校本資優教育：
資優教育學校網絡 2023/24
經驗分享會（中學）（新辦）

數學教育:在第一層全班式教學中滲入資優教育三元素 – 數列和畢氏定理

校本經驗分享

保良局蔡繼有學校

李澤林老師

20.06.2024

分享內容

2023-2024資優網絡學校計劃的課堂設計及實施

1)中三級數列

2)中一級畢氏定理

2023-2024資優網絡學校計劃的課堂設計及實施

https://www.edb.gov.hk/tc/curriculum-development/curriculum-area/gifted/resources_and_support/school_network/index.html

課堂一:中三級數列

- Lesson objective:

Learn how to find the nth term of linear, quadratic, cubic and geometric sequences

學生實例:

探究學習,小組互相合作和討論

Po Leung Kuk Choi Kai Yau School	Name: ( )
Date:	Class:
Investigation worksheet 01 (Y9 IMaths)	Unit: Sequences

Investigation worksheet 01

Aim:

- To investigate how to work out the general term of linear sequence.

Part 1: Linear sequences

All linear sequences have the general term of $u_n = an + b$.

Consider the linear sequence $u_n = 3n + 2$.

The first few terms of this sequence are:

$u_1 = 3(1) + 2 = 5$
$u_2 = 3(2) + 2 = 8$
$u_3 = 3(3) + 2 = 11$
$u_4 = 3(4) + 2 = 14$
$u_5 = 3(5) + 2 = 17$

n	1	2	3	4	5
u_n	5	8	11	14	17

We can construct a **difference table** to display the sequence like the following, and we include a row for the first difference $\Delta 1$ which is the difference between successive terms of the sequence.

n	1	2	3	4	5
u_n	5	8	11	14	17
$\Delta 1$	3	3	3	3	

The first difference $\Delta 1$ of this sequence $u_n = 3n + 2$ is **3**.

If we compare to the general term $u_n = an + b$, the "a" value is **3**.

Setter: Mr. F LI Page 1

1. For each of the linear sequence below, complete the difference table and fill in the blanks:

(a) $u_n = 6n + 1$
$u_1 = 6(1) + 1 = 7$
$u_2 = 6(2) + 1 = 13$
$u_3 = 6(3) + 1 = 19$
$u_4 = 6(4) + 1 = 25$
$u_5 = 6(5) + 1 = 31$

n	1	2	3	4	5
u_n	7	13	19	25	31
$\Delta 1$	+6	+6	+6	+6	

The first difference $\Delta 1$ of this sequence $u_n = 6n + 1$ is +6.

If we compare this sequence and general term $u_n = an + b$, the "a" value is 6.

(b) $u_n = -4n + 5$
$u_1 = -4(1) + 5 = 1$
$u_2 = -4(2) + 5 = -3$
$u_3 = -4(3) + 5 = -7$
$u_4 = -4(4) + 5 = -11$
$u_5 = -4(5) + 5 = -15$

n	1	2	3	4	5
u_n	1	-3	-7	-11	-4
$\Delta 1$	-4	-4	-4	-4	

The first difference $\Delta 1$ of this sequence $u_n = -4n + 5$ is -4.

If we compare this sequence and general term $u_n = an + b$, the "a" value is -4.

2. **(i)** For linear sequences above, what did you notice about the first difference $\Delta 1$ row?

The first difference is the same ~~with~~ as differences between each term.

(ii) What did you notice about the value of the first difference $\Delta 1$ and the "a" value of the general term?

the values are the same

Setter: Mr. F LI Page 2

3. **(a)** Complete the following difference table for the sequence 6, 13, 20, 27, 34, …

n	1	2	3	4	5
u_n	6	13	20	27	34
$\Delta 1$	+7	+7	+7	+7	

(i) Is 6, 13, 20, 27, 34, … a linear sequence? Explain your reason.

Yes,

(ii) For the general term $u_n = an + b$ of this sequence, what is the "a" value?

$u_n = 7n + b$

(iii) **Using your result from previous part** and use the given fact $n = 1, u_n = 6$, work out the value of b.

$7(1) + b = 6$
$\therefore b = -1$

(iv) The general term of this sequence 6, 13, 20, 27, 34, … is:

$u_n = 7n - 1$

(b) **(i)** Complete the following difference table for sequence 40, 37, 34, 31, 28, ...

n	1	2	3	4	5
u_n	40	37	34	31	28
$\Delta 1$	-3	-3	-3	-3	

(ii) Work out the general term using the difference table above.

