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Foreword

Welcome to the 26th issue of the School Mathematics Newsletter (SMN).

The School Mathematics Newsletter (SMN) is for mathematics teachers. It serves as a channel of communication on mathematics education in Hong Kong. This issue involves various articles written by academics, curriculum officers and teachers in different areas, including academic insightful views on mathematics education, suggestions of effective strategies in learning and teaching of mathematics about STEAM education, computationally enhanced mathematics education, online interactive mathematics lessons and articles related to application of mathematics, algorithms and artificial intelligence, etc.

In the existing education system, mathematics teachers are faced with the tremendous challenge of teaching students of very different abilities, motivations and aspirations. To meet this challenge, mathematics teachers need to equip themselves with necessary mathematical skills and teaching strategies to cope with different teaching situations. We do hope that all readers will find the content of this issue informative and stimulating.

The Editorial Board of SMN wishes to express again its gratitude to all contributors, and also to the fellow colleagues of the Mathematics Education Section who have made great efforts in publishing the SMN Issue 26.

SMN provides an open forum for mathematics teachers and professionals to express their views on the learning and teaching of mathematics. We welcome contributions in the form of articles on all aspects of mathematics education. However, the views expressed in the articles in the SMN are not necessarily those of the Education Bureau. Please send all correspondence to:

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目錄

1. Integration of Programming, Problem Solving and Recreational Mathematics for a Computationally Enhanced Mathematics Education

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Introduction

Computational thinking (CT) is a powerful cognitive tool for solving problems, designing systems, and understanding human behavior by drawing on concepts fundamental to computer science (Wing, 2006). It is helpful not only in maintaining competence in a technological society but also in supporting development in higher-order skills such as critical thinking, analysis, and scientific inquiry for the Science, Technology, Engineering, Mathematics (STEM) disciplines. Surrounding this, calls for incorporating CT into mathematics education are rapidly increasing. However, the mere presence of computers in the classroom does not ensure their effective use or quality education. Structural changes in the curriculum and instructions are needed to take full advantage of using CT to teach mathematics and problem solving.

Our work builds on the first author's previously developed conception of "learning as Making" (Ng & Chan, 2019； Ng & Ferrara 2020) to envision a computationally enhanced mathematics education. In particular, we were interested in digital making as a form of artefact construction through screen-based programming to support mathematical problem-solving in fundamental ways. Digital Making involves students' active creation of digital artefacts through block-based programming. It promotes 21st century learning skills and transforms mathematical problem-solving from merely using formulae and performing arithmetic calculations procedurally (Ng & Cui, 2021； Weng et al., 2022a). Rather, CT concepts and practices such as sequences, variable-naming, abstraction, algorithmic thinking, decomposing, and iterating are highlighted during problem-based digital making activities, through which scientific inquiry, mathematical thinking, and engineering design can also be exhibited as integrated STEM learning.

In the past two years (2020-2022), the research team as led by the first author has made significant efforts and progress in designing problem-based digital making tasks with content appropriate to senior primary and junior secondary mathematics curricula in Hong Kong. These lessons have been implemented in one public education institution (i.e., The Hong Kong

Academy of Gifted Education) and two secondary schools totaling over 120 students. In this paper, we share some detailed lesson plans in which we integrated programming, problem solving and recreational mathematics for a computationally enhanced mathematics education in a class of Secondary 1 to Secondary 3 students. Our goal is to reflect on the success and challenges in the implementation of the lessons, as well as to provide pedagogical suggestions for educators for future implementations of integrating CT in mathematics learning.

We begin with describing our lesson design and rationale in terms of teaching CT in the context of recreational mathematics and problem solving, followed by providing our observations on what students did and learned during the lesson implementation, drawing on some student solutions and projects as examples. We end with discussing some challenges experienced gathered through the year as well as pedagogical suggestions for better integrating the elements of programming, problem solving and (recreational) mathematics for integrated STEM learning in future practices.

Lesson Design and Rationale

In this article, we describe our implementation of five lessons with the following themes: Introduction, Numbers & Algebra, Data Handling, Geometry, and Finale. Throughout the five lessons, we chose to adopt Desmos Classroom to collect students' responses and to monitor their progress whenever necessary. In each lesson, one of the authors would be the main speaker and the rest would be teaching assistants, in the hopes of gaining insightful observation from multiple perspectives. To achieve "low floor, high ceiling" learning, we also included extension problems in the design to achieve differentiated instruction.

Tryouts

Since not all students have programming background, our first lesson aimed at equipping students with the necessary skills. We started by introducing some mini-games to demonstrate the function of each command, such as "if … then …", the use of "variables", and the concept of "events". For example, students will learn how to move the sprite (i.e., characters in the program) using an "if... then..." statement with arrow keys, and command the cat to fire balls at the bat in order to gain points through the use of "variables".

Figure 1: Shooting Game

After getting a sense of Scratch programming, the students customized their shooting games by adding personal elements to the games. Through this, they demonstrated the open-ended nature of digital making in offering opportunities for the students to apply their programming skills in different ways and creatively (see also Weng et al., 2022b).

Numbers and Algebra

The next lesson introduced three activities: Prime Number Tester, Euclidean Algorithm, and the Count from 21 Game. The first one tests whether an input integer is a prime number； and the second one utilizes an algorithm to find the greatest common divider (GCD). The concept of Arrays (or Lists) was also established in this lesson. The overall teaching strategy for the first two activities was to guide students in understanding the program specifications, decompose the problem into smaller steps, and be able to read codes that were already given to them in the program. We also designed opportunities for learning by leaving some code blocks blank and provided suitable guidance when students tried to fill in the blanks on their own (see Figure 2).

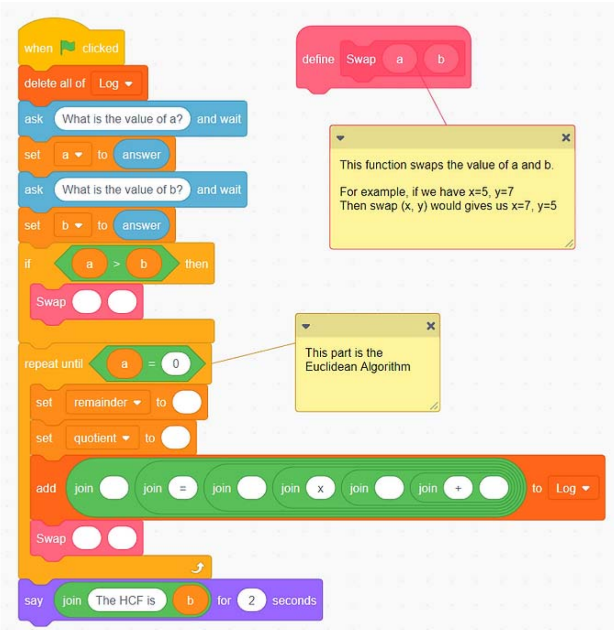

Figure 2: An illustration of missing parts of a program

For the third activity, we decided to adjust our teaching strategy. Again, using game-based learning, we guided the students to create a program related to a game called Count from 21. Count from 21 is a game where two players take turns to cross out one or two numbers (starting from 21, see Figure 3), and whoever crosses '1' wins the game. In this task, the students were prompted to design programs in which a user plays the game with it, but it's always the computer who wins.

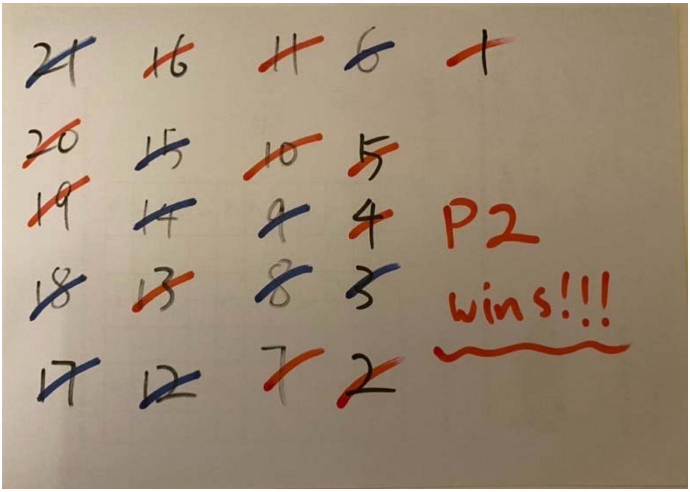

Figure 3: Count from 21

To understand the game thoroughly, we let students play with their groupmates and come up with a winning strategy on their own. In doing so, they would discover the mathematics behind the winning strategy, which was always to call a number which is a multiple of three. Then, they would have to complete the master program on their own with a few specifications. Such design is very engaging because the students first physically experienced the game played and then were challenged to devise a way that would always win. Overall, the three tasks called upon the students to use both their knowledge and discovery about numbers, as well as their developing programming skills to solve problems.

Data Handling

We then turned our focus on simulation in this lesson. Specifically, we chose three very well-known games to address the concept of experimental probability, namely Rock-Paper-Scissors, Dice Rolling, as well as Dart Throwing.

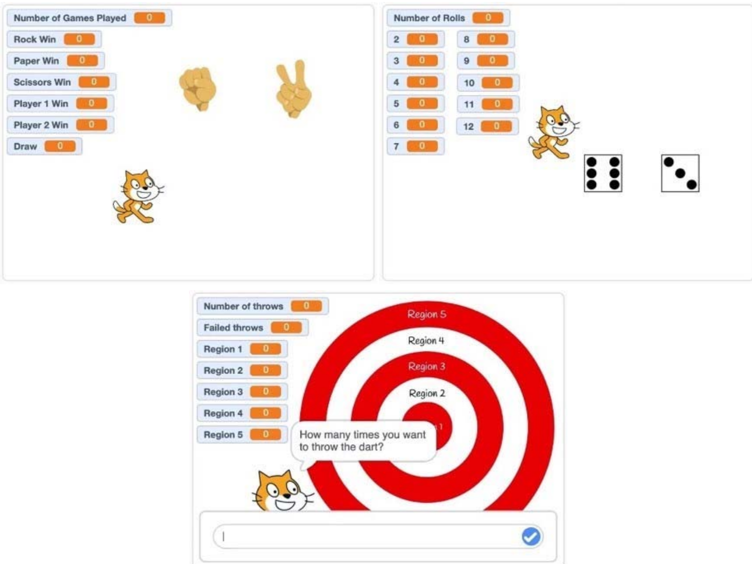

Figure 4: Snapshots of Simulation Games

To reinforce their programming and problem-solving skills, students discussed with their partners and designed the simulations based on the requirements given. In addition, we asked some extended questions to enhance their statistical and probability-related conceptions, such as the highest occurrence of events, expected values, etc. The following are some examples of questions used:

- 'If we play the rock-paper-scissors game 1000 times instead of 100 times, what do you expect to observe?', and
- 'If we roll 6 dice instead of 2 dice, describe what you think the distribution graph will look like?'.

Through these questions, students were encouraged to make further observations and uncover the statistical characteristics such as the distribution behind the events. Therefore, we intended not only to use programming for solely generating and recording the results, but we also aimed at developing students' statistical sense and thinking throughout the lesson. Ultimately, the students gained an understanding about the advantages of using the "random" function for simulating random events in programming contexts versus hands-on simulations in real-life. They became appreciative to computational means of solving problems related to random events.

Geometry

This lesson consists of three activities: Rotational Symmetry, Tessellation and Sierpiński Triangle. During the first two activities, geometric properties were introduced in a traditional sense and then applied in a programming manner. To begin, students learned to draw figures using a "Pen" extension in Scratch program (Figure 5). It was expected that students could draw an assortment of shapes with different repetition and number of sides using the "repeat" and "change by 1" codes. Indeed, the students were quick to learn how to draw in Scratch, while visualizing different geometric figures by the way (i.e. distance and angle of turn) the "Pen" moved in their designed program. As such, the students interpreted the meaning of geometrical concepts from the program, such as which kinds of regular polygon can tessellate and which cannot (Figure 6), and the degree of rotational symmetry of a figure.

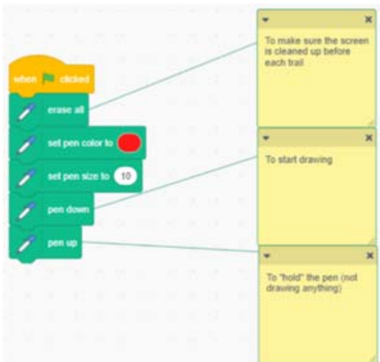

Figure 5: A snapshot of Pen function

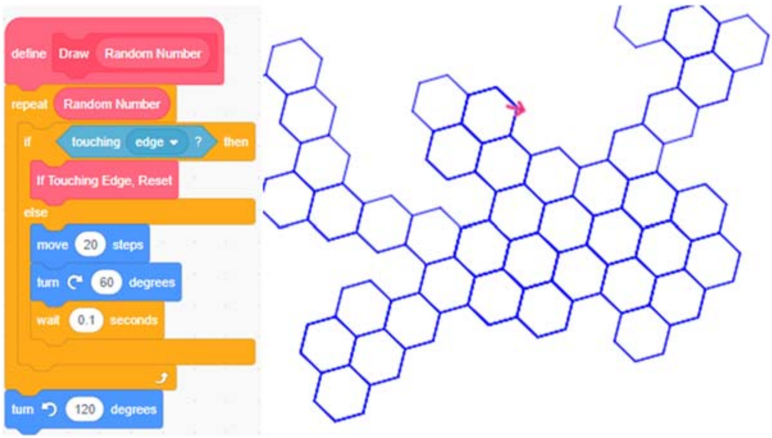

Figure 6: Program for Tessellation

The third program serves to develop an additional knowledge beyond the school curriculum for students, namely the idea of recursion, in the sense of "a function that calls itself". This is a challenging task for students to apply what they have learnt in this and previous lessons in order to create a program with a "myBlock" and "nested loop". Given that this task was challenging, the students tested and debugged their programs with their partners. Through solving this task, we intended for the students to make sense of "myBlock" as a function mathematically in the sense of "a machine with an input and output" and to advance their programming knowledge of "nested loop" by putting one "repeat" within another.

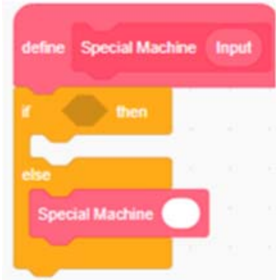

Figure 7: An example of recursion in Scratch

The Finale

For the final lesson, we hope to (1) consolidate the students' CT skills, and (2) immerse them in the real-world mathematical problem-solving in a programming context. Hence, we designed and adopted a more challenging task, the Water Jug Problem, with the following set up.

Suppose there are 3 jugs which do not have any measurement indicators. It is given that the volume of Jug A and B are m and n respectively with $gcd(m, n) = 1$, and that of Jug C is $m+n$. Initially only Jug C is filled with water. Suppose a desired volume is input by the user (an integer from 1 to $m+n$), students were required to create a program which informed the user how to obtain the desired volume by pouring the water among the three jugs.

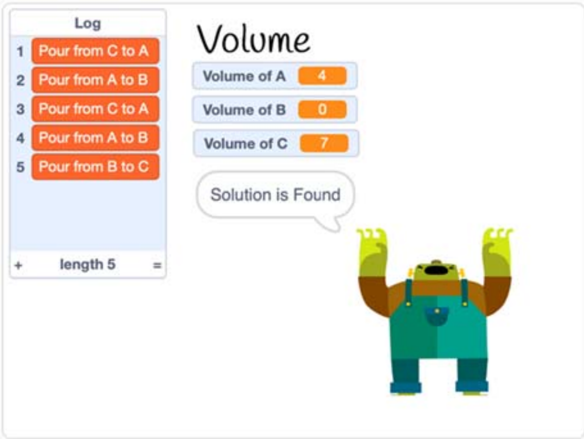

Figure 8: A snapshot of a sample program

We first let students formulate the algorithms on their own by playing with our "trial program", in which they could easily observe the behaviors of the volumes. This goal-oriented problem in which students were first shown the digital artefact, or the end product, was a pedagogical design very useful for teaching programming, for the students would be well informed what their programs should behave in the course of programming. Then, upon some mini-presentation, students started to work on the program. Extension questions were adopted to guide students in revising their program as well as their mathematical and computational thinking.

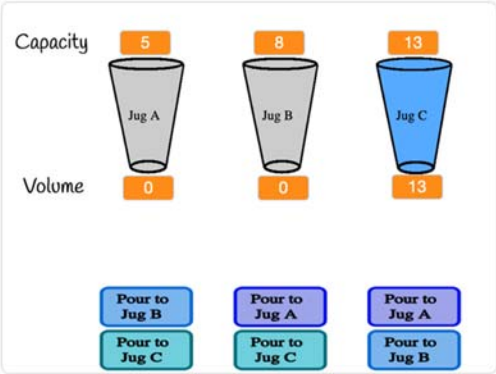

Figure 9: A snapshot on our "trial program"

Observation and Students work

Based on students' performance in the five lessons and their classwork, we made the following observations. First, as Scratch programming enables rapid generation of numerical and geometric examples, it helps students to visualize mathematical relations and patterns. For example, in non-programming environments, students often find it difficult to hypothesize the probability distribution graph of throwing two dice at one time and finding its sum. With the help of Scratch, students were able to observe the results of 100 or more throws, thus uncovering that the event of obtaining the sum of "7" has the highest occurrence and hence highest probability among all possible events. These activities would not be possible in conventional mathematics classrooms. In other words, Scratch facilitates generation of mathematical examples, thus enabling observation of mathematical relations and patterns.

Second, the use of Scratch also enhances students' problem-solving skills. As mentioned, CT enables one to understand and solve problems by drawing on concepts that are fundamental to computer science (Wing, 2006). In the Water Jug problem, for instance, students were given opportunities to understand and solve the problem from the perspective of programming. Aligned with previous studies, they performed CT as a mode of problem solving while applying various Scratch commands, such as variables, my blocks, to express their solutions in the programming environment (Ng & Cui, 2021), and they developed positive problem-solving disposition such as willingness to persist in solving the problem (Ng et al., 2021). As a result, students develop better understanding of the logical relationship between various parts of the solution and various mathematical concepts, such as the concept of function.

Third, we observed that the students were highly engaged and motivated compared to conventional classrooms. With appropriate lesson and task design, the students experienced hands-on, personal, and goal-oriented learning experiences in the "learning as making" environment (Ng & Chan, 2019; Ng & Ferrara 2020). This resulted in the phenomenon that almost all students were actively engaged throughout the whole program. Meanwhile, students were able to understand and solve more difficult and open-ended mathematics problems with the use of Scratch than the ones they were familiar in a traditional mathematics classroom. Therefore, our problem-based digital making instructional design enabled teachers to introduce mathematical problems with multiples entry and exit points, into the mathematics classroom. With such hands-on experiences, we observe that students were more willing and able to engage in each lesson, as well as participate in the activities collaboratively.

However, we also note that some students would get confused by the programming syntax easily. In those tasks where multiple loops, conditionals (e.g., “repeat”, “if… then… else…”, etc.), variables or multiple sprites were needed (e.g., Euclidean Algorithm and Rock-Paper-Scissors), students might not be well equipped to solve the problem by thinking of how it can be decomposed into smaller steps and by designing an algorithm that executes the steps, hence resulting in errors. For example, some students programmed their solution to allow an input of 0 for the task of Count from 21, which is not acceptable since the game only accepts 1 or 2 from each player. In spite of that, it should be noted that once guidance was provided by teaching assistants, such issues were resolved, and students were becoming more and more confident in applying CT to participate in the activities, as evidenced by their responses in the extension questions.

