的學生；每次上課，老師向全班講授後，指派小組一起工作，務求使每一組員理解及掌握教學內容，約3至5堂後，老師舉行一個短測。短測中每一學生自行作答，暫不容許小組討論。老師評定短測成績後，與以往的成績比較，所進步的分數視為該生的積點。每次短測後，老師都計算每一學生的積點，進步較多的應加讚許；同時老師又為每一組計算其總積點，這樣各組又可作為一個競賽。

小組遊戲比賽法的分組和形式都與小組成就分部法相同，分別之處在於比賽方法，每次短測各學生都會與不同能力的其他同學成為一組，而每組得分是獨立計算的。

組員互助法則較強調個別學習，各學生皆有其個別的進度，如遇困難便找其他組員協助。老師雖準備不同進度的練習紙給不同能力的學生；並按個別學生的學習成效予以讚賞，最後才評定每組的總體進步情況。

(Ⅱ)在小六數學科推行合作學習

(一)計劃的目的

在大學兼讀的學習中，最令筆者深刻的是自從九年免費及普及教育施行後，本港的教育體制中有很多未能配合以作應有的改善，特別是課程和教學兩方面是十分需要改進的。在課程方面，筆者作為一個批評者，對教育當局提出過不少的意見；在教學方面，筆者是一個教學前線工作者，實應參與建設。而這個在小六數學科試行合作學習計劃，是筆者於九二年的嘗試，實踐一個較新的教學概念——合作學習。

在六年級下學期最後的階段中，筆者感到學生的數學水平參差很大，因此希望利用合作學習計劃，複習一些在小學數學中的重要課題，藉著各組中水平較高的學生協助能力較低的同學，以便他們到中學階段時，數學的能力不至於相距太遠，而對於能力高的同學，一方面可溫故知新，一方面可發展與人溝通及組織的能力。所以，此計劃的目的——從學生角度來看是：將學生分組推行合作學習，複習小學階段中主要的數學基本課題，使各同學可溫故知新，以準備升中。

（二）重組教學內容

筆者在決定於六年級數學科推行一個為期四週的合作學習計劃時，已了解到在這一個月中要忘記現有的課本。為了擬定複習小學階段的主要數學科基本課題，筆者細心閱讀小學數學科的課程綱要，當中書末的「小學數學科課程流圖」是最有參考價值。再加上翻閱多個出版社的小五及小六數學課本，筆者定了下列學習範圍：代數、小數、分數、百分數、統計圖及面積、體積和容量。

為了較系統地安排以上課題，筆者將四週分為四個階段，當中每一階段以一個短測作為小結；而每一階段的課題取一至兩個主要學習範圍，及揉合了部份的圖形計算，詳情請參看圖一。

（三）學生的分組

不少美國學者指出在採用學生小組——成就分部法（Student Teams and Achievement Divisions）時，各小組應包括不同學業表現的學生。

第一階段	第二階段	第三階段	第四階段
小數加減	正比例	分數	百分率
小數乘除	反比例	分數加減	百分率應用
因數分解	方程式	分數乘除	圓形圖
質數與合成數	速率	面積	容量
H.C.F.及L.C.M.	直線圖像	圓面積	體積
複合棒形圖			

（圖一）

在推行合作學習的班別中，共有35名學生，為了平分各組組數，筆者擬定了全班分 7 組，每組各有 5 名學生。在合作學習推行的第一天，筆者首先介紹此計劃的目的，並在獲得學生的認同後，將全班學生按其在上學期的數學科總平均分由高至低分成 5 組能力，由最高能力的七位同學在每一能力組別中選取一同學作為其組員，這樣可使此能力高的同學成為當然的組長，在分組時有較高的影響力和最大的責任。與此同時，因各組員大都是由組長主動挑選，故此組長和組員間有較好的關係，這有助於提高各組內的合作。

(四)評估與獎勵

根據 R.E.Slavin 在 Small-group Instruction 一文中，建議計算學生的積點是短測得分減去基礎分(base score)，而基礎分則是學生以往的總平均分減5分，這樣學生便較容易得到成功的機會。在每次短測的得分超過基礎分 1分便算得到一個積點，一個學生最高可得十個積點。筆者是採用此評估方法計算學生每次的成果，而每個學生所得的積點之和便是該組的積點。

在推行時，筆者在每次短測中，除批改每一學生的得分外，並即時計算其積點，然後向學生公佈；以便各學生得知其進度。為了減少各學生的壓力，筆者只公佈每組的積點而並非公佈每一學生的積點；與此同時，筆者也會公佈每次滿十個積點的學生，以便他們獲得多些稱許。

至於獎勵方面，筆者向全班七組宣稱得積點較高的四組可獲獎勵，而全班積點最多的十個學生可分別獲得獎狀一份。

(五)計劃的成果

在四次短測中，平均成績較期中試有進步的有16位同學（佔全班45.7%）；而累積十個積點以上的同學，共有24人（佔全班68.6%）。證明了這個合作學習確能令學生在數學學習有進步，及使大部份學生得到稱讚和榮譽。

此外，為了了解學生對此計劃的反應，筆者在計劃結束時做了一個學生問卷調查，當中超過七成的學生認為自己的數學

科成績有改善；而超過一半的學生覺得改善了與小組同學的關係。至於學習自信心和態度方面都有超過四成的學生感到有改善。

總體而言，學生是認定了小組合作的重要，知悉了學習是可透過同學間互助而達成，這點是筆者認為最重要的。在九年免費強迫教育推行後，同學間的競爭是減少了很多，至於如何引入合作的重要給學生，合作學習便是一個最好的方法。

(Ⅲ)小結

事實上，筆者在數學教學中並非一個專家，但有幸在進修中接觸到一些較新的教學法，而合作學習又確實有其值得推廣之處。故此，筆者大膽地將自己試行的實驗計劃精簡後向數學教學的教師作報告，寄望大家多多批評指正。

最後，本港有關合作學習(Cooperative learning)的中文書籍和資料十分匱乏，筆者恐自行翻譯後有損原學者的理念，故在引用時採用英文表達，敬希各位老師見諒。

參考資料

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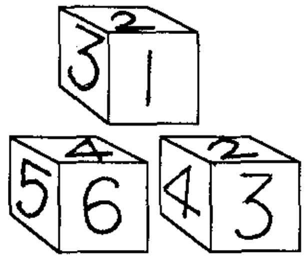
A Probability Game



In a game, there are three bags : A, B and C, each of which contains 3 dice. The numbers on the 6 faces of each die are given as follows:

	Bag A	Bag B	Bag C
1st die	0,0,4,4,4,4	3,3,3,3,3,3	2,2,2,2,6,6
2nd die	2,3,3,9,10,11	0,1,7,8,8,8	5,5,6,6,6,6
3rd die	1,2,3,9,10,11	0,1,7,8,8,9	5,5,6,6,7,7

A man Y randomly selects a bag and randomly draws a die from it. Another man X then chooses a bag from the remaining two and again draws a die in the same way.The dice are thrown. If the number on X's die is greater than the number on Y's die, then X wins; otherwise X loses. It could be found that the probability

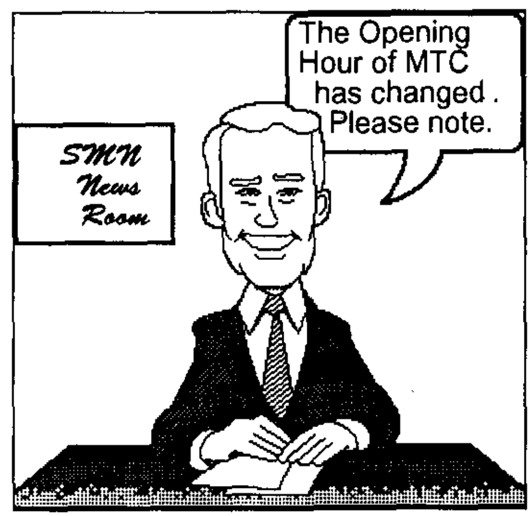
that X wins if Y chose a die from A and X chose a die from C is

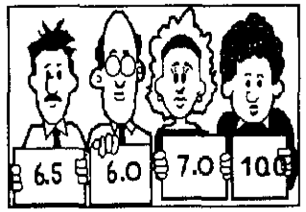
$\frac{180}{324}$.

(i) What can you conclude from this probability ?

(ii) If you were Y, which bag would you choose in order to maximize your chance to win ?

***   ***   ***





通達學習法之理論及實踐經驗談

張志鴻
彩雲聖若瑟小學下午校

一、前言

在本港，通達學習法相信仍是一個十分陌生的名字。雖然，在1990年教育統籌委員會第四號報告書中曾有提及，但亦只是寥寥數語而已（見p.82）。它究竟是甚麼東西？本文會先對有關理論作一個簡單介紹，然後，再就一間小學的實踐經驗和各位分享，希望拋磚引玉，藉此引起各位興趣及得到一些積極回應，以改進教學成效。

二、通達學習法理論簡介

通達學習法(Mastery Learning)又稱為掌握學習法或精熟學習法，是由布魯姆

(Benjamin Bloom)在1968年所提出的。其目的是針對學生的個別差異，而在教學上作出適切的安排，使所有學生都能學得好——即學生能學得更快、學得更有效和學得更有自信。

布魯姆認為無論「好」的與「差」的學習者和「快」的與「慢」的學習者皆可以透過學校情境加以改變。如果提供每個學生最適切的學習情境，則大多數的學生在學習能力、學習速率和進一步學習的動機方面，都變得非常近似。

他認為教師質素對於整個教學成效來說，其實只佔一個較輕的比重，而最重要的，卻是其他的兩個因素——學生的認知起點行為和情意起點特性。所以教師和學校應設法找出每個學生學習某一教材的認知起點行為和情意起點特性，而以此為基礎，進行包括「回饋—校正」的個別化學習策略，即可改善或消除學生學習結果的個別差異，使學得「慢」的學生像學得「快」的學生一樣快速學習。

「通達學習法」的策略是先將一學年或一學期的教材，依課程目標，細緻地劃

分為若干個小目標，再由淺入深地把教材按目標編排好；並且讓學生清楚地明白他們要學的是什麼。由於學生能知道他們要學的是什麼，便可以消除他們對課程的疑慮，從而減低他們的「必需的學習時間」。

教師在開始授課之前，首先會讓學生做一個診斷測驗(Diagnostic Test)以判斷哪些教材是所有學生該學的，以及應學到何種程度。在完成一個單元的授課後，教師會給予學生一個「形成測驗A」(Formative Test A)。這通常要求學生能有80或85分方為及格。這個測試基本上亦是一個診斷工具，用以評估學生是否能達到「通達水平」及幫助教師計劃校正活動，以彌補起始教學(Original Instruction)時的錯誤。