$U_n = -3n + b$
$-3(1) + b = 40$ ← substitution
$-3 + b = 40$
$b = 43$
$\therefore U_n = -3n + 43.$

Setter: Mr. F LI Page 3

4. **(a)** Is 4, 9, 18, 31, 48, ... a linear sequence?

No, because the difference between each term is different.

You may use the difference table to help you:

n	1	2	3	4	5
u_n	4	9	18	31	48
Δ1	+5	+9	+13	+17	
	+4	+4	+4		

學生匯報：挑戰學生能否解釋到背後的原因，評論和分析

(b) Find the next two terms of the sequence.

4, 9, 18, 31, 48, 69, 94

Po Leung Kuk Choi Kai Yau School	Name: ( )
Date:	Class:
Investigation worksheet 02 (Y9 IMaths)	Unit: Sequences

Investigation worksheet 02

Aim:

- To investigate how to work out the general term of quadratic and cubic sequences.

Part 2: Quadratic sequences

All quadratic sequences have the general term of $u_n = an^2 + bn + c$.

Consider the quadratic sequence $u_n = 2n^2 - n + 3$.

The first few terms of this sequence are:

$u_1 = 2(1)^2 - (1) + 3 = 4$
$u_2 = 2(2)^2 - (2) + 3 = 9$
$u_3 = 2(3)^2 - (3) + 3 = 18$
$u_4 = 31$
$u_5 = 48$
$u_6 = 69$

Again, we can construct a **difference table** to displace the sequence, and this time we include an extra row for the for the **second difference Δ2** which is the difference between consecutive terms of the first difference row.

n	1	2	3	4	5	6
u_n	4	9	18	31	48	69
Δ1	5	9	13	17	21	
Δ2	4	4	4	4		

The **second difference Δ2** of this sequence $u_n = 2n^2 - n + 3$ is **4**.

If we compare to the general term $u_n = an^2 + bn + c$, the "a" value is **2**.

5. For each of the linear sequence below, complete the difference table and fill in the blanks:

(a) $u_n = n^2 + 3n + 3$

$u_1 = (1)^2 + 3(1) + 3 = 7$ $\quad u_4 = (4)^2 + 3(4) + 3 =$ _____

$u_2 = (2)^2 + 3(2) + 3 = 13$ $\quad u_5 =$ _____

$u_3 = (3)^2 + 3(3) + 3 = 21$ $\quad u_6 =$ _____

n	1	2	3	4	5	6
u_n	7	13	21			
Δ1						
Δ2						

The **second difference Δ2** of this sequence $u_n = n^2 + 3n + 3$ is _____.

If we compare this sequence and general term $u_n = an^2 + bn + c$, the "a" value is _____.

(b) $u_n = -3n^2 + 2n - 7$

n	1	2	3	4	5	6
u_n						
Δ1						
Δ2						

The **second difference Δ2** of this sequence $u_n = -3n^2 + 2n - 7$ is _____.

If we compare this sequence and general term $u_n = an^2 + bn + c$, the "a" value is _____.

6. **(i)** For quadratic sequences above, what did you notice about the **second difference Δ2** row?

(ii) What did you notice about the value of the **second difference Δ2** and the "a" value of the general term?

Scaffolding 越來越少

7. **(a)** Complete the following difference table for the sequence 2, 0, 0, 2, 6, …

n	1	2	3	4	5
u_n	2	0	0	2	6
Δ1		−2	0	+2	+4
Δ2		+2	+2	+2	

(i) Is 2, 0, 0, 2, 6, … a quadratic sequence? Explain your reason.

Yes, because the second differences are the same.

(ii) For the general term $u_n = an^2 + bn + c$ of this sequence, what is the "a" value?

$u_n =$ 1 $n^2 + bn + c$

(iii) Using your value of a and work out the value of b and of c.

$\begin{cases} 2 = (1)^2 + b(1) + c \quad \text{— (1)} \\ 2 = (4)^2 + b(4) + c \quad \text{— (2)} \end{cases}$

From (1), $2 = 1 + b + c$

$b + c = 1$

$c = 1 - b$ — (3)

From (2), $2 = 16 + 4b + c$

$4b + c = -14$ — (4)

Sub (3) into (4),

$4b + 1 - b = -14$

$3b = -15$

$b = -5$

sub $b = -5$ into (3)

$c = 1 - (-5)$

$= 6$

$\therefore b = -5,\ c = 6$

(iv) The general term of this sequence 2, 0, 0, 2, 6, … is:

$u_n =$ $n^2 - 5b + 6$

8. **Use the same method as Q7, find the general term of the following sequence:**