Reflection and Future Directions

In this era of technological advancement, there has been a call for nurturing students' capabilities of solving complex real-world problems. To achieve so, students are expected to develop higher-order skills such as critical thinking, analysis, and scientific inquiry for the STEM disciplines. In particular, CT enables one to understand and solve problems from the perspectives of computer science (Wing, 2006). In other words, students equipped with CT can implement information technology into problem-solving with higher efficiency and precision when compared to conventional mathematics learning and teaching. Furthermore, programming enables students to observe patterns and simulations that cannot be done in conventional mathematics classrooms, thus creating personal and mathematical meanings to them. As such, we believe these lessons could serve as an alternative pedagogical approach in the teaching of secondary mathematics in the future.

We also see that the current curriculum for STEM in Hong Kong are not yet mature such that most teachers find teaching STEM difficult. Prior research evidenced that the barriers of developing STEM are availability of corresponding lesson designs, the support and resources, the tasks and processes of implementing STEM education, and teachers' professional development (Geng et al., 2019; Lin et al., 2021). To further improve the design and practice of STEM curriculum in Hong Kong, it is essential to explore lesson designs and instructional plans enabling teachers' effective integration of elements from the STEM disciplines into mathematics classrooms. This includes lessons that effectively incorporate the interplay between mathematical and computational thinking (Cui & Ng, 2021). However, not all mathematics teachers had received relevant training in related fields; let alone having a background on programming. To this end, we are encouraged by the fact that despite some of the co-authors had little background related to programming initially, all of us were able to learn, develop, and implement engaging lessons for this project. Therefore, we believe that it is worthwhile for in-service teachers and panels to consider the possibility of integrating some elements of CT into mathematics classrooms, so that students can experience the importance of CT and information technology in the process of learning mathematics and problem-solving, as well as to appreciate the importance of STEM education.

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2. 數學與 STEM 教育的融合：數學教師多面睇

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1. 引言

作為一種統整的教育模式，STEM 教育融合了科學（science）、科技（technology）、工程（engineering）和數學（mathematics）四個不同的學習範疇，旨在強化學生在各範疇的知識學習，並強調跨範疇的融合和應用，注重培養學生各種高階能力和 21 世紀所需的技能 （Bybee, 2010）。STEM 教育的理念最早於 1986 年由美國國家科學基金會提出，以推動社會培育未來所需的數理人才為宗旨。隨後，此理念逐漸在全球受到廣泛重視（National Research Council, 2012），甚至啟發了 STEAM（加入了 Art）、STREAM（增加了 Reading 和 Art）乃至「STEM+」等理念，在教育界中蔚然成風。由於在科學研究領域和從事數理科技工作的女性人數相對男性較少，不同地方包括香港社會仍存有女性不擅長理工科的刻板印象。故而，在學校基礎教育推行 STEM 教育，也將有助縮減男、女學生在數理學習上的差距，達到處理學習差異的目的（UNESCO, 2016）。

2015 年，香港特區政府在《施政報告》中首次明確提出推動 STEM 教育（課程發展議會，2015）。在政府的引導和支持下，香港中小學踏上了探索、實施 STEM 教育之路。在數

學教育領域中，STEM 教育成為學校數學課程更新的發展焦點之一，以期培養學生成為科學、科技和數學的終身學習者，並能在不同世代中，應對科技急速發展帶來的種種改變和挑戰（課程發展議會，2017，頁 4）。在落實 STEM 教育方面，以整合的課程模式、跨學科的方式進行，理應最為理想。然而，學科主導的教學模式仍是香港學校課程的主要形式。就單一學科教學而言，如何能將學科教學與推行 STEM 教育合理、有效地結合起來，是擺在教師面前的現實問題。

數學是 STEM 的重要組成元素。要將數學的學科知識揉合在 STEM 教育當中，這不僅取決於教師的教學實踐，更重要的還關乎教師如何看待 STEM 教育，以及他們如何處理數學教學和 STEM 教育之間的關係。教師持有的教學信念和對學科本質的看法會影響其教學實踐，進而影響著學生的學習（Zhang & Wong, 2015）。故此，瞭解教師對 STEM 的看法以及他們在推動 STEM 教育過程中遇到的種種挑戰，對於實踐 STEM 教育尤其重要。因此，研究人員深入訪問了 9 位在學校參與 STEM 教育的數學教師，旨在透過他們的教學經歷，瞭解前線教師在推行 STEM 教育時真正的關注點和遇到的挑戰，以為今後在學校推行 STEM 教育帶來啟示。

參與訪問的 9 位受訪教師中，4 位任教小學數學，5 位任教中學數學。他們均擁有數學教師專業文憑，教學經驗介乎 5

年至 15 年不等，當中超過一半老師任職數學科主任。訪問的問題主要涵蓋四個方面—(一) 教師們如何看待 STEM 的教育目標、(二) 教師們推行 STEM 教育時的實施方式、(三) 教師們如何理解數學學科和 STEM 教育的關係，以及 (四) 教師們在實踐 STEM 教育的過程中所遇到的困難和挑戰。本文主要從整體上總結和概括受訪教師們的看法，不具體區分和詳細介紹每一位教師的意見。

2. STEM 教育的目標

在訪問中，教師們普遍認同政府推行 STEM 教育這一政策。對於 STEM 教育的價值，受訪教師均認為學校推行 STEM 教育，既有益於學生的個人發展，也對香港整體社會發展有價值（圖 1）。

就個人發展方面，教師們均認同 STEM 教育除了重視跨學科學習、培養學生積極的學習態度外，也著重培養學生的共通能力，比如解難能力、協作能力、創造力等。對於 STEM 教育能否照顧學生的學習差異，受訪教師的意見不一。有老師認為推行 STEM 教育需結合學生的個人學習特點和風格（如有些學生喜歡動手操作，有些學生則喜好安靜學習），教育方式並不能以一式走天涯。至於男女學生的學習差異方面，受訪老師都沒有提及。

在社會發展方面，國際社會普遍認同 STEM 教育對科技創新和人類社會發展帶來的深遠影響（English, 2016）。事實上，科技實力既為硬實力，也是軟實力，對地區發展有著至關重要的作用。透過 STEM 教育，學校不僅培養了學生對科學創新的興趣，也為他們提供了未來升學的方向。放眼未來，STEM 的教與學更為社會栽培了具備科技素養的青年，營造了促進社會創科發展的濃厚氛圍，對提升地區的競爭力亦十分重要。

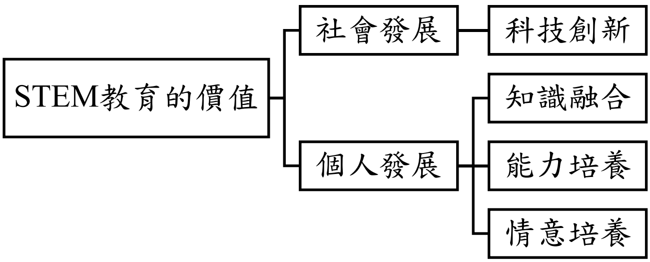

圖 1. STEM 教育的價值

3. STEM 教育的實施模式

目前常見的三種推行 STEM 的方式為多學科（multidisciplinary）、跨學科或科際整合（interdisciplinary）和超學科（transdisciplinary），融合程度依次遞增（圖 2）。這一分法結合了 STEM 教育的特點，並借用了課程研究中統整課程的理論（Drake & Burns, 2004）。多學科指處理一個主題或問題時結合或涉及多個學科。跨學科指兩個或以上學科的融合，學科間存在交叉內容。但這並不等同於簡單的

學科相加，而是指學科間的邊界依然存在，彼此又有著相互的影響。超學科同樣指兩個或以上學科的融合，但更著重打破學科之間的藩籬，強調在真實情境中重新整合這些學科。

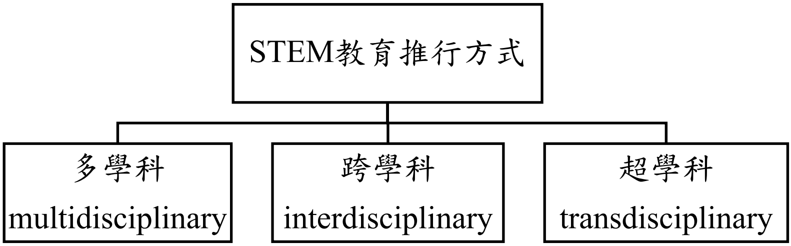

圖 2. STEM 教育的推行方式

受訪教師在各自學校普遍採用跨學科方式進行 STEM 教育。學生在各學科課堂學習了相關的課題內容後，在課後的 STEM 活動中綜合應用各科知識。雖然受訪教師均認為超學科是最為理想的 STEM 推行方式，但考慮到重新整合學科的難度、課程安排等因素，超學科僅被少數學校採用，並且一般以獨立的 STEM 課堂形式進行。針對不同主題，學生在 STEM 課堂中直接學習、運用相關知識。

4. 數學與 STEM 教育的關係

從學科特點來看，數學和 STEM 有著毋庸置疑的密切關係。不過，在實踐的過程中，這種密切關係並不容易被凸顯出來。針對數學與 STEM 教育的關係，受訪教師的看法可大致分為三種（圖 3）：

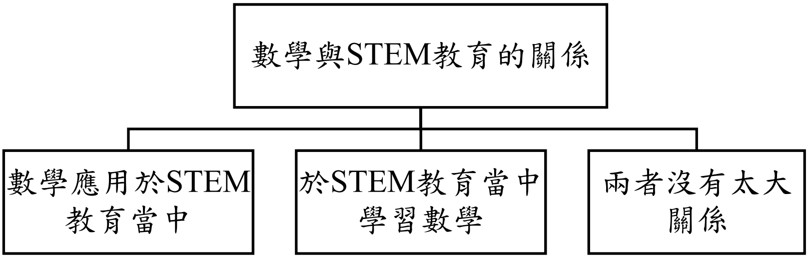

圖 3. 數學與 STEM 教育的關係

受訪教師普遍認同「數學知識能夠應用於 STEM 教育」，當中以統計學和幾何學的相關知識應用最為廣泛。例如，在其中一位受訪小學老師提及，學生在「夏威夷小結他製作」的課題中，需應用幾何學知識去構造零件；另一位受訪中學老師則提到，學生就需應用統計學的知識在一環保課題中製作圖表、計算並分析數據。

除此之外，部分受訪教師也提出了「在 STEM 教育當中學習數學知識」的看法。例如，小學生通過參與編程活動，提前學習了中學的坐標系相關概念；而製作輪椅課題，則令中學生學習到正規數學課程以外的內容。

不過，部分教師也有感「數學與 STEM 教育的關係不大」。在當前的 STEM 教育中，數學的參與度並不高，往往以輔助的角色出現。例如，小學 STEM 活動的課題多以常識科為主，而中學 STEM 活動的課題偏重科學。事實上，並非只

有香港需面對 STEM 教育中學科間不平衡的難題。在 2014 年溫哥華舉行的 STEM 會議上發表的 141 篇常規論文中，45%有關科學，12%有關科技，9%有關工程，16%有關數學（English, 2016）。可見，儘管數學是 STEM 中其他理工學科（如工程、科技和科學）的基礎，但在實際教學中，以數學主導的 STEM 活動並不常見（Couso & Simarro, 2020）。

5. 數學與 STEM 教育融合的挑戰

在探索數學學科如何與 STEM 教育融合的過程中，數學教師面臨著不少困難和挑戰。在應試教育的社會大環境下，統測和考試為在校內廣泛推行 STEM 教育帶來了挑戰。數學教師有迫切的學科教學任務，學生則需要集中精力應對考試。這種現象在高中年級，師生需面臨大考（比如公開試）這一衝刺階段時尤為明顯。雖然校方能夠感受到 STEM 教育對學生的創造力、思維、應用能力等方面的幫助，但學生在考試中表現出眾仍然是校方、家長、學生最熱切的期望。在統籌課程、課時安排時，STEM 教育很多時候被迫讓路：既未被納入正規的課程體系中，也缺乏統一的評價方案，最終成為忙碌學習中的調劑品，例如考試後的 STEM 活動周或者平時的課外活動內容。

除來自考試壓力的影響，數學教師在校內還遇到其他不同的挑戰。雖然近年香港特區政府為中小學提供了不同程度的資金支持，也開辦了不少的公開講座、工作坊和展覽，但校方對 STEM 教育的支持力度、教學人員的協調、STEM 教學內容的選取才是真正影響著 STEM 教育實施的關鍵因素。受

到新冠疫情的影響，校方的教育發展重心亦會隨客觀教學環境、面授課時數等轉移。在校內推行 STEM 教育可能從初期備受重視的跨學科教育，轉變為流於形式的任務（如「為做而做」）。對教師而言，課堂教學之外的 STEM 教育從某種程度上也加重了教師的教學負擔。時間、人力安排、技術培訓等都依賴著教師的積極主動性。而以數學為核心的 STEM 教育研究、培訓等的總體不足，也給數學老師帶來選題的困難── 如何以數學內容為主題設計 STEM 課堂對數學教師而言仍是一大難題。

6. 結語與展望

儘管面臨多種挑戰，在探索 STEM 教育的路上，受訪教師們仍迎難而上，甚至分享了一些應對經驗。例如，為爭取校方支持，有受訪教師曾通過努力帶領學生一起「做出成績」，來獲得校方更多的支持和投入；為協調時間、人力，有受訪教師在所任教的學校組織常規 STEM 會議、成立 STEM 教學小組、組織教師參加相關的教學培訓等。這些具體的措施，著實鼓舞著每位教育工作者奮勇向前。不過，受訪數學教師並未針對 STEM 教育中的學科不平衡提出建議。

在缺乏適當的課程框架和資源的情況下，若要提升數學在 STEM 教育中的作用和重要性，一個可行的做法是將現時的課外 STEM 活動和課堂教學結合起來，讓學生多接觸一些真實情境的問題，引導學生學會以數學模型化（modelling）的思維方式來思考現實情境中的問題；幫助學生培養出能真

正應用於生活的解難能力（Kertil & Gurel, 2016；Tezer, 2019)；令學生「看到、知道、明白」在 STEM 教育中，數學元素的要義與應用價值。然而，並非所有教師均熟悉和具備這樣的數學建模概念，要實踐這樣的教學，我們還需對從事 STEM 教學的人員（無論是主修數學還是非主修數學）進行專業培訓。

經過幾年的實踐探索，香港不少學校在推行學校為本的 STEM 教育過程中，漸漸走出了自己的方向。他們從最初的成果導向（以製作出某個物件為目標)，慢慢聚焦於在活動過程中，培養學生思考、解難、創造等能力；從學習他人經驗、進行聯校合作，走向在自己校內制定並開展 STEM 課程的中期、長遠發展計畫。

近期，香港特區政府開始強調在 STEM 的學習中加入藝術元素，推行 STEAM 教育。藝術元素的增加或能令 STEM 教育增添一些人文性，啟發思考科技產品以外的種種可能和應用。在政府投放大量資源、外圍環境配合的條件下，學校和教師在設計 STEM/STEAM 學習和相關活動（抑或將來其他新的課程發展主題）時，除旨在提升學生動手能力、對數理學科的興趣外，也應格外審慎，避免相關學習流於體驗玩樂。教師應當著眼於如何促進學生在學科、跨學科知識方面的理解，並提升學生的解難能力和思維發展。此外，要實踐 STEM 教育的初衷目標，STEM/STEAM 的學習機會應盡可能讓學

校全體學生受惠，包括不同性別、不同能力和不同族群的學生。而在學與教的互動中，教師也能借助這股改革熱潮提升自身的專業能力，教學相長。

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3. 概念、程序、做數

黃毅英

退休數學教育工作者

從一項研究談起

為應對「大眾數學」，在「回到基本」運動前夕，美國全國數學督導員議會（National Council of Supervisors of Mathematics，1977）提出，「學習解決問題是研讀數學之主要目的」[1]。無論我們怎樣詮釋「問題解決」，「做數」始終在學校數學學習中佔有重要的地位。但成功「做妥」一道數學題有不同途徑及手法。筆者在課程發展處委託的一項研究中，就曾利用開放題訪問了小三及小六學生，探討他們解決數學問題的策略，得出了一些有趣的結果（Wong，Marton，Wong 和 Lam，2002）[2]。

按「關鍵字」看看它屬於哪個課題（甚至教科書中的哪一課）是常用的策略—估計屬於某一課後，便看該課有哪幾道公式，然後逐一代入試試。甚至利用核對單位的手段。其中有一例，題目涉及「圓」，學生就記起它屬於某一課，其中有兩道公式：圓周和圓面積；而題目給出的資料有 cm^2，故此

1 「…learning to solve problems is the principal reason for studying mathematics」。

2 這是早年研究，近年應該有不少改進。

判斷應該不是圓周公式,那就一定是圓面積公式了！由學生自己口中所說的，解決數學問題的一般策略包括：

※把題目看過並選出最方便之法；
※嘗試曾學過之法；
※看屬於哪課題，代入數字希望得到答案；
※看過題目，抽出數字試作＋、－、×、÷，再看問些甚麼；
※看看題目涉及哪些數字,抽出這些數字,看看它們之間有何關係，一步步的簡化就可得到答案了；
※了解題目，看問甚麼，然後列出式子；
※首先列出有關的式子，然後慢慢算下去；
※先看看問些甚，看前前後後有何關係，然後再試列式。

他們也會探尋似是如非的蛛絲馬跡,去解決數學問題。例如對於

> 有 26 盞紅燈，有 14 盞黃燈，每五盞掛一排可以掛幾排？

有學生說：「由於題目說 5 個一排，故不是×便是÷，但×則數字太大,所以必是÷」。又一學生說：「由於題目說 26 盞燈，又有 14 盞燈，然後 5 盞一排，故必為加法後除法。」

另一問題：

張明知道在班級裏有百分之 50 的學生喜歡玩籃球，有百分之 50 的學生喜歡玩乒乓球，他就認為百分之一百的學生都喜歡玩籃球或乒乓球。您認為張明說得對嗎？請解釋，如有需要，可以用繪圖說明，也可以舉例說明。

有學生說：「由於題目提到分組，故一定是除法」。又有學生說：「由於題目提到 50%，故必是百分比題目，但看看又不像，必屬『IQ 題』[3]題」。

又一題：

李萍告訴她弟弟李俊她在數學課上所做的遊戲。

李萍說：「李俊，今天我在數學課上用了積木方塊。當我把積木分成 2 個一組時，還多出 1 個積木方塊；當我把積木分成 3 個一組時，還是多出一個積木方塊；當我把積木分成 4 個一組時，仍然還是多出 1 個積木方塊。」

李俊問：「你總共有多少積木方塊？」

李萍給她弟弟的答案可能是甚麼呢？

有一位學生說：「因題目說數字不為 2 整除，故必是奇數，故我逐個奇數去試，最後得出 13」。另一學生說：「我依稀記得這類題目不是最大公約數便是最小公倍數，但因它涉及

[3] 所謂「腦部急轉彎」。

2、3、4，必然不是最大公約數，太大了，故此必為最小公倍數」。

有些學生反映，會「抽出數字，試作＋、－、×、÷，再看看哪些答案較似」。又或（胡亂）設待找的答案為未知數（x），看能不能弄到一道方程出來。

概念 — 程序

這樣不求甚解當然並不理想，宛如 Cronbach（1955）所說的「學生只是依法泡製，並不比打字員把此稿件打出需要更多的工作」[4]（頁 194），何況上面的一些學生策略連依法泡製也談不上。但究竟概念，程序和做數之間起著甚麼關係呢？

早年大家把「程序」與「概念」對立起來（dichotomise），尤以 Skemp（1976）的觀點為人所津津樂道，他認為「程序性知識」（procedural knowledge）根本不能算得上理解（understanding）。於是「先概念後運算」變成流行的說法。

漸漸地，學者開始把關注擴充至「*p* 和 *p*」的討論。其中一個 *p* 是「結果」（product），包括（學習）「內容」（content）。另一個 *p* 是「過程」（process），包括「過程能力」（process

[4] "…the pupil merely applies a skill, doing no more problem-solving than the typist does in copying off this manuscript".