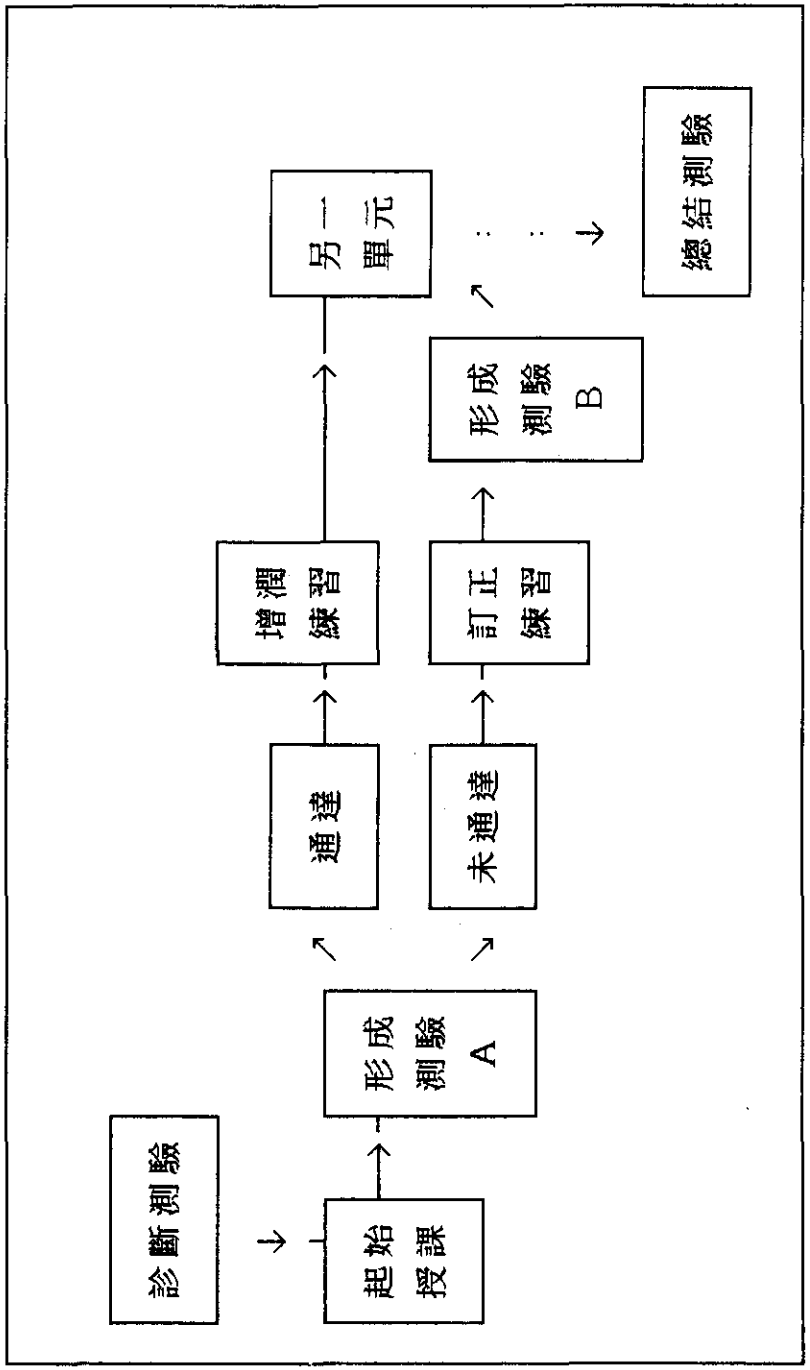
如果有部分學生不能達到這個水平，教師是會為這些學生設計一些訂正練習(Corrective Exercises)，其目的是讓學生能有機會鞏固他們學得不好的地方。在做訂正練習之前，教師是會透過一些個別化的教學方式，使學生能夠有信心和能力去應付下一個測試——「形成測驗B」。這個測驗的形式和深淺與上一個測驗類似。通常大部份的學生都可以通過這次測驗的。

至於已通過形成測驗的學生，教師會為他們預備一些增潤練習(Enrichment Exercises)，使他們可以獲得更深一層的知識或達到更高的教學目標。

當所有學生都已完成這些練習或測試後，教師才會進入另一個單元的授課。這授課的過程見圖二所示的流程圖。

在完成所有單元的教學後，教師會給予學生一個總結性測驗(Summative Test)。這個測試除了作為對學生的等級評定外，其主要目的是要評估學生在整個學習過程中所達到的通達程度。而總結性測驗和形成測驗都屬於標準參照測驗(Criterion-referenced tests)(簡稱C.R.T.)。

由於「通達學習法」採用一套「回饋—校正」策略和C.R.T.的評估，故此能提供更好的教學品質，給予學生更多的獲得成功的機會，所以能夠提升他們的學習態度和學習速率(毛連塭 1987)。



（圖二）

三、「通達學習法」之實踐初探

甲、試驗原因：

筆者在1990年六月間，曾分析過本校該年度的香港學科測驗資料，發現一至六年級的數學科成績中，以三年級的成績最差。當時，我們與幾間小學的教師和校長，加上中文大學教育學院的導師和對通達學習有經驗的教師，共同策劃和推行，目標是透過通達學習，以改善三年級的數學科成績。

乙、進行過程：

在進行「通達學習」的試驗計劃之前，有關的學校校長和教師，都曾參加由中文大學教育學院導師為這個試驗計劃而舉辦的研討會及工作會議，以認識「通達學習」的理論和實施步驟。然後，我們分為若干個工作小組，分頭設計和編寫教學計劃（包括選擇適合的課程和教材、釐訂教學目標、擬定診斷測驗、形成測驗、校正練習、增潤練習及總結測驗等）、教案和教具等。

由1990年九月開始，我們分別在學校進行了該項試驗計劃。

四、實施「通達學習」的初步檢討：

在實行「通達學習」試驗計劃的八個月後，我們希望能分別從學生的學業成績（用校內成績來評估）和給他們回答的問卷中，獲知這個計劃的試驗成效。以下是本校的試驗結果：

甲、學業成績：

本校一至三年級均採用活動教學法，每班學額為 35 人。一至三年級均不以成績分班，所以本班（3E班）學生在試驗計劃前的成績和其他三年級學生都只屬一般（請參看圖三）。

由1990年九月至1991年五月底，本校一共進行過三次測驗和三次考試。在試驗計劃前，即二年級下學期期終試時，3E班學生的成績和其他三班學生的成績都相當平均，其中達到80分或以上（即「通達標準」）的學生只佔約四分之一，而70分或以下之學生亦佔約四分之一，其餘的學生約佔半數。

成績	試驗前	%	八個月後	%
80 分或以上	10人	28.6	27人	77.1
70-79 分	16人	45.7	8人	22.9
70 分以下	9人	25.7	0人	0.0
本班平均成績	※ 74.1分		※※83.6分	
其他三班平均成績	※ 70.7分		※※75.3分	

※為二年級下學期期終試成績
※※為三年級三次測驗及三次考試之平均成績

（圖三）

但在試驗計劃的八個月後，可以發現3E班學生的成績比以前確有很大的進步。而且有幾個地方是特別值得注意的：

1.其平均成績已達「通達標準」:83.6分；

2.80分或以上的學生竟有 27人 (77.1%)，與布魯姆理想中的教學要求——學生達到高水準的學習成就，80% (參考書目十一第二十二頁)，相距不遠；

3.學生間的成績雖然仍有差距，但距離已大為縮小，但其他三班學生的成績卻仍有頗大距離，這方面形成一個強烈對比；

4.該班其中有三名學生的成績是在60分以下的，但經過「通達學習」之後，成績均已有顯著進步。

從以上的學業成績來看，可以發現，進行「通達學習」的學生（實驗組）和非進行「通達學習」的學生（控制組），是有分別的。

乙、學習態度：

雖然從進行「通達學習」的該位教師口中，可以知道學生的學習態度是相當正

面的，本校亦進行過一次學生意見調查，結果如下：

(1)覺得在本班學習是快樂的。 (91.4%)

(2)對數學科的學習興趣是濃厚的。 (88.6%)

(3)對每次學習單元的學習目標是清楚的。 (80%)

(4)認為每次數學練習都是由淺入深的。 (100%)

(5)有信心應付形成測驗A(94.2%)及形成測驗B。 (97.1%)

(6)認為在每個單元後做測驗是有幫助的。 (88.6%)

(7)喜歡做增潤練習。 (80%)

(8)適應教師在通達學習的授課方法。 (91.4%)

(9)採用通達教學法後，做數學的速度比從前快(77.1%)，而且做功課時，大部份或完全不需要家人或成人協助。 (82.9%)

(10)對完成考試具有信心。　(94.2%)

(11)認為教師與學生的關係很密切。(74.3%)

(12)認為教師在通達學習中有個別指導的機會。(97.1%)

(13)覺得本班的功課量適中。　(74.3%)

(14)贊成其他科目都採用通達教學法。(68.6%)

(15)贊成全校都採用通達教學法.(62.9%)

(16)對取得通達證書表示興奮。　(88.6%)

(17)對取得通達榮譽證書表示興奮。(88.6%)

從以上的結果顯示，學生對於學習的態度是相當積極及正面的，對於學習的自我觀念也很強。這個結果顯示學生經過「通達學習」後，其「情意的起點特性」已達到一個相當理想的程度。根據布魯姆的理論，這因素是佔影響學習成就的四分之一成效。

五、結語

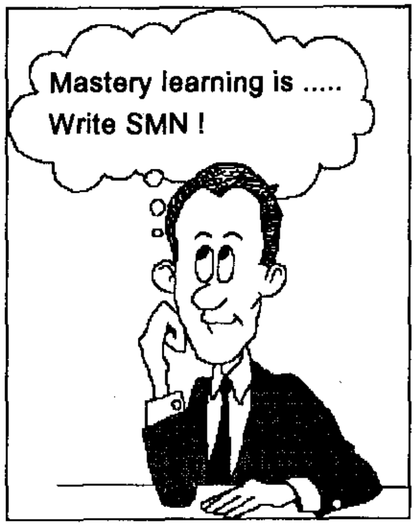
綜合上文「通達學習」理論與策略和實踐成果等的探討，加上在外國也有大量的有關研究，已證明「通達學習」具有不容忽視的效果，尤其是對於中小學的學生為然。所以，「通達學習」的確是針對時弊的一劑「良藥」。不過，它並不是「萬應靈丹」，誠如毛連塭所言：

「精熟學習當然不是教育上的萬靈藥。它無法解決教師必須面對的所有問題。但是，在不同的國家不同的教育階段中已經證實一點：實施精熟學習可以幫助更多的學生學會老師所教的內容，使得老師對學生發生更強而更有力的影響。」（參考書目十一第六頁）

所以，教育工作者不應單以一種教學方法或策略去處理所有的教材或教學目標，而應配合其他的教學法或策略以增加「通達學習」的成效。

在本港，實踐「通達學習」的小學，現時只是剛踏出了第一步。雖然我們作了一些檢討，但基於很多因素，例如時間上

的配合、研究的方法及其他變項，如不同學校的師資、試卷、校長的領導方式、學校教學情境、學校氣氛、學生學能及其他因素等，都會影響研究結果。事實上，這只是一個初步檢討，還有待進一步的探索。不過，值得高興的是各參與試驗計劃的學校，對初步的試驗結果均感到滿意。我們期望將來有更多的學校自願參加有關的試驗計劃及對有關教學策略提出更多及更有建設性的意見。



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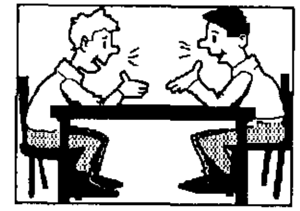
<u>淺談ＴＴＲＡ</u>



