Yan Chai Hospital Law Chan Chor Si College

Name: ____________________ (   )
Class: ____________________

XX Lemon Tea (250 ml)

This is an interesting problem which demonstrates the power of Mathematics in the **daily life applications**. This shows that Mathematics is not just textbook exercise.

Step 1

Ask the students to measure the dimensions of a packet of XX Lemon Tea (250 ml).
The answers (approximate) are listed as follows:

Length = ____________________ cm
Height = ____________________ cm
Width = ____________________ cm

Step 2

Ask the students why the dimensions must be ______________________________.
The possible answer is that the manufacturer tries to
minimize ____ under the **constraint that the volume must be** ______________________________.

Step 3

Let the length, height and width be y cm, $1.6y$ cm and x cm respectively.

(Remarks: The ratio $\dfrac{\text{height}}{\text{length}} = 1.618033989...$ which is the golden ratio)

For the sake of simplicity, we use 1.6 as an approximated value.

Step 4

From 3, we have $y \times 1.6y \times x = 250$. (*)

Using (*), prove that $\text{length} = \dfrac{25}{2\sqrt{x}}$ cm, $\text{height} = \dfrac{20}{\sqrt{x}}$ cm and $\text{width} = x$ cm.

Step 5

Let the total area be $A\text{ cm}^2$.

(a) Prove that $A = 2\times\left(\dfrac{25}{2\sqrt{x}}+x\right)\left(\dfrac{20}{\sqrt{x}}+x\right)$.

(b) Prove that $A = 2x^2 + 65\sqrt{x} + \dfrac{500}{x}$.

Step 6

Ask the students to find $\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\left(2x^2 + 65\sqrt{x} + \frac{500}{x}\right)$ and use differentiation to find the minimum value of A and determine the corresponding dimensions of the packet of lemon tea. Make sure you check the answers by means of the first derivative test or the second derivative test.