

# Market and Efficiency\*

May 3, 2012

## 1 Efficiency of Competitive Equilibrium

### 1.1 Gains from trade and efficient allocation

- Two-person-two-good *exchange* economy
  - Persons  $A$  and  $B$
  - Goods 1 and 2
- $A$  has endowment  $E^A = \{e_1^A, e_2^A\}$  (Figure 1),  $B$  has endowment  $E^B = \{e_1^B, e_2^B\}$  (Figure 2).
- Write  $E_1 = e_1^A + e_1^B$  and  $E_2 = e_2^A + e_2^B$ . See Figure 3.
- A feasible allocation comprises of any consumption bundles for the two individuals,  $C^A = \{c_1^A, c_2^A\}$  and  $C^B = \{c_1^B, c_2^B\}$ , such that

$$c_1^A + c_1^B \leq E_1,$$

$$c_2^A + c_2^B \leq E_2,$$

See Figure 4.

- We are looking for allocations  $\{C^A, C^B\}$ , which is preferred by  $A$  over  $E^A$  and by  $B$  over  $E^B$ . See Figure 5 for the set of all such allocations. Which particular allocation will be chosen depends on the “trading mechanism” adopted by the two individuals. One such allocation, denoted as  $PO$  in Figure 6, deserves special attention. At  $PO$ , the ICs of the two individuals are tangent to each other. Moving away from  $PO$  to any other allocation must involve either  $A$  or  $B$  being made worse off relative to  $PO$ . This is the case since the set of allocations preferred by  $A$  and the set preferred by  $B$  are *disjointed*.

---

\*References: Chapters 30 and 33 of Hal Varian, *Intermediate Microeconomics: A Modern Approach*, 6th edition, Norton.