趙勝強

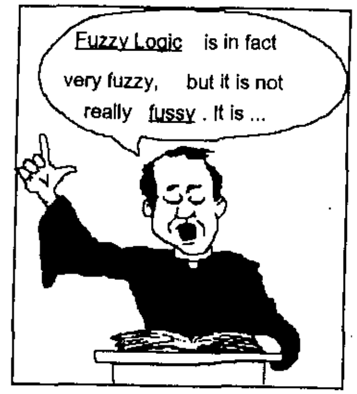
我是一個數學教師，所任教的學校今年參加了ＴＴＲＡ試驗計劃。在試驗當中，我發覺到ＴＴＲＡ確能準確地衡量學生的表現，並協助我更有效地去幫助學生取得更佳的學習成果。試行此計劃時，教師需要重新組織教學內容施教，同時針對學生的長處和弱點而加以強化和改善，而學生亦可清晰知道所學到的知識。因此，學生在學習上更顯積極，更能朝著學習目標前進。

在課業設計上，同級施教的教師可以互相協助及改善。當然，設計和討論是需要花時間的，但是，所花的時間和工作量，只是較往日的增加了百分之十到十五之間。如果將來減少了每班的人數，工作就會較為輕鬆。

在學習上，學生的確比前「活」了許多，如果能處理適當的話，秩序仍是良好的；而成績好的學生又能指導成績較差的，如是者，在一段短時間內成績較差的又再跟上了。

至於評估方面，這確是一個大難題。在某一階段，某個或某組學生的成績是優異的，但當轉入另一階段後，同一個學生的成績卻低落了。但再轉入第三個階段後，該學生又回復成績優異。考慮到自己的教學是否適當，學生是否有外在因素的影響，評估就變得難了！希望有關方面能夠給予指導和幫助。

總的來說，ＴＴＲＡ計劃是好的！這計劃越早推行，越能知道困難所在，使將來更能掌握。我認為一面去做一面去改善——這是現在、將來的做法。至於說是以學生做實驗品，我是反對的！只要教師是真真正正的去做，適時改善，ＴＴＲＡ計劃的成功是指日可見的。



甚麼是快思邏輯？

水戈木

一、模糊邏輯簡介

快思邏輯，其正式學名為模糊邏輯(Fuzzy logic*)，是由美國自動控制論教授沙達洛菲（L.A. Zadeh）於1965年所創立。它的產生不僅打破了傳統邏輯的規限，而且更為電腦模仿人類思考的研究方面，帶來重大的突破。

在傳統的邏輯運算上，我們對每一個概念，都要作出清晰而又準確的表達。例如，我們可以說：「他身高1.84米」，而不能說：「他的身裁高大」。可是，在現代日益複雜的社會裡，要對每一件事物進行精確的描述，根本辦不到或沒有這個精要，因此，模糊邏輯就應運而生。

模糊邏輯並不需要對每一件事情進行精密的描述，我們只要對一些句子提出一個「可靠性」的百分比就足夠。例如：「『他身裁高大』的可靠性是60%」，表示以他的身高，有百分之六十的人，認為他是「高大」的，這裏並不需要深究，他到底是多少米高。這方法雖然降低了對事物描寫的精確度，但卻為一些複雜的訊息，提供了一個簡明又可行的描述方法。難怪模糊邏輯一經提出後的三十年間，就廣受歐、美、甚致中國的數學家重視和研究，並在自動控制、系統分析、知識描述、語言加工、圖象識別、訊息複制、醫學診斷、經濟管理等研究上，有明顯和實際的成果，亦為電腦科學的發展，提供了強而有力的工具。最近，更有生產商，將模糊邏輯的應用技術，引入家庭電器之中，相信會對我們日後的生活質素，帶來進一步的提高。

※註：fuzzy一字解釋為「模糊的」、「形狀不清楚的」。

二、習作：模糊集合及運算

Let X be an ordinary set.

Definition A fuzzy set on X is a function

$$\underline{A} : X \to [0, 1],$$

and the set of all fuzzy sets on X is given by

$$F(X) = \{\underline{A} \mid \underline{A} : X \to [0, 1]\}.$$

Let $\underline{A}, \underline{B} \in F(X)$. Then $\underline{A} \subset \underline{B}$ if $\underline{A}(x) \le \underline{B}(x)$ for all $x \in X$.

Then $\underline{A} = \underline{B}$ if $\underline{A}(x) = \underline{B}(x)$ for all $x \in X$.

Define $\phi : X \to [0, 1]$ s.t. $\phi(x) = 0$ for all $x \in X$.

$\underline{X}: X \to [0, 1]$ s.t. $\underline{X}(x) = 1$ for all $x \in X$.

Then (1) $\phi \subset \underline{A} \subset \underline{X}$

(2) $\underline{A} \subset \underline{A}$

(3) If $\underline{A} \subset \underline{B}$, $\underline{B} \subset \underline{A}$, then $\underline{A} = \underline{B}$.

(4) If $\underline{A} \subset \underline{B}$, $\underline{B} \subset \underline{C}$, then $\underline{A} \subset \underline{C}$.

Define $(\underline{A} \cap \underline{B}) : X \to [0, 1]$ s.t. $(\underline{A} \cap \underline{B})(x) = \min\{\underline{A}(x), \underline{B}(x)\}$,

$(\underline{A} \cup \underline{B}) : X \to [0, 1]$ s.t. $(\underline{A} \cup \underline{B})(x) = \max\{\underline{A}(x), \underline{B}(x)\}$,

and $\underline{A}^c : X \to [0, 1]$ s.t. $\underline{A}^c(x) = 1 - \underline{A}(x)$, for all $x \in X$.

Properties of Fuzzy Set Operations

(1) $(\underline{A}\cup\underline{B}) = (\underline{B}\cup\underline{A})$; $(\underline{A}\cap\underline{B}) = (\underline{B}\cap\underline{A})$,

(2) $(\underline{A}\cup\underline{B})\cup\underline{C} = \underline{A}\cup(\underline{B}\cup\underline{C})$; $(\underline{A}\cap\underline{B})\cap\underline{C} = \underline{A}\cap(\underline{B}\cap\underline{C})$,

(3) $\underline{A}\cup(\underline{B}\cap\underline{C}) = (\underline{A}\cup\underline{B})\cap(\underline{A}\cup\underline{C})$;

$\underline{A}\cap(\underline{B}\cup\underline{C}) = (\underline{A}\cap\underline{B})\cup(\underline{A}\cap\underline{C})$,

(4) $\underline{A}\cup(\underline{A}\cap\underline{B}) = \underline{A}$; $\underline{A}\cap(\underline{A}\cup\underline{B}) = \underline{A}$,

(5) $\underline{A}\cup\underline{A} = \underline{A}$; $\underline{A}\cap\underline{A} = \underline{A}$,

(6) $(\underline{A}^c)^c = \underline{A}$,

(7) $\underline{X}\cap\underline{A} = \underline{A}$; $\underline{X}\cup\underline{A} = \underline{X}$,

$\underline{\phi}\cap\underline{A} = \underline{\phi}$; $\underline{\phi}\cup\underline{A} = \underline{A}$,

(8) $(\underline{A}\cup\underline{B})^c = \underline{A}^c\cap\underline{B}^c$; $(\underline{A}\cap\underline{B})^c = \underline{A}^c\cup\underline{B}^c$

Remark : In general, it is not necessarily true that

$$\underline{A}\cup\underline{A}^c = \underline{X} \quad \text{or} \quad \underline{A}\cap\underline{A}^c = \underline{\phi}$$

although it is true in our ordinary set theory.

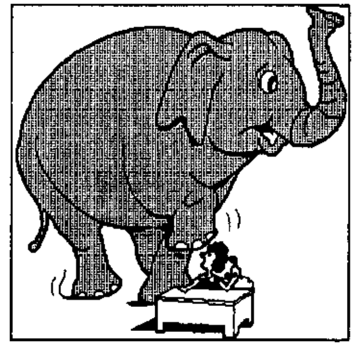
Consider $X=[0,1]$ and $\underline{A}(x) = x$ for all $x\in X$.

Then $(\underline{A}\cup\underline{A}^c)(½) = (\underline{A}\cap\underline{A}^c)(½) = ½$ and hence

$$\underline{A}\cup\underline{A}^c \neq \underline{X} \quad \text{and} \quad \underline{A}\cap\underline{A}^c \neq \underline{\phi}.$$

從以上習作可見，模糊集合運算與傳統集合運算的最大分別，就是模糊集合運算並不滿足互補律。事實上，在許多實際問題中，大量存在著模稜兩可的情形。因此，模糊集合運算就更能反映事物的客觀狀態。

Role of Panel Chairman in Remedial Teaching of Mathematics in Secondary School



Lau Ming Yee,
Mathematics Department,
Sir Robert Black College of Education

Remedial teaching was first introduced to Hong Kong secondary schools in September 1982. In order to facilitate the operation of remedial programs in subjects of Chinese and English at the junior levels, two additional graduate teachers were provided. An additional graduate teacher and two non-graduate teachers were respectively provided in September 1983 and September 1986, to carry out remedial teaching in other subjects, to help in career guidance, to promote extra-curricular activities and social services in the schools. Since then, many secondary schools in Hong Kong have started remedial teaching of mathematics, again, mostly at junior levels.

The main objective of running remedial programs in the subject of mathematics is to give additional help to students, who are relatively falling behind and cannot catch up with the rest of the class. It is believed that through small group teaching, the mathematics class teacher could help these students develop positive attitude towards the subject, to regain confidence in themselves and ultimately to have the motive to learn mathematical skills and to appreciate the beauty of mathematics.

Regarding the implementation of remedial teaching in secondary school mathematics, the role of the Panel Chairman becomes more important. The success of running the remedial programs, to some extent, depends on his effort and leadership. It is most desirable that he himself takes part in the actual teaching of remedial classes. In doing so, he can realize the basic difficulties that the other class teachers are facing every day. With the cooperation of other panelists, his first task is to make a careful diagnosis of students' general strengths and weaknesses in mathematics as well as their needs. On the basis of the result of the diagnosis tests, an appropriate mathematics programme for the whole school can be designed. Hereby, teachers can gather together to work out the common core and optional syllabuses catering for the needs of both the mainstream and the remedial classes. Meanwhile, the panel chairman should ensure that there is adequate coordination across levels to provide a smooth progression and continuity for students of different ability groups.

The coordination between teachers of the mainstream classes and those of the remedial classes is vital. The panel chairman should arrange regular meetings, both formal and informal ones. Through these meetings, they can design the scheme of work, select suitable core and optional items for teaching, share experience in the use of teaching aids, formulate and carry out policies on how to set examination and test papers. Last but not the least, teachers can share and interchange their own experiences on every aspect of mathematics teaching.