(a) −5, 4, 19, 40, 67, …

n	1	2	3	4	5
u_n	-5	4	19	40	67
Δ1		+9	+15	+21	+27
Δ2		+6	+6	+6	

$u_n = 3n^2 + bn + c$

$\begin{cases} -5 = 3 + b + c \quad \text{— (1)} \\ 4 = 12 + 2b + c \quad \text{— (2)} \end{cases}$

From (1), $b + c = -8$

$c = -8 - b$ — (3)

From (2), $2b + c = -8$ — (4)

sub (3) into (4),

$2b - 8 - b = -8$

$b = 0$

sub $b = 0$ into (3),

$c = -8$

$\therefore b = 0,\ c = -8$

$\therefore u_n = 3n^2 - 8$

(b) −2.5, −3, −4.5, −7, −10.5, …

n	1	2	3	4	5
u_n	-2.5	-3	-4.5	-7	-10.5
Δ1		-0.5	-1.5	-2.5	-3.5
Δ2		-1	-1	-1	

$u_n = -\frac{1}{2}n^2 + bn + c$

$\begin{cases} -\frac{5}{2} = -\frac{1}{2} + b + c \quad \text{— (1)} \\ -3 = -2 + 2b + c \quad \text{— (2)} \end{cases}$

From (1), $-2 = b + c$

$c = -2 - b$ — (3)

From (2), $-1 = 2b + c$ — (4)

sub (3) into (4),

$-1 = 2b - 2 - b$

$1 = b$

$b = 1$

sub $b = 1$ into (3)

$c = -2 - 1$

$= -3$

$\therefore b = 1,\ c = -3$

$\therefore u_n = -\frac{1}{2}n^2 + n - 3$

Part 3: Cubic sequences

All cubic sequences have the general term of $u_n = an^3 + bn^2 + cn + d$.

9. Use the difference tables below for cubic sequences $u_n = 3n^3 - 7n^2 + n$, $u_n = n^3 + 2n^2 - 3n + 4$ and $u_n = -2n^3 + 5n - 3$, and answer the following questions:

(i) From the difference table, what can you conclude about cubic sequence? Δ3 are the same.

(ii) How can we find out the "a" value of the general term $u_n = an^3 + bn^2 + cn + d$ using the difference table? Is there any relationship with the "difference"? Substitude numbers (4 terms)

$u_n = 3n^3 - 7n^2 + n$

n	1	2	3	4	5	6
u_n	-3	-2	21	84	205	402
Δ1		+1	+23	+63	+121	+197
Δ2		+22	+40	+58	+76	
Δ3			18	18	18	

$u_n = n^3 + 2n^2 - 3n + 4$

n	1	2	3	4	5	6
u_n	4	14	40	88	164	274
Δ1		+10	+26	+48	+76	+110
Δ2		16	22	28	34	
Δ3			6	6	6	

$u_n = -2n^3 + 5n - 3$

n	1	2	3	4	5	6
u_n	0	-9	-42	-111	-228	-405
Δ1		-9	-33	-69	-117	-177
Δ2		-24	-36	-48	-60	
Δ3			-12	-12	-12	

學生後期自己發現有更快的方法

10. Find the general term for the sequence:

15, 27, 35, 33, 15, ... ∴ cubic sequence.

n	1	2	3	4	5
u_n	15	27	35	33	15
Δ1	+12	+8	−2	−18	
Δ2	−4	−10	−16		
Δ3	−6	−6			

smart way

$6a = -6$

$a = -1$

$12(-1) + 2b = -4$

$-12 + 2b = -4$

$2b = 8$

$b = 4$

$-7 + 12 + c = 12$

$c = 7$

$-1 + 4 + 7 + d = 15$

$d = 5$

$\therefore u_n = -n^3 + 4n^2 + 7n + 5.$

$u_n = -n^3 + bn^2 + cn + d$

$-1 + b + c + d = 15$

$b + c + d = 16$

$-8 + 4b + 2c + d = 27$

$4b + 2c + d = 35$

$-27 + 9b + 3c + d = 35$

$9b + 3c + d = 62$

$b + c + d = 16$

$-\ 4b + 2c + d = 35$

$-\ 9b + 3c + d = 62$

$-12b - 4c = -81$ ③

Sub ④ into ③,

$-12\left(\frac{35}{6} - \frac{2}{9}c\right) - 4c = -81$

$-70 + \frac{8}{3}c - 4c = -81$

$c = \frac{33}{4}$

$b = 4$

$d = \frac{15}{4}$

$-64 + 16b + 4c + d = 33$

$16b + 4c + d = 97$ — ①

$-125 + 25b + 5c + d = 15$

$25b + 5c + d = 140$ — ②

$9b + 3c + d = 62$ — ③

① − ② − ③,

$-18b - 4c = -103$ — ④

$b = \frac{\cancel{-}103 + 4c}{\cancel{-}18}$

$b = \frac{35}{6} - \frac{2}{9}c$

$\left(\frac{8}{3} - 4\right)c = -11$

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思考題

Extension

11.