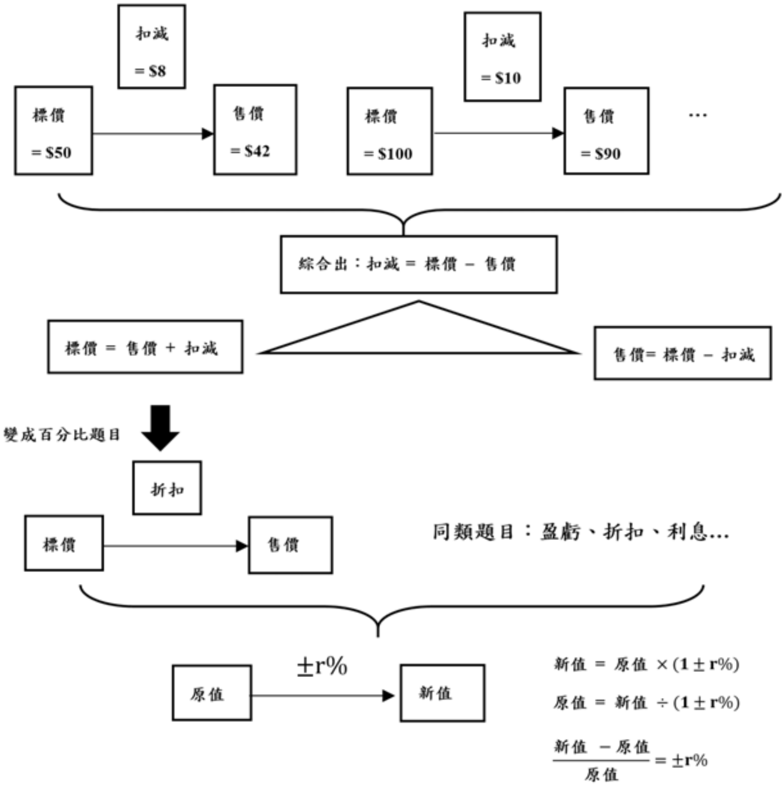
ability），就是今天所說的「高層次思維能力」。在第二次國際數學教育研習班中，學者們得出兩者均應兼顧及作適當平衡的結論（Howson 和 Wilson，1986）。不過筆者認為平衡還是不夠，我們更應進一步探討如何把兩者連繫起來，「怎樣在教授數學知識之同時，以之作為培養深層能力的基礎」（黃毅英，1995，頁 71）。在一方面，Sfard（1991）等人開始提出的「程序」、「概念」兩者密不可分，「操作規則」是「概念」中不可分割的一部分（又見黃毅英，2015 及黃毅英、張僑平，2014）。另一方面，Gray 和 Tall（1994），源於資訊科技教學的討論，亦提出了 Procept（Procedure – Concept）的概念。筆者〈數學化過程與數學理解〉一文（黃毅英，2007）便有略述。

其實問題可以很簡單。理解的提升往往是透過「碰壁–反思–進階」進行（就是朱熹等哲學家所說的「疑–悟」：見 Wong，2006）。比如做數學題，新手固然可以盲目套用公式，就算老手如我們，遇到諸如分數相加，雖然我們懂得背後概念，在實際操作時也只會用常規程序去做，這就是 Kerkman 和 Siegel（1997）所說的「最速（解決問題）策略」（fastest strategy）（又見黃毅英，2007，2019）。直至情境變複雜了（由整數至小數，由數字至符號……），原有策略再不管用，就會透過種種辦法（這些辦法包括反回基本原理，就是他們所說的「後備策略」backup strategy。又或類比等），轉向找尋（統攝較低層次法則的）高層次法則（Scandura, 1977，又見黃毅

英，1990）。這些方式做熟了又變成另一種常規。如此循環向上。這些程序其實已經涉及概念性理解（又見 Pierie 和 Kieren，1989，1992）。所以常規程序完全有發展到高層次法則的可能。

Baroody，Reil 和 Johnson（2007）更提出「深層程序」（deep procedure）這個概念。原來程序可以深層的。究竟甚麼是「深層程序」？除了在操作程序（know how）時亦「知其所以然」（know why）外，Star（2005）等提出和連繫性（connectedness）有密切關係[5]。例如一個人掌握了很多程序如「連加」、「加與減互逆」、「乘」等等之後，又能連繫起來，那就很可能涉及概念性理解了。例如把折扣、折舊、盈虧連繫起來，知道它們都是百分數增減的一種，百分數增減程序是其通則；又從熟習這些程序之後，了解原價、百分率、新價三者知其二必能找出餘下一個；得出了基本關係就可把兩個百分數加減組合起來（如生產價$\xrightarrow{+\%}$標價$\xrightarrow{-\%}$折扣價），問題即可「照辦煮碗」的照做……。這些都可以是從操作程序衍生的概念性理解，也就是「深層程序」。

[5] 理解（understanding）和腦海中智性網絡的關係在 Hiebert 及 Carpenter（1992）的經典，以至 Resnick 和 Ford（1981）都已有基本的解說。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±r%
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原值
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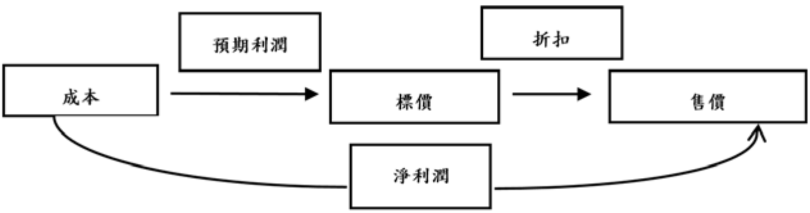
複合題目：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圖一

這和 Ausubel（1963）所說的「逐步分野」（progressive differentiation）和「綜合性再認」（integrative reconciliation）有相似之處（詳見 Bell，1978）。

怎樣促成「深層程序」？Rittle-Johnson 和 Star（2007）便提出「透過比較及尋找共通點」（from compare to commonality）去促成，這和「變式教學」有密切關係。透過題型的變，可掌握從中不變的通則，亦透過變，製造上面所說般「碰壁」，就是讓學生原有常規策略用不著而迫使尋找更高一層的策略。如果能作深層反思，做過一大堆數後確可把概念和技巧組織起來。當然，若「死記硬背」、「盲操瞎練」做過「題海」仍只是一部超級計算器吧了！而透過日常練習，只要適當引導，完全可以透過「深層程序」達到「概念理解」（見黃毅英、郭觀麟，2019）。

如上所述，這涉及變式／變異的概念。中外學者都指出「做數」是否帶出效果端視重複演練當中能否佈置有系統的變化（systematic introduction of variation），又是否能在變化中綜合出（放諸不同情境不變的）通則（Biggs，1994；Gu，Marton 和 Huang，2004；Marton，Watkins 和 Tang 1997；Wong，2004，2017）。變式教學在內地已經流行了很多年，甚至有學者認為這是中國人數學成績優異其中的一個原因（Huang 和 Leung，2006；Wong，2013）。內地流行有不同的變式，我們曾經綜合出四種（見下）。不過很多文章集中

論述各種的變式，而沒有特別聚焦在推行變式之前，是要先進行一個課題分析：找出哪些現實情境須提升到抽象層面（「歸納變式」）？哪些概念或技巧跨度比較大，需要鋪墊？哪些概念或技巧需要轉移到另一個處境(例如由整數除法推到分數除以整數、分數除以分數等：「廣度變式」及「深度變式」）？又練習題如何能夠一層接一層的讓學生遞升(「應用變式」）？詳見黃毅英、林智中、孫旭花（2006）；黃毅英、林智中、陳美恩、王豔玲（2008）。

每所學校，甚至補習社都會給學生海量的習題，不見得有甚麼「秘題」、「密卷」。何以有些人能事半功倍，有些做了一大堆數仍重複犯錯？除了推諉到「天聰」外，能否有系統地佈置變化題，讓學生經歷「深層程序」，並適當引導學生綜合出通則，使他們達到真正的理解可能是關鍵。

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4. 新常態下的遙距互動數學課堂

蘇志峰

沙田蘇浙公學

前言

在 2022 年初，新冠疫情突然變得嚴峻，確診人數倍數上升，情況令人擔憂。在一月下旬，教育局宣佈全港中小學暫停面授課堂，學界再一次進入遙距課堂的日子。透過之前多個月來混合學習模式(Blended Learning)的訓練，筆者及學生亦已習慣使用電子學習工具「Desmos Classroom」進行互動課堂，是次疫情來襲，亦正好讓筆者進一步測試使用 Desmos Classroom 在完全遙距的網課下，到底能否提升網課的互動，提升網課的學與教效能，達至真正的「遙距學習」，而非只有教師自說自話的「遙距講課」。從是次全面網課的經驗來看，使用 Desmos Classroom 一方面能有效幫助學生進行「同步教學」(synchronize learning)，另一方面能讓老師從學生的回應中得到回饋，從而改善教學。

甚麼是 Desmos Classroom

Desmos 是由 Eli Luberoff 於 2011 年創立，其理念最初是創造一個「更好的圖像計算機」(Build a better Graphing

Calculator)，並啟發學生的創意及對數學的熱情[1]。至於 Desmos Classroom 則是 Desmos 圖像計算機之後，另一個出色的電子學習工具，教師可以自行制作教件，並可透過 Pacing 的方式，限制老師和學生看同一個頁面資料，並且學生可以在此回應老師設立的問題。其功能與大家一般所認識的電子學習工具 Nearpod 或 Pear Deck 相似，當中最重要的是學生和老師可以使用很多與數學有關的工具，而且老師還可以透過預設的儀表板 (Dashboard)，了解學生的學習情況，甚至可以即時知道每一個學生的回應，這些工具在遙距課堂，發揮了重要性的作用。

Desmos Classroom 功能介紹一：使用 text + math input + sketch 功能

Desmos Classroom 本身是針對數學教學而設計，因此功能上有很多對學習數學非常有用的設置。詳細的功能介紹，老師們可以輕易在網絡上找到相關教學影片，在此，筆者就無謂多費唇舌，畢竟影片的介紹會比文字的介紹詳盡。在此，筆者想集中介紹幾個筆者常用的工具。圖中所示為筆者其中一個教件[2]，當中筆者使用了顯示文字的「text」，讓學生輸入公式的「math input」，以及讓學生作繪圖的「sketch」。

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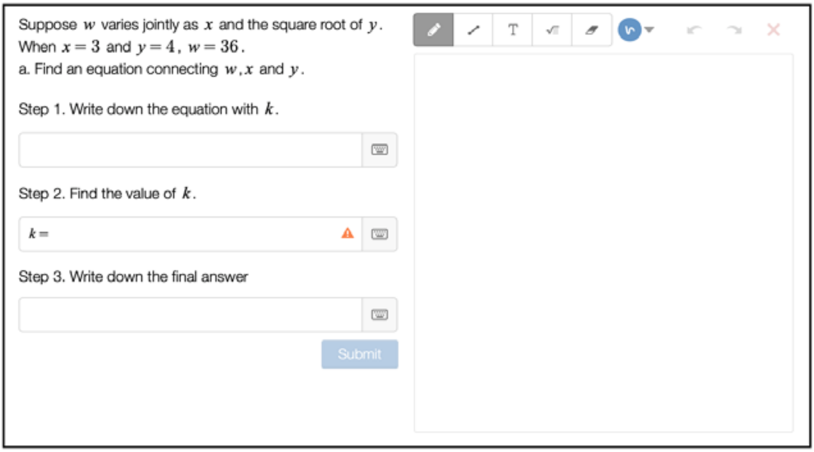
[1] https://artofproblemsolving.com/blog/articles/building-a-better-graphing-calculator-with-eli-luberoff

[2] https://teacher.desmos.com/activitybuilder/custom/624c15f37e0cec05210df99c

在這個教件中，學生已學習了正變及反變，而本教件則是聯變的例子，以及一些相關的應用題。課堂設計方，筆者讓學生先輸入有變分常數的公式，計算並輸入變分常數的值，最後寫出答案，所以當中用上了三個「math input」。在圖一中，右方的「sketch」，則是讓學生寫下草稿，也就是計算變分常數的過程，用意一方面是希望透過此方式了解學生計算步驟，另一方面是不讓學生有「掛機」的機會。

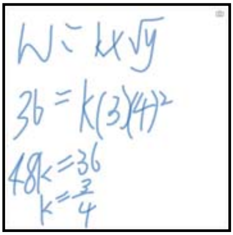


圖一

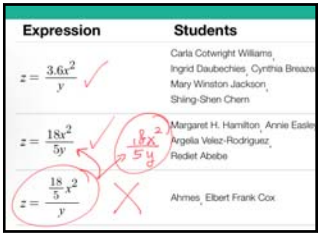
在老師的介面，可以看見學生的回應，如圖二。在網課時，筆者會在 Zoom 中展示此頁，讓學生了解班中其他同學的情況。老師可以利用 Desmos Classroom 中的「匿者模式」(anonymous mode)，讓學生的名字由各大數學家名字取代，老師可以知誰人得到正確答案，而學生則知道自己是否正確。由於學生需要寫上自己的草稿，老師可以透過「sketch」

的版面，立即知道同學答案的原因在哪裡，詳情可以參考圖三。

圖二



圖三

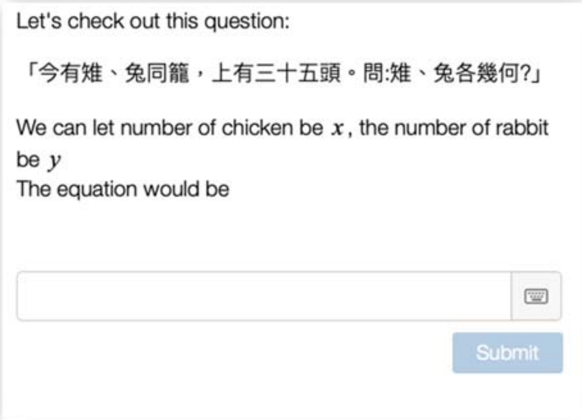


圖四

另外，使用 Desmos Classroom 的另一個好處，就是可以在學生作答後提供即時的回饋，我們可以參考圖四的例子。作為老師，當然明白如果變分常數是一個分數，學生會寫出 $\frac{\frac{18}{5}x^2}{y}$ 是常見的事。不過，如果能在學生犯了這個毛病時，立即作出提醒，相信學生會更投入課堂，畢竟課堂互動確實是讓學生投入課堂的原素，這個部分亦可以彌補遙距課堂之下，缺乏師生互動之間的不足。

Desmos Classroom 功能介紹二：善用 math input 讓學生思考多個答案

筆者在網課期間，亦跟中二的同學討論二元一次方程。在引入的部分，筆者使用了雞兔同籠的問題，並嘗試讓同學明白，一條二元一次方程有無限多個解。而當中的過程很有趣，也讓筆者在「同步教學」上有更多的反思。



圖五

在教件的設計上，基本上與聯變的教件使用同一個套路，使用了「text」作題目，並使用「math input」要求學生回應，如圖五所示。

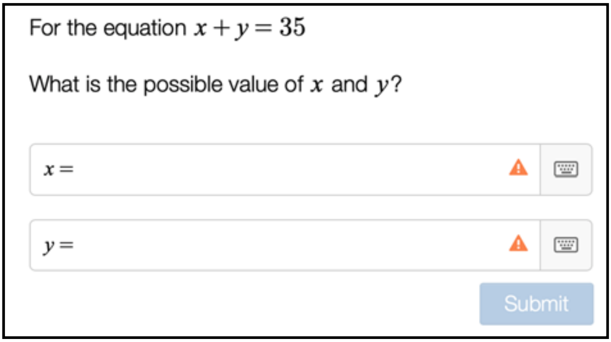
作為老師當然明白中二的學生極有可能寫出 $x = 17.5$，$y = 17.5$ 之類的答案，因為自小學起，他們總是以「數字」作為正確答案。所以，如果他們寫不出方程式，筆者並不感到意外。然而當有同學寫出「數字」時，筆者就會使用暫停功能，提示寫下數字的同學，這裡需要的是方程式，是有 x 有 y 有等號（=）的方程式。

結果顯示，$x + y = 35$ 當然為數最多，有個別同學寫下 $\frac{x}{2} + \frac{y}{4} = 35$ 也是預期之內，最離奇的是有一位同學寫下了

$x+y+long=35$。這樣筆者百思不得其解，整段文字當中，哪裡來一個 long？

當大部分同學都完成作答，筆者當然要提醒答錯同學，例如 $\frac{x}{2}+\frac{y}{4}=35$ 的同學需注意雞兔的總數與腳的數目無關。接著便詢問寫下 $x+y+long=35$ 的同學，甚麼是 long，他的回應是「long 就是籠呀，有雞，有兔，有籠嘛！」

怎麼會想到，我們腦海中的「雞兔同籠問題」，在他的眼中卻是「雞、兔、籠」的問題。慶幸的是，因著 Desmos Classroom，筆者能在下一個討論前解釋清楚同學對此問題的誤解。記得最初提及混合模式學習時，曾有人提過混合模式學習就是「同步教學」和「非同步教學」，而網課就屬於同步教學一類。但如果沒有課堂互動工具，在網課時不能收集同學的回應，我們怎能做到同步？誠然，在實體的課堂之中，老師們能夠觀察學生的眼神，了解同學是否明白；但在網課上，同學若對課題存有誤解，確實在難以讓老師知悉。久而久之，老師變得會自說自話，所謂的同步教學亦名存實亡。所謂同步，當中其實需要老師與學生之間的互相確認，Desmos Classroom 在此能透過收集學生的答案，讓老師了解同學是否「同步」，唯有真正的「同步教學」，方能讓學生在網課中仍能跟上學習的節奏。



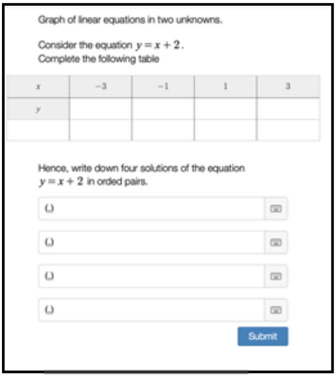
圖六

這個教件的第二步，就是 讓同學寫下所有可能解，如圖六所示。想當然有不少同學寫下 $x = y = 17.5$，這時候，筆者也提醒同學，答案只能有一對嗎？

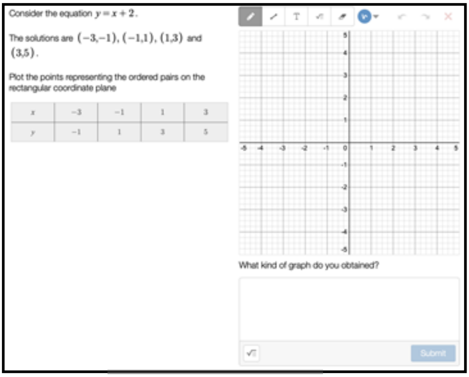
然後，更多的答案就湧現，筆者也樂意將他們的答案展示在 Zoom，讓他們了解一個事實，28 位同學就有 28 組不同的答案，這也是與過往不同的地方，一元一次方程是就有一個答案，但在二元一次方程的世界，答案可以是有很多個的，可以是點數、分數、負數，當中有無限個解。