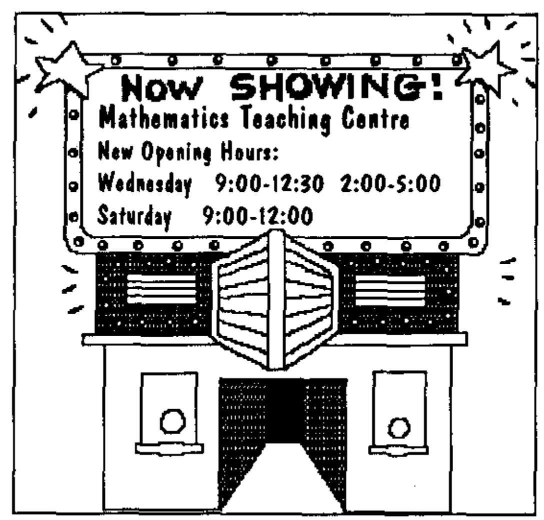
Teachers in Hong Kong are sometimes lonely. They need both professional and moral supports. It is, therefore, one of the role of the panel chairman to offer professional advice to the teachers engaged in remedial teaching. He should have mutual understanding with the class teachers. He should also try to keep himself well informed of the actual running of the remedial classes.

He should be ready to give help to his fellow colleagues. He should try his best to provide administrative support for teaching strategies. This can be done through the purchase of relevant books, references and audio-visual aids.

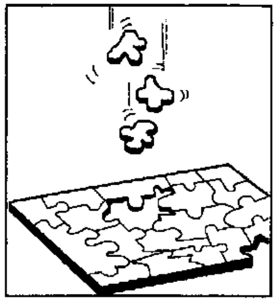
In practice, there are many problems associated with remedial teaching that have no ready solutions. However, the panel chairman at these times can provide the moral support to his colleagues. He can show his full understanding and concern, he can recognize the good work and the contribution of the teachers and he should show his care by taking part in the exploration for possible solution to the encountered problems.

Rome was not built in one day. To carry out remedial teaching of mathematics in secondary schools is a difficult task. However, with the best lead by the panel chairman, we are marching towards our goal.

*** *** ***



How Students Solved It



CHEUNG Pak-hong
Department of Curriculum Studies
The University of Hong Kong

In the summer of 1992 I had a chance to meet about sixty senior secondary students selected as trainees for the Hong Kong team to participate in the International Mathematical Olympiad (IMO) 1993. Before they left the short meeting I distributed to each one ten problems to solve within five weeks. These problems were taken from *Quantum*, an American students magazine of mathematics and science based on *Kvant* published in Russia. Most of these problems are much easier than IMO problems and demand only knowledge of mathematics below Secondary 5. Whereas textbook and public examination 'problems' can usually be solved by applying routine algorithms, these problems are not straightforward and require exploration with insight and perseverance. Only about one-third of the trainees sent in solutions to one or more problems. The small number of responses reflected that the majority of these victors of examinations were not interested in solving genuine problems in mathematics. However, a number of the solutions submitted demonstrated a high standard of problem-solving by some students. In this article, I shall try to discuss the strategies,

based mainly on students' ingenious solutions, to successfully solving these problems. Readers are encouraged to try to solve the problems before reading the solutions and comments.

The Problems

1. The lengths of CB and CA of ΔABC are 10 and 15 respectively . The bisector of $\angle C$ meets AB at D. Prove that the length of CD is less than 12.

2. Prove that any non-negative integer n can be represented in the form

$$n = \frac{(x+y)^2 + 3x + y}{2}$$

with non-negative integers x and y, and that such a representation is unique.

3. A quadrilateral is inscribed in a parallelogram whose area is twice that of the quadrilateral. Prove that at least one of the quadrilateral's diagonals is parallel to one of the parallelogram's sides.

4. Three frogs are playing - what else? - leapfrog. When frog A jumps over frog B, it lands at the same distance from B as it was before the jump (and, naturally, on the same line AB.) Initially the frogs are located at three vertices of a square. Can any of them get to the fourth vertex after several jumps?

5. On straight lines AB and BC containing two sides of a parallelogram ABCD, points H and K are chosen so that the triangles KAB and HCB are isosceles (KA = AB, HC = CB.) Prove that the triangle KDH is also isosceles.

6. (a) When a number N is multiplied by 8, the sum S(N) of its digits can, for some N, decrease (for example, S(75) = 12, whereas S(8X75) = S(600) = 6). Prove that it can't decrease by a factor of more than 8. In other words, prove that

$$\frac{S(8N)}{S(N)} \geq \frac{1}{8}$$ for any natural number N.

(b) What are the other natural numbers k for which a positive c_k can be found such that

$$\frac{S(kN)}{S(N)} \geq c_k$$

for any natural number N? What's the greatest suitable value of c_k for a given k?

7. A pair of jeans with a total area of 1 have five patches on them. The area of each patch is not less than 1/2. Prove that there are two patches such that the area of their common part is not less than 1/5.

8. Two congruent circles intersect at points A and B. Two more circles of the same radius are drawn : one through A, the other through B. Prove that the four points of the paired intersection of all four circles (other than A and B) are the vertices of a parallelogram.

9. Positive integers are written at points of a line segment according to the following rule : at the first step two 1's are written at the ends of the segment; at the second step their sum 2 is written in the middle; at each subsequent step the sum of every pair of neighbouring numbers (obtained from the previous steps) is written in the middle of the segment between them. How many 1992's have been written at the 1992nd step?

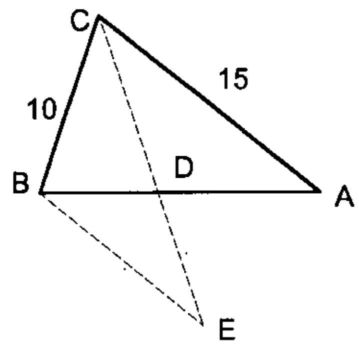
10. A lion rushes about a circus ring with a radius of 10 m. It runs 30 km along a broken line. prove that the sum of the angles of all the turns on its route is greater than 2998 radians.

Comments and Solutions

PROBLEM 1

This problem is easy for those who use trigonometry. However, trigonometry is seldom useful for IMO problems. The following solution, based on the one proposed by **HA Lik** of **Pui Ching Middle School** (S6), is a beautiful one .

In a triangle ABC let CD be the bisector of angle ACB,
AC = 15, BC = 10.



Draw a line through B parallel to AC and intersecting CD produced at E. Angles BEC and ACD, and hence BCE, are equal. Therefore, triangle BCE is isosceles. So BE = 10.

Because of the similarity of triangles ACD and BED,
CD/CE = 15/(10+15) = 3/5.

But CE < BC + BE = 20. This gives CD < (3/5) x 20 = 12.

A similar approach is to construct a line through D parallel to AC and intersecting BC at F. Try to solve the problem this way. How about drawing lines parallel to BC instead of AC?

How can one think of such a solution? Let's try to learn a lesson from it:

What is the special feature of the ***given conditions***?
A pair of ***equal angles***.
What is our ***goal***?. An ***inequality on lengths.***

Our strategy is to find a route from the given conditions to the goal.

Is there any property of triangles involving equal angles in different triangles and the ***sides*** of these triangles?

How about ***similar triangles***?

Are there any similar triangles in the given figure?

If no, can we ***construct*** two by adding a line?

What kind of line will produce angles equal to the given ones and hence produce similar triangles?

If you are stuck at some stage, why not ***work backwards***?

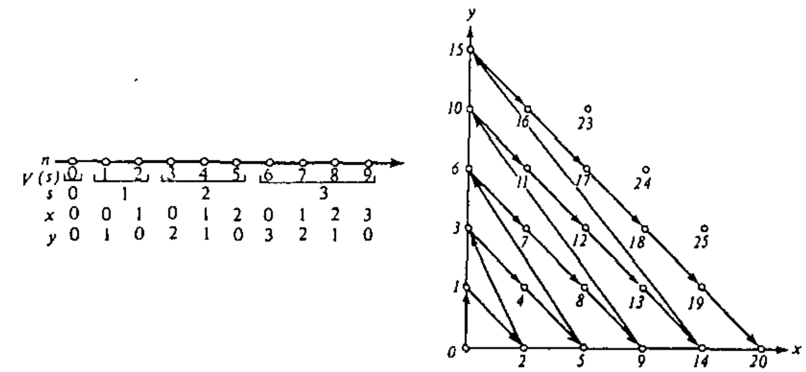
What inequality on lengths of sides of triangles is the most obvious to you?

Have you ever heard about the ***triangle inequality*** *- the sum of lengths of any two sides of a triangle is always greater than the length of the third side?*

Try to bridge the gap between the ***subgoals*** established by our strategy and solve the whole problem. Even if you are through, try to spend sometime thinking about ***alternative solutions*** and possibilities of ***modifying*** the given problem to cover more ***general*** situations, if appropriate.

PROBLEM 2

Although some students were able to solve this problem, their solutions are long and often the necessity to prove the ***uniqueness*** of the representation was overlooked. It is too naive to expect IMO-type problems on natural numbers can be solved by simple induction only. Instead of starting with induction, why not **substitute the first few pairs of non-negative integers into the given expression *systematically* to search for a pattern**? After some trials, the following patterns should be obvious:



Based on our pattern, we can proceed to present our solution :

Let us set the sum s = x + y > 0. Then the set V(s) of values assumed by

$$\frac{(x+y)^2+3x+y}{2}=\frac{s^2+s}{2}+x,$$

when x varies from 0 to s, consists of all the integers from $(s^2+s)/2$ to $(s^2+s)/2+s$, each of them assumed ***once***.

Now let us notice that the last number of V(s) and the first number of V(s+1) are consecutive .

$$\frac{s^2+s}{2}+s+1=\frac{(s+1)^2+s+1}{2}$$

Therefore, the sets V(s) cover all the non-negative integers *n* without overlaps or gaps. Since any *n* gets into one and only one of the sets V(s), it can be represented in the required form. Also, s, x and y = s - x are determined by *n* uniquely.

PROBLEM 3

This problem is easy for those students who use trigonometry and algebra to produce clumsy solutions. However, two nice approaches were given by some students. **LAU Wing Hon** of **Christian Alliance S. C. Chan Memorial College** (S6), **SIU Yiu Hang** of **Tang King Po School** (S7) and **LEE Wai Fun** of **St. Mark's School** (S7) solved it by first ***assuming*** that both diagonals of the quadrilateral are ***not parallel*** to any side of the parallelogram and then deduced from this assumption a ***contradiction*** to the given relationship between the ***areas*** .

Suppose the diagonals of the quadrilateral are both ***not*** parallel to the sides of the parallelogram. Construct lines *KP*, *LQ*, *MR* and *NS* parallel to the sides of the parallelogram. Depending on the positions of the vertices of the quadrilateral, the different situations may be represented by the following cases in general :

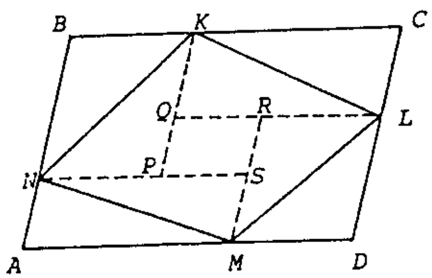


Figure 1

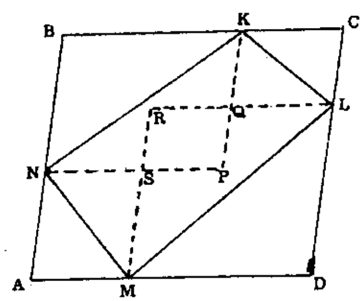


Figure 2

Since the area of a parallelogram is always bisected by any one of its diagonals, we have:

in case 1 (figure 1) , area of parallelogram *ABCD* = 2 x area of quadrilateral *KLMN* - area of parallelogram *PQRS*;

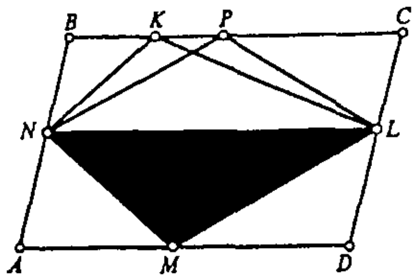
in case 2 (figure 2) , area of parallelogram *ABCD* = 2 x area of quadrilateral *KLMN* + area of parallelogram *PQRS*.