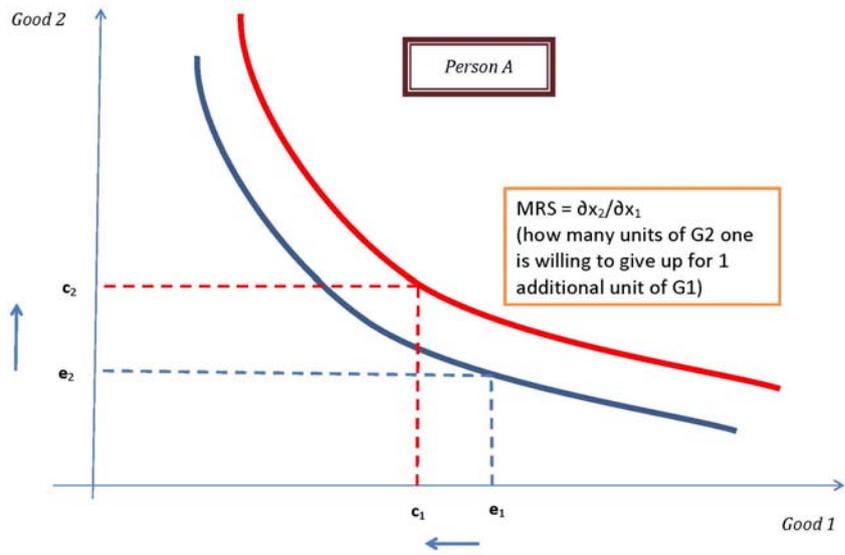


Figure 1: A's endowment and preferences

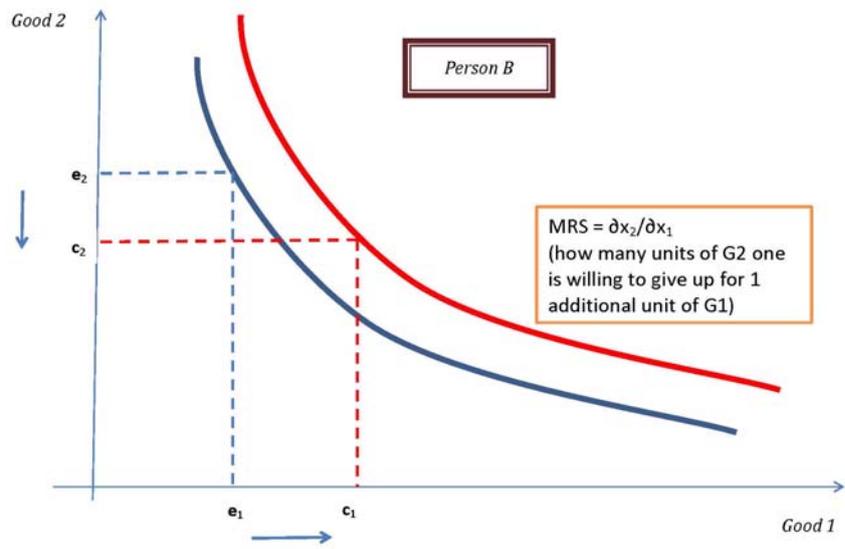


Figure 2: B's endowment and preferences

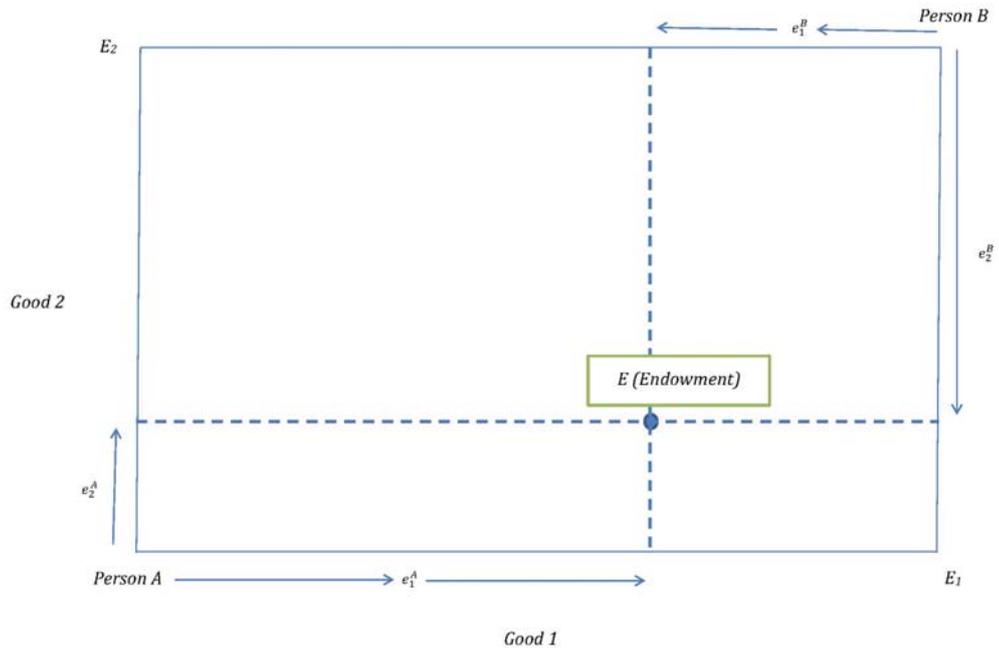


Figure 3: The Edgeworth Box

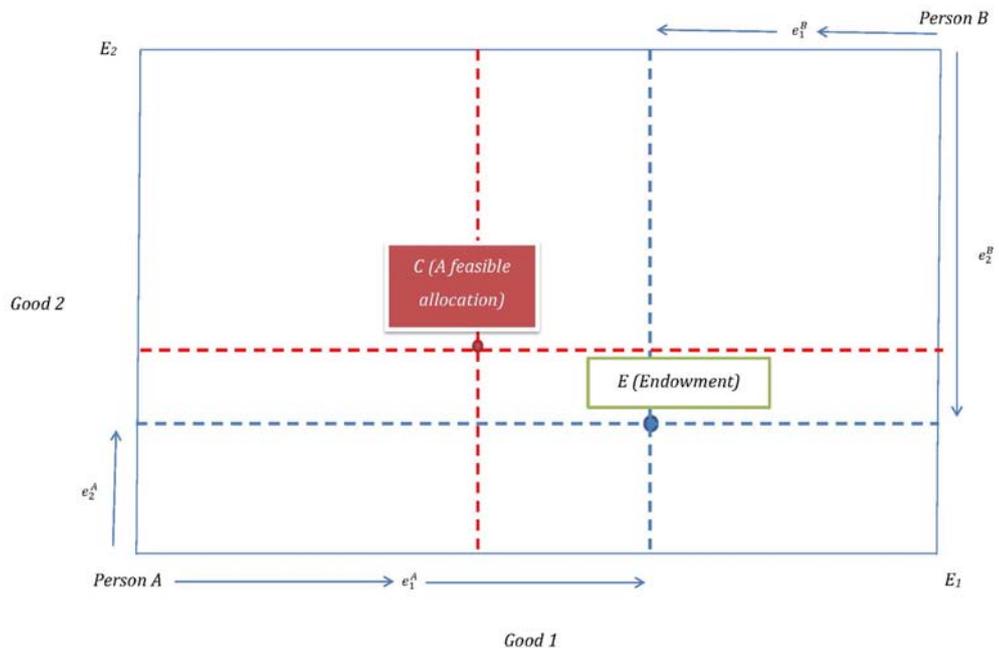


Figure 4: Feasible Allocations

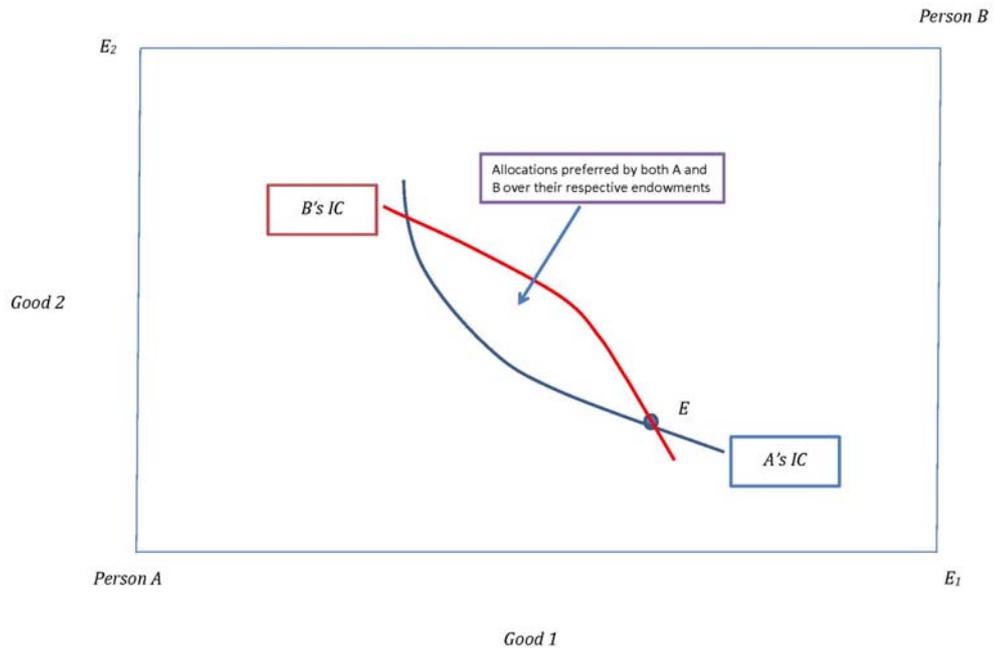


Figure 5: Set of allocations preferred over endowment

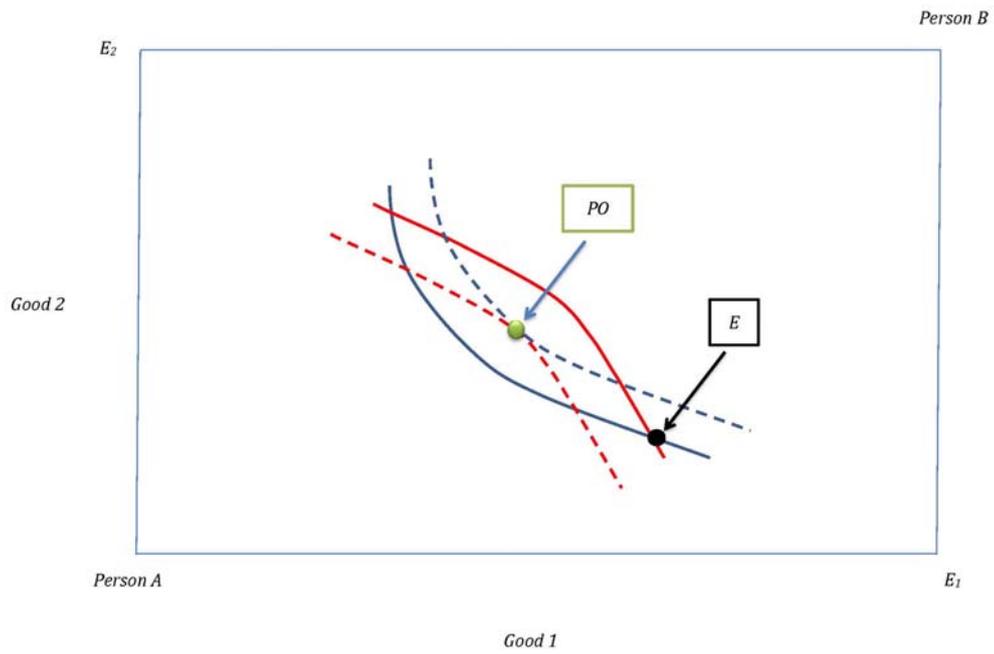


Figure 6: Pareto Optimal Allocation

- An allocation such as  $PO$  in Figure 6 is known as a Pareto Optimal ( $PO$ ) or simply efficient allocation. There are various (equivalent) definitions for  $PO$  allocations:
  - There is no way to make all people involved better off.
  - There is no way to make some individual better off without making someone else worse off.
  - All of the gains from trade have been exhausted.
  - There are no mutually advantageous trade to be made.
- A still yet important characteristic of an efficient allocation is that where the two ICs are tangent to each other in the Edgeworth Box, the  $MRS$ s of the two individuals are equal at the allocation.
- Suppose that at a certain allocation,  $MRS_A = 4$  whereas  $MRS_B = 2$ . Recall that here we define  $MRS = \partial x_2 / \partial x_1$ . Then, in this example,  $A$  is willing to acquire one unit of good 1 at the expense of 4 units of good 2. Meanwhile,  $B$  is willing to cut back the consumption of good 1 by 1 unit to obtain 2 units of good 2. A trade between the two in which  $B$  sells a unit good 1 to  $A$  in exchange for anywhere between 2 to 4 units of good 2 must make both individuals better off. A  $PO$  allocation in which the consumption of both goods are non-zero for both individuals must be where the  $MRS$ s are equal.
- If in an efficient allocation,  $A$  and/or  $B$ 's consumption of good 1 or 2 is equal to zero, however, the  $MRS$ s of the two individuals need not be equal. The same principle applies to any *first order conditions* in the discussion that follows. The equality conditions are necessary conditions for optimum only in *interior solutions* in which the choice variables concerned take on non-zero values.
- Once we think about the definitions of efficient allocations above more carefully, it should be immediately clear that there is no reason to expect that such an allocation is unique. Compare the two allocations
  - $C^A = \{E_1, E_2\}$  and  $C^B = \{0, 0\}$ ,
  - $C^A = \{0, 0\}$  and  $C^B = \{E_1, E_2\}$ .

