Gifted Education School Network 2022/23
KLA/ Cluster: Secondary Mathematics
Lesson Design

Acknowledgement: This lesson example was adapted/adopted from the tryout by Mr Julian Tam of Yan Chai Hospital Law Chan Chor Si College

Key Learning Area:	Mathematics Extended Part (Module 2)
Level:	Secondary 4
Topic	Application of Differentiation
Learning Objectives:	For average students: ● Students can find the maximum and minimum values of a function ● Students can solve the problems relating to rate of change, maximum and minimum For gifted/more able students: ● Students can design the package of the drink with the minimum amount of material used when the volume is given
Prior Knowledge of students:	● find the derivatives of functions involving algebraic functions, trigonometric functions, exponential functions and logarithmic functions ● find the second derivative of an explicit function
Highlights of this lesson example:	● This lesson example demonstrates the merits of exploratory task as a learning tool. ● This lesson example gives the students an opportunity to apply the mathematical knowledge. ● The less able students start analysing the problem with simple situation while; ● the gifted/more able students can learn deeper through the designing activities.
Strategies employed:	In order to cater for the specific learning needs of students in a mixed ability class, differentiation strategies could be employed to maximise the learning outcomes of students. Differentiated instruction helps less able students to get ready for the lesson and gives challenges to the gifted/more able students.

Flow of Learning Activities	Rationale and Tips for Implementation
1) Investigation ● Ask the students to measure the dimensions of a packet of XX Lemon Tea (250 ml). The answers (approximate) are listed as follows: Length = ____________________ cm Height = ____________________ cm Width = ____________________ cm	✧ Ask the students why the dimensions must be in this ratio. ✧ The possible answer is that the manufacturer tries to minimize the total surface area under the constraint that the volume must be constant.
2) Development ● Let the length, height and width be y cm, $1.6y$ cm and x cm respectively.	✧ The ratio $\dfrac{\text{height}}{\text{length}} = 1.618033989...$ which is the golden ratio ✧ For the sake of simplicity, we use 1.6 as an approximated value.
3) Exploration ● From the above, we have $y \times 1.6y \times x = 250$. (*) ● Using (*), prove that $\text{length} = \dfrac{25}{2\sqrt{x}}$ cm, $\text{height} = \dfrac{20}{\sqrt{x}}$ cm and $\text{width} = x$ cm.	**For gifted/more able students:** ✧ Teacher can raise problems like the following to arouse the awareness of the linkage between mathematics and daily life: ➢ As the dimensions of the rectangle in the front view (height and length) are in golden ratio, why those in the side view (length and width) are not in golden ratio? ➢ If the width is independent of the length and height, what does it depend on?
4) Consolidation and Stretching the potential ● Let the total area be A cm^2. (a) Prove that $A = 2 \times \left(\dfrac{25}{2\sqrt{x}} + x\right)\left(\dfrac{20}{\sqrt{x}} + x\right)$.	✧ Ask the students to derive these expressions. ✧ This may be a little bit difficult as the total area is not just the sum of the area of the six faces. You had better unfold the packet of lemon tea in the class and the students will understand immediately once they see the nets of the packet.

(b) Prove that $A = 2x^2 + 65\sqrt{x} + \frac{500}{x}$.	
5) Making conclusion and extending the problem ● Ask the students to find $\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\left(2x^2 + 65\sqrt{x} + \frac{500}{x}\right)$ and use differentiation to find the minimum value of A and determine the corresponding dimensions of the packet of lemon tea. Make sure you check the answers by means of the first derivative test or the second derivative test.	**For average students:** ✧ Complete the first derivative test ✧ When students are solving for the roots of the first derivative, they will encounter the following equation: $8x^3 + 65x^{\frac{3}{2}} - 1000 = 0$. Teachers may give hint for students to transform it into a simple quadratic equation. **For gifted/more able students:** ✧ Teachers may encourage them to apply the second derivative test to check for the extremity. ✧ Teachers may ask them to design another package of the dimensions being another ratio or of a shape other than cuboid such that the total surface area is minimum when the volume is given.