In both cases we arrive at contradictions to the given condition that the area of parallelogram *ABCD* is twice the area of quadrilateral *KLMN*. Hence at least one of the quadrilateral's diagonals is parallel to one of the parallelogram's sides.

CHAN Tsz Lung of **La Salle College** (S6), **CHAN Tsz Ho** of **Ying Wa College** (S6) and **HA Lik** of **Pui Ching Middle School** (S6) used another approach to solve the problem : they ***assumed*** that ***one*** diagonal of the quadrilateral is ***not parallel*** to the sides of the parallelogram and then deduce from this assumption, using the given relationship between ***areas***, that the other ***diagonal*** must be parallel to two opposite sides of the parallelogram.

If a diagonal of the inscribed quadrilateral, say *KM*, is parallel to a side of the parallelogram, we are done. Otherwise we mark the point *P* on *BC* such that *PM* is parallel to *AB*.

Since the area of triangle *PMN* is equal to half of the area of parallelogram *PMAB*, and the area of triangle *PML* is similarly equal to half of the area of parallelogram *PMDC*, the area of quadrilateral *PLMN* is therefore equal to half the area of parallelogram *ABCD*.



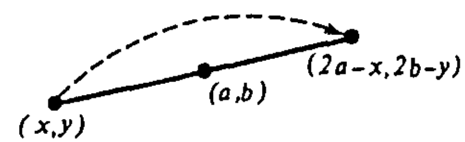
Thus the quadrilaterals *KLMN* and *PLMN* have the ***same area***. Subtracting the triangle *LMN* from both of these quadrilaterals, we get two triangles, *LNK* and *LNP*, with the ***common base*** and ***equal areas***. From this it follows that BC is parallel to the diagonal *LN*.

PROBLEM 4

This problem was solved by **TSUI Ka Hing** of **Queen Elizabeth School** (S6), **LAW Hiu Chung** of **Wah Yan College, Kowloon** (S7), **CHAN Tsz Lung** of **La Salle College** (S6) and **CHAN Tsz Ho** of **Ying Wa College** (S6). Some others attempted to present a valid approach but their presentations were long-winding, confusing and focussing only on the initial jumps instead of the general case. Mathematics is not just computation. You have to learn to communicate your ideas logically and concisely to others using words. The strategy

to solve this problem is firstly ***representing the given conditions by suitable mathematical conventions*** - coordinates in this case; then trying to establish a condition on the positions of ***any*** frog before and after a jump over another frog at ***any*** position. The following solution is similar to those proposed by the few successful students .

The answer is no. To prove it, let us introduce a coordinate system on the plane such that the initial positions of the frogs get the coordinates (0, 0), (1, 0) and (0, 1). When a frog sitting at (x, y) jumps over a frog at (a, b), it stands at the point (2a-x, 2b-y). So the ***parities*** (oddness and evenness) of a frog's coordinates do not change after a jump. At the start each frog had at least one ***even*** coordinate. Therefore, none of them can hit a point with two ***odd*** coordinates, in particular the point (1, 1) - that is, the fourth vertex of the square.



PROBLEM 5

The plainest solution to this problem is to show that triangles *AKD* and *CDH* are congruent (*AD* = *HC*, *AK* = *CD*, ∠ *DAK* = ∠ *DCH*). Almost all students who submitted any solution solved the problem this way. It is my fault to include such a below-level 'exercise' in the problem set.

PROBLEM 6

TSUI Ka Hing of **Queen Elizabeth School** (S6) solved this difficult problem and presented a lengthy but perfect solution without any mistake. I shall present here an improved version of his solution. To solve this problem, it is worthwhile for us to ***explore a special case*** first : *S(kN)* = *S(N)*. It should not be too difficult to observe that powers of 10 are suitable choices for ***k***; among them 1000 is the least one divisible by 8. This gives us :

$$S(N) = S(1000N) = S(125 \times 8N).$$

If we can prove that $S(125 \times 8N) \le S(125) \times S(8N)$, or more generally, that $S(AB) \le S(A)S(B)$ for any two natural numbers A and B, then we may use the fact that $S(125) = 8$ to complete the proof for (a). We shall prove this crucial inequality by using a hill-climbing process - we shall prove the following inequalities in succession, the proof of each of the inequalities (2)-(4) depends on the validity of the preceeding one :

(1) $S(A+B) \le S(A)+S(B)$,

(2) $S(A_1+A_2+\ldots+A_n) \le S(A_1)+\ldots+S(A_n)$,

(3) $S(nA) \le nS(A)$,

(4) $S(AB) \le S(A)S(B)$.

Here comes our solution for part (a) :

We denote by $[a_n a_{n-1} \ldots a_o]$ a natural number with the 10^k-th digit equal to a_k (k= 0, 1, 2, ..., n) .

Let $A = [a_n a_{n-1} \ldots a_o]$ and $B = [b_m b_{m-1} \ldots b_o]$ be any two natural numbers. Without loss of generality, we assume that $m \le n$ and hence that $B = [b_n b_{n-1} \ldots b_{m+1} b_m b_{m-1} \ldots b_o]$ where

$$b_n = b_{n-1} = \ldots = b_{m+1} = 0.$$

For each i = 1, 2, ..., n, there exists an integer c_i (= 0 or 1) such that

$d_i = a_i + b_i + c_{i-1} - 10 c_i$ is the 10^i- th digit of A + B,

$d_0 = a_0 + b_0 - 10c_0$ and $d_{n+1} = c_n$ (c_0 = 0 or 1).

$$\begin{aligned} S(A+B) &= \sum_{i=0}^{n+1} d_i \\ &= \sum_{i=0}^{n} a_i + \sum_{i=0}^{n} b_i + \sum_{i=0}^{n} c_i - 10\sum_{i=0}^{n} c_i \\ &= S(A) + S(B) - 9\sum_{i=0}^{n} c_i \\ &\le S(A)+S(B) \end{aligned}$$

Assume, for some natural number k, any k natural numbers A_1, A_2, ..., A_k satisfy the inequality :

$$S(A_1+A_2+...+A_k) \le S(A_1)+...+S(A_k)$$

Then, for any $k+1$ natural numbers $A_1, A_2, ..., A_k, A_{k+1}$,

$$\begin{aligned} S(A_1+A_2+...+A_k+A_{k+1}) &\le S(A_1+A_2+...+A_k)+S(A_{k+1}) \\ &\le S(A_1)+...+S(A_k)+S(A_{k+1}) \end{aligned}$$

It follows by induction that, for any natural numbers $A_1, A_2, ..., A_n$,

$$S(A_1+A_2+...+A_k+A_n) \le S(A_1)+...+S(A_k)+S(A_n)$$

As a particular case of this inequality, for any natural numbers n and A,

$$S(nA) \le nS(A)$$

Finally, since $A = [a_n a_{n-1} \cdots a_o]$,

$$\begin{aligned} S(AB) &\le S(a_n 10^n B)+S(a_{n-1}10^{n-1}B)+...+S(a_0 B) \\ &= S(a_n B)+S(a_{n-1}B)+...S(a_0 B) \\ &\le a_n S(B)+a_{n-1}S(B)+...+a_0 S(B) \\ &= S(A)S(B) \end{aligned}$$

Now the required inequality follows :

$$\begin{aligned} S(N) &= S(1000N) \\ &= S(125x8N) \\ &\le S(125)\ S(8N) \\ &= 8S(8N) \end{aligned}$$

For (b), re-examination of the case $k = 8$ in (a) leads us to ***generalise*** it to any number which divides a power of 10, that is, numbers of the form $2^m 5^n$ where m and n are non-negative integers. The argument is similar to that in (a).

For any natural number N and any non-negative integers m and n,

$$\begin{aligned} S(N) &= S(10^{mn} N) \\ &= S(2^n 5^m 2^m 5^n N) \\ &\le S(2^n 5^m) S(2^m 5^n N) \end{aligned}$$

$$\frac{1}{S(2^n 5^m)} \le \frac{S(2^m 5^n N)}{S(N)}$$

Furthermore, for $N = 2^n 5^m$,

$$\frac{S(2^m 5^n 2^n 5^m)}{S(2^n 5^m)} = \frac{S(10^{m+n})}{S(2^n 5^m)} = \frac{1}{S(2^n 5^m)}$$

Hence, for $k = 2^m 5^n$, $1/S(2^n 5^m)$ is a suitable value of c_k and this is also the greatest possible value.

As there are no other obvious choices for k, it is natural for us to conjecture that for any $k = 2^m 5^n q$, where q ($\ne 1$) is ***coprime*** (having no common factor except 1) with 10, the ratio *S(kN)/S(N)* can be made arbitrarily small by choosing suitable values for N. If this conjecture is wrong, our effort in 'proving' it may then lead us to discover other choice of k for which a suitable c_k exists. Since, $S(kN) \le S(2^m 5^n)S(qN)$, it suffices for us to consider the simpler case $k = q$. We wish to find a sequence $\{N_n\}$ of natural numbers (dependent upon q) such that $S(qN_n)$ is *constant* while $S(N_n)$ *increases without bound*. The cruicial question is : How can we find $\{N_n\}$?