Both should be efficient allocations and there is no reason to consider one to be more desirable than the other on efficiency grounds.

- If an efficient allocation is one where the set of allocations preferred by one individual and the set by the other individual are disjoint, any one of the allocations on the green curve in Figure 7 are efficient. The set of all efficient allocations is known as the Pareto set and the green curve in Figure 7 is known as the contract curve.

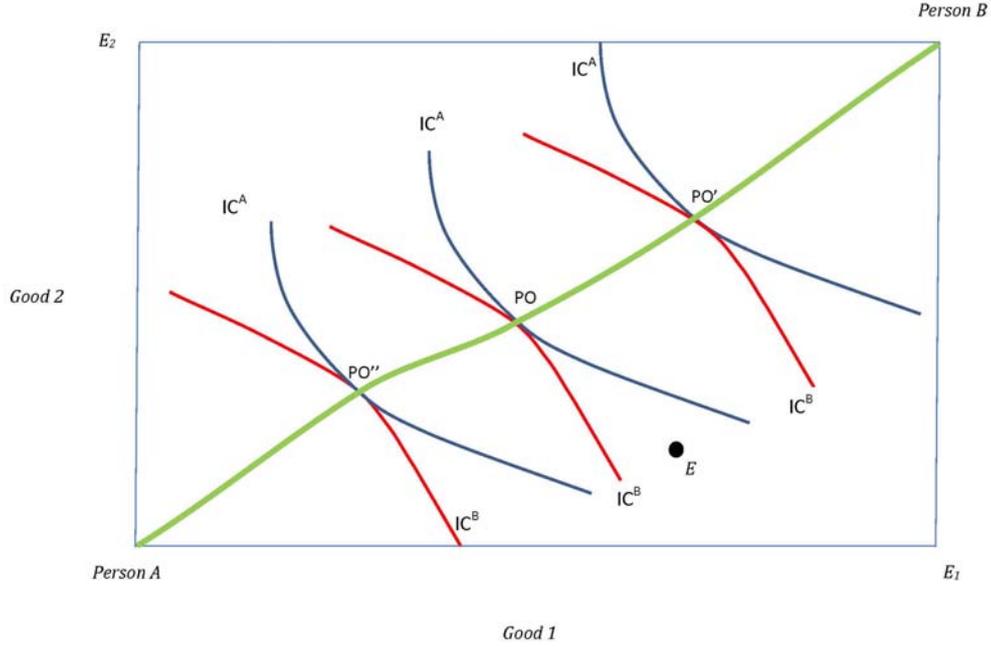


Figure 7: The Pareto set and the contract curve

## 1.2 Competitive equilibrium

- Suppose the two individuals can buy and sell as many units of the two goods as they desire in the *market* at prices  $p_1$  and  $p_2$ , respectively.
- $A$  can add to her endowment of good 1 ( $e_1^A$ ) by buying additional units in the market. Then her consumption of the good ( $c_1^A$ ) is the sum of her endowment and the amount bought. Conversely, she can sell part (or all) of her endowment of good 1. In this case,  $c_1^A$  is equal to the difference between  $A$ 's endowment and the amount sold. It is easiest if we use  $x_1^A$  to denote both (net) demand and supply, where a positive  $x_1^A$  denotes the amount bought in the market and a negative  $x_1^A$  for the amount  $A$  sells in the market. In this case,  $c_1^A = e_1^A + x_1^A$ , and that

$$p_1 c_1^A = p_1 (e_1^A + x_1^A).$$

A similar equation holds for good 2,

$$p_2 c_2^A = p_2 (e_2^A + x_2^A).$$

Summing the two equations yields

$$p_1 c_1^A + p_2 c_2^A = p_1 (e_1^A + x_1^A) + p_2 (e_2^A + x_2^A). \quad (1)$$

Now, how much  $A$  can afford to pay for the purchase of a good in the market should be constrained by how much  $A$  earns from selling the other good. That means that  $p_1x_1^A + p_2x_2^A = 0$ . Then (1) becomes

$$p_1c_1^A + p_2c_2^A = p_1e_1^A + p_2e_2^A. \quad (2)$$

This is  $A$ 's budget constraint, with the LHS denoting “consumption expenditure” and the RHS “income” derived from the sale of her endowment. The budget constraint defines the set of  $C^A = \{c_1^A, c_2^A\}$  that  $A$  can “afford”. In particular, more of one good means less of another, and that her endowment  $E^A$  must be a member of the budget set.

- Notice that in (2), proportional increases in the prices of the two goods leave no effect on the budget set. All that matters is relative price. In this connection, we can designate a good, say good 2, as the *numeraire* of the economy by *normalizing* its price to one. The budget constraint reads

$$\rho c_1^A + c_2^A = \rho e_1^A + e_2^A, \quad (3)$$

where  $\rho = p_1/p_2$  denotes the price of good 1 “relative” to the price of good 2, i.e., how many units of good 2  $A$  has to give up to obtain an additional unit of good 1. At a higher  $\rho$ , one has to pay for her purchase of good 1 with more units of good 2.

- In Figure 8, the budget constraint is a straight line with a slope equal to  $\rho$  that passes through the endowment of  $A$ . In utility maximization,  $A$  chooses the bundle at which an IC is just tangent to the budget constraint, or at where  $MRS = \rho$ . In this particular case,  $A$  sells some of her endowment of good 1 in exchange for more of good 2.
- $B$ 's utility maximization can be analyzed in an analogous manner. Then, when both individuals face the same relative price  $\rho$ , they must buy and sell up to the point where they attain the same  $MRS$  in consumption.
- In Figure 9, we depict how many units of both goods the two individuals want to buy and sell at some particular relative price  $\rho$ . Given that for each of the two good, the amount one individual tries to sell is not equal to the amount the other individual wants to buy, neither market is in equilibrium. Saying that equilibrium fails to obtain is the same as saying that the allocation  $\{C^A, C^B\}$  does not constitute a feasible allocation, given the total endowment of the two goods between  $A$  and  $B$ .
- A competitive equilibrium ( $CE$ ) allocation is a vector of prices (in case of the 2-good economy, a relative price  $\rho$ ), in which