Desmos Classroom 功能介紹三：善用「Overlay」工具

二元一次方程的第二部分，就是讓同學繪畫直線的圖像。學生會先計算 3-4 個不同的 x 所對應的 y 值。頁面的設計如圖七所示。

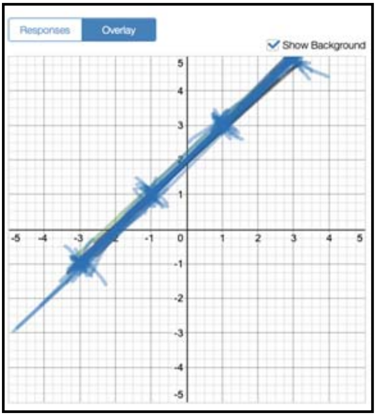


圖七



圖八

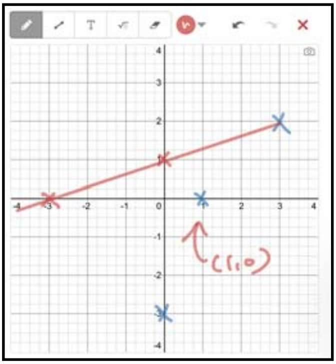
然後，他們需要在圖八的頁面畫上有關的圖像。Desmos Classroom 的 Sketch 工具有一個很出色的功能，就是在老師的頁面，可以將學生的答案重疊(overlay)起來，如圖九所示，讓老師們可以容易看到同學的學習成果。



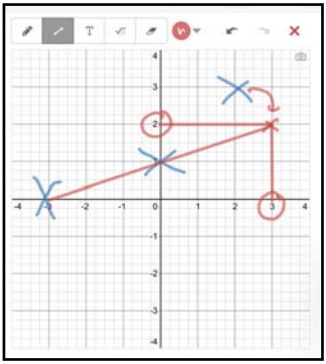
圖九

由於網課時，所有同學都需要在這裡繪畫自己的圖像，而他們幾乎是不可能和其他同學互相討論，所以老師如能在 Zoom 顯示出同學們的作答，其實更有利學生的學習，因為這正正可以彌補，在網課期間，同學之間互相不能討論、參考的情況。

由於所有同學的繪圖均即時顯示在老師的介面，老師亦可以和所有同學一同討論一些同學的「作品」。例如在圖十中（註：藍筆是學生的回答，紅筆是老師的回應），同學明顯將 x 和 y 的坐標調亂了，以致本該屬於 (0,1) 的地方畫了在 (1,0) 之上，而 (–3, 0) 的地方則畫了在 (0,–3) 之上。



圖十



圖十一

然而圖十一的情況則剛好相反，同學畫對了在軸上的點，反而將 (3,2) 的地方畫了在 (2,3) 之上，這正好也值得和同學們一同討論，促進學生之間的互動。

總結一：善用不同電子學習工具進行網課

在這段時間的網課中，Desmos Classroom 成為一個不可或缺的工具，但這並不代表他可以做到所有的東西，我們仍需其他電子學習工具完成其他工作。在此段期間，Google Classroom 仍會使用收發功課，Moodle Assessment Module 仍是筆者用作電子評估的工具，有需要時，GeoGebra 仍是一個有效展示平面幾何的電子學習工具。作為數學老師，如一位筆者非常敬重的前輩所述，如何適時、適地、適用不同電子教學工具改善學與教，這個是老師的教學專業所在。

總結二：在實體課建立學習習慣的重要

另一方面，這次學生能快速進入網課使用 Desmos Classroom，有賴於同學們在實體課已建立了良好的學習習慣。筆者本年度的上學期，還是有半天的面授課堂時，便已經與學生使用 Desmos Classroom。學生一直學習使用 Desmos Classroom 進行一些學習活動。當然，在實體課中，老師可以有很多不同的方法與學生互動，故此並不需要高度依賴 Desmos Classroom 收集學生數據。不過，當學生平日在實體課時知道如何使用，亦知道老師會將其結果發放在螢幕上，這個便會成為他們學習的習慣。當他們在網課中見到相同的畫面，便會自然進入學習模式，而非被其他東西分散注意力。

總結三：「同步教學」才是真正的「教」和「學」

在是次遙距授課的個多月中，筆者大部分時間均使用 Desmos Classroom 作為課堂工具，以幫助同學學習，也讓筆者了解同學的進度，盡可能與同學「同步」，讓同學在網課期間能投入課堂。在這段時間，筆者深深體會使用電子學習工具確能提升課堂的互動，特別是在缺乏面對面即時交流的情況下，透過電子學習工具收集學生回應更顯重要。另一方面，正因收集了學生的回應，筆者更明白在網課期間更應放慢腳步，多用不同的渠道收集學生的數據，因為這些數據正正反映了學生的情況。我們也知道，當學生發現自己總是跟不上老師的速度，帶來的多是沮喪和失望。而當我們能與他們同步，學生才能真正的「學」，我們的「教」才更有意義。

5. 三角學在天地測量之應用

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前言

近年學界提倡 STEM 教學，鼓勵學生結合科學、科技、工程及數學知識，以靈活思維，解決日常生活上的問題。三角學（Trigonometry）是一門實用學科，有著悠久的歷史，古人憑此知識，進行大地以至天文方面的測量工作。本文試論三角學可以如何配合手機或平板電腦軟件，融入 STEM 教學，提高學生學習數學的興趣。

三角測量

如欲測量某目標點與觀測者的距離，三角測量（Triangulation）是一常用方法。如圖 1，觀測者在 A 點及 B 點，希望測量與目標點 C 之距離。假設基準線 AB = c 的長度、以及角 A 和角 B 大小已知，則可由正弦公式（Sine Formula）分別求出 AC = b 和 BC = a 的長度：$a = c\frac{\sin A}{\sin C}$ 和 $b = c\frac{\sin B}{\sin C}$ 。若由目標點 C 作垂線至基準線 AB，則垂線長度 $d = c\frac{\sin A \sin B}{\sin(A+B)}$ 。

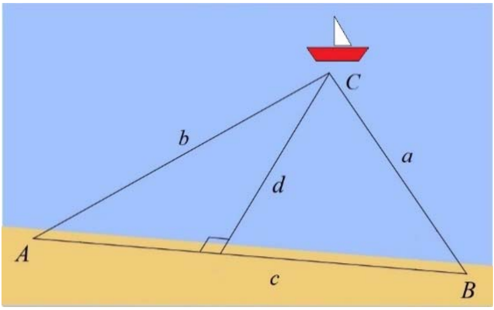


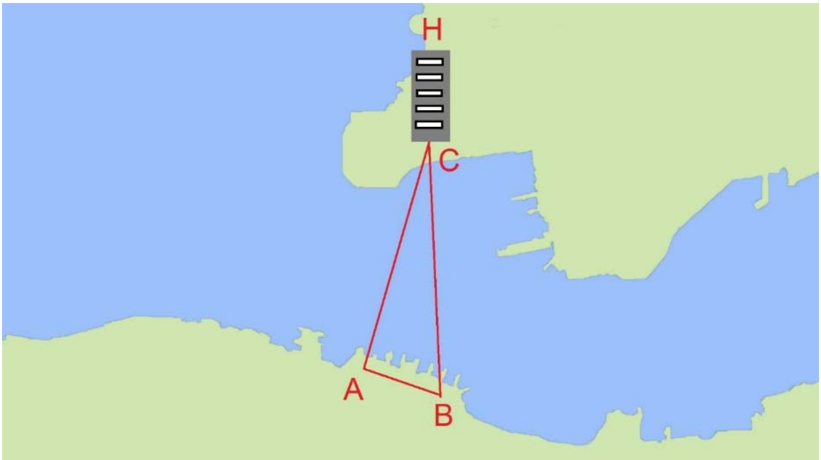
圖 1：三角測量示意

三角測量有廣泛實用價值，我們可以為學生安排戶外測量活動，體驗如何運用三角學，解決生活問題。以前從事測量工作，往往需使用專業儀器，但現今智能手機及平板電腦功能齊備，安裝用於測量的移動軟件（Mobile App）後，把相機鏡頭瞄準目標點，測量軟件便會顯示目標點的羅盤方位、仰角等資訊，如圖 2 所示。要測量兩點的距離，我們可使用計步器（Pedometer），這些移動軟件會利用全球定位系統（GPS），記錄觀測者的步行距離和移動軌跡。

以下分享實例，介紹如何利用三角測量，在中環海濱一帶，進行西九龍環球貿易廣場（ICC）的測量。如圖 3，設 A 為三號（愉景灣）碼頭及四號（南丫島）碼頭之間的某點，B 為六號（坪洲／梅窩）碼頭及七號（天星）碼頭之間的某點，C 為環球貿易廣場，是目標點。我們使用測量軟件，測得 AC 方位為 005°，AB 方位為 107°，因此∠CAB=102°；又測得 BC 方位為 356°，BA 方位為 287°，因此∠CBA=69°。另一

方面，使用計步器，測得基準線 AB 長度為 310 米。在三角形 ABC 中運用正弦公式，可得 AC 長度為 1850 米，BC 長度為 1938 米。測量軟件亦會顯示與目標點的仰角，因此我們亦可估算環球貿易廣場的高度。設 H 點為大廈頂點，則 ΔACH 為直角三角形。我們在 A 點測得仰角 $\angle CAH$ 為 15.1°，由正切（Tangent）關係，算得大廈高度 CH 為 499 米。

圖 2：移動軟件的測量介面和功能



圖表 3：在中環海濱從事西九龍環球貿易廣場的測量

由於測量軟件不是專業測量儀器，因此以上測量結果只是估算值。比對公開資料，環球貿易廣場的高度為 484 米。要求中學生有效消除測量誤差，未免超出其水平；但本著 STEM 教學理念，我們可以提問，希望學生結合所學到之科學知識，尋找可能出現測量誤差的原因。以下筆者提出數項誤差來源供參考：

1. 觀測者拿著手機或平板電腦進行測量時，手部或身體擺動，導致誤差。我們可以把手機或平板電腦置放在平穩的腳架，提高測量精度。
2. 中環海濱觀測點，以及環球貿易廣場，各地點可能有不同的海拔。亦可在計算中，考慮觀測者的身高。

3. 從事大尺度的測量，少許角度改變，可以導致大幅度的計算誤差。某些測量軟件支援放大（Zoom In）功能，我們可以把相機影像放大，便能更好地瞄準目標點。
4. 測量可能受附近磁場影響。依筆者經驗，在中環海濱的堤岸進行測量，羅盤方位可以有逾 20°之誤差。若移至離開堤岸的空曠地方，測量結果會較準確。
5. 空氣在不同的高度，有不同的密度和溫度，由此產生大氣折射（Atmospheric Refraction），因此測量仰角時所瞄準的方向或有誤差。最極端的情況，便是海市蜃樓。
6. 由於地球是球形，因此從事大範圍的測量，須用球面三角學（Spherical Trigonometry），平面三角學就不適用。

周日視差與日地距離之測量

三角測量，除了可用於測量山川大地外，在天文學中，亦可用於測量地球與星體的距離。如圖 4，觀測者觀測某一星形物件，在該物件遙遠的後方有紅色及藍色正方形。若觀測者由 A 點移至 B 點，則星形物件看起來會由藍色正方形移至紅色正方形，天文學稱此為視差（Parallax）。測量 AB 距離及視差大小，便可算出觀測者與星形物件的距離。

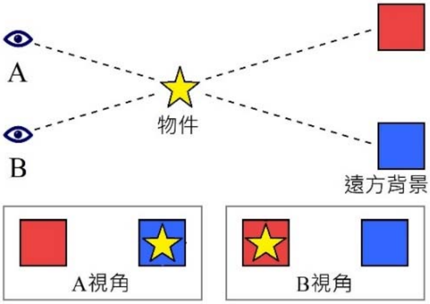


圖 4：視差示意

由於地球繞軸自轉，觀測者相對地球中心的位置隨時而變，觀測星體時便會有視差，天文學稱此為周日視差（Diurnal Parallax）。如圖 5，設 M 是觀測者，E 是地球中心， S 是星體，角 p 便是周日視差。設星體 S 相對於觀測者天頂方向 Z 形成夾角 q，r 是地球半徑，則由正弦公式，可算出地球中心至星體的距離 $d = r\frac{\sin(180°-q)}{\sin p} = r\frac{\sin q}{\sin p}$。若星體S位於觀測者 M 的地平線，$q = 90°$，周日視差達到最大值 p_{max}，此稱為地平視差（Horizontal Parallax），即 $d = \frac{r}{\sin p_{max}}$。

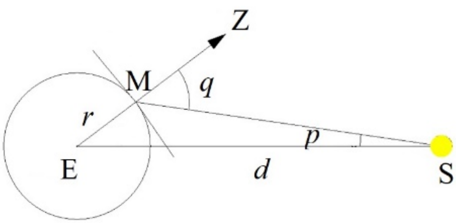


圖 5：周日視差

早於公元前 2 世紀，古希臘天文學家喜帕恰斯（或譯伊巴谷，Hipparchus，約前 190 年—約前 120 年）便發現周日視差，這是科學史上的重大成就。他測得太陽的地平視差是 7’，由此算出太陽與地球的距離，為地球半徑的 490 倍。另一位古希臘天文學家托勒密（Claudius Ptolemy，約 100 年—約 170 年）改太陽地平視差為 3’，即日地距離是地球半徑的 1210 倍。托勒密的觀測結果對後世有深遠影響，古代歐洲、印度及阿拉伯天文學家或會自行測算太陽地平視差，但所取數值皆與托勒密相差不大。直至 17 世紀，發現行星運動三定律（Kepler's Laws of Planetary Motion）的著名天文學家開普勒（Johannes Kepler，1571 年—1630 年）才對此質疑，認為太陽地平視差，應該不會超過 1’，即日地距離，比之前想像的來得遙遠。太陽地平視差的今測值為 8.8”，即日地距離，約為地球半徑的 23400 倍。

周年視差與恒星距離之測量

通過測量視差，亦可推算恒星與地球的距離。只是恒星比太陽更為遙遠，即使地球半徑亦顯得太短，我們須另覓更長的基準線。地球除了繞軸自轉外，亦會繞日公轉，公轉時位置改變，因此亦會產生視差，天文學稱此為周年視差（Annual Parallax）。如圖 6， 地球 E 圍繞太陽 S 公轉，便會產生周年視差，恒星 P 看起來好像在背景星空 B 移動。由於日地距離已知，測量周年視差，便可推算恒星與地球的距離。

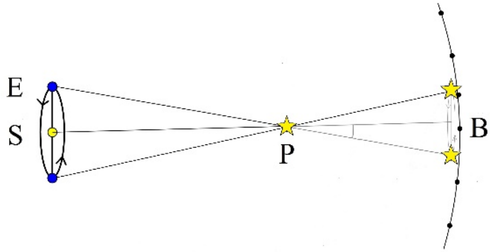


圖 6：周年視差

1543 年，著名天文學家哥白尼（Nicolaus Copernicus，1473 年—1543 年）在臨終前出版《天體運行論》（*De Revolutionibus Orbium Coelestium*），正式發表日心說。然而當時日心說面對種種駁議，其中之一是假如地球真的繞太陽運行，那麼應該可觀測到恒星的周年視差。儘管理論可行，但恒星距離地球實在太遠了，在 19 世紀前，天文學家皆未能測得任何恒星的周年視差。直至 1837-1839 年間，天文學家才成功測得織女星（天琴座 α）、天鵝座 61 和南門二（半人馬座 α）三顆恒星的周年視差，數值皆小於 1”，可見測量難度之高。哥白尼提出日心說將近 300 年後，日心說終於得到無可置疑的確認。

結語

由本文可見，三角學有廣泛的實際用途。三角學固然可用於大地測量，進行測遠、測高的工作，但遠至太陽系，甚至某些恒星，三角測量法亦同樣適用。倘若三角學能與 STEM 教

學結合，安排學生使用測量軟件，作戶外實地測量，當可加深學生對三角學的理解，提高其學習興趣。

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6. 淺嚐人工智能：結合數學與 Python 的暑期工作坊

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前言

日新月異的科技發展，擴大了人們對運算的殷切需求，各式越來越強大的計算機亦應運而生。我們對下一代的教育也應該與時並進，課堂中涉及編程的數學課在本地多所大學皆有舉辦。至於本地中學課程，香港考試及評核局亦有一張清單列出便攜式計算機供考生選擇。儘管考試制度的更新仍稍為落後於科技的發展，課堂以外的活動卻為我們提供更大的想像空間及實踐機遇。

二〇二二年暑假，作者之一應聖保羅書院潘維凱老師所邀，嘗試規劃及製作了一個創新工作坊，為約四十名中學生提供一個淺嚐人工智能算法的獨特體驗。項目旨在讓同學涉獵更多當今世界發生的新事物，先瞭解將來可作的職業抉擇，更甚者可由此定立志向。具體內容揉合了一些基礎數學知識，並包括一些預先編寫的 Python 代碼。工作坊共有三節，每節兩小時，並分為兩班，每班約二十人於八月已在學校電腦室順利舉行。工作坊時間緊逼，有賴香港大學 STEM 實習計劃，我們招募得另外三位來自數學系及計算機科學系的學

生協助主持活動並作一些展示準備,三位亦即本文的其他作者。

但開風氣不為師,項目大部分素材及源代碼已發佈於 GitHub 檔案庫 https://github.com/HanlunAI/ATasteOfDeepLearning,並以 MIT 授權條款開放予同工使用,旨在拋磚引玉,還望不吝指教。謹此我們亦向學校籌劃項目的老師及積極參與的學生致以謝意。

前事表過,言歸正傳,本文將簡單報告一下工作坊的執行經過。

基礎

工作坊並不希望把一般同學排除在外,故活動於報名之初已言明參與者不一定需要有編程經驗。學生當中,有參加過其他 Python 工作坊的,也有首次接觸 Python 的。我們採用了 Colaboratory(簡稱 Colab)構作一系列雲端運行的免費互動 Python 筆記本貫徹整個活動。學生只需在瀏覽器打開筆記本,然後按快捷鍵「Shift+Enter」即能運行預先寫好的代碼,省卻了傳統設置編程環境的冗長程序。一如其他編程工作坊,我們由輸出字串「Hello World」開始,這個過程可以使學生熟習 Colab 環境。

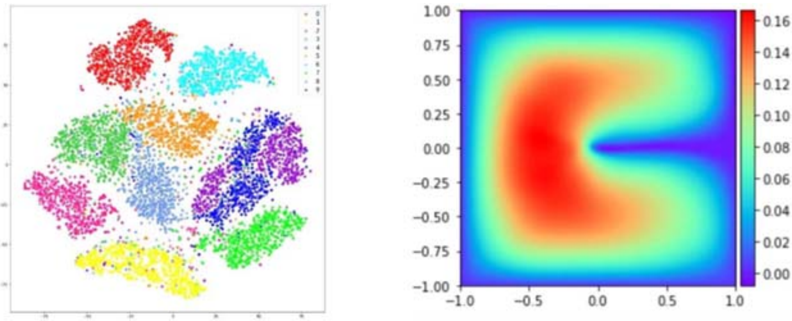

圖 1

工作坊目標遠大，過程中亦涉及不少進階課題。我們主要著重直觀意象（intuition）及學習動機（motivation）。為了給同學留下深刻印象，我們以圖 1 進行了第一次互動，他們需細看圖案然後找出左邊與右邊的異同。同學表現相當雀躍，大部分在討論後都能夠指出：兩者同為色彩斑斕，差異之處則在於左邊是離散的色彩， 而右邊就如光譜般產生連續的變化。

讀者若已熟悉兩者的意義，當知道左邊來自數據集分類問題，一般可以使用人工智能的深度神經網絡處理；右邊就來自偏微分方程的邊值問題，一般可以使用傳統數值分析方法處理。而工作坊的同學只須辨認出兩者分別涉及分類（categorical）變量及連續（continuous）變量即可進入下一環節。