Instead of guessing wildly, why don't we use some property which distinguishes q from integers of the form $2^m 5^n$? We know that whereas the *reciprocals* of $2^m 5^n$ can always be represented as *finite* decimals, reciprocals of our q's will always be equal to *recurring* decimals in the form $0.a_1a_2 \ldots a_m a_1 a_2 \ldots a_m \ldots$, which is in turn equal to

$$\frac{[a_1a_2\ldots a_m]}{99\ldots9} \quad (= \frac{[a_1a_2\ldots a_m]}{10^m - 1}) .$$

This property of q, which will be proved in our solution, leads us to the equality

$$q[a_1a_2\ldots a_m] = 10^m - 1.$$

Naturally, we may consider the sequence $[a_1a_2\ldots a_m]$, $[a_1a_2\ldots a_m a_1a_2\ldots a_m]$, $[a_1a_2\ldots a_m a_1a_2\ldots a_m a_1a_2\ldots a_m]$, ... as a choice for $\{N_n\}$.However, a quick check reveals that for $N_n = [a_1a_2\ldots a_m] \mathrm{x} (10^{(n-1)m} + 10^{(n-2)m} + \ldots + 1)$, $S(qN_n) = 9mn$, which increases with n. But don't be disappointed. We are very close to a suitable choice for $\{N_n\}$. Note that

$S(qN_n + 1) = S(10^{mn}) = 1$, which is a constant. Should we then consider the alternative

$$N_n = [a_1a_2\ldots a_m] \mathrm{x} (10^{(n-1)m} + 10^{(n-2)m} + \ldots + 1) \ +1 \ ?$$

The rest is straightforward and let us now present our ***formal*** *solution.*

Consider any natural number q(> 1) coprime with 10.
By using the pigeonhole principle, we conclude that there exists two numbers in the form $10^n - 1$, n being a non-negative integer, having equal remainders when divided by q. Denote these two numbers by $10^s - 1$ and $10^t - 1$, where s > t. Their difference $10^t(10^{s-t} - 1)$ is then divisible by q. Denote $s - t$ by m and $(10^m - 1)/q$ by R. For any natural number n, let

$$N_n = R(10^{(n-1)m} + 10^{(n-2)m} + \ldots + 10^m + 1) + 1$$
$$= \frac{10^{mn} - 1}{q} + 1.$$

Since $R < 10^m - 1, S(N_n) = (n-1)S(R) + S(R+1) > (n-1)S(R)$.
On the other hand, $S(qN_n) = S(10^{mn} - 1 + q) = 1 + S(q-1) = S(q)$ since q < 10^m and the last digit of q is non-zero.
Finally,

$$\frac{S(qN_n)}{S(N_n)} < \frac{S(q)}{(n-1)R} \to 0 \ \text{ when } n \to \infty .$$

For any natural number $k = 2^m 5^n q, S(kN) \le S(qN)S(2^m 5^n)$ and hence the ratio $S(kN)/S(N)$ can also be made arbitrarily small. Therefore only natural numbers in the form $2^m 5^n$ are suitable values for k.

PROBLEM 7

No student could solve this problem. The solution is, in fact, not difficult to arrive at by first ***expressing the given conditions in suitable mathematical conventions :***

Let $x_k (k = 0, 1, 2, 3, 4, 5)$ be the area of the part of the jeans that is covered by exactly k patches. Then the area of the jeans is

$$A_0 = x_0 + x_1 + x_2 + x_3 + x_4 + x_5 = 1.$$

The sum of the areas of the 5 patches is

$$A_1 = x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 \ge \frac{5}{2},$$

the area of the n-fold intersection of patches being counted here n times.

The sum of the areas of the $\binom{5}{2}$ (= 10) paired intersections is equal to

$$A_2 = x_2 + \binom{3}{2}x_3 + \binom{4}{2}x_4 + \binom{5}{2}x_5$$
$$= x_2 + 3x_3 + 6x_4 + 10x_5$$

Our purpose is to show that $A_2/10 \ge 1/5$, that is, $A_2 \ge 2$. Obviously, this inequality can only come from $A_1 \ge 5/2$, and probably with the help of the equality $A_0 = 1$ as well. A natural response would be to consider $2A_1 - 3A_0$:

$$2A_1 - 3A_0 = -3x_0 - x_1 + x_2 + 3x_3 + 5x_4 + 7x_5 \le A_2$$

but

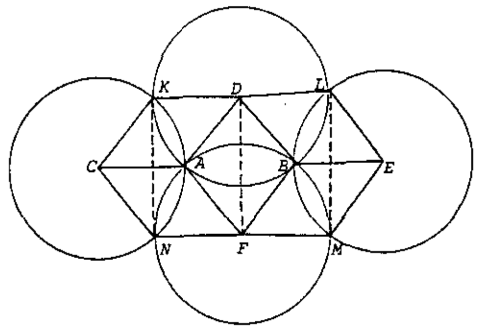
$$2A_1 - 3A_0 \ge 2 \times \frac{5}{2} - 3 \times 1 = 2.$$

Hence $A_2 \ge 2$ and thus at least one of the 10 possible paired intersections has an area not less than $2/10 = 1/5$.

PROBLEM 8

This is an easy problem and a number of students solved it in different ways. The first solution is based on those proposed by **LAM Hin Shun** of **STFA Leung Kau Kui College** (S6), **LAU Wing Hon** of **Christian Alliance S. C. Chan Memorial**

College (S6) and **KU Chong Man** of **Ying Wa College** (S5). In each of the solutions, we shall denote the centres of the given circles by *C, D, E, F*, and their pairwise intersection by *K, L, M, N*.

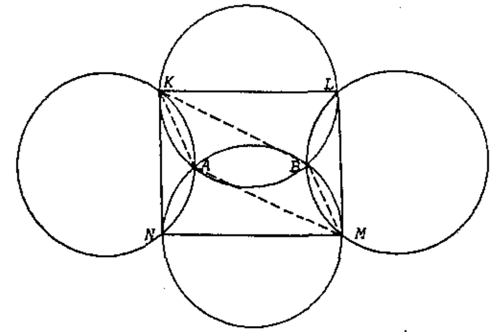


Since the circles are congruent, quadrilaterals *KDAC* and *NFAC* are rhombuses. ***KD,CA***, and ***NF are parallel and equal***, and thus quadrilateral *KDFN* is a parallelogram. Consequently, ***KN*** and ***DF are equal and parallel***.

Similar argument on two other rhombuses *LDBE* and *MFBE* will lead to ***LM = DF and LM//DF***.

Therefore ***LM*** and ***KN are equal and parallel***, and hence quadrilateral *KLMN* is a parallelogram.

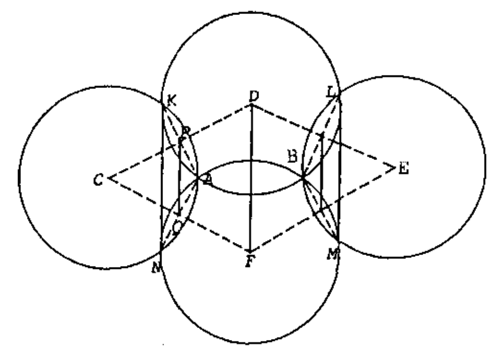
The second solution is based on those suggested by **CHEUNG Kwok Koon** of **SKH Bishop Mok Sau Tseng Secondary School** (S5), **LI Tsan Hang** of **Ying Wa College** (S5), **CHAN Tsz Lung** of **La Salle College** (S6) and **CHAN Tsz Ho** of **Ying Wa College** (S6).



By analogy with the equality of angles subtended by the same chord at points of the **same** segment, angles subtended by the same chord at points of **congruent** segments are also equal. Hence $\angle AKN = \angle AMN$, $\angle AKB = \angle AMB$, and $\angle BKL = \angle BML$.

Therefore the opposite angles *LKN* and *LMN* of quadrilateral *KLMN* are equal. Similarly, the opposite angles KLM and KNM are also equal and so quadrilateral *KLMN* is a parallelogram.

A third solution is suggested here for your comparison.



Segment *CD* cuts the common chord *AK* of circles *C* and D at its midpoint *P*. Since the circles are congruent, *P* is also the midpoint of *CD*.

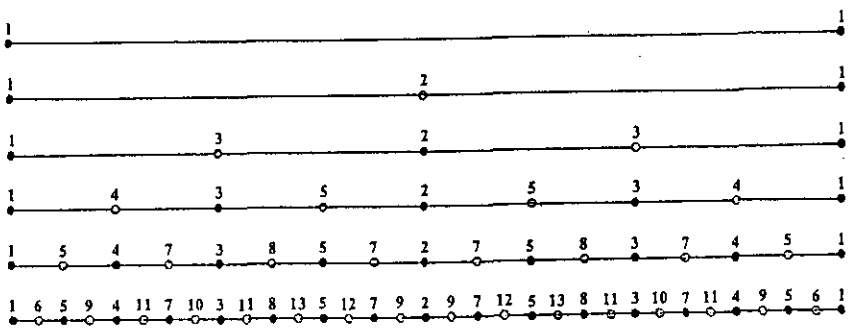
Similarly, segment *CF* and the common chord *AN* of circles C and F have the same midpoint *Q*.

It follows that *PQ* is a midline of both triangles *CDF* and *AKN*, so ***DF*** = 2***PQ*** = ***KN and DF//PQ//KN*** .Replacing *C* with E in this reasoning , we know that ***DF and LM*** are equal and parallel. Thus ***KN and LM are equal and parallel*** and we can conclude that *KLMN* is a parallelogram.

PROBLEM 9

No students could solve this problem, which demands the solver to ***replace*** 1992 ***by smaller natural numbers*** first. This is followed by ***searching for patterns***, ***generalising*** and ***proving***.

Writing out the first few lines of numbers resulting from successive steps of our process, we obtain the following table :



Clearly all ***new*** numbers that appear in the kth line of the table - that is, at the *k*th step of the process - are greater than the ***new*** numbers that appeared in the (*k*-1)th step, which in turn are greater than the ***new*** numbers from the (*k*-2)th step, and so on. So the new numbers writen down at the *k*th step are all not less than *k*. This means that the *n*th line of our table contains all numbers n that ever appear on the segment in the course of our process.

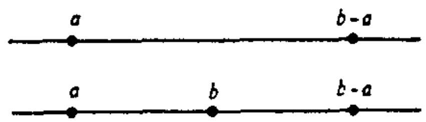
Let us try to find how many of these n's there are.

Every time we write down a number *n* according to our rule, between the neighbouring numbers *a* and *n* - *a* we have a pair of numbers *a* and *n* in that line of the table, where $a < n$ and obviously $n-a < n$. Conversely, if a pair (*a, n*), $a < n$, occurs in the table - that is, *n* stands to the immediate right of a in some line - then the number *n* of this pair (and not *a*) appears for the first time at this position. So the number of *n*'s in the nth line is equal to the number of pairs (*a, n*), $a < n$, that occur in our table.

A careful study of the table reveals that, for $2 \le n \le 6$, a pair (a,n) occurs in our table if and only if a and n are ***coprime.***

We shall now prove that every pair (a, b) of ***coprime*** numbers a and b occurs in the table ***exactly once***, while every other pair does not occur at all. We shall do it by induction over s = a + b :
For s = 2 the statement is obviously true - the only pair with s = 2 is (1, 1).

Let the statement be true for all pairs (a, b) such that a + b < s, and let us consider a pair (a, b) such that a + b = s. Since the table is symmetrical with respect to its vertical midline, we can assume a < b. Pair (a, b) occurs in the table if and only if pair (a, b-a) occurs in the preceding line.



But a + (b - a) = b < s, so by the induction hypothesis our statement is true for (a, b-a), and common divisors of the numbers a and b-a are the same as those of a and b. Therefore, the statement is true for (a, b), and we are done.

So the number of 1992's written after the 1992nd step is the same as the number of positive integers coprime to 1992 and less than 1992. The prime divisors of 1992 are 2, 3 and 83. By the Principle of Inclusion-Exclusion, the desired number is given by 1992 - A + B - C where:

$$A = \frac{1992}{2} + \frac{1992}{3} + \frac{1992}{83} = 1684,$$

$$B = \frac{1992}{2\times 3} + \frac{1992}{2\times 83} + \frac{1992}{3\times 83} = 352$$

$$\text{and } C = \frac{1992}{2\times 3\times 83} = 4.$$

The final count is 1992 - 1684 + 352 - 4 = 654.

Finally, the number of 1992's written after the 1992nd step is 654.