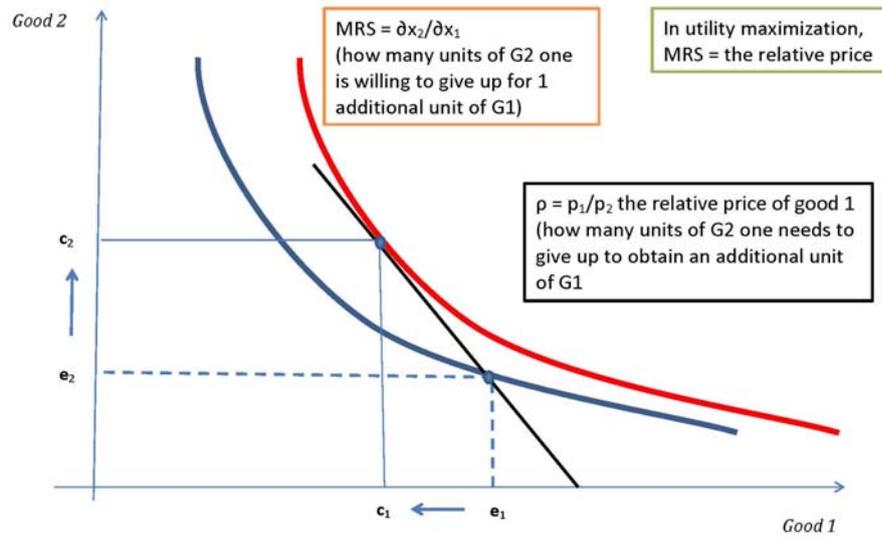


Figure 8: Utility maximization subject to the budget constraint

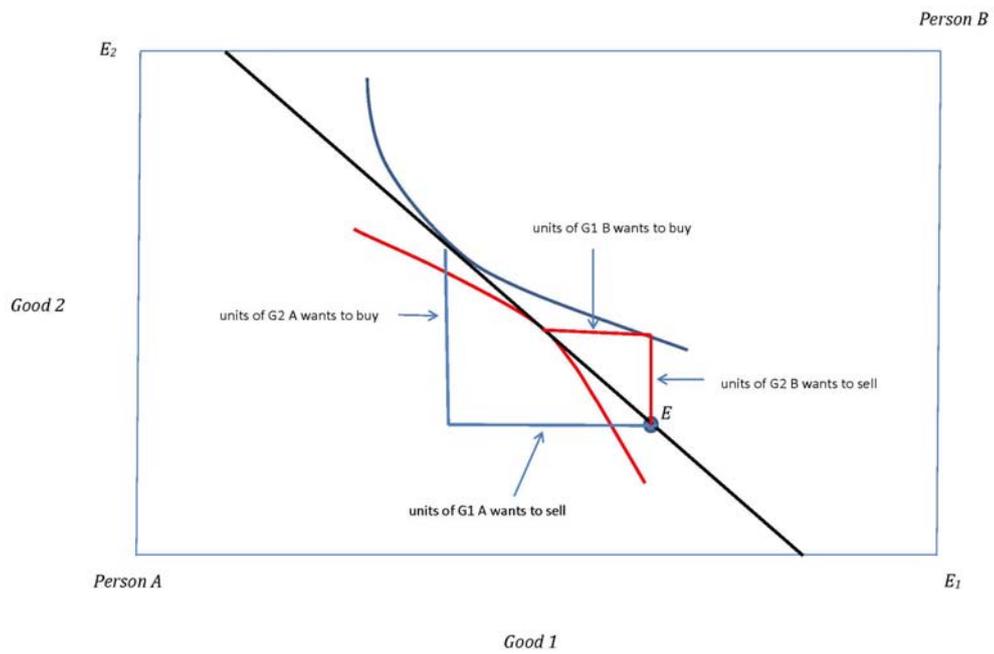


Figure 9: Non-competitive equilibrium allocation

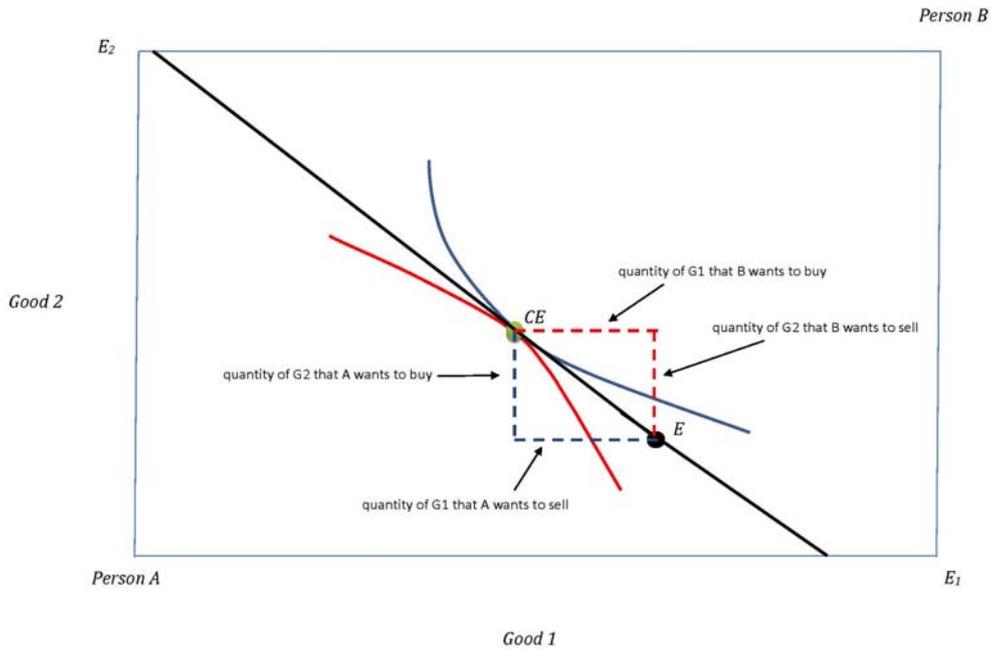


Figure 10: Competitive equilibrium

- Each individual maximizes utility subject to his/her budget constraint.
  - The demand for and the supply of each good are equal.
- Figure 10 depicts the competitive equilibrium in our 2-person-2-good economy. In utility maximization, each individual chooses to consume at where  $MRS = \rho$ . Since they face the same prices, their  $MRS$ s are equal. For the market of both goods to be in equilibrium,  $C^A$  and  $C^B$  must be located at the same point in the Edgeworth box. In other words, a  $CE$  allocation, by construction, is a feasible allocation.

### 1.3 The efficiency of competitive equilibrium

- Clearly, the allocation  $CE$  in Figure 10 is efficient, where the allocations preferred over the  $CE$  allocation by  $A$  and  $B$  are disjoint.
- **The First Welfare Theorem:** Any competitive theorem with a complete set of markets is Pareto Optimal, given local non-satiation at the  $CE$  allocation.
- The theorem is much more general than the analysis in Figure 10 for the 2-person-2-good economy with interior optimum. A general proof proceeds as

follows: First consider an arbitrary feasible allocation

$$\{D^A, D^B\} = \{\{d_1^A, d_2^A\}, \{d_1^B, d_2^B\}\}.$$

Since the allocation is feasible,

$$d_1^A + d_1^B = e_1^A + e_1^B, \quad (4)$$

$$d_2^A + d_2^B = e_2^A + e_2^B. \quad (5)$$

We continue to denote the  $CE$  allocation as  $\{C^A, C^B\} = \{\{c_1^A, c_2^A\}, \{c_1^B, c_2^B\}\}$ . Now, suppose contrary to the First Welfare Theorem,  $\{C^A, C^B\}$  is not  $PO$  because it is Pareto-dominated by the  $\{D^A, D^B\}$  allocation. That means that for example,  $A$  prefers  $D^A$  over  $C^A$  and  $D^B$  is at least as good as  $C^B$  for  $B$ . Given the  $CE$  prices, as summarized by  $\rho$ , if  $A$  actually prefers  $D^A$  over  $C^A$ ,  $A$  would have chosen  $D^A$  over  $C^A$  if not for  $D^A$  not being affordable. Hence, it must be the case that

$$\rho d_1^A + d_2^A > \rho e_1^A + e_2^A, \quad (6)$$

if  $D^A$  is preferred over  $C^A$  by  $A$ . Next, if  $D^B$  is also preferred over  $C^B$  by  $B$ , an inequality analogous to the above will hold. But  $B$  can be just indifferent between  $D^B$  and  $C^B$ . In this case, if the  $D^B$  allocation costs less than  $B$ 's income at price  $\rho$ ; i.e.,

$$\rho d_1^B + d_2^B < \rho e_1^B + e_2^B, \quad (7)$$

$C^B$  cannot be the utility-maximizing choice of  $B$  at price  $\rho$ . If  $B$  consumes  $D^B$  instead of  $C^B$ , she is just indifferent. When  $B$  chooses  $D^B$ , there is surplus purchasing power leftover, which will allow her to acquire additional units of good 1 and/or good 2. Given local non-satiation,  $C^B$  is not utility maximizing for  $B$  at price  $\rho$  if (7) holds. Thus, if  $D^B$  is at least as good as  $C^B$  for  $B$ ,

$$\rho d_1^B + d_2^B \geq \rho e_1^B + e_2^B. \quad (8)$$

To proceed, sum over the LHSs and RHSs of (6) and (8),

$$\rho (d_1^A + d_1^B) + (d_2^A + d_2^B) > \rho (e_1^A + e_1^B) + (e_2^A + e_2^B).$$

The final step is to substitute (4) and (5) into the LHS of the above,

$$\rho (e_1^A + e_1^B) + (e_2^A + e_2^B) > \rho (e_1^A + e_1^B) + (e_2^A + e_2^B).$$

This is a logical inconsistency, which implies that the presumption of the  $CE$  allocation being dominated by a given feasible allocation that begins the analysis cannot be true.