For quartic sequence $u_n = an^4 + bn^3 + cn^2 + dn + e$, what do you think the relationship will be for the "a" value and the fourth difference?

The fourth difference will be ~~the same~~ Δ4 divided by

12. Did you notice any shortcut when using "Difference method" to work out the general term? E.g. Do we always have to draw and complete the whole table out?

13. The method introduces in this worksheet is one of the ways to work out the general term of a linear/quadratic/cubic sequence. Can you think of any alternative way?

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Nth Term of Cubic Sequence

We know that a given sequence is a cubic sequence if the third difference is a constant.

The nth term of the cubic sequence will be of the form:

$$an^3 + bn^2 + cn + d$$

We can derive the formulas for a, b, c and d

	n = 1	n = 2	n = 3	n = 4
Term	$\boxed{a+b+c+d}$	$8a+4b+2c+d$	$27a+9b+3c+d$	$64a+16b+4c+d$
1st Difference	$\boxed{7a+3b+c}$	$19a+5b+c$	$37a+7b+c$	
2nd Difference	$\boxed{12a+2b}$	$18a+2b$		
3rd Difference	$\boxed{6a}$			

$6a$ = 3rd difference

$12a+2b$ = 1st term of 2nd difference

$7a+3b+c$ = 2nd term – 1st term

$a+b+c+d$ = 1st term

Adapted from: https://www.onlinemathlearning.com/quadratic-sequences.html

學生創作: 每組學生創作一個數列, 讓其他組找出nth term (迷你比賽)

學生實例:

1. 76, 149, 223, 299, 377

2. 2.5, 3.75, 5.625, 8.4375

3. 1.5, 8.5, 22.5, 43.5, 71.5

1, 7, 20, 39, 64, 95, 132, 175

32128.0273, 384348.63, 286016.1771, 508182.8472, 793848.6125

讓學生分享創作的過程,給特別讚揚

FFFFFCBA

~~[illegible]~~, 620, 2658, 6B6C, E366

Oh no...

學生實例:

1. $U_n = n^2 + 69n + 7$ (quadratic sequence)

2. $U_n = \frac{3}{2}n$ (linear sequence)

1. 76, 149, 223, 299, 377

2. 2.5, 3.75, 5.625, 8.4375

T_1 T_2 T_3

1 7 20 39 64 95 132 175

+7 + 13 + 19 + 25 + 31 + 37 + 43

6 6 6 6 6 6

$U_n = an^2 + bn + c$

$a = 3$

$1 = 3 + b + c$

$-1 \times \boxed{b + c = -2} = -b - c = 2$

$7 = 3(2)^2 + 2b + c$

$7 = 12 + 2b + c$

$\boxed{2b + c = -5}$

general Term

$\boxed{U_n = 3n^2 - 3n + 1}$

$U_2 = 3(2)^2 + (2)(-3) + 1$

$12 + -6 + 1$

$= \boxed{b = -3}$ $c = 1$

$b + c = -2$

$-3 + c = -2$

$c = -2 + 3 = 1$

1, 7, 20, 39, 64, 95, 132, 175

分享相關數學知識，讓數學能力高的學生能課後自行理解

(Further readings) Interesting facts about general term for:

- Linear (arithmetic) sequence

$$u_n = dn + (a_1 - d)$$

"a_1" is the first term
"d" is the common difference

- Quadratic sequence

$$u_n = \frac{d}{2}n^2 + \left(a_2 - a_1 - \frac{3d}{2}\right)n + 2a_1 - a_2 + d$$

Even more interesting:
You don't have to use "a_1" and "a_2", you can use like "a_3" and "a_4", it should work!