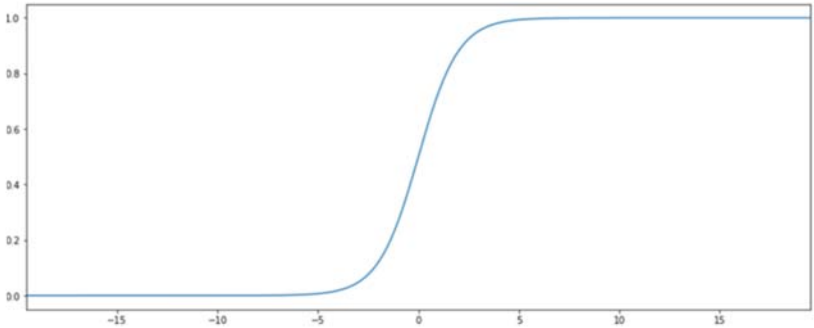

圖 2

邏輯斯諦函數（logistic function）是一種可用來繫起連續變量與二元分類變量的 *S* 型函數，一般分類變量的處理亦可由此推廣所得。工作坊下一環節就讓學生透過預先準備的代碼繪製邏輯斯諦函數的圖像（見圖 2），並透過簡單的代碼微調，實現圖像的變換。

工作坊亦討論到另一種常見 S 型函數：正態分佈的累積分佈函數，這亦是作者之一與該校合作另一工作坊的主線，在此就不細表了。

至此，我們已享受到 Python 諸多套件的便利：NumPy 支援多維陣列運算；Matplotlib 提供可視化工具；SciPy 提供大量特殊函數及數值法。將來涉及機器學習部分則可考慮 Scikit-learn，至於深度學習框架亦有 TensorFlow、PyTorch 等常見選項。工作坊在短短六小時內故然無法一一探究，但由此起，有興趣的同學將來就有了自行探索的方向。

經典

若要處理連續變量間的關係，一個經典方法是線性迴歸（linear regression）。工作坊援引陳泳昌《淺說人工智能之數學方法》[1] 一文預測樓價模型的例子，並按其建議，使用 Python 的 NumPy 套件示範矩陣運算，處理了六個數據點的線性迴歸。繼而回應該文提及的大量數據問題：如何有效處理數十萬數據點的線性迴歸。

考慮到我們並未要求學生事先已認識矩陣，實際執行時我們採取了與該文不盡相同的處理手法。我們首先透過故事引入了三個重要的數學概念：馬可夫鏈（Markov chain），蒙地卡羅方法（Monte Carlo experiment）以及貝葉斯定理（Bayes' Theorem）。馬可夫鏈可以透過考慮以下問題來理解：某人在三個處所 S_1, S_2, S_3 之間進行隨機漫步；對於 $1 \leq i, j \leq 3$，設其漫步前出現於 S_i 的概率為 x_i，其由處所 i 漫步至（或停留於）處所 j 的概率為 $p_{i,j}$；學生即可動手計算漫步後其出現於 S_i 的概率 y_i。參與工作坊的同學大部分可以使用全概率公式求得答案。透過有系統地記錄計算過程，同學亦順利認識了向量與矩陣的乘法：

$$(x_1 \;\; x_2 \;\; x_3)\begin{pmatrix} p_{1,1} & p_{1,2} & p_{1,3} \\ p_{2,1} & p_{2,2} & p_{2,3} \\ p_{3,1} & p_{3,2} & p_{3,3} \end{pmatrix} = (y_1 \;\; y_2 \;\; y_3)。$$

學生繼而可思考多次隨機漫步後其出現在各處所的概率，並進一步了解一些矩陣乘法的特性。透過這個故事介紹基礎線性代數對參與工作坊的同學非常有效，亦留下深刻印象，或許也值得同工參考。（題外話： 工作坊亦提到有早期搜尋器是基於馬可夫鏈實現的。）

至於蒙地卡羅方法其實是一些電腦模擬統計方法的統稱，我們在 Colab 上示範如何以大量的隨機實驗求得未知數。工作坊亦簡單說明了技術的進步如何改變我們處理問題的手段。

而貝葉斯定理就幫助我們處理先驗概率與後驗概率之間的關係。我們舉例在不同假想環境下作某疾病檢測，始於不同的先驗概率（prior probability），得到一次呈陽性報告後再計算染病的後驗概率（posterior probability）也會有所不同。圖 3 展示其中一種情況，左右分界標記染病的先驗概率為 0.1%，而檢測準確度為 99%，則得到陽性報告後的後驗概率為左邊紅色長方形佔全部紅色面積的比例，僅為約 9%。可能由於當時疫情仍然持續，同學們對這個故事表現相當熱情。

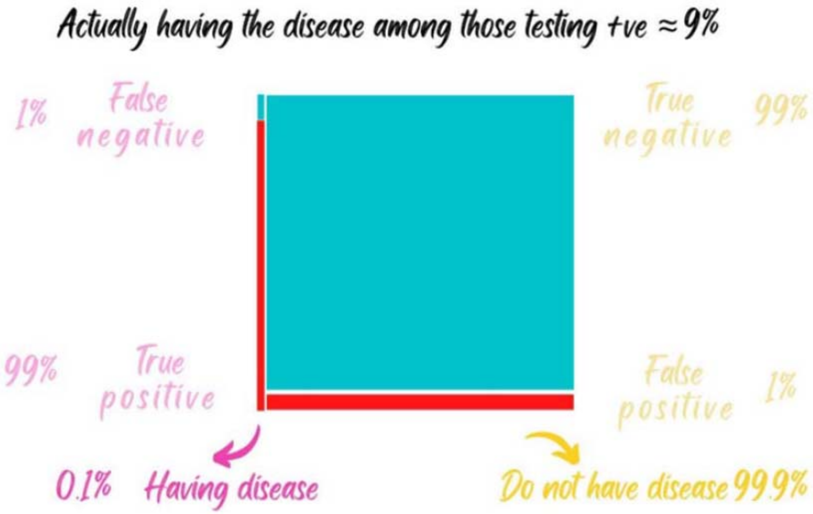

圖 3

工作坊至此，我們已積累了足夠的手段去處理一百萬個數據點的線性迴歸問題。我們隨即展示 Metropolis-Hastings 算法，一種經典的馬可夫鏈蒙地卡羅方法 MCMC（Markov chain Monte Carlo）解決問題。科學實驗中常涉及的最佳擬合線，其實就是一個自變量與應變量的線性迴歸。考慮到電腦可更進一步提供三維數據的可視化結果，我們選擇以最佳擬合平面示範兩個自變量與應變量的線性迴歸。學生在這部分只需運行預先寫好的代碼：首先制定兩個自變數作為 KPI（key performance index），然後混合線性關係和噪音生成一百萬個數據；並構作輔助函數，計算線性迴歸模型參數在小批量數據下的先驗概率及後驗概率等；隨機分小批處理生成的數據，每個批次都參考馬可夫鏈及貝葉斯定理，按

一定的概率決定更新抑或保留模型參數；最後按蒙地卡羅方法整合各批所得的模型參數為答案。於是我們便得到頗為亮麗的結果：線性迴歸模型幾乎與生成數據的線性關係平面重疊（見圖 4）。

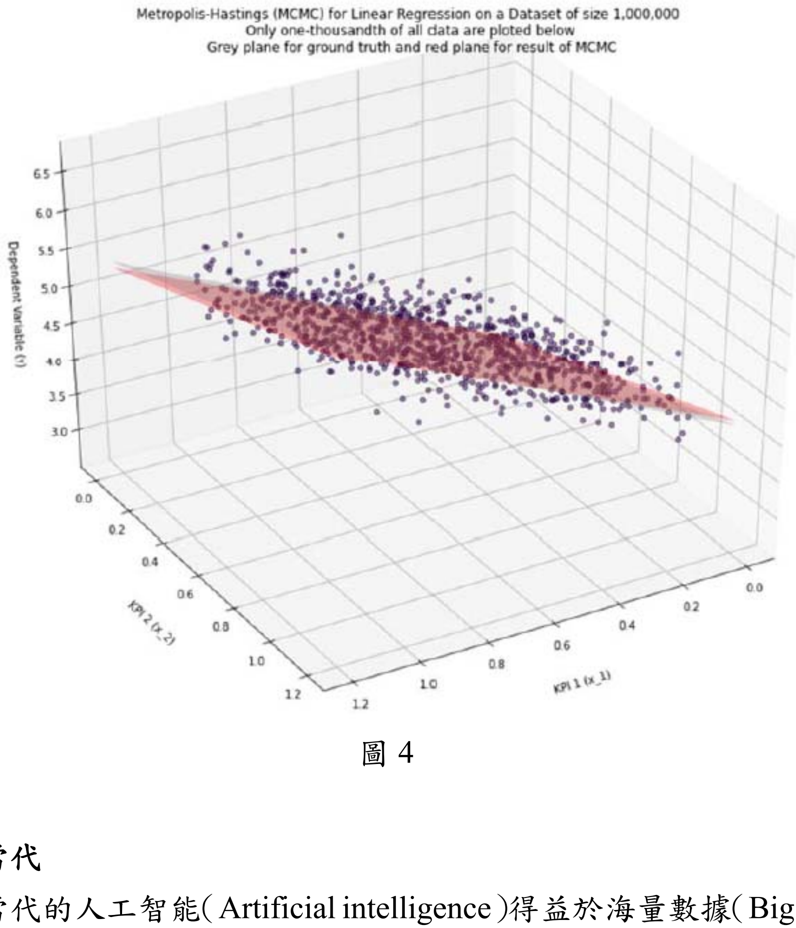

圖 4

當代

當代的人工智能（Artificial intelligence）得益於海量數據（Big data）以及算力（Computing power）的配合，促進了一系列

名為深度神經網絡（Deep neural networks）的算法發展。深層神經網絡是由多層人工神經網絡複合而成，當中的一層可以理解為涉及眾多參數的線性關係與非線性關係的複合。對於分類問題，深層神經網絡的目標是以較低誤差預測每個輸入數據的標籤。

MNIST 數據庫（Modified National Institute of Standards and Technology dataset）為例，輸入數據為手寫 0 至 9 數字圖案，希望神經網絡輸出預測的標籤能對應圖案的數字。前文圖 1 的左側就是使用 *t*-SNE（*t*-distributed stochastic neighbor embedding）可視化一些 MNIST 數據樣本的結果，每個圖案對應嵌入空間中的一點，標籤則以色彩來表達。

經過訓練的神經網絡可以頗為準確地完成 MNIST 標籤任務。所謂訓練就是透過迭代調整參數以有效減低預測誤差的過程。大量數據分批處理以及判斷參數應否更新的過程可堪與前文經典的 Metropolis-Hasting 相提並論。只是神經網絡的迭代操作一般會使用梯度下降方法，以多元微積分中的鍊式法則處理複合關係的參數變化。有關這方面的理論解說，可以在先前提到陳泳昌的文章找到，在此我們就不複述了。透過使用 TensorFlow 的套件，可以方便執行上述操作過程。這也是工作坊的一個重要里程碑，如果有更多時間，我們甚至可以讓同學嘗試不同的神經網絡複合結構。

雖然標籤對人而言可能只是感知層次的任務，但對一些電腦程式而言卻恰恰是自動化流程的最後一里路。工作坊以一款自動解數獨（Sudoku）的擴增實境 AR（augmented reality）程式為例說明狀況。數獨是一種為人熟知的約束滿足問題，在 9 乘 9 個方格中，有些方格上印有 1 至 9 的數字，也有些空格， 玩家需要在方格上填上數字使得每行每列以及一些特定的 3 乘 3 方陣中 1 至 9 全部數字僅出現一次。這個程式涉及幾個工序：一、從鏡頭獲取數獨問題；二、分辨 81 個方格有哪些數字經已填上；三、解約束滿足問題；四、把解投影到原來的畫面上製造 AR 效果。這幾個工序中，除了第二步外，在早年就有不錯的解決方案；第二步雖然只涉及感知任務，但只有近年神經網絡普及後才有較理想的解決方案。這個感知任務與 MNIST 有少許微細不同（沒有 0，但有空格），不過無傷大體。最後的工作就是要蒐集足夠多的數據訓練神經網絡，以準確標籤每個方格（是空格抑或 1 乘 9 中的其中一項），難題在於需要連續進行 81 次標籤，且須全部準確方可。於圖 5 可見同學們雀躍地使用程式的境況。

圖 5

前瞻

來自香港大學的同學也在工作坊結尾介紹了一些學術界的前沿發展。原來圖 1 右邊是泊松方程（Poisson's equation）的邊界值問題，現在也有一種基於深層神經網絡的數值解，方法名為 Deep Ritz method [2]。另一篇文章論及數據驅動方法（data-driven approach）與基於第一原理方法（first-principle-based approach）兩者的發展沿革 [3]，也很值得注意。至於業界的前沿發展，我們也分享了一些不同範疇應用人工智能的案例。

匆匆數周，幸而工作坊由籌劃到完成一切皆大致順暢。今日回眸，其實很多篇幅都與前事緊扣，時機成熟自然水到渠成。以內容為例，工作坊有不少部分俱與中學數學延伸課程中的單元一 [4] 與單元二 [5] 不謀而合：代數領域中有矩陣；統計領域中有概率分佈，期望值和方差，正態分佈，進階概率中的貝葉斯定理；微積分領域中有求導法，二階導數等。他朝再辦，興許我們有更多時間對各課題作更深入的討論，並能惠及更多莘莘學子。

工作坊以蕭文強教授最近提出的 THAMES（Technology - Humanity - AI/Arts - Mathematics - Engineering - Science）一詞 [6] 作結，並籲同學謹記科學技術的發展終究應為人所用，為人服務。

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7. Sharing of experience on using Jupyter notebook for teaching

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Introduction

Jupyter notebook (https://jupyter.org/) is a popular web-based platform for Data Science. It provides an environment for interactive and exploratory computing. Users can present the ideas and methods as a web document with the codes that generate the results.

Due to its document-centric nature, notebooks are also useful for teaching and learning (https://jupyter4edu.github.io/jupyter-edu-book/). In this article, we share some experience on using notebooks in an interdisciplinary general education course "Playing Sound". This course covers the topics from different fields including Physics of the sound waves, Mathematics of the musical tunings, and computer coding for the sound generation and analysis. We wrote the notebooks for the demonstrations in lectures and computer laboratories, and also the weekly exercises. For example, Figure 1 shows a demonstration of sound (traverse wave) propagation by an interactive animation.

Figure 1: An interactive animation in Jupyter notebook

Available Platforms

Jupyter notebook for a standalone computer is freely available to download. However, it is necessary to install the toolchain of computer language, e.g. Python, beforehand. A simple solution is to install Anaconda (https://www.anaconda.com/) which includes Jupyter notebook, Python, and other computer languages.

It is more convenient to use Jupyter notebook on online website without installation of any software. In this case, students can access the notebooks by their mobile phones or tablets. The following are two common platforms which are free to use:

1. Google Colab (https://colab.research.google.com/)
2. JetBrains Datalore (https://datalore.jetbrains.com/)

If technical supports are available, installation of Jupyter notebook system into a server computer is a good option. We installed "The Little JupyterHub" (https://tljh.jupyter.org /en/latest/) into our own server so that we have the full control of the system. The result was very satisfactory. The creation of user accounts and the distribution of notebooks can be highly simplified. However, due to the change of policies, we were not allowed to run our own server.

The current notebooks of the course "Playing Sound" were implemented in Datalore。

Typical Example

Figure 2 shows a section of a notebook to demonstrate the relation between the frequency and the pitch of sound. Basically, a Jupyter notebook is a list of GUI elements called cells. There are three types of cells: the markdown cell for text, the widget cell for GUI input, and the code cell for computer language (e.g., Python) statements. Markdown is a simple way to specify the formats of text. For example, the section title is the markdown "# Frequency and Pitch" where the "#" sign denotes heading. The description of this section can be found in the cell below the title.

The next cell is a widget cell called "slider". User can drag the slider to different positions to specify the value of a variable "frequency" in this example. The widget cells make a notebook much easier to use, especially for the students who are not familiar with computer languages. Without the widget cells, user needs to modify the computer codes in order to change the results. Widget cells, like slider, provide a user friendly environment where the user need not touch the computer codes.

In the code cell below the slider, there are two Python statements. The first statement generates a sinusoidal wave with the frequency specified in the slider and the duration one second. It utilises a Python module called ThinkDSP (https://github.com/AllenDowney/ThinkDSP). The next statement converts the sinusoidal wave to a sound recording. The widget with a play button and time slider is the result of the execution of the code cell. User can perceive the connection between the frequencies and the pitches of sound using the notebook.

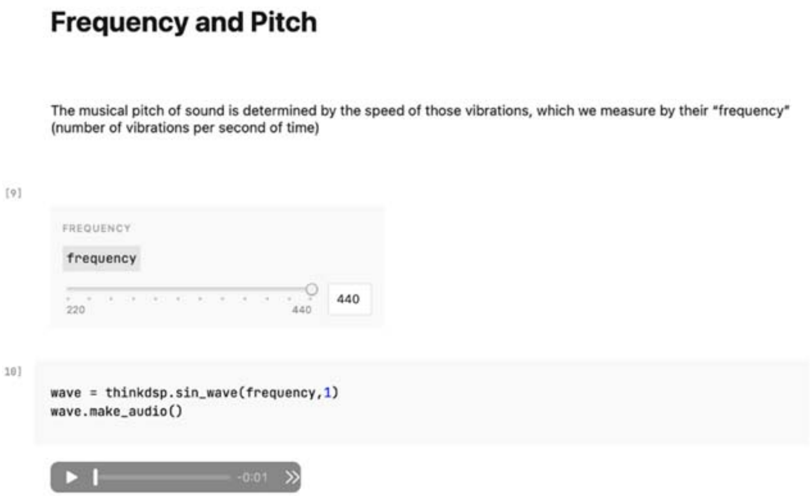

Figure 2: A typical section in Jupyter notebook

Possible Usages in Teaching Mathematics

The following are some ideas of the usages of Jupyter notebook for teaching mathematics:

1. Some students may have the difficulties to visualise the relations between the parameters and the graph of a

function. An interactive plot of a function with the parameters specified by some sliders can be implemented.

2. Jupyter notebook with Python can be used for the illustration of 3D geometric shapes. Figure 3 shows a 3D harmonograph implemented by the students of the course “Playing Sound”. The orientation of the object can be changed by dragging the graph. The interactive 3D plots coupled with the widget and markdown cells should be helpful for the students with the difficulties to visualise 3D objects.

3. SymPy is a Python library for symbolic mathematics. It can be used to solve equation or find derivative and anti-derivative. When it is used with Jupyter notebook, teachers can provide illustration of the ideas in Calculus. It can also be used as computer algebra system (CAS) for students to find the correct answers of Calculus exercises.

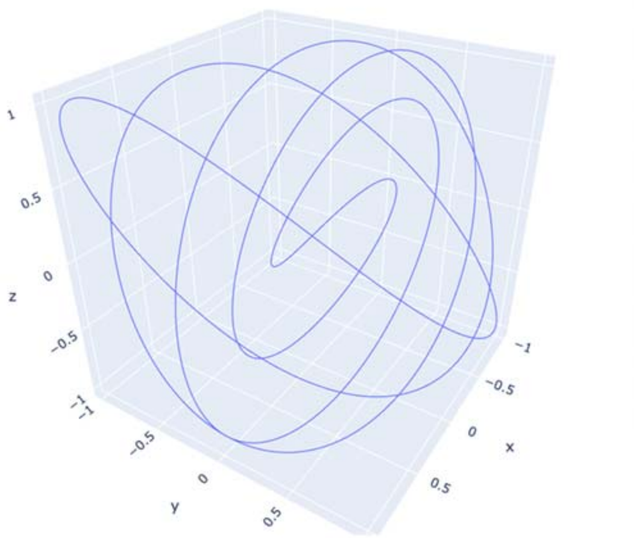

Figure 3: 3D Harmonograph

Limitations and Problems

Some basic knowledge of computer programming are needed to write a notebook. Although Python is considered beginner-friendly and easy to understand and use, novice may need to take around one or two months to learn the language.