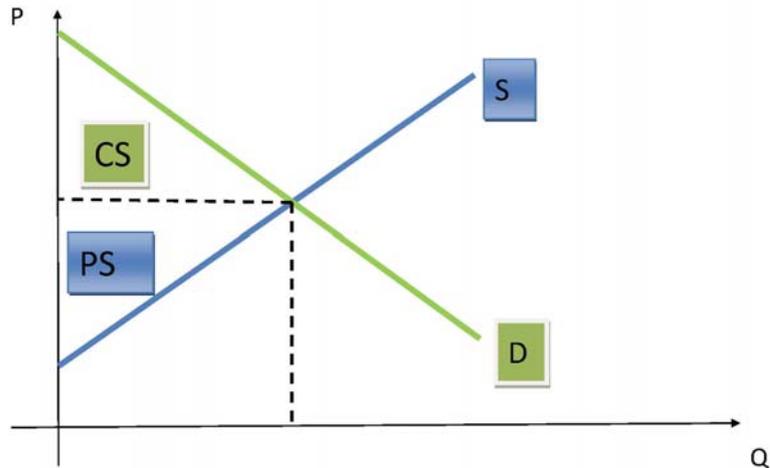


Figure 11: CS, PS, and market equilibrium

## 2 Maximizing surplus in partial equilibrium

- In the usual partial equilibrium analysis of a single market, with a downward-sloping demand curve and an upward-sloping supply curve, we argue that in equilibrium, the sum of consumer's and producer's surplus is maximized, as shown in Figure 11. Then, the market outcome is said to be efficient. How might we connect this result with the First Welfare Theorem?
- Take the market for good 1 for example.  $P$  on the vertical axis of Figure 11 is then taken as the relative price  $\rho = p_1/p_2$ . In utility maximization, both individuals should consume at where  $MRS = \rho$  holds. At a very low  $\rho$ , each consumes at the same small  $MRS$ . With diminishing marginal utility, a person's  $MRS$  is small when he or she consumes a large quantity of the good – such a large quantity that should exceed either individual's endowment of good 1. Then, both  $A$  and  $B$  are net demanders of good 1, so that the supply of the good in the market is zero. In Figure 11, at this low  $\rho$ , the quantity on the market demand curve is equal to  $c_1^A + c_1^B - e_1^A - e_1^B$ . No matter,  $\rho$  on the demand curve is equal to the common  $MRS$  of the two individuals, measuring how many units of good 2 each is willing to give up to obtain an additional unit of good 1. At a larger  $\rho$ , utility maximization calls for consuming at the same larger  $MRS$  and smaller quantities of good 1 for both individuals. The net demand declines and we are moving up along the demand curve.
- At a sufficiently large  $\rho$ , someone's utility-maximizing choice of good 1 can fall below her endowment of the good. Given that in our example, good 1

is relatively abundant in  $A$ 's endowment, the first individual to experience a negative net demand for good 1 should be  $A$ . She is still consuming at  $MRS = \rho$ , just as  $B$  does. But the quantity on the market demand curve at this point comes only from  $B$ 's net demand  $c_1^B - e_1^A$ . Meanwhile, the market supply of good 1 becomes positive and the quantity is given by  $A$ 's positive net supply  $e_1^A - c_1^A$ .

- To interpret the relationship between market price  $\rho$  and the location of the supply curve, it is useful to start with a  $\rho$  where the condition  $MRS = \rho$  holds just at  $c_1^A = e_1^A$ . In this case,  $A$ 's supply of the good to the market is equal to 0. This is the vertical intercept of the supply curve in Figure 11, which measures how many units of good 1, zero in this case, the marginal supplier  $A$  will sell. This happens because by selling a unit of good 1,  $A$  can acquire  $\rho$  units of good 2, whereas raising the consumption of good 2 by  $\rho$  units while cutting the consumption of good 1 by a unit just leaves  $A$  as well off as before since  $MRS = \rho$  to begin with.
- Next, consider a slightly larger value for  $\rho$ . If  $A$  continues to consume at  $c_1^A = e_1^A$ , her  $MRS$  remains unchanged and now falls below  $\rho$ . Of course,  $A$  is better off selling the last unit of good 1 at price  $\rho$  than to consume the unit. Indeed, she should continue to sell good 1 (in exchange for good 2) until her  $MRS$  has risen to the new higher level of  $\rho$ . When  $A$  is indifferent between selling the last unit of good 1 or keeping it for own consumption,  $MRS = \rho$ . In other words, the revenue generated from selling the last unit of good 1, in the form of  $\rho$  units of good 2, is just enough to compensate  $A$  for consuming one unit less of good 1. Price on the supply curve measures how much the marginal supplier needs to be compensated to sell the last unit. In our 2-person-2-good economy and for the market for good 1, this compensation is in units good 2, which is the numeraire good of the economy.
- In this economy, we know that around the market equilibrium,  $A$  should be a net supplier of good 1 and  $B$  a net demander. Then, the supply curve measures  $A$ 's willingness to sell good 1 for units of good 2 and the demand curve measures  $B$ 's willingness to buy good 1 at the expense of good 2. At the equilibrium quantity, the  $MRS$ s of the two are equal and efficiency attains.
- In general, the demand curve in a market measures the willingness to pay ( $MRS$ ) in units of some numeraire good of the marginal buyer and the supply curve the willingness to sell in exchange for units of the same numeraire good of the marginal seller. In an exchange economy, the willingness to sell is one and the same as the  $MRS$ . In an economy with production, this willingness to sell is the  $MC$  measured in units of the same numeraire good in which the willingness to pay is measured.

## 3 Market Failure

### 3.1 Market power

- In a *CE*, each individual buys and sells each and every good at the given market price. This is reasonable only when no one's market activity in any market is important enough to affect the market price. The free market is not expected to be competitive otherwise. In non-competitive markets, there are agents whose buy and sell orders are sufficiently sizeable to impact market prices in non-negligible manners and such agents should rationally take advantage of the market powers in their possession.
- In Figure 12, the *CE* and *PO* allocation, point *CE*, is the equilibrium market outcome only when both *A* and *B* take the market price  $\rho$  as given in arriving at their respective utility-maximizing choices. What if one individual, say *A*, is not a price-taker but rather can choose to transact with *B* at any price that she deems desirable. *B* remains a price-taker, and so whatever price *A* selects, *B* chooses her consumption bundle where the budget line is tangent to the highest IC attainable. The figure shows that with a higher  $\rho$ , *B* can only choose smaller amounts of both goods 1 and 2 (point *M*). This happens because the decline in the relative price of good 2 has a disproportionate and negative effect on her purchasing power, given that good 2 is relatively abundant in her endowment. Clearly then *B* is worse off given a higher relative price for good 1. At the same point *M*, *A* is better off however. Although the allocation *M* is not *PO* with the ICs of the two cutting each other, it is an allocation preferred by *A* over the *PO* and *CE* allocation.
- One question comes to mind: How come *A* is not choosing a bundle at which the budget constraint is just tangent to her highest IC? She should do just that in utility maximization? The answer to the question is at the heart of the way in which market power is distortionary. Utility maximization taking prices, and therefore the location of the budget constraint, as given assumes price-taking behavior. In Figure 12, as a price-maker, *A* understands that when moving away from *M* in a north-westerly direction to cut back her consumption good 1 while raising her consumption of good 2, the market supply of good 1 will increase and that the demand for good 2 increase. The relative price for good 1  $\rho$  must then decline. This decline in  $\rho$  can make *A* worse off. To sell an additional unit of good 1 in return for additional units of good 2 must involve all previous units to be transacted at the new less favorable rate of exchange. The calculations will no longer be based on the simple comparison of *MRS* and the market price for the last unit bought and sold.
- A basic result in microeconomic theory is that perfect price discrimination is not distortionary. In the Edgeworth box analysis, if *A* can practice perfect