- Cubic sequence

u_n

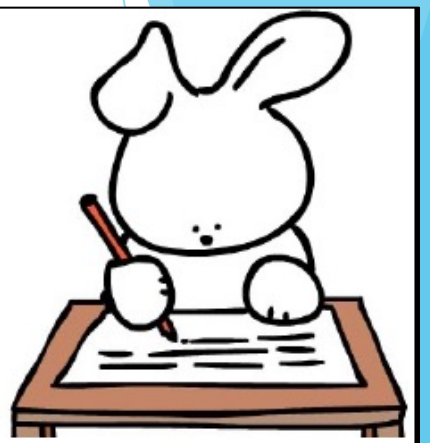
$$= \frac{d}{6}n^3 + \left(\frac{a_1 - 2a_2 + a_3}{2} - d\right)n^2 + \left(\frac{11d}{6} - \frac{5a_1 - 8a_2 + 3a_3}{2}\right)n + (3a_1 - 3a_2 + a_3 - d)$$

Adapted from: https://mathsodology.wordpress.com/2020/12/26/1-linear-quadratic-and-polynomial-sequences/

課後挑戰，有趣的家課挑戰數學能力高的學生

TAKE HOME CHALLENGE:

$$1 + 4 + 9 + 16 + 25 + \ldots + 1600$$



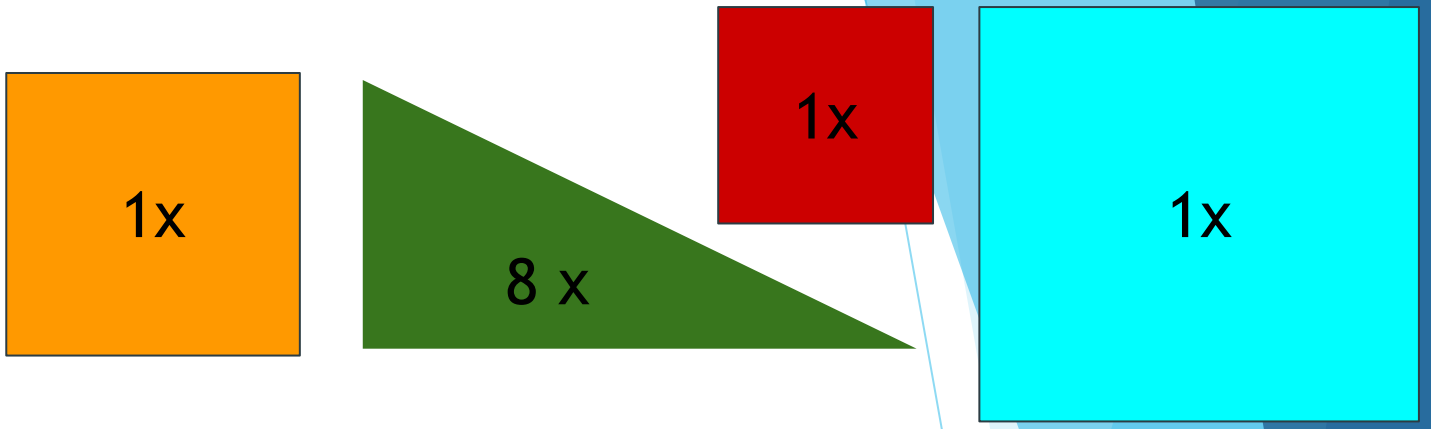
課堂二: 中一級畢氏定理

- Lesson objective:

Understand and apply Pythagoras' theorem

手動活動去引入和提升學生的興趣

Challenge 1



You will be split into groups.

Each group will be given a set of shapes.

培養團隊合作精神, 學生從教其他的同學中學習

You will be given 10 minutes, working in your groups to achieve the objective.

Your objective:

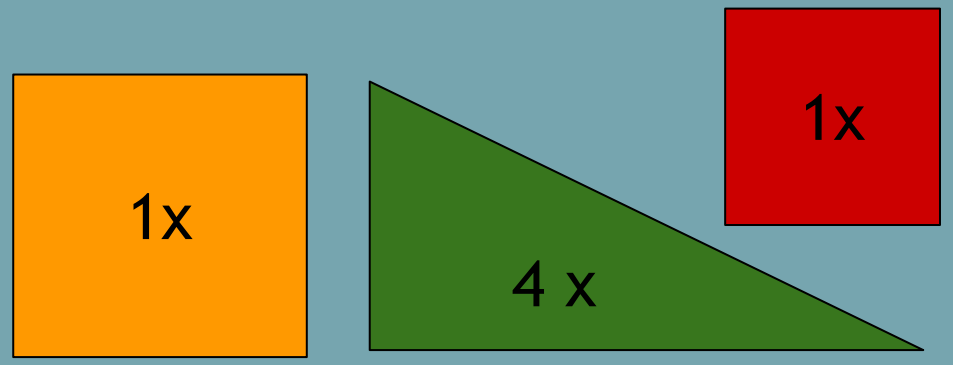
Without any overlap, and by using ALL of the shapes provided, create 2 squares. Find the relationship between the area of the 2 squares.

