Application of Mathematics in STEAM

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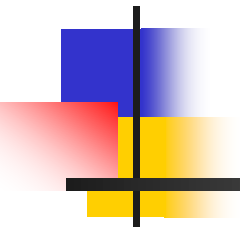
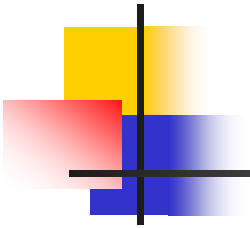


This is an interesting problem which demonstrates the power of Mathematics in daily life applications. This shows that Mathematics is not just textbook exercise.



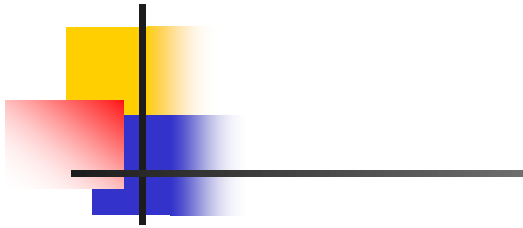
Our aim is to derive the dimensions of the packet of lemon tea by minimizing the total surface area and assuming that height/length is the golden ratio.

Step 1

Ask the students to measure the dimensions of a packet of Vita Lemon Tea (250 ml).

The answers (approximate) are listed as follows:

Length = _____ cm
Height = _____ cm
Width = _____ cm



Step 2

Ask the students why the dimensions must be _____.
The possible answer is that the manufacturer tries to
minimize _____ under the **constraint that the volume must be** _____.

Step 3

Let the length, height and width be y cm, $1.6y$ cm and x cm respectively.

(Remarks: The ratio $\frac{\text{height}}{\text{length}} = 1.6$ which is the golden ratio)

(Remarks: The ratio $\frac{\text{height}}{\text{length}} = 1.618033989...$ which is the golden ratio)

For the sake of simplicity, we use 1.6 as an approximated value.

Step 4

From 3, we have $y \times 1.6y \times x = 250$. (*)

Using (*), prove that length $= \dfrac{25}{2\sqrt{x}}$ cm , height $= \dfrac{20}{\sqrt{x}}$ cm and width $= x$ cm .


BEST BEFORE
DD.MM.YY
TX22 037192
11.02.24
此日期前最佳
日.月.年
X
2
Y
1.6Y
每100毫升
Per 100mL
55千卡/kcal
0克/g
0克/g
0克/g
0克/g
13.6克/g
13.6克/g
10毫克/mg
250mL(毫升)
4 891028 164456
FSC

Step 5

Let the total area be $A\,\text{cm}^2$.

(a) Prove that $A = 2\times\left(\frac{25}{2\sqrt{x}}+x\right)\left(\frac{20}{\sqrt{x}}+x\right)$.

(b) Prove that $A = 2x^2 + 65\sqrt{x} + \frac{500}{x}$.

Ask the students to derive this expression. This may be a little bit difficult as the total area is not just the sum of the area of the six faces. You had better unfold the packet of lemon tea in the class and the students will understand immediately once they see the real thing.

Step 6

Ask the students to find $\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\left(2x^2 + 65\sqrt{x} + \frac{500}{x}\right)$ and use differentiation to find the minimum value of A and determine the corresponding dimensions of the packet of lemon tea. Make sure you check the answers by means of the first derivative test or the second derivative test.

$$\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\left(2x^2 + 65\sqrt{x} + \frac{500}{x}\right) = 4x + \frac{65}{2}x^{\frac{-1}{2}} - \frac{500}{x^2} = 0$$

$$4x^3 + \frac{65}{2}x^{\frac{3}{2}} - 500 = 0$$

$$4(x^{\frac{3}{2}})^2 + \frac{65}{2}x^{\frac{3}{2}} - 500 = 0$$

$$x^{\frac{3}{2}} = 7.833041444$$

$$x = 3.944151753$$


$\frac{d^2A}{dx^2} = 4 - \frac{65}{4}x^{\frac{-3}{2}} + \frac{1000}{x^3}$

Further Topics for investigation