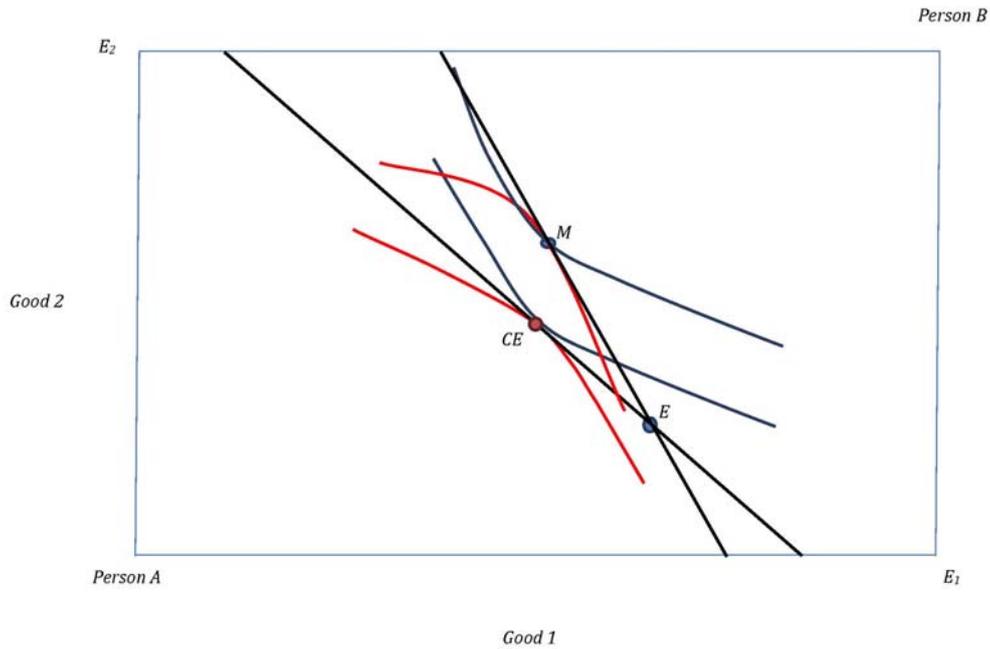


Figure 12: Monopoly price distortion

price discrimination on  $B$ , she will trade each unit of good 1 for good 2 with  $B$  at a different price, equal to  $B$ 's  $MRS$  for that particular unit, so that  $B$  is just indifferent between dealing and not dealing with  $A$  for each and every unit exchanged. In this case, a trade is profitable to  $A$  as long as  $A$ 's  $MRS$  exceeds that of  $B$ . When all profitable trading opportunities are exhausted, the two  $MRS$ s must be just equated and the allocation is  $PO$ . See Figure 13.

## 3.2 Externality

### 3.2.1 Consumption externality

- Two restaurant patrons:  $A$  and  $B$  and two commodities: money and smoke. Smoke is a good to  $A$  and a bad to  $B$ . The quantity of smoke can vary between 0 and 1, and that the same quantity must be consumed by the two persons.  $A$  and  $B$  each has an endowment of money equal to 100. In Figure 14, the quantity of smoke is plotted on the vertical axis. This quantity is the quantity that affects the welfare of both individuals.  $A$  is made better off when the quantity increases; she is moving to a higher IC. Meanwhile,  $B$  suffers; she is moving to a lower IC.
- Suppose  $A$  can freely fill the restaurant by as much smoke as possible. We say that  $A$  has the property rights to how much smoke she can fill the restaurant

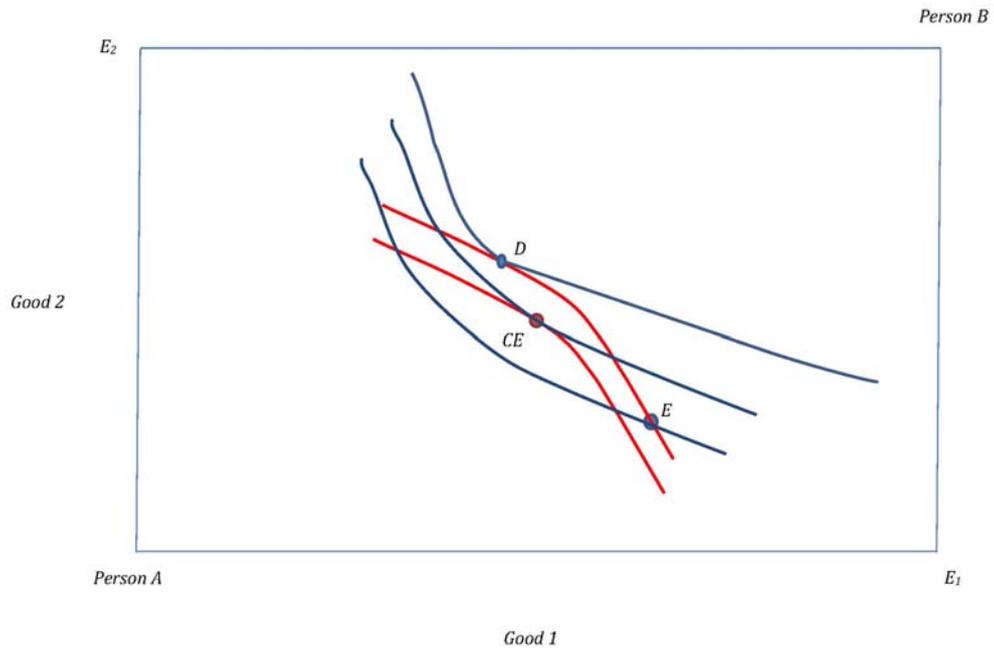


Figure 13: Perfect price discrimination

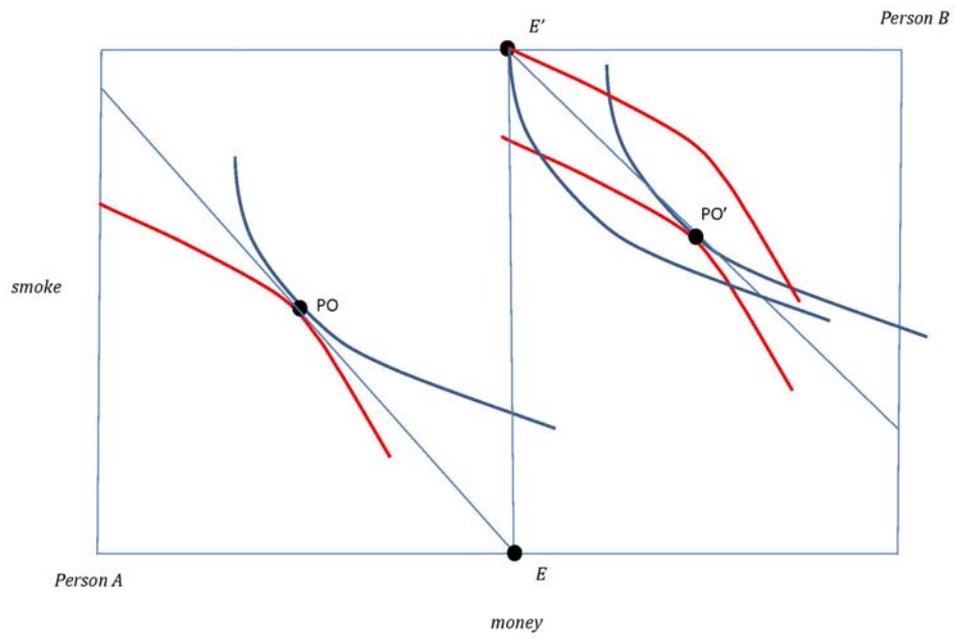


Figure 14: Consumption externality

with. Point  $E'$  denotes the initial allocation. As the ICs of the two individuals intersect each other at  $E'$ , the allocation is not efficient. An efficient allocation, point  $PO'$ , is preferred by both individuals over their respective endowments. This underscores the result that unfettered consumption externality almost always causes inefficiency.