Without any overlap, and by using ALL of the shapes provided, create 2 squares. Find the relationship between the area of the 2 squares.

幫助學生去開始

Hint

Build the first square with:



教材與 PPT 有聯繫

學生實例:

同時讓學生去報告

$\cancel{(a+b)^2 = c^2 \times \frac{ab}{2} \times 4}$

$(a+b)^2 = (a+b)^2$

square 1:

$a^2 + b^2 + \frac{ab}{2}(4)$

$= a^2 + 2ab + b^2$

square 2:

$(a+b)^2$

$= a^2 + 2ab + b^2$

The area of

$\therefore$ square 1 = square 2.

$\cancel{a \times b}$

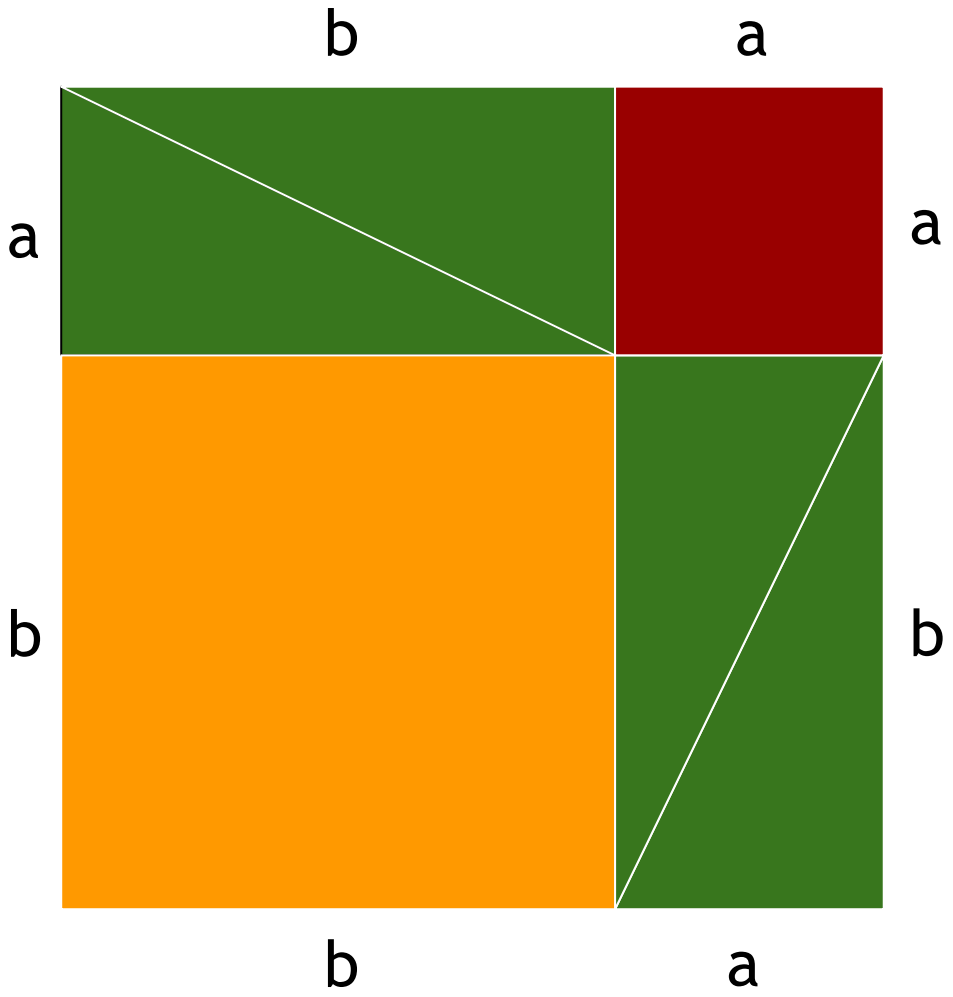
$\cancel{(ab)^2 = (ab)^2}$

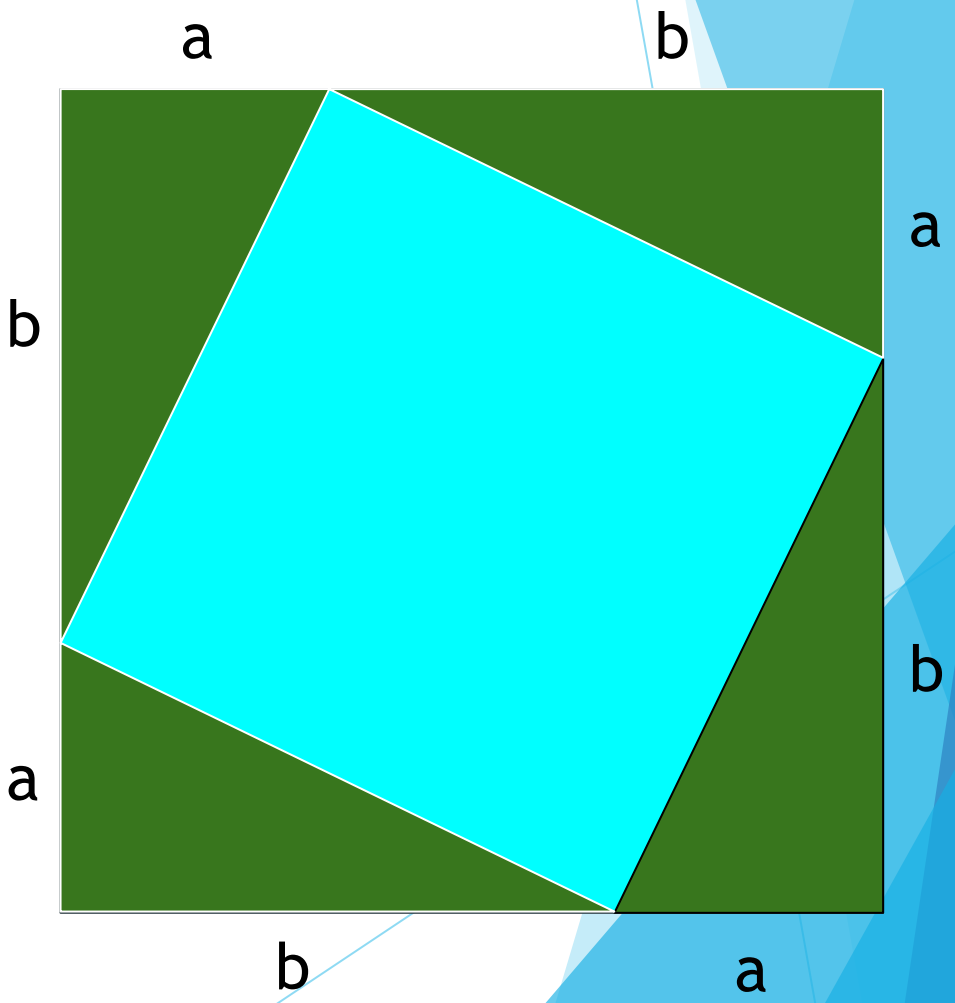
$(a+b) \times (a+b) = (a+b) \times (a+b)$