In general, Jupyter notebooks are quite computational demanding. Using the free web platforms (Colab and Datalore), sometimes the execution can be slow. Moreover, there is no simple way to distribute the notebooks files to students efficiently.

Using the standalone version may have the same problems if the computer processing power is not good enough. Moreover, Anaconda is bundled with many software and occupies a lot of disk space.

Conclusion

Jupyter notebook provides a relatively simple way to create document-centric and interactive environment for students to understand and test various ideas. When it works with Python, which has a large number of free packages available, there are many possibilities to explore.

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8. 中學數學與 PageRank 演算法

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2021/22 年度數學教育組借調教師

起草本篇文稿時，筆者剛好看到一則網絡新聞:「Facebook 母公司 Meta 運用演算法「隨機」解僱 60 員工」(蘇家華 22.08.2022 ezone)。內容指出由於企業業績不佳，公司需要節省成本，因此決定裁員。這則新聞引起我注意的地方是「演算法」的使用。演算法就是為了解決問題或達到某種目標而依次進行的程序。在現今社會，演算法的應用很廣泛，如組織的人事任命、以及決定某人是否可以接受財務放貸等事情。但是，你可能會發現演算法的確可能出現錯誤、或偏誤。演算法究竟是誰編寫的?是誰為演算法擷取數據源?編寫演算法的人會否抱著既有的成見、在編寫演算法或執行時出現偏差、影響最後的結果。筆者的電腦知識有限，無法逐一評審不同演算法的好壞。筆者只是想帶出一個訊息，大家在現實生活中非常受演算法的影響，在這方面添加一點認知，對大家，尤其是年輕讀者，益處更大，因演算法的應用在將來的生活層面只會越來越多。

讀者們就讀小學期間可能聽過、或學過「輾轉相除法」，用它來找出兩個整數的最大公因數。它其實是演算法的一種，亦稱為歐幾里得算法(Euclidean Algorithm)。讀者可於互聯網上尋找相關短片，短片中會透過 “磁磚鋪排" 和 "直式

運算" 兩種方式來詳述這個演算法的操作步驟，解釋如何可以得到兩個數之間的最大公因數。其內容並不是本篇文章的重點所在，所以不作詳述。歐幾里得算法可以用來找最大公因數、判斷兩數是否互質的特性，常常被應用在密碼學領域，如 RSA 加密演算法。

回到本文的主題，若今天讀者需要於互聯網上尋找資料，我相信很多人也會採用 Google 這個搜尋器，原因可能是較容易找到所想要的資料，究竟搜尋器內的搜尋引擎怎樣決定找到的資料的排序呢？排在前列的當然是 Google 認為是最相關的搜尋結果。Google 有多項重要的演算法來協助搜尋引擎運作，當中 PageRank 演算法是 Google 對網站進行排名的其中一種演算法。本文嘗試運用中學數學知識，解說 PageRank 演算法的運作原理。

PageRank 演算法的概念是一個網頁如果得到其他人的「連結引用」，那理論上這個網頁就越重要，其 PageRank 值會較高，亦代表在搜尋結果中有更高的排名。情況就好像學者在研究時參考及引用的學術論文，論文被引用的次數越多，其內容的可信度越高、品質越好，在學術界的地位越重要。

一個網站的 PageRank 值由下列三個因素決定：

- **連結數量**：如果一個網站被其他網站連結，可能會吸引更多用戶點擊。

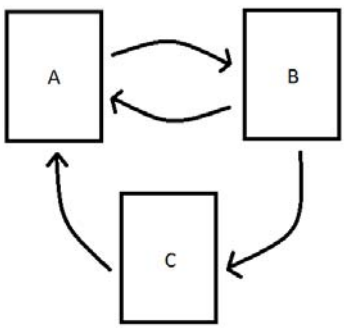
- **連結強度**：訪問這些連結的頻率越高，網站的流量就越高。
- **連結來源**：被其他高等級的網站連結時，也會提升網站本身排名。

跟著，我們嘗試簡化 PageRank 值的計算要求，以一個較簡單的情況，述說如何運用數學知識找到各網頁的 PageRank 值。為方便說明，我們以三個網頁作示例，解說 PageRank 值的計算原理。

進行簡化版 PageRank 值的計算

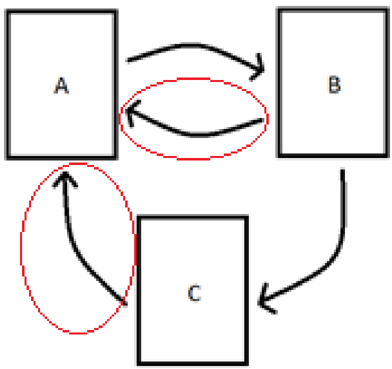
一個網站的 PageRank 值，等於「所有連結到該網頁的網頁之 PageRank 值除以網頁的導出連結數」的總和，再通過「迭代」(iteration)去迫近其最終理論值。

假設現在只有 A、B 及 C 三個網頁來展示 PageRank 的計算。該三個網頁的連結如下圖所示：

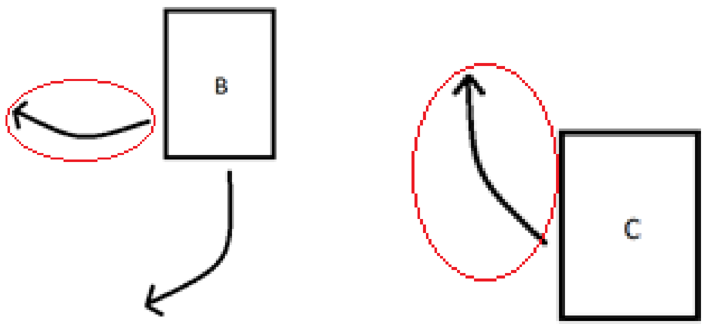


步驟一：確認各網頁的連結關係及 PageRank 計算公式：

設網頁 A 於第 n 次迭代的所得 PageRank 值以 $PR_n(A)$ 來表示。以網頁 A 為例子，網頁 A 得到網頁 B 及網頁 C 的連結。



因此，在計算網頁 A 的 PageRank 值時，需用到網頁 B 及網頁 C 原來的 PageRank 值。再個別觀察網頁 B 及網頁 C：



網頁 B 有 2 個導出連結，而網頁 C 只有 1 個導出連結，故在迭代時網頁 A 的 PageRank 公式如下：

$$\begin{aligned} PR_{n+1}(\mathrm{A}) &= \frac{PR_n(\mathrm{B})}{2} + \frac{PR_n(\mathrm{C})}{1} \\ &= \frac{PR_n(\mathrm{B})}{2} + PR_n(\mathrm{C}) \end{aligned}$$

用相同的方法，我們也可得出網頁 B 及網頁 C 的 PageRank 公式

網頁	PageRank 公式
A	$PR_{n+1}(\mathrm{A}) = \frac{PR_n(\mathrm{B})}{2} + PR_n(\mathrm{C})$
B	$PR_{n+1}(\mathrm{B}) = PR_n(\mathrm{A})$
C	$PR_{n+1}(\mathrm{C}) = \frac{PR_n(\mathrm{B})}{2}$

步驟二：重複進行計算以得出每次迭代的 PageRank 值

我們首先設定 $\frac{1}{\text{網頁總數}}$ 為各網頁的 PageRank 值的初始值。然後利用各網頁的 PageRank 公式重複進行 PageRank 值的計算，直至每個網頁的 PageRank 值沒有進一步的變化。

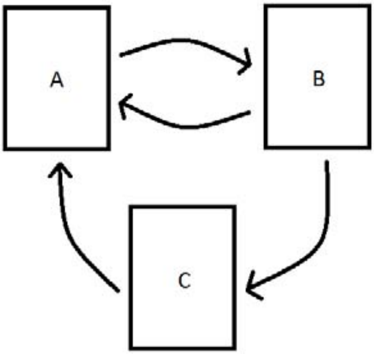
2) 下表展示了在迭代 1 時的計算，試完成由迭代 2 至迭代 4 時的計算。

	PageRank 值				
網頁	迭代 0	迭代 1	迭代 2	迭代 3	迭代 4
A	$\frac{1}{3}$	$\frac{1}{3}\div 2+\frac{1}{3}=\frac{1}{2}$			
B	$\frac{1}{3}$	$\frac{1}{3}$			
C	$\frac{1}{3}$	$\frac{1}{3}\div 2=\frac{1}{6}$			

以上的方法只是每次迭代時求出各網頁的 PageRank 值，如果我們需要求出最終理論值，可以怎樣做呢？

利用線性方程組計算簡化版 PageRank 值

繼續考慮以下情況：



已知 $\begin{cases} PR_{n+1}(\mathrm{A}) = \dfrac{PR_n(\mathrm{B})}{2} + PR_n(\mathrm{C}) \\ PR_{n+1}(\mathrm{B}) = PR_n(\mathrm{A}) \\ PR_{n+1}(\mathrm{C}) = \dfrac{PR_n(\mathrm{B})}{2} \end{cases}$，

若各網頁 PageRank 的最終理論值存在，則在足夠多次數的迭代後，

$$\lim_{n\to\infty} PR_n(\mathrm{A}) = \lim_{n\to\infty} PR_{n+1}(\mathrm{A}) = PR(\mathrm{A})$$

$$\lim_{n\to\infty} PR_n(\mathrm{B}) = \lim_{n\to\infty} PR_{n+1}(\mathrm{B}) = PR(\mathrm{B})$$

$$\lim_{n\to\infty} PR_n(\mathrm{C}) = \lim_{n\to\infty} PR_{n+1}(\mathrm{C}) = PR(\mathrm{C})$$

故我們可直接解以下方程組：

$$\begin{cases} PR(\text{A}) = \dfrac{PR(\text{B})}{2} + PR(\text{C}) \\ PR(\text{B}) = PR(\text{A}) \\ PR(\text{C}) = \dfrac{PR(\text{B})}{2} \end{cases} \Rightarrow \begin{cases} PR(\text{A}) - \dfrac{PR(\text{B})}{2} - PR(\text{C}) = 0 \\ PR(\text{A}) - PR(\text{B}) = 0 \\ \dfrac{PR(\text{B})}{2} - PR(\text{C}) = 0 \end{cases}$$

$$\begin{pmatrix} 1 & -\frac{1}{2} & -1 \\ 1 & -1 & 0 \\ 0 & \frac{1}{2} & -1 \end{pmatrix} \begin{pmatrix} PR(\text{A}) \\ PR(\text{B}) \\ PR(\text{C}) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

通過高斯消去法：

$$\begin{pmatrix} 1 & -\frac{1}{2} & -1 \\ 1 & -1 & 0 \\ 0 & \frac{1}{2} & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -\frac{1}{2} & -1 \\ 0 & -\frac{1}{2} & 1 \\ 0 & \frac{1}{2} & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -\frac{1}{2} & -1 \\ 0 & -\frac{1}{2} & 1 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -\frac{1}{2} & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} PR(\text{A}) - \dfrac{PR(\text{B})}{2} - PR(\text{C}) = 0 \\ PR(\text{B}) - 2PR(\text{C}) = 0 \end{cases}$$

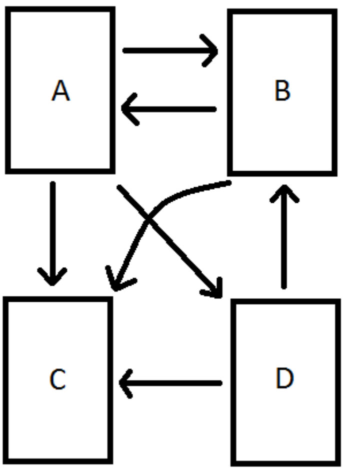
$$\Rightarrow \begin{pmatrix} PR(\mathrm{A}) \\ PR(\mathrm{B}) \\ PR(\mathrm{C}) \end{pmatrix} = PR(\mathrm{C}) \cdot \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

因為 $PR(\mathrm{A}) + PR(\mathrm{B}) + PR(\mathrm{C}) = 1$，故 $PR(\mathrm{C}) = \frac{1}{5}$，即

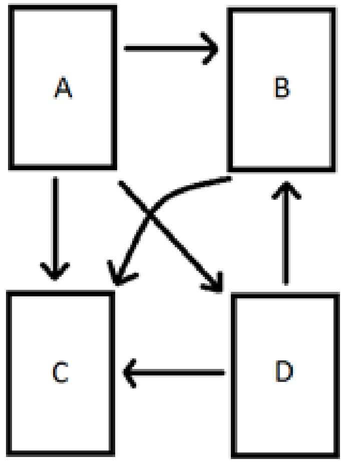
$$\begin{pmatrix} PR(\mathrm{A}) \\ PR(\mathrm{B}) \\ PR(\mathrm{C}) \end{pmatrix} = \begin{pmatrix} \frac{2}{5} \\ \frac{2}{5} \\ \frac{1}{5} \end{pmatrix}$$。

讀者可運用類似的方法，根據網頁的連結情況，求取各網頁 PageRank 的最終理論值。

1)

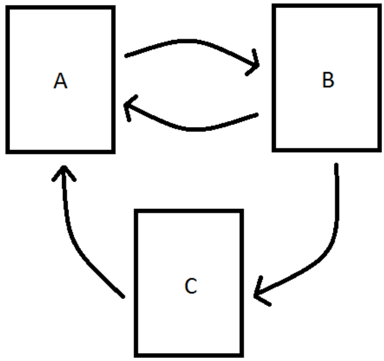


2)*



(延伸學習) 實際的 PageRank 公式

問題二告訴我們，簡化版 PageRank 值的計算有其缺憾，在一些情況將得到平凡解，沒法得出網頁的搜尋順序。因此，實際的 PageRank 公式有一個常數部分，以確保計算出來的 PageRank 值不會歸零。有興趣的同學可以在網上尋找相關資料。如以下所示:



以上三個網頁的 PageRank 值為以下方程組的解。

$$\begin{cases} PR(\mathrm{A})=\dfrac{0.15}{3}+0.85\left[\dfrac{PR(\mathrm{B})}{2}+PR(\mathrm{C})\right] \\ PR(\mathrm{B})=\dfrac{0.15}{3}+0.85\left[PR(\mathrm{A})\right] \\ PR(\mathrm{C})=\dfrac{0.15}{3}+0.85\left[\dfrac{PR(\mathrm{B})}{2}\right] \end{cases} \Rightarrow \begin{cases} PR(\mathrm{A})=0.05+0.85\left[\dfrac{PR(\mathrm{B})}{2}+PR(\mathrm{C})\right] \\ PR(\mathrm{B})=0.05+0.85\left[PR(\mathrm{A})\right] \\ PR(\mathrm{C})=0.05+0.85\left[\dfrac{PR(\mathrm{B})}{2}\right] \end{cases}$$

$$\Rightarrow \begin{cases} PR(\mathrm{A})-0.425PR(\mathrm{B})-0.85PR(\mathrm{C})=0.05 \\ 0.85PR(\mathrm{A})-PR(\mathrm{B})=-0.05 \\ 0.425PR(\mathrm{B})-PR(\mathrm{C})=-0.05 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & -0.425 & -0.85 & 0.05 \\ 0.85 & -1 & 0 & -0.05 \\ 0 & 0.425 & -1 & -0.05 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -0.425 & -0.85 & 0.05 \\ 0 & -0.63875 & 0.7225 & -0.0925 \\ 0 & 0.425 & -1 & -0.05 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & -0.425 & -0.85 & 0.05 \\ 0 & 1 & -\dfrac{578}{511} & \dfrac{74}{511} \\ 0 & 1 & -\dfrac{40}{17} & -\dfrac{2}{17} \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & -0.425 & -0.85 & 0.05 \\ 0 & 1 & -\frac{578}{511} & \frac{74}{511} \\ 0 & 0 & -\frac{10614}{8687} & -\frac{2280}{8687} \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & -0.425 & -0.85 & 0.05 \\ 0 & 1 & -\frac{578}{511} & \frac{74}{511} \\ 0 & 0 & 1 & \frac{380}{1769} \end{array}\right)$$

$$\therefore \quad \begin{pmatrix} PR(\text{A}) \\ PR(\text{B}) \\ PR(\text{C}) \end{pmatrix} = \begin{pmatrix} \frac{703}{1769} \\ \frac{686}{1769} \\ \frac{380}{1769} \end{pmatrix}$$

$$\begin{pmatrix} PR(\text{A}) \\ PR(\text{B}) \\ PR(\text{C}) \end{pmatrix} = \begin{pmatrix} 0.397 \\ 0.388 \\ 0.215 \end{pmatrix}$$

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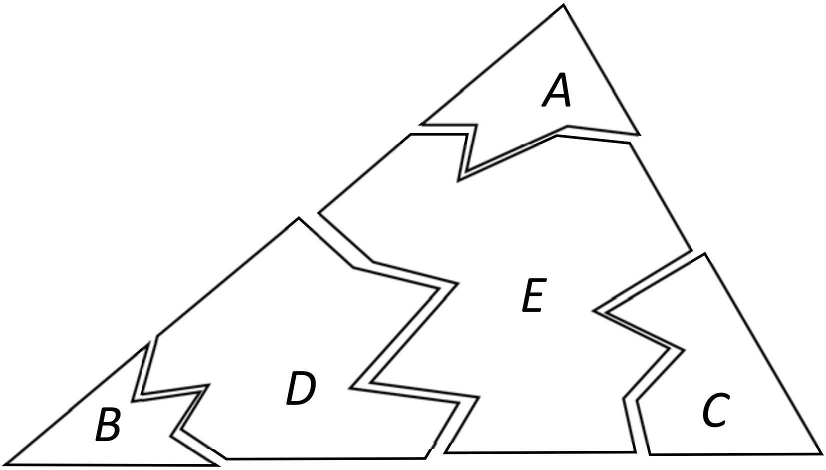
9. 全等概念的再思

程國基

數學教育組

小明不小心將一塊三角形的玻璃打碎成 A、B、C、D 和 E，共 5 塊碎片(圖一)。可否只帶其中一塊碎片去玻璃行找師傅再切一塊和原來形狀大小完全一樣的三角形玻璃？若可，請問應帶哪一片？



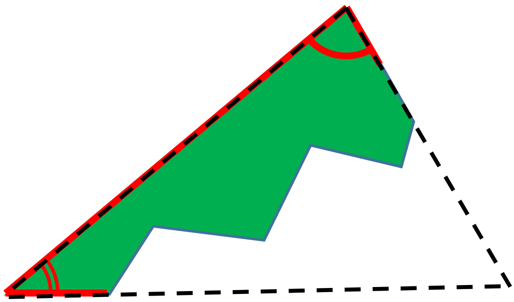
圖一

其實以上的問題涉及全等三角形的判別情況。大家都應知道：如果兩個三角形的三對對應邊和三對對應角相等，則這兩個圖形可被稱為全等圖形。如何判別兩個三角形全等？初中學生應學會運用以下判別條件：