- On the other hand, if  $B$  has the rights to a smoke-free environment, the initial allocation lies at point  $E$ , which, like point  $E'$ , is generally an inefficient allocation. In this case, the efficient allocation which is also preferred by both individuals over their respective endowments is at point  $PO$ .
- Suppose the endowment is at  $E'$ , and that there is a competitive market for smoke in which  $A$  can sell her rights to smoke in return for money while  $B$  can buy her rights to a smoke-free environment by paying money. In equilibrium, how much  $A$  sells must be equal to how much  $B$  buys. Not surprisingly, the competitive equilibrium allocation coincides exactly with the efficient allocation. This is an instance of the First Welfare Theorem. Clearly, the same conclusion is reached if the endowment is at  $E$ .
- The problem of externality then is one of missing market – externality is distortionary and causes inefficiency insofar as the market for the commodity that causes the externality does not exist.
- If any possible exchanges are merely between 2 persons, it is hard to imagine how a competitive market can arise to facilitate those exchanges. Ronald Coase, however, argued that as long as property rights are well defined, the two individuals have incentives to improve their welfare by moving away from the inefficient endowment. For example, if  $E$  represents the endowment point, Coase argues that the two individuals will bargain their way through to the efficient allocation  $PO$ . Before the allocation  $PO$  is agreed upon, gains from trade are yet to be exhausted.
- A version of the so-called Coase Theorem posits that the particular assignment of property rights is immaterial to the eventual efficient allocation to be reached, so long as the rights are clearly defined. This is evidently false in the analysis in Figure 14. The efficient allocation that will be reached is, by all means, sensitive to the assignment of rights. In Figure 15, however, all efficient allocations entail the same quantity of smoke, which means that no matter where the endowment lies, the bargaining should lead to the same quantity of smoke to be created. This happy coincidence is due to the assumption in Figure 15 that each individual's  $MRS$  is independent of the amount of money in the consumption bundle, so that the tangency of the two ICs is always at the same quantity of smoke. People often refer to this as the assumption of the absence of income effect.

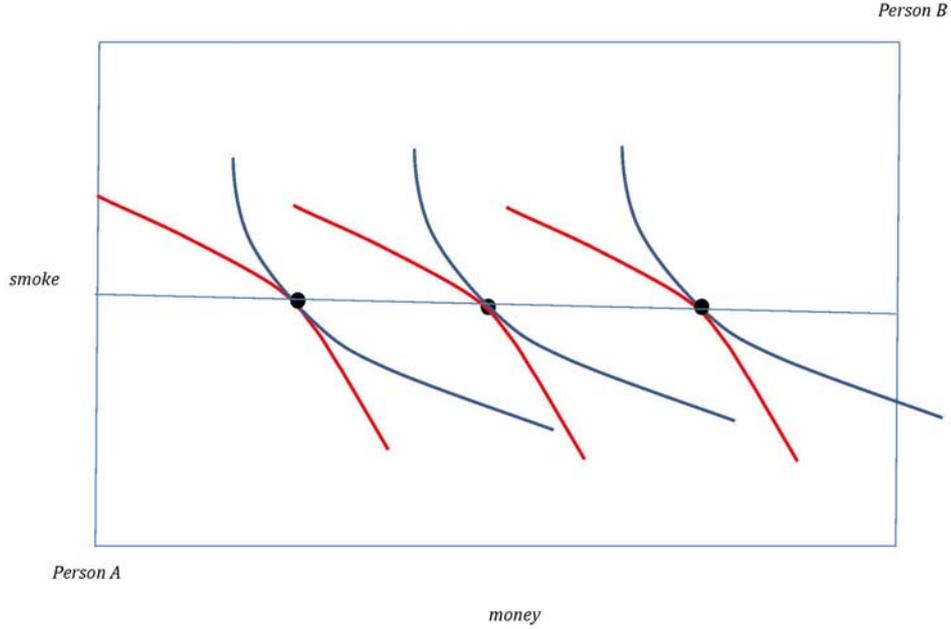


Figure 15: The Coase Theorem

### 3.2.2 Production externality

- Firm  $S$  produces steel  $s$  and a certain amount of pollution  $x$ , which it dumps into a river. Its cost function is  $c_s(s, x)$ , which increases with  $s$  but declines with  $x$ . Firm  $F$  is a fishery located downstream affected by the pollution dumped by  $S$ . The firm's cost function is  $c_f(f, x)$ , where  $f$  denotes the amount of fish caught, and that  $c_f$  is increasing in both  $f$  and  $x$ . Let  $p_s$  and  $p_f$  be the prices of steel and fish, respectively. In profit maximization,  $S$  chooses  $s$  and  $x$  to satisfy,

$$p_s = \frac{\partial c_s(s, x)}{\partial s}, \quad (9)$$

$$0 = \frac{\partial c_x(s, x)}{\partial x}, \quad (10)$$

whereas  $F$ 's profit-maximizing choice of  $f$  satisfies,

$$p_s = \frac{\partial c_f(f, x)}{\partial f}. \quad (11)$$

In maximizing profit,  $S$  will choose to dump pollution  $x$  to the river up to the point where its total cost is minimized, given its choice of  $s$ . In the interim, as  $x$  rises,  $F$ 's total cost increases and profit declines.

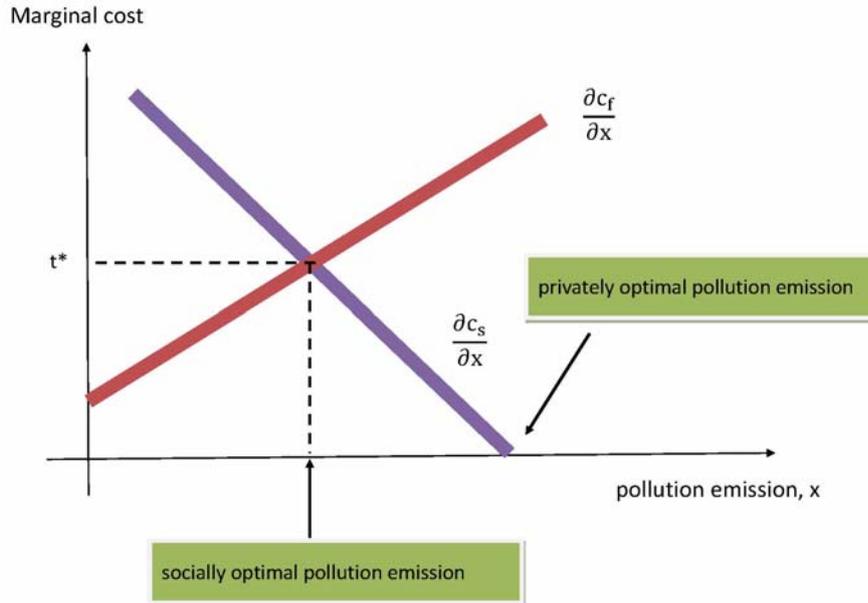


Figure 16: Private versus socially optimal pollution emission

- A production externality exists to the extent that  $F$  cares about the amount of  $x$  but has no control over its quantity, whereas  $S$ , in choosing the amount of pollution to emit, only cares about its own profit. For efficiency,  $S$  should also consider the impact of its pollution emission on  $F$ 's cost of production. Firm  $S$ 's *social cost* of production should be made up of by how much  $F$ 's cost increases as a result of its pollution emission, in addition to its private cost  $c_s(s, x)$ . In this regard, the choices in the competitive market as dictated by the conditions in (9)-(11) are generally inefficient to the extent that  $S$  ignores part of its social cost of production.
- For efficiency,  $S$  should take into account the effects of its pollution emission, not just on its own private cost, but also on the cost faced by  $F$ . That is, for efficiency,  $S$ 's choice of  $x$  should be made to satisfy