$\cancel{a^2}$

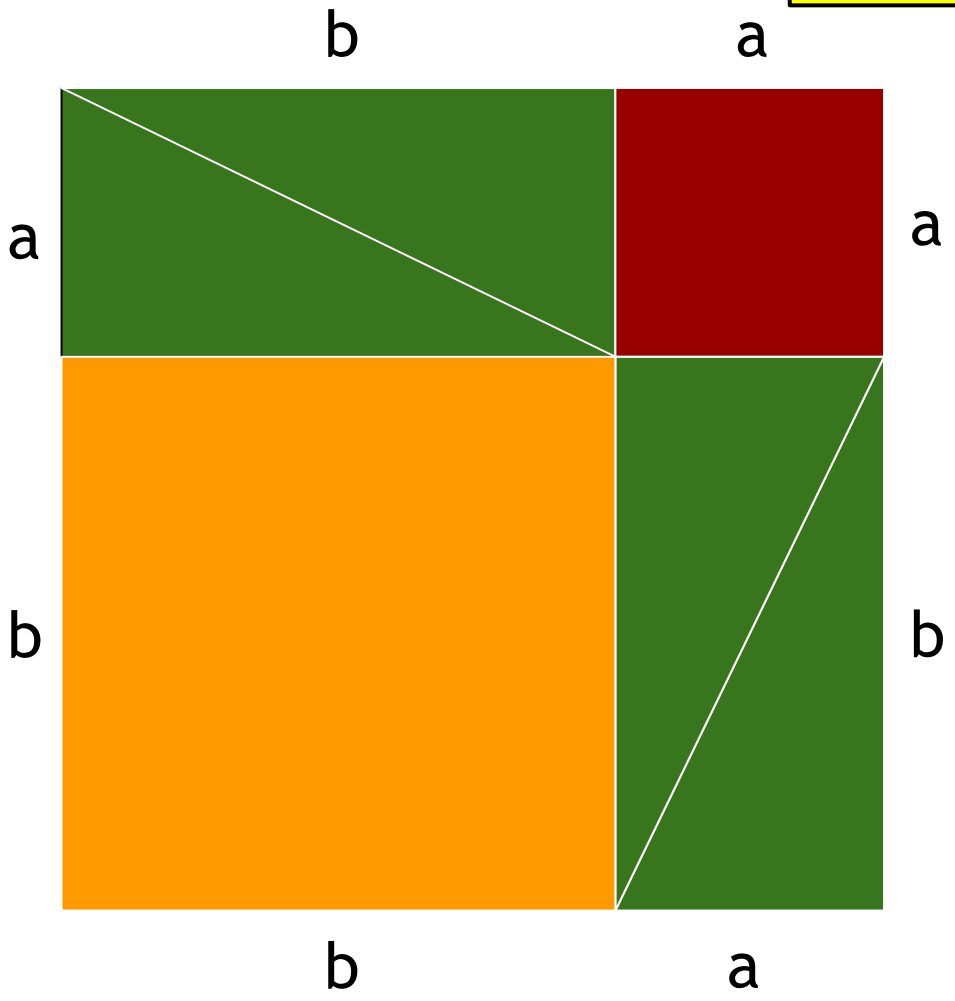
$(a+b)^2 = (a+b)^2$

Challenge 1 Answer

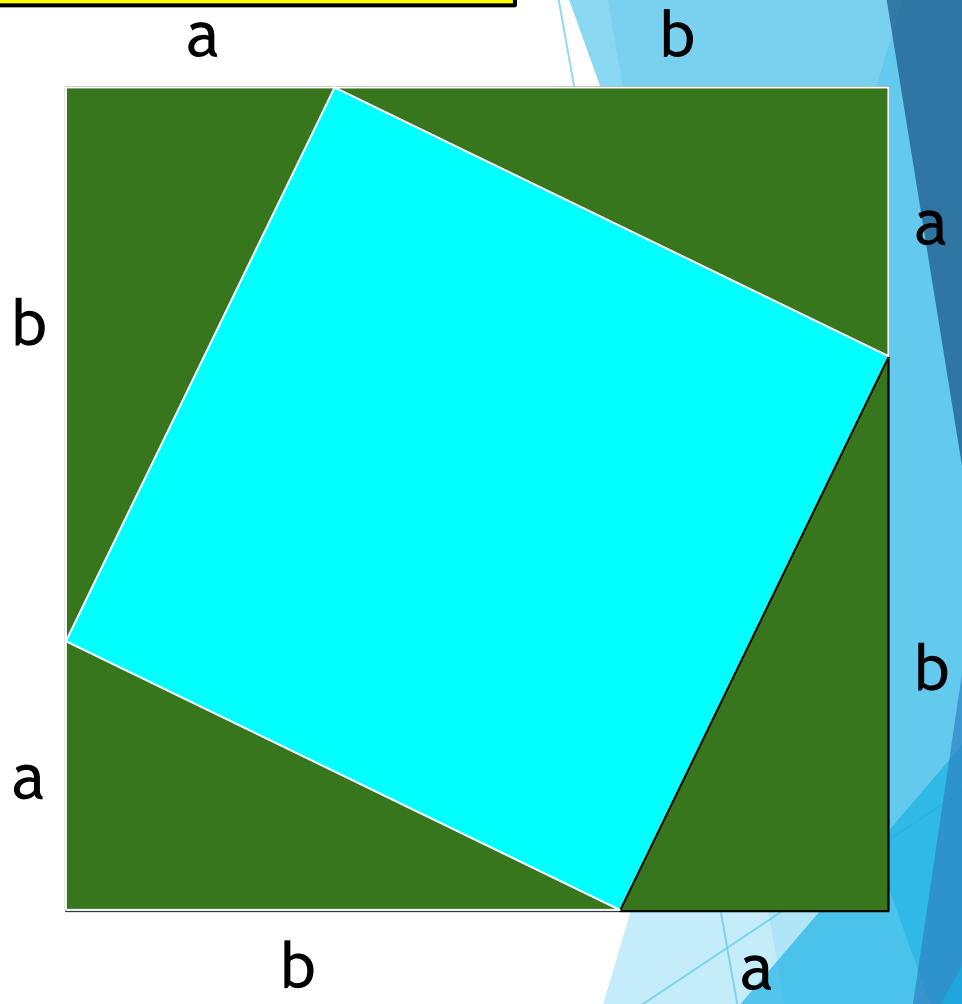