SAS、SSS、ASA、AAS 和 RHS

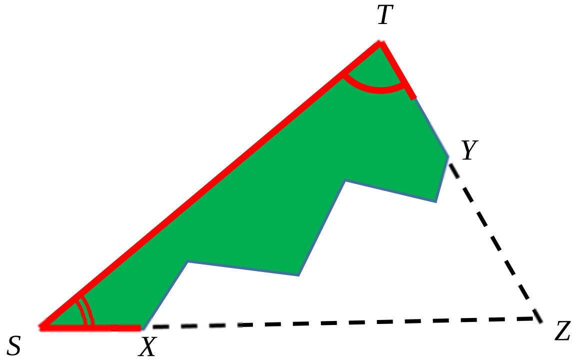
現在先考慮一個簡單的情況：

圖二綠色部分為被打碎的三角形玻璃（虛線部分）的其中一塊碎片，若取這碎片給玻璃行的師傅再切一塊與原來形狀大小完全一樣的三角形玻璃，這是可行的。為什麼？

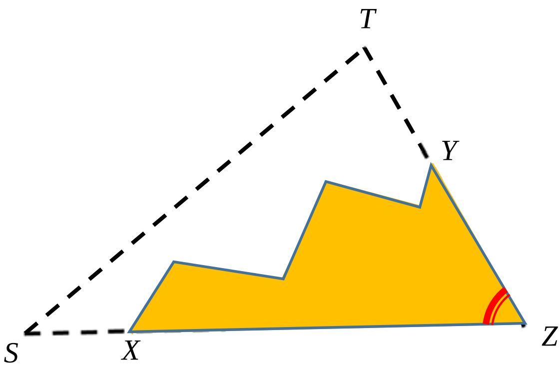


圖二

因為可運用全等三角形判別條件 ASA 以確定 SX 和 TY 延長相交於 Z 的 ΔSTZ 和原來三角形玻璃的形狀大小完全一樣。



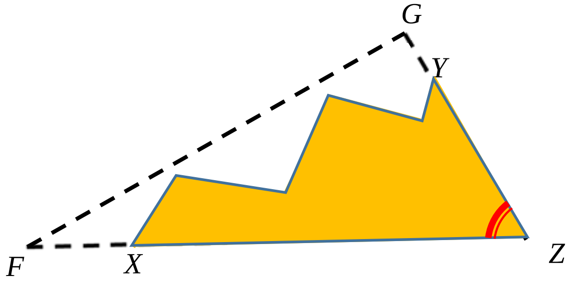
若小明取另外一塊碎片給玻璃行的師傅嘗試再切一塊和原來 ΔSTZ 形狀大小完全一樣的玻璃，如圖三，是否可行？為什麼？



圖三

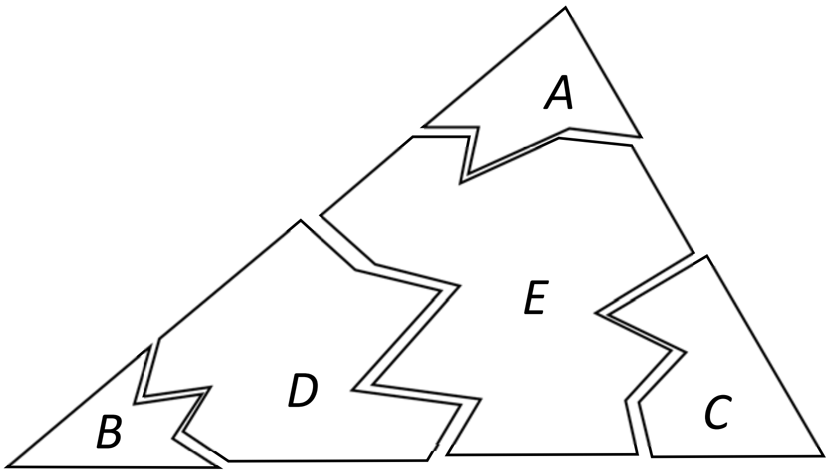
因為僅僅 $\angle Z$ 可以確定，對於 ZX 和 ZY 只是原來三角形玻璃的兩邊的部分。若將 ZX 延長到 F 和 ZY 延長到 G，由於

FG 的長度可變，因此所形成的 *FGZ* 有機會和 *STZ* 不全等的。如圖四。



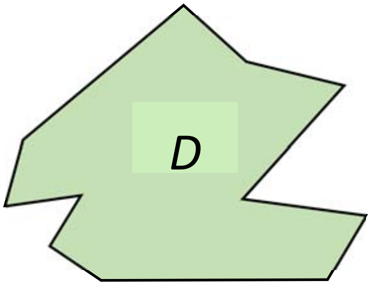
圖四

現在回到最初的情況：

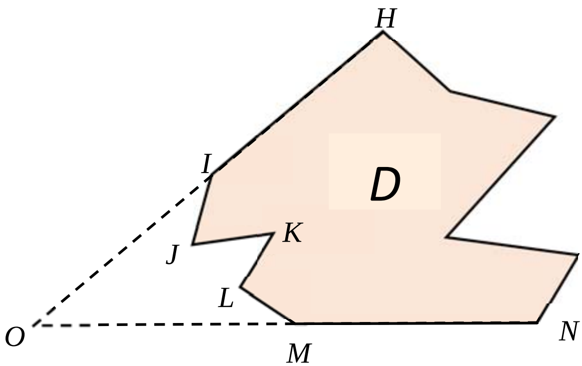


從之前那簡單的情況，可知道若只運用碎片 A，B 或 C 都不能保證切出來的玻璃和原來的玻璃形狀大小完全一樣。

對於碎片 D，

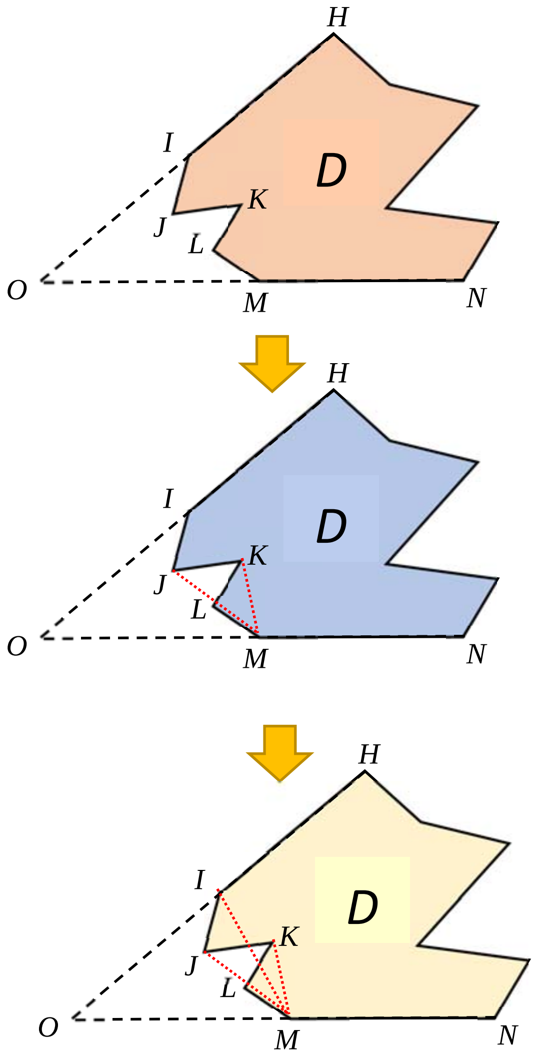


若將 HI 和 NM 延長相交於 O，$\angle IOM$ 的值是可以確定的（因 $\angle HIJ$，$\angle IJK$，反角 $\angle JKL$，$\angle KLM$ 和 $\angle LMN$ 不變）。



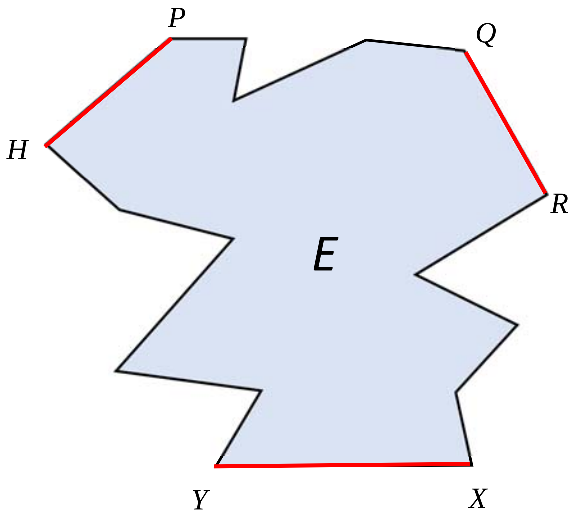
單就 $\angle IOM$ 的值是固定的，可否判斷 OI 和 OM 的長度不變？可能我們會覺得 O 的位置可以確定，並用此來肯定 OI 和 OM 的長度不變。想一想，這塊碎片會被拿到不同地方找

師傅，O 的位置真的可以真確定？這個時候，我們可運用全等三角形的判別條件和性質得 KM，JM，IM 的長度不變，從而也得出 OI 和 OM 的長度固定。

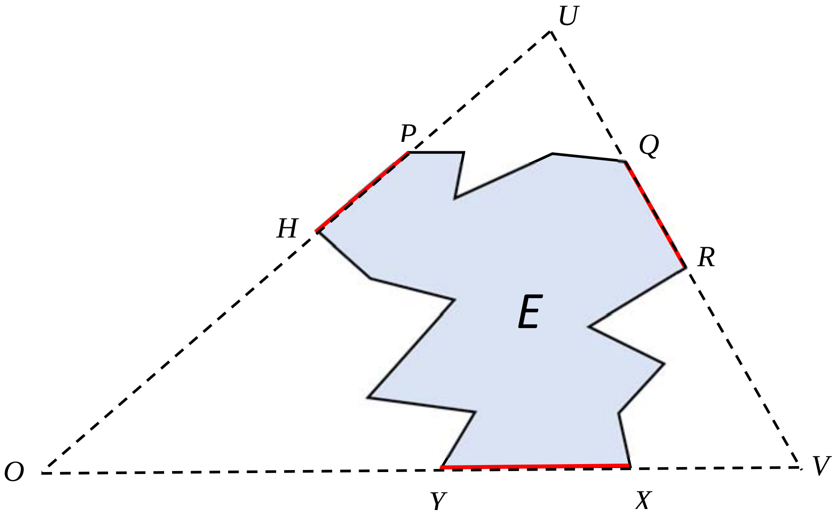


然而這個情況和碎片 A，B 或 C 的情況一樣，也就是說，只運用碎片 D 都不能保證切出來的玻璃和原來的玻璃形狀大小完全一樣。

對於碎片 E 又如何?



已知 HP，QR 和 XY 是原來三角形玻璃的三邊的部分。
若將 HP，QR 和 XY 兩邊延長並且兩兩相交於 O，U，V。如圖五。

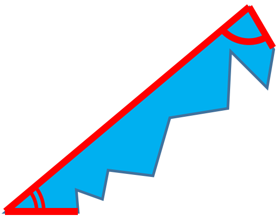
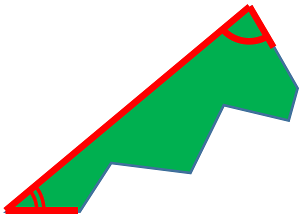
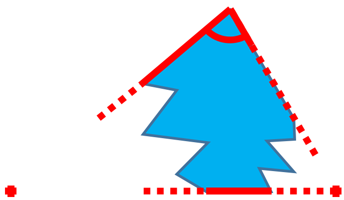
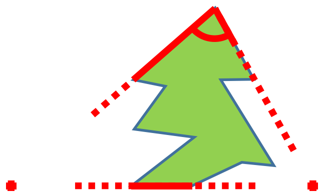


圖五

對於 HP 和 XY，可運用碎片 D 的情況得出 OH 和 OY 的長度固定。同樣地，對於 HP 和 RQ，QR 和 YX，可運用碎片 D 的情況得出 UP 和 UQ，VX 和 VR 的長度固定。因此，ΔSTZ 的形狀大小完全固定不變。若運用碎片 E，小明能保證切出來的玻璃和原來的玻璃形狀大小完全一樣。

從小明打碎玻璃的問題可以看到判斷兩個三角形是否全等，可不用以整個三角作比較，只要運用三角形適當的部分亦可知道兩個三角形是否全等。

反思問題：能否從兩片碎片去推測原本三角形玻璃是否全等？為什麼？

情況一： 從圖 A 和圖 B 的兩塊碎片的原來三角形玻璃是否全等？為什麼？若否，可加什麼相同的資料可使原來兩塊三角形玻璃是否全等？
 圖 A  圖 B
情況二： 從圖 C 與圖 D 的兩塊碎片的原來的三角形玻璃是否全等？為什麼？若否，可加什麼相同的資料可使原來兩塊三角形玻璃是否全等？
 圖 C  圖 D

10. Promote LaC and STEAM Education in Primary Mathematics

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Promotion of reading to learn in schools is one of the effective teaching strategies in learning Mathematics. While language teachers focus more on the teaching of reading strategies and skills, Mathematics teachers should encourage students to apply the relevant skills, and broaden their knowledge and exposure through reading materials in subject matters. This paper reveals a study conducted in different primary schools over the past years and shares teachers' experience in effective use of reading strategies to strengthen students' mathematical & STEAM knowledge and foster their generic skills such as self-study, reading, communication and problem-solving skills. Data was collected in the form of classroom observations, field notes, documents and students' annotated work. To encourage student's learning through reading in Mathematics classrooms, teaching strategies such as group presentation, reading competition, annual book fair as a platform for promoting

reading in Mathematics and STEAM education, student's reflective journals and book reports will be reviewed.

INTRODUCTION

A well-known and essential learning strategy for life-long learning is 'learning through reading'. Promoting a reading culture among students is therefore one of the key tasks in the curriculum reform with the aim to strengthen students' learning capabilities, especially in the subject of Mathematics. According to the Progress in International Reading Literacy Study (PIRLS) results at 4th Grade in 2007 (Baer, J., Baldi, S., Ayotte, K., and Green, P., 2007), Hong Kong students ranked the second among the forty-five countries/regions all over the world. As compared to the result published in 2001, Hong Kong students have a significant improvement. In the 2016 result at 4th Grade (Mullis, I.V.S., Martin, M.O., Foy, P., & Hooper, M., 2017), Hong Kong students are still at the position of top-performing countries/regions. Below is captured to show situation and scores of those top-performing countries/regions:

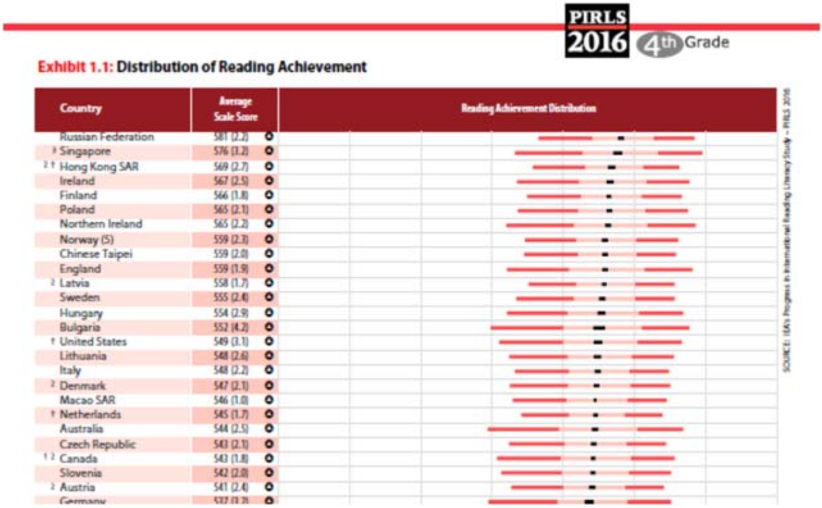
PIRLS 2016 4th Grade

Exhibit 1.1: Distribution of Reading Achievement

Country	Average Scale Score	Reading Achievement Distribution
Russian Federation	581 (2.2)	
² Singapore	576 (3.2)	
² † Hong Kong SAR	569 (2.7)	
Ireland	567 (2.5)	
Finland	566 (1.8)	
Poland	565 (2.1)	
Northern Ireland	565 (2.2)	
Norway (5)	559 (2.3)	
Chinese Taipei	559 (2.0)	
England	559 (1.9)	
² Latvia	558 (1.7)	
Sweden	555 (2.4)	
Hungary	554 (2.9)	
Bulgaria	552 (4.2)	
† United States	549 (3.1)	
Lithuania	548 (2.6)	
Italy	548 (2.2)	
² Denmark	547 (2.1)	
Macao SAR	546 (1.0)	
† Netherlands	545 (1.7)	
Australia	544 (2.5)	
Czech Republic	543 (2.1)	
† ² Canada	543 (1.8)	
Slovenia	542 (2.0)	
² Austria	541 (2.4)	
Germany	537 (3.2)	

SOURCE: IEA's Progress in International Reading Literacy Study – PIRLS 2016

(Source: PIRLS 2016 International Results in Reading, p.20)

Regarding "Multiple Comparisons of Average Reading Achievement" for the above table, Hong Kong's good performance is captured and listed below (p.22):

> "… *Hong Kong SAR was another top performer, with achievement similar to or higher achievement than all the countries except the Russian Federation* …"

When this paper was written, the PIRLS 2021 result has not yet been released. Hence, the latest data cannot be obtained yet, but the following change captured from its website is highlighted below for information and reference:

PIRLS 2021: Transitioning to Digitally Based Assessment

PIRLS 2021 offers the PIRLS assessment of literary and informational reading in a digital format, presenting reading passages and items as an engaging and visually attractive experience that motivates students and increases operational efficiency.

Having PIRLS in an electronic environment enables joining it with the ePIRLS assessment of online informational reading. With PIRLS all electronic, countries can administer the full PIRLS reading assessment - PIRLS Literary and Informational as well as the ePIRLS Online Informational - as one seamless, digitally based endeavor. It is also possible for countries to administer the paper-based version of the PIRLS assessment of literary and informational reading.

(PIRLS International Research Program: http://www.pirls.org/)

Thus, the reading culture and practice are changing in the world and in Hong Kong as well. For Mathematics in Hong Kong nowadays, reading helps students develop thinking skills, enrich knowledge, enhance language proficiency and broaden life experience; and promotion of reading to learn in schools is one of the effective teaching strategies in learning mathematics. While language teachers focus more on the teaching of reading strategies and skills, Mathematics teachers should encourage students to apply the relevant skills, and broaden their knowledge and exposure through reading materials in subject matters. Reading across the Mathematics curriculum needs to be strengthened and a whole school approach should be adopted to share good practices and nurture a reading culture within the school (CDC, 2002). Students should be encouraged, as early as possible, to make full use of the school libraries to read a wide variety of materials (e.g. mathematics story books & history in mathematics) apart from textbooks or reference books for achieving different learning targets and hence lifelong, independent learning. In response to the changing needs of society, the rapid development of science and technology, the results of international studies on our education system, as well as views of stakeholders, the Mathematics Education KLA curriculum (CDC, 2017) is developed in a direction to extend the existing strengths, to enhance students' learning progression and to align with the focal points of ongoing renewal of school curriculum. The focal points that connect with the development of the Mathematics Education KLA include STEAM education, information technology in education, Language across the Curriculum (LaC), etc.