The Principle of Inclusion - Exclusion states that :

If $A_1, A_2, \ldots, A_n$ are finite sets, then

$$|A_1 \cup A_2 \cup \ldots \cup A_n|$$

$$= \sum|A_i| - \sum|A_i \cap A_j| + \sum|A_i \cap A_j \cap A_k| - \ldots + (-1)^{n-1}|A_1 \cap A_2 \cap \ldots \cap A_n|.$$

Here $|S|$ denotes the number of elements of S.

If we denote by A_1, A_2 and A_3 respectively the sets of multiples of 2, 3 and 83 not exceeding 1992, then:

$$A = |A_1| + |A_2| + |A_3|,$$

$$B = |A_1 \cap A_2| + |A_1 \cap A_3| + |A_2 \cap A_3|$$

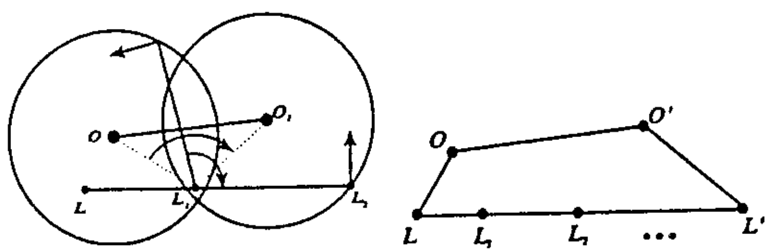
$$\text{and } C = |A_1 \cap A_2 \cap A_3|.$$

Thus the number of positive integers coprime to 1992 and less than 1992 is 1992-$|A_1 \cup A_2 \cup A_3|$

PROBLEM 10

No students gave satisfactory solution to this problem, which can be solved in the following way:

Denote the segments of the lion's route by $x_1, x_2, x_3, \ldots, x_k$ and the angles of its turns by $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{k-1}$ (the number of turns is 1 less than the number of segments). Let us ***straighten the route*** by turning it successively around the ends of segments x_i through angles α_i to make each x_{i+1} the extension of x_i, and rotating the ring together with the segments.



Instead of a broken line we get a straight segment LL', 30000 m long consisting of the smaller segments $LL_1 = x_1,\ L_1L_2 = x_2, \ldots, L_{k-1}L' = x_k$.

Now let us follow the path of the ring's center O during successive rotations. The first rotation around L_1 takes O into point O_1 such that $L_1O_1 = L_1O \le 10$ and $\angle OL_1O_1 = \alpha_1$. Therefore $OO_1 < 10\alpha_1$. Similarly, the distance from O_1 to the next position

O_2 of the center is less than $10\alpha_2$, and so on. It follows that the distance between the first and the last positions of point O can be estimated as:

$$OO' < 10(\alpha_1 + \alpha_2 + \ldots + \alpha_{k-1})$$

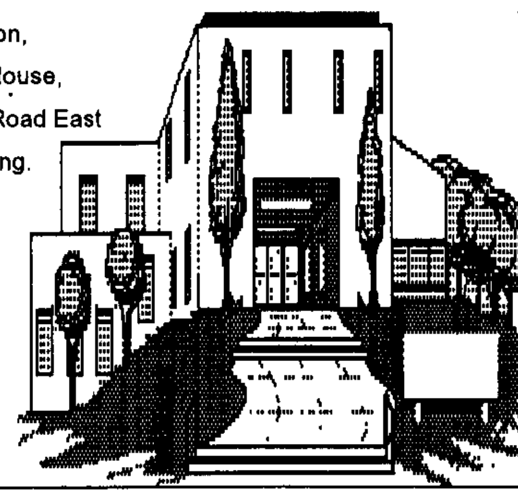
On the other hand, the inequality $LL' \le LO + OO' + O'L'$ yields

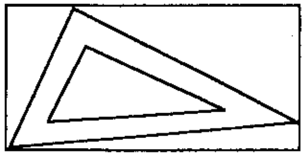
$$\begin{aligned} OO' &\ge LL' - LO - O'L' \\ &\ge 30000 - 10 - 10 \\ &= 29980. \end{aligned}$$

So $\alpha_1 + \alpha_2 + \ldots + \alpha_{k-1} > 2998$ radians.

Our New Address

Mathematics Section,
12/F., Wu Chung House,
197-221 Queen's Road East
Wanchai, Hong Kong.





Pythagorean Numbers

Did Pythagoras prove the theorem that bears his name? The proof of the proposition is attributed to Pythagoras (540 B.C.) by various writers, including Proclus (460), Plutarch (1st century), Cicero (c. 50 B.C.), Diogenes Laentius (2nd century) and Athenaews (c. 300). No one of these lived within four centuries of Pythagoras, so that we have only a weak evidence to believe that Pythagoras was the first to prove the theorem. However, Pythagoras is credited with the discovery of a simple method of finding Pythagorean numbers, (a, b, c) which satisfy the equation

$$a^2 + b^2 = c^2 .$$

The method is :

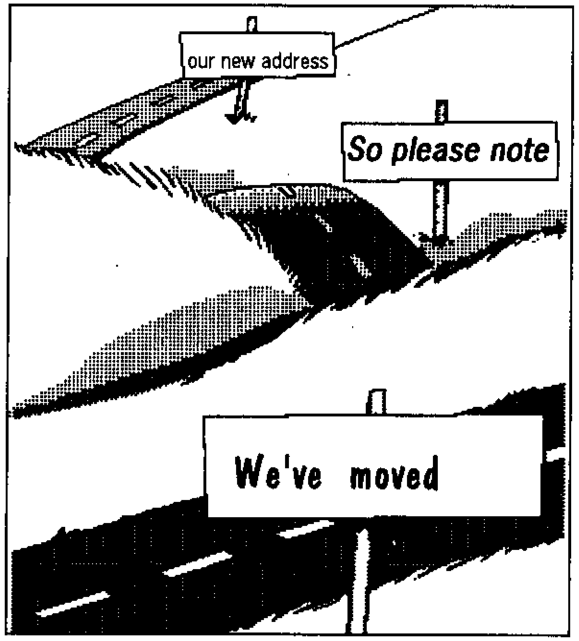
(1) Choose any positive odd integer ***a*** larger than 1 and square it.
(2) Express the square as the sum of two consecutive integers ***b*** and ***b***+1.
(3) Then ***a, b*** and ***b+1*** form a Pythagorean Triple.

E.g., take a = 3, then $a^2 = 9 = 4 + 5$
Then (3, 4, 5) is a Pythagorean Triple.

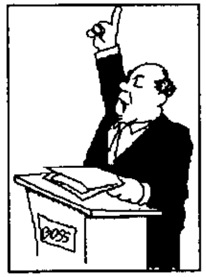
Moreover, one can prove that if (a, b, c) is a Pythagorean Triple in which a<b<c, there is one and only one such (a, b, c) if a is prime.

Also, if (a, b, c) is a Pythagorean Triple, is it always true that
(i) ***a*** x ***b*** x ***c*** is a multiple of 60?
(ii) one of the numbers a, b, c is divisible by 4?

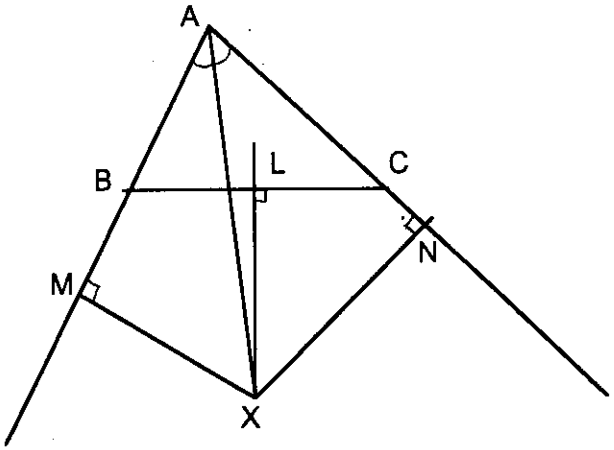
*** *** ***



Any triangle is isosceles !

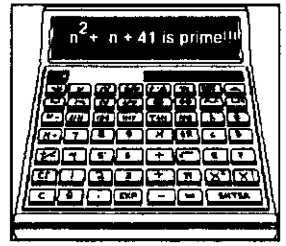


In ΔABC, the angle bisector of $\angle BAC$ and the perpendicular bisector LX of BC meet at X. XM and XN are perpendiculars from X to AB and AC respectively.



We can prove that $\Delta AMX \cong \Delta ANX$, $\Delta BLX \cong \Delta CLX$. Consequently, AM=AN, MX=NX and BX = CX. Again $\Delta BMX \cong \Delta CNX$ and so, BM = CN. Finally, AB = AM-BM = AN - CN = AC and

ΔABC is isosceles !!!



A Formula for Generating *Primes*

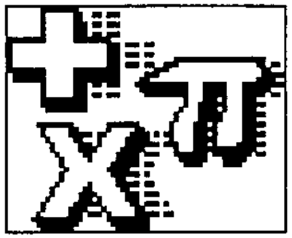
Mathematicians have long dreamed of finding a formula which generates all prime numbers or at least generates prime numbers only.

Leonhard Euler (1707-1783) played around with the simple formula $n^2 + n + 41$. For n=0 to n=39, the formula works perfectly well; but for n=40, the formula fails.

Many mathematicians tried to find one such formula but unluckily all of them failed. Then, some began to look for better formulae which could generate either more prime numbers or mostly prime numbers.

Making use of some fast computing machines, people found that Euler's simple formula was surprisingly good. For numbers smaller than 10 million which are generated by the formula, 47.5% of them are prime. For values of n under 2398, almost 50% of the generated numbers are prime. And for values of n under 100, the formula yields 86 primes and only 14 composite numbers .

MORE ABOUT DIVISION OF POLYNOMIALS



TSE Ping-nam,
NG WAH COLLEGE - SECONDARY

In junior secondary mathematics teaching, the normal teaching sequence on basic operations of polynomials will be addition, subtraction, multiplication and division. For each of the first three operations, students are usually introduced two working methods - horizontal and vertical modes, as I often call them. For instance, in the addition of the two polynomials $2+5x-7x^2$ and $2x^2-9x+4$, students may adopt one of the following modes :

Horizontal mode

$$\begin{aligned}(2+5x-7x^2)+(2x^2-9x+4) &= (-7x^2+2x^2)+(5x-9x)+(2+4)\\ &= -5x^2-4x+6\end{aligned}$$

Vertical mode

$$\begin{array}{rrrr} & -7x^2 & +5x & +2 \\ +) & 2x^2 & -9x & +4 \\ \hline & -5x^2 & -4x & +6 \end{array}$$

However, when dealing with division of polynomials such as finding the quotient and the remainder when $6x^2 - 2x + 1$ is divided by $3x + 2$, nearly all students perform the division by using a vertical mode called long division as shown below :

Vertical mode

Quotient = $2x - 2$

Remainder = 5

$$\begin{array}{r} 2x-2 \\ 3x+2\overline{)6x^2-2x+1} \\ \underline{6x^2+4x\quad} \\ -6x+1 \\ \underline{-6x-4} \\ 5 \end{array}$$

Could we, as in the case of addition, have a similar horizontal mode for the division operation? The following tries to illustrate a possible horizontal operation for division of simple polynomials. It starts with the following fact that

Dividend = Quotient x Divisor + Remainder.