$$0 = \frac{\partial c_x(s, x)}{\partial x} + \frac{\partial c_f(f, x)}{\partial x}, \quad (12)$$

in place of (10). The first term on the RHS is typically negative, denoting by how much  $S$ 's cost falls as emission increases, whereas the second term is typically positive, denoting by how much  $F$ 's cost rises in the interim. Efficiency obtains when the cost saving enjoyed by  $S$  and the cost increase that  $F$  suffers are equal at the margin. Figure 16 illustrates.

- How might the problem of the production externality in this example be resolved? Suppose the two firms merge to form a steel mill-fishery conglomerate.

The merged firm then should choose  $s$ ,  $x$ , and  $f$  to maximize the joint profit of steel and fishery production. The profit-maximizing choice of  $x$  for the merged firm is one and the same as (12). We say that the externality is internalized.

- Absent the merger, efficiency can be restored if  $S$  were made to pay for its social cost of production by means of a pollution tax. Suppose that for each unit of  $x$  produced,  $S$  has to pay an amount  $t$  to the tax authority. Then its profit-maximizing choice of  $x$  should satisfy

$$0 = \frac{\partial c_x(s, x)}{\partial x} + t. \quad (13)$$

In this case,  $S$ 's choice of  $x$  best balances the benefit of a lower cost of production and the cost of a heavier tax burden. But what is the appropriate level of  $t$ ? Clearly, the optimal *Pigouvian* tax is given by

$$t = \frac{\partial c_f(f, x)}{\partial x};$$

i.e., the externality that it imposes on  $F$ . The practical difficulty of implementing the optimal Pigouvian tax is that one has to know what the optimal level of pollution emission is. If that is known, a perhaps simpler alternative is just to regulate  $S$  to pollute that much.

- Of course, at a deeper level, the problem of production externality, like the problem of consumption externality, can be thought of as the problem of missing market. Think of a market in which  $S$  can sell its rights to pollute at a given price  $q$ , whereas  $F$  can buy the rights to a pollution-free river at the same price  $q$ . When the market clears, optimal pollution emission follows. In this argument, we assume  $S$  possesses the property rights to pollute. Conversely, the same market mechanism works just fine if it is  $F$  that possesses the rights to a clean environment and may choose to sell part of its endowment away at some market price, whereas  $S$  has access to the same market to buy the rights to pollute. The different property rights assignment should result in the same efficient pollution emission as the Coase Theorem should apply where the marginal cost and benefit of pollution does not depend on the “wealth” positions of the two players.

## 4 Income redistribution

- In Figure 17, if the endowments of the  $A$  and  $B$  are located at  $E$ , the equilibrium and efficient allocation that would be reached is at point  $PO$ . Efficient notwithstanding, the allocation can be deemed to be unjust with  $A$ 's consumption of both goods at substantially higher levels than those of  $B$ . In general,

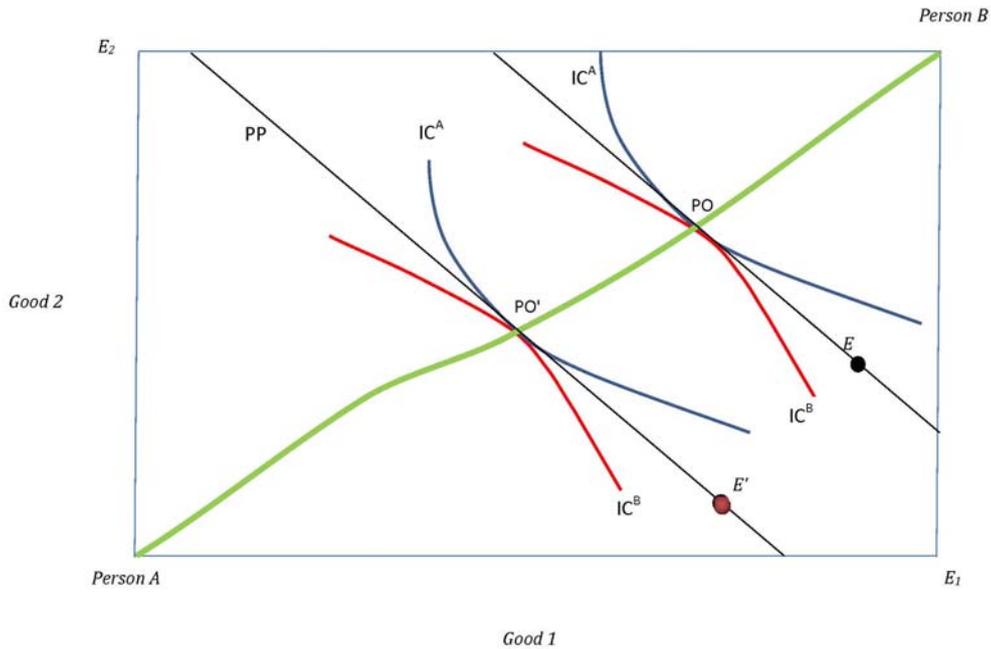


Figure 17: The Second Welfare Theorem

an efficient allocation in the free market can be rather inequitable with few individuals amassing disproportionate amounts of the real income (purchasing power) available. A role for the government to intervene in the free market to redistribute income can be justified on equity grounds.

- An alternative efficient allocation in Figure 17, at point  $PO'$ , may be considered more desirable than the free market outcome  $PO$  on equity grounds. Now, if the government can redistribute the endowments of the two goods among  $A$  and  $B$  to any point on the line labelled  $PP$ , say point  $E'$ , the free market, via competition, will reach precisely the efficient and arguably more equitable allocation  $PO'$ .
- **The Second Welfare Theorem:** Any Pareto optimal allocation can be supported as a competitive equilibrium allocation via some appropriate redistribution of endowments if people's preferences are convex.
- The theorem says that the goals of efficiency and equity need not be contradictory in principle. Society can reach efficient as well as equitable outcomes via the market mechanism as long as the government can tax people's endowment and redistribute the purchasing power appropriately.
- The more subtle problem is that taxing endowments may not be as straight-

forward as it sounds. In reality, taxes are typically imposed on people's market activities. Income tax is based on one's labor supply but not on labor endowment. Profit tax depends on the amounts of factor inputs the owners choose to supply firms but not on the factor endowments of the owners. That is, in reality, taxes are almost always proportional to factor incomes, rather than lump-sum. Taxes that come closest to true taxes on endowments in reality are compulsory military services. Such a tax, however, can serve no meaningful income redistribution purpose. Non-lump-sum taxes, such as labor income and profit taxes, can certainly serve to redistribute income. The problem is that such taxes, insofar as they add to or subtract from the prices at which people transact in the market, often drive wedges among people's *MRS*s and therefore are almost always distortionary. In practice, more often than not, a more equitable allocation than the prevailing free market outcome may only be attained by compromising efficiency to a certain extent.