A Pilot Scheme - Reading to Learn (Primary Mathematics)

To achieve the mentioned-above, for the past years, Reading to Learn (Primary Mathematics) was a longitudinal study of the pilot scheme conducted by the Mathematics Education Section, Education Bureau. The teacher seconded in 2018/19 (Teacher A) and another teacher seconded in 2021/22 (Teacher B) from Kowloon Women's Welfare Club Li Ping Memorial School worked in the Mathematic Education Section to serve as teacher researchers to participate and assist in conducting the pilot scheme in those school years respectively. The teacher researchers served an important role, and one of their major tasks was to conduct the scheme and collect the data. After teacher A finished her secondment and returned to her own school, she joined the scheme to promote LaC in her school. While working in the section, teacher B served as the key person of the scheme and conducted school visits for the participating primary schools to collaborate with teachers to design, develop and try run the reading activities including students' reading competition/sharing, annual book fair as a platform for promoting reading in Mathematics and STEAM education, Reading Day, etc. For data collection, they were collected through classroom observation (Zoom mode due to the COVID pandemic), document analysis, teachers' discussion and students' annotated work collected. The qualitative approach is mainly adopted to conduct the scheme. In addition, a partnership amongst the teacher researchers and school teachers of primary schools was well established in order to generate and develop different reading activities with authentic situation on the Mathematics lessons. For the analysis on collected school-based reading materials/passages or developed reading activities, this study aimed to explore (i) how the reading activities can arouse students' reading/learning motivation and enhance their mathematical interest/ability; and (ii) how students can apply their mathematical knowledge and STEAM skills to solve the issues raised in the reading activities in mathematics lessons. For the deliverables of the scheme for each school year, a book list on reading mainly including Mathematics and STEAM education were generated and uploaded to the web as below

https://www.edb.gov.hk/attachment/tc/curriculum-development/kla/ma/res/pri/PriMathBkList.pdf

for schools' use and reference, to well understand how to promote LaC and STEAM Education in primary Mathematics in the scheme, the following schools' experiences captured in different school years are to show and illustrate on how to achieve the scheme aims mentioned above.

Schools' Experiences Sharing (2020/21) - Reading Activities

During the 2018/19 school year, Teacher A actively participated in the Reading to Learn Pilot Scheme and assisted in the development on school-based reading plan & activities in Mathematics of the participating schools. After she completed her secondment, she encouraged her school head to join the Pilot Scheme in the next two school years so as to further promote the LaC in her school, especially in Mathematics and STEAM education. In 2020/21 school year, her school reading strategies included the following

Reading Strategies	Preparation / Implementation
Theme-based Reading	• Teachers selected different themes, e.g. STEAM to arouse and strengthen students' interest among different subjects. • Book fair was the original plan. However, due to COVID-19 pandemic, books sharing on display board was adopted on STEAM Day.
Reading Activities	• Reading materials were displayed in the class for students' easy access.

Reading Strategies	Preparation / Implementation
	• During school holidays, students needed to read the materials/essays/tailor-made video clips distributed by their teachers. And, students needed to write the reading report for sharing with classmates during the lesson later. • Through Reading Award Scheme, students were encouraged to foster their reading interest in the areas of LaC/STEAM.
Promotion of Books	• For those promoted books, teachers provided tailor-made video clips and sound files for their students (due to COVID-19 pandemic & no face-to-face lessons in school). (Figure 1)
Reading Day	• Teachers identified some interesting topics such as paper folding, curve stitching, 2-D shapes, etc. as the focus of the Reading day. After reading the related materials, students were required to make their models accordingly. It could make students feel satisfactory with the product (the

Reading Strategies	Preparation / Implementation
	Platonic solid "dodecahedron") through learning from reading. (Figure 1)
Book Reading Sharing	• After students' book reading activities had arranged in first & second school terms respectively, they needed to share in the class. Due to COVID-19 pandemic, Zoom lesson was in first school term (Figure 2). And in the second school term, a face-to-face lesson could be conducted and students were eager to share their reading experience with other classmates.
Promotion of Reading Atmosphere	• Teachers designed and developed "pocket size reading" essays for students with less reading ability so as to enhance their reading capacity gradually and steadily (Figure 3). • Different and simple essays were put on display board in the classroom so as to arouse students' reading motivation and interest (Figure 3).

	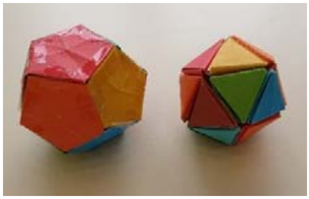
video clip for promotion of book	Students' Work (Platonic solid "Dodecahedron")

Figure 1

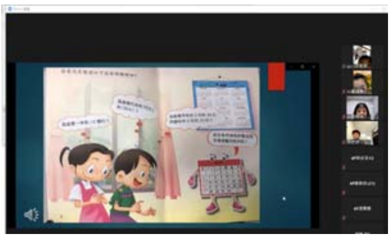	
Illustration on Reading materials (Zoom lesson)	Sharing Reading experience among students (Zoom lesson)

Figure 2

	
“pocket size reading” essay	Essays on Display Board for students

Figure 3

In spite of the suspension of face-to-face lessons for some periods in the 2020/21 school year, the planned reading activities were smoothly conducted. Apart from the above, Teacher A also generated the school-based LaC/STEAM reading book list for their students at each level from primary one to six (Appendix 1), which can be a good reference for other schools.

Schools’ Experiences Sharing (2021/22) - Reading Plan with Mathematics/STEAM

In the 2021/22 school year, although the COVID-19 pandemic still affected the school operation, the Reading to Learn Pilot Scheme kept on running. Another teacher (Teacher B) was seconded and assisted in the development on school-based reading plan and activities in Mathematics of the participating schools. While face-to-face sharing presentation discussion, Zoom lesson observation and school visits with lesson observation were conducted, two pieces of valuable experience generated would be worth sharing for teachers' reference. The first highlight was the school-based annual reading plan with STEAM elements embedded (Reading Plan_steam), and the other was a Zoom reading lesson in STEAM. For the Reading Plan_steam, one participating school designed a number of different reading activities and implemented throughout the whole school year, including "Preparation for the reading materials and list focusing on Maths/STEAM", "Purchasing more Maths/STEAM books", "Reading activities for the whole school year", "Introducing the Maths/STEAM books for students", "Web surfing for Maths/STEAM reading materials and promotion", "Sharing of reading results", etc. For the details, the Appendix 2 is referred. The aforesaid Reading Plan_steam was comprehensive and viable for implementation. Even under the impact of COVID-19, the majority of the proposed activities were smoothly conducted. For the second highlight, a Zoom reading lesson with STEAM elements embedded was conducted in junior primary level in early 2022 during the period of suspension of face-to-face classes of primary schools. A school teacher extracted different content with STEAM elements from a book and illustrated one by one in the lesson. Students were guided to read the simple and short materials displayed in the PowerPoint slides in the lesson and answer teachers’ questions posed (Figure 4).


PowerPoint Slide developed by teacher for introducing the Reading materials (Zoom lesson)

Students answered teacher’s question after her illustration (Zoom lesson)

Promotion of different STEAM books

Figure 4

Towards the end of the Zoom lesson, the teacher provided and promoted different STEAM books for her students so as to encourage and foster their reading interest and ability (Figure 4). At the review meeting with the teacher, she expressed that the school had already purchased a number of Maths/STEAM books for their students and some of them were especially for primary two students. During the first school term, the Panel Chairperson produced video clips to share with students on Maths/STEAM books in their reading activities, and students were asked to finish the related worksheets/reading reports. A Book Fair focusing on Maths/STEAM reading materials was rearranged from the first school term to second school term due to the COVID-19 pandemic. For the above, the school's practices generated were worth referencing by other schools.

Reflection on the Pilot Scheme

The Pilot Scheme has been conducted for a number of years and lots of valuable experience has already generated and delivered for schools' reference and use. However, for the recent years, the COVID-19 pandemic made the previous arrangement change a lot. Teachers' reflection from the above shared cases were provided below so that schools could promote LaC and STEAM education in primary Mathematics effectively through reading.

- Increase the number and variety of Mathematics books in schools - Through this project, teachers had the opportunity to re-examine the existing Mathematics books in schools, and explored the digital reading materials in the web so as to purchase some new books/e-books that suit students' level and interests.

- Increase students' attention to reading Mathematics books and develop the habit of reading Mathematics/STEAM books - The school used posters and short videos to promote teachers' favorite Mathematics/STEAM books, which successfully attracted students' attention to reading Mathematics/STEAM books and the school encouraged students to continue borrowing Mathematics/STEAM books through the Reading Rewards Program to cultivate their habit of reading Mathematics/STEAM articles.

- Increase students' mathematical knowledge and generic skills - Most students could complete the reading reports or worksheets carefully. During the sharing session, students were able to prepare materials carefully and prepare some questions to help their classmates better understand the contents of the book. The whole activity allowed them to gain knowledge, value judgments and learning attitudes.

- Nurture students’ reading motivation and interest - Through the experience of implementing the annual reading plan, teachers knew more about which type of Mathematics/STEAM books students were interested in. Therefore, it could help teachers purchase/select some interesting Mathematics/STEAM books that students could read easily so as to broaden students' knowledge.

- Enhance the professional development of teachers - In the process of planning, implementing and reviewing the whole pilot scheme, teachers believed that their professional development could be facilitated by interacting with the support of the Mathematics Education Section. Teachers understood how to plan various reading activities in line with the objectives of the Mathematics curriculum, and integrate the "Mathematics Book Reading Plan" into their learning, such as how to use reading chapters, arrange reading activities that suit students' levels, and cultivate their habit and interest in reading Mathematics/STEAM books. Teachers raised that the pilot scheme enabled them to observe students' performance in sharing reading results and participating in reading days, and enhancing student engagement. It was more effective than simply recommending reading experiences by teachers.

Concluding Remark

Since "Reading ro learn" is one of the four key tasks of the curriculum, the school in this case study has always listed the promotion of reading as one of the key development items, hoping to cultivate students' interest in reading and establish good reading habits. However, most schools arrange reading activities in Chinese and English subjects, and the promotion of "learning by reading" in Mathematics is sometimes neglected. Therefore, schools lack experience in finding resources and promoting them. Through collaboration with the Mathematics Education Section of the Education Bureau, such as writing an implementation plan, formulating work items, formulating success criteria and implementing reading activities, letting them understand the value of the plan and the details of its implementation. Due to the epidemic, some plans have not yet been implemented. It can be expected to promote the atmosphere of reading Mathematics/STEAM books. This paper aims to contribute to issues on promoting LaC and STEAM Education in primary Mathematics by presenting a pilot scheme with schools’ experiences and good practices generated. For the current trend “ePIRLS assessment of online informational reading", it would be the direction of future research to investigate on how to promote it in schools by strengthening the LaC which students can more deeply appreciate the joy of reading Mathematics/STEAM books, particularly for those reading resources in a digital format provided in an electronic environment.

References

[1] Baer, J., Baldi, S., Ayotte, K., and Green, P. (2007). The Reading Literacy of U.S. Fourth-Grade Students in an International Context: Results from the 2001 and 2006 Progress in International Reading Literacy Study (PIRLS) (NCES 2008–017). National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education. Washington, DC.

[2] Curriculum Development Council (2002). Mathematics Education, Key Learning Area Curriculum Guide (Primary 1 – Secondary 3), Hong Kong: The Printing Department.

[3] Curriculum Development Council (2017). Mathematics Education Key Learning Area Curriculum Guide (Primary 1 – Secondary 6) (2017), Hong Kong SAR: Education Bureau.

[4] Ina V.S. Mullis. (2019). PIRLS 2021 ASSESSMENT FRAMEWORKS: Introduction. In I.V.S. Mullis & M. O. Martin (Eds.), PIRLS 2021 Assessment Frameworks. TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College and International Association for the Evaluation of Educational Achievement (IEA)

[5] Mullis, I.V.S., Martin, M.O., Foy, P., & Hooper, M. (2017). PIRLS 2016 International Results in Reading. TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College and International Association for the Evaluation of Educational Achievement (IEA)

[6] PIRLS International Research Program: http://www.pirls.org/

Appendix 1

學校 LaC / 數學 / STEAM 圖書目錄

2020-2021 年度數學科跨課程閱讀

請各科任就本學年的進度，編排各級的跨課程閱讀內容。

年級	課題	書名
P1	1 至 10 各數、數數活動	大自然的合奏
		小柏做蛋糕
		老鼠童軍團
		森林裏
	排次序、順數和倒數	迷離大屋
	10 以內的加法	天上來的寶物
		老伯伯與兔子
	立體圖形	認識金字塔
		神奇畫板變變變
	報時	精靈的奇幻旅程
	貨幣	錢幣的由來
P2	垂直線	垂直線和平行線
	時和分	精靈的奇幻旅程
	角柱和圓柱	認識金字塔
	全年的日子	月份的故事
	貨幣	錢
		我想買恐龍
		一塊錢流浪記
	方向	幫媽媽送禮物
P3	秒	為何要爭分奪秒
	分數	魔數小子－噢！被搬！
P4	2、5、10 的整除性	妖怪有多少？
	公倍數和最小公倍數	魔幻城堡
	面積	土地神
	小數	巧遇小數點
P5	八個方向	尋找方向
		古代的指南針
	體積	聰明的烏鴉
P6	容量和體積(一)	數學家的傳奇歷險記 2—阿基米德
	百分率的應用	「孤寒」店主

Appendix 2

XXX 學校
從閱讀中學習(小學數學) 試行計劃（2021/22）
「推行計劃」

推行級別：小二
計劃目標：1. 培養學生對閱讀數學和 STEM 書籍的興趣及習慣；
2. 營造校內學習數學的良好氛圍；
3. 優化校內數學和 STEM 圖書館藏。

項目	策略/工作	推行時間表	所需資源	評估方法	成功準則	負責人
1. 編寫閱讀材料一覽表	1.1 檢視現行學校圖書館內的數學和 STEM 閱讀材料，編寫目錄，讓教師在設計學習活動時，能參考上述書目，以促進從閱讀中學習。	• 2021 年 10 月	• 數學和 STEM 圖書	• 檢視數學和 STEM 圖書目錄表	• 完成編寫數學圖書目錄表	• 圖書館主任
	1.2 於學期終結時，在數學和 STEM 圖書目錄上標示出老師曾推介及同學喜愛的數學圖書。	• 2022 年 6 月	• 數學和 STEM 圖書目錄	• 檢視書目中老師曾推介及同學喜愛的數學和 STEM 圖書	• 完成檢視工作	• 圖書館主任
2. 增加數學圖書	2.1 增加校內圖書館數學和 STEM 圖書館藏的 X 本，為配合小二數學閱讀，增添約 Y 本適合小二的數學和 STEM 圖書。	• 2021 年 12 月	• 圖書館數學和 STEM 圖書	• 統計圖書館添置的數學和 STEM 圖書數量	• 完成添置圖書 X 本	• 圖書館主任 • 科主任

1

項目	策略/工作	推行時間表	所需資源	評估方法	成功準則	負責人
3. 閱讀活動	3.1 鼓勵閱讀：安排學生進行讀書活動 a. 於二年級課室內增設「讀書角」，把曾閱讀的數學和 STEM 圖書名稱和日期記錄(在《數學小博士 PASSPORT》內)。	• 2021 年 12 月	• 數學和 STEM 圖書 • 閱讀紀錄表/passport	• 觀察學生表現	• 學生能主動閱讀數學和 STEM 圖書及表現有興趣	• 科主任 • 科任老師
	b. 學生於長假期借閱數學和 STEM 圖書，全年共 X 本，並完成 Y 份閱讀報告。(Y 可小於 X)	• 2021 年 12 月 • 2022 年 2 月 • 2022 年 4 月	• 數學和 STEM 圖書 • 閱讀報告(可參考附件二)	• 檢示閱讀報告	• 學生認真完成閱讀報告	• 科主任 • 科任老師
	c. 於「讀書角」張貼 X 份與二年級數學和 STEM 課題有關的圖書簡介，學生閱讀後，完成數學周的 XX 活動(問答活動，解難活動)。(可參考附件一，讓學生閱讀輕松的電子書或實體書，回答數學或 STEM 問題和培養正面態度，例如動物各有習性，參觀時應注意避免驚擾動物，好好愛護牠們，以配合學校培養學生品德的關注事項)	• 2022 年 4 月：張貼簡介 • 2022 年 5 月：進行活動	• 與數學和 STEM 課題有關的圖書簡介 • 活動工作紙	• 觀察學生表現 • 檢示活動工作紙	• X%學生參與活動並認真完成	• 科主任 • 科任老師
	3.2 獎勵計劃：(數學小博士 PASSPORT) a. 學生在本學年完成閱讀 X 本數學或 STEM 圖書與 Y 張數學和 STEM 篇章工作紙，並記錄在《數學小博士 PASSPORT》內可獲(獎品/獎狀)	• 2021 年 12 月至 2022 年 6 月	• 數學和 STEM 圖書 • 數學和 STEM 篇章	• 檢視數學和 STEM 篇章工作紙和紀錄表	• X%學生能獲得(獎品/獎狀)	• 科主任 • 科任老師

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項目	策略/工作	推行時間表	所需資源	評估方法	成功準則	負責人
			工作紙 • 紀錄表/passport • 獎品/獎狀	/passport		
	b. 配合數學周，學生完成各項閱讀活動後，在《數學小博士 PASSPORT》內進行閱讀活動記錄可獲(獎品/獎狀)。	• 2022 年 5 月	• 《數學小博士 PASSPORT》活動冊 • 獎狀	• 檢視活動冊	• X%學生能完成活動冊，並投入參與活動	• 科主任 • 科任老師
	3.3 閱讀工作紙 a. 科任以短片/現場與學生進行數學或 STEM 圖書分享 X 次，學生完成閱讀工作紙。	• 2021 年 12 月	• 閱讀工作紙	• 檢視閱讀工作紙	• 學生對介紹的數學或 STEM 圖書感興趣並認真完成閱讀工作紙	• 科任老師
	b. 圖書館主任在圖書課期間與學生進行數學或 STEM 圖書分享 X 次，學生完成閱讀工作紙。	• 2022 年 3 月				• 圖書館主任
4. 數學圖書推介	4.1 在書展中增加數學和 STEM 類書籍的展覽	• 2022 年 1 月	• 數學圖書和 STEM	• 檢視書展情況	• 完成書展 • 學生主動選取數學和 STEM 圖書	• 圖書館主任
	4.2 科主任及科任老師定期向學生推介數學或 STEM 圖書，全年共 X 次	• 2021 年 12 月 • 2022 年 4 月 • 2022 年 6 月	• 數學或 STEM 圖書	• 檢視老師曾推介的數學或 STEM 圖	• 完成推介活動 • 學生對推介	• 科主任 • 科任老師

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項目	策略/工作	推行時間表	所需資源	評估方法	成功準則	負責人
				書 • 觀察學生表現	的數學和 STEM 圖書感興趣	
	4.3 於數學周設置數學和 STEM 圖書展板，推介數學和 STEM 圖書	• 2022 年 5 月	• 數學和 STEM 圖書推介內容	• 檢視數學周展板情況 • 觀察學生表現	• 完成數學周展板 • 學生主動閱讀展板內容	• 圖書館主任
5. 瀏覽網上數學或 STEM 閱讀材料/閱讀篇章	5.1 閱讀數學或 STEM 篇章 各級學生閱讀校內數學報(於操場壁佈展示)及網上數學或 STEM 短文 X 篇後，完成活動作紙，並紀錄在《數學小博士 PASSPORT》內。	• 2022 年 2 月	• 校內數學報 • 網上數學或 STEM 短文 • 活動工作紙 • Passport	• 觀察學生表現 • 檢視活動工作紙	• 學生表現有興趣 • 學生認真完成閱讀工作紙	• 數學科主任 • 科任老師
6. 閱讀成果分享	6.1 分享讀後感或推介好書：「Book Salesman」 a. 小二學生於數學課中向同學分享數學或 STEM 圖書，全年 2 次。	• 2022 年 2 月 • 2022 年 6 月	• 數學和 STEM 圖書	• 觀察學生表現	• 學生認真閱讀數學和 STEM 圖書內容和向同學分享	• 科任老師

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