Hence,

$$\begin{aligned}\frac{\text{Dividend}}{\text{Divisor}} &= \frac{\text{Quotient x Divisor}}{\text{Divisor}} + \frac{\text{Remainder}}{\text{Divisor}}\\ &= \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}\end{aligned}$$

So when $6x^2 - 2x + 1$ is divided by $3x + 2$, it can be done as follows :

<u>Horizontal mode</u>

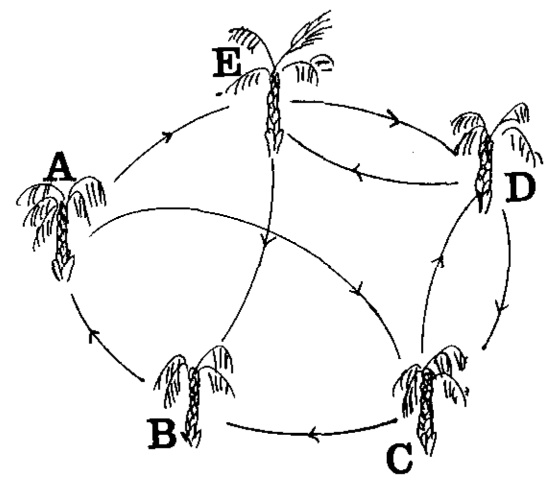
$$\frac{6x^2 - 2x + 1}{3x+2} = \frac{2x(3x+2) - 4x - 2x + 1}{3x+2}$$
$$= 2x + \frac{-6x+1}{3x+2}$$
$$= 2x + \frac{-2(3x+2)+5}{3x+2}$$
$$= 2x - 2 + \frac{5}{3x+2}$$

Quotient = $2x - 2$ Remainder = 5

The reasons why the above treatment is seldom found in current junior secondary mathematics textbooks are possibly due to more working steps involved and the necessary manipulation with divisor in the numerator. Nevertheless it is pedagogically desirable to include the above in our teaching for at least we can give our students a whole, and complete picture of the two modes for each of the four basic operations of polynomials.

Tracking the Monkey

Each arrow in the following diagram represents a jump of a monkey from one tree to another. No jump is to be repeated.



1.From which tree does the monkey start ?

2.At which tree does the monkey stop ?

3.How many possible paths are there ?

幾個數學問題

梁念生

上月在葛量洪教育學院舉行的初中數學賽決賽，有幾個問題的結果值得再加探討，現將學生答案寫出，方便各同工跟進。這次比賽共有 26 隊參加，參賽者是中三學生，他們都是初賽中的優勝者或歷屆決賽優勝學校的學生，故數學成績應較好。題目全部中英對照，以照顧中英文中學的學生。

第一題要討論的題目如下：

In a university there are six times as many students as professors. If there are S students and P professors, find the relation between P and S.

某大學裏每六個學生就有一名教授。現有S 個學生及P 名教授，求P 與S 的關係。

這是一題非常著名的題目，外國有很多研究（如 Clement,1982;Clement, Lochhead, & Monk, 1981）。研究者將這題目問大學工程系的學生，結果祇有37% 的學生答對，因他們將P 及S 當做教授及學生而不是教授及學生的人數。但我們這次數學比賽的結果如下：

答案*	選答隊數	選答百分率
A. S = 6P	15	58%
B. 1:6	5	19%
C. P = 6S	4	15%
D. P = S/7	1	4%
E. S = 5P	1	4%

* A 為正確答案

很明顯，香港的學生答對比率高得多（取 B. 作答案的學生可能答對，但因表達不清楚而不算正確），筆者猜想有兩個可能的原因：第一是這班學生程度較高，第二是這類中文題目不易誤導學生。若單給中文或英文題目與中英文學校學生而作比較，可能有些特別的結果。

第二題要討論的題目如下：

Yesterday your father asked you what was the temperature. You said, "The temperature is y^y degrees Celsius where y is an integer." Find y.

昨天你的父親問你氣溫是多少度。你回覆說：「溫度是攝氏y^y度，y 是整數。」求 y。

這題是測試學生的常識，根本不難計算，但這次數學比賽得到的結果如下：

答案*	選答隊數	選答百分率
A. 3	13	50%
B. 沒有回答	4	15%
C. 5	3	11%
D. 1	2	8%
E. $0 \le y \le 3$	1	4%
F. 27	1	4%
G. 2	1	4%
H. 30	1	4%

* A 為正確答案

從結果顯示，我們還需努力教導學生不要祇忙於計算，更要仔細分析。

第三題要討論的題目如下：

Solve the following equation:

$$(x - 1)^{x-1} = 1$$

解下列方程：

$$(x - 1)^{x-1} = 1$$

這題的目的是要測試學生會否把 0^0 當做一個數，比賽結果如下：

答案*	選答隊數	選答百分率
A. 2	16	61%
B. 1	6	23%
C. 沒有回答	2	8%
D. 1 or 2	1	4%
E. $(x-1)^x$	1	4%

* A 為正確答案

答對的百分比還不錯，希望中三老師能更進一步，提醒學生為什麼 0^0 不是一實數。以下是一個提議：

根據定義，我們有

$$a^3 = a.a.a;$$
$$a^4 = a.a.a.a, ...$$

容易看出，我們有

$$a^p/a^q = a^{p-q} .... (*)$$

這裏 $a \neq 0$，p，q 為正整數且 $p > q$。數學上，很多時我們會將一些以前沒有定義的加上合適的定義，使數學結果擴展，令內容較完整或方便計算，但擴展的其中一個原則是不會引起矛盾。若將 (*) 的 $p > q$ 的條件改為 $p \geqslant q$，可定義 $a^0 = 1$ $(a \neq 0)$ 而 (*) 還是成立的。但若再將 $a \neq 0$ 的條件放寬的話，得到 $0^0 = 1$。而從另一角度看，$0^3 = 0$；$0^2 = 0$；$0^1 = 0$；很自然地會擴展成 $0^0 = 0$。那麼 0^0 同時等於 0 及 1，這就引起矛盾。故我們不把 0^0 當做一個數。

一九九三年六月

參考資料

1. Clement, J. (1982). Algebra word problem solutions: Thought processes underlying a common misconception. <u>Journal for Research in Mathematics Education, 13</u>, 16-30.

2. Clement, J., Lochhead, J., & Monk, G. (1981). Translation difficulties in learning mathematics. <u>American Mathematical Monthly, 88</u>, 286-290.

For Your Information

1. International Mathematical Olympiad

The 33rd IMO was held from 10 to 21 July in Moscow, Russia. The contest was on 15 and 16 July 1993. It was the fourth year for the Hong Kong Team to participate in the event.

The Hong Kong Team won a Silver Medal, two Bronze Medals and two Honourable Mention. Among the 56 participating countries, Hong Kong Team was at the twenty-sixth place in the competition.

The 34th IMO would be held in Istanbul, Turkey in July 1993. Members of the Hong Kong Team had been selected . The six members were:

CHAN Tsz-ho	(Ying Wa College)
CHU Hoi-pan	(Chong Gene Hang College)
LAM Chi-wai	(Chuen Yuen College)
LIN Kwong-shing	(Tsuen Wan Government Secondary School)
TSUI Ka-hing	(Queen Elizabeth School)
YUNG Fai	(Chuen Yuen College)

The Prize Giving and Flag Presentation Ceremony was held on 19 June 1993.

2. IMO Snapshots

The Mathematics Section planned to publish seven quarterly issues of "International Mathematical Olympiad Snapshots". The first three issues had been published and sent to schools in January 1993, April 1993 and July 1993 respectively.

3. IMO(1994) Logo Design Competition

There were a total of 409 entries in the IMO(1994) Logo Design Competition. The champion design would be used as the IMO(1994) Logo. The champion was **WONG San-keung** of **Queen Elizabeth School**. The First Runner-up was **POON Tak-nai** of **Shau Kei Wan Government Secondary School** and the Second Runner-up was **FUNG Ho-yin** of **Ma On Shan Tsung Tsin Secondary School**.

4. Asian Pacific Mathematical Olympiad (APMO)

The Hong Kong Team won three Silver Awards, four Bronze Awards and three Honourable Mentions in the APMO held this year. The three Silver Award Winners were **TSUI Ka-hing** of **Queen Elizabeth School**, **CHAN Tsz-lung** of **La Salle College** and **LIN Kwong-shing** of **Tsuen Wan Government Secondary School.**

5. The Tenth Hong Kong Mathematical Olympiad (HKMO)

184 secondary schools participated in the Tenth HKMO. After the heat events, 40 schools were selected to enter the final event which was held on 13 February 1993 at the hall of the Northcote College of Education.

The Prize giving Ceremony was held after the Final Event. The Assistant Director of Education, **Mr. C. L. HO** and the Principal of Northcote College of Education, **Mrs. A. S. K. CHENG** were the guests of honour and they presented the trophies and prizes to the winners.

The Champion of the competition was **Clementi Secondary School**. The First Runner-up was **Ying Wa College** and the Second Runner-up was **Pui Kiu Middle School**.

The Champion of the Poster Design Competition for the Tenth HKMO was **MOK Hoi-yan** of **Tsuen Wan Government Secondary School**. The First Runner-up was **Jacqueline CHAN**

of **Leung Shek Chee College** and the Second Runner-up was **WONG Yik-bun** of **St. Stephen's College, Stanley**.

The heat event of the Eleventh HKMO was scheduled to be held on 11 December 1993.

6. HKMO Poster Design Competition(92-93)

The adjudication of the HKMO Poster Design Competition (92-93) had been completed by the Judging Panel of the Competition. The Champion was **TSE Yuen-ling** of **Queen's College Old Boy's Association Secondary School**. The First Runner-up was **LAU Ka-yan** of **Queen's College Old Boy's Association Secondary School** and the Second Runner-up was **YU Man-ha** of **Lung Kong World Federation School Limited Lau Wong Fat Secondary School**. The champion poster will be reproduced and issued to all secondary schools and the three winning posters together with the twelve meritorious posters will be exhibited at the Northcote College of Education during the Final Event of the Eleventh HKMO.

7. Mathematics Teaching Centre

The opening hours of the Mathematics Teaching Centre had been rescheduled as

Wednesday	9:00a.m. - 12:30 p.m.
	2:00p.m. - 5:00 p.m.
Saturday	9:00a.m. - 12:00 noon

A TTRA Resource Corner in Mathematics had been set up in the Mathematics Teaching Centre.

8. Change of Address

The new address of the Mathematics Section had been changed to

Room 1207, Wu Chung House,
197-221 Queen's Road East,
Wanchai, H.K.

From the Editor

I would like to express my gratitude to those who have contributed articles and also those who have given valuable comments and suggestions to the newsletter

The SMN cannot survive without your contribution. You are, therefore, cordially invited to send in articles, puzzles, games, cartoons, etc for the next issue. Anything related to mathematics education will be welcomed. We particularly need articles on sharing teaching experience, classroom ideas, teaching methodology on particular topics ,organization of mathematics clubs and even the organization, administration and co-ordination of the mathematics panel. Please write to the SMN (with your contact address included please) as soon as possible and the address is

The Editor,
School Mathematics Newsletter,
Mathematics Section,
Room 1207, Wu Chung House,
197-221, Queen's Road East,
Wanchai, Hong Kong.

For information or verbal comments and suggestions, please contact the editor on 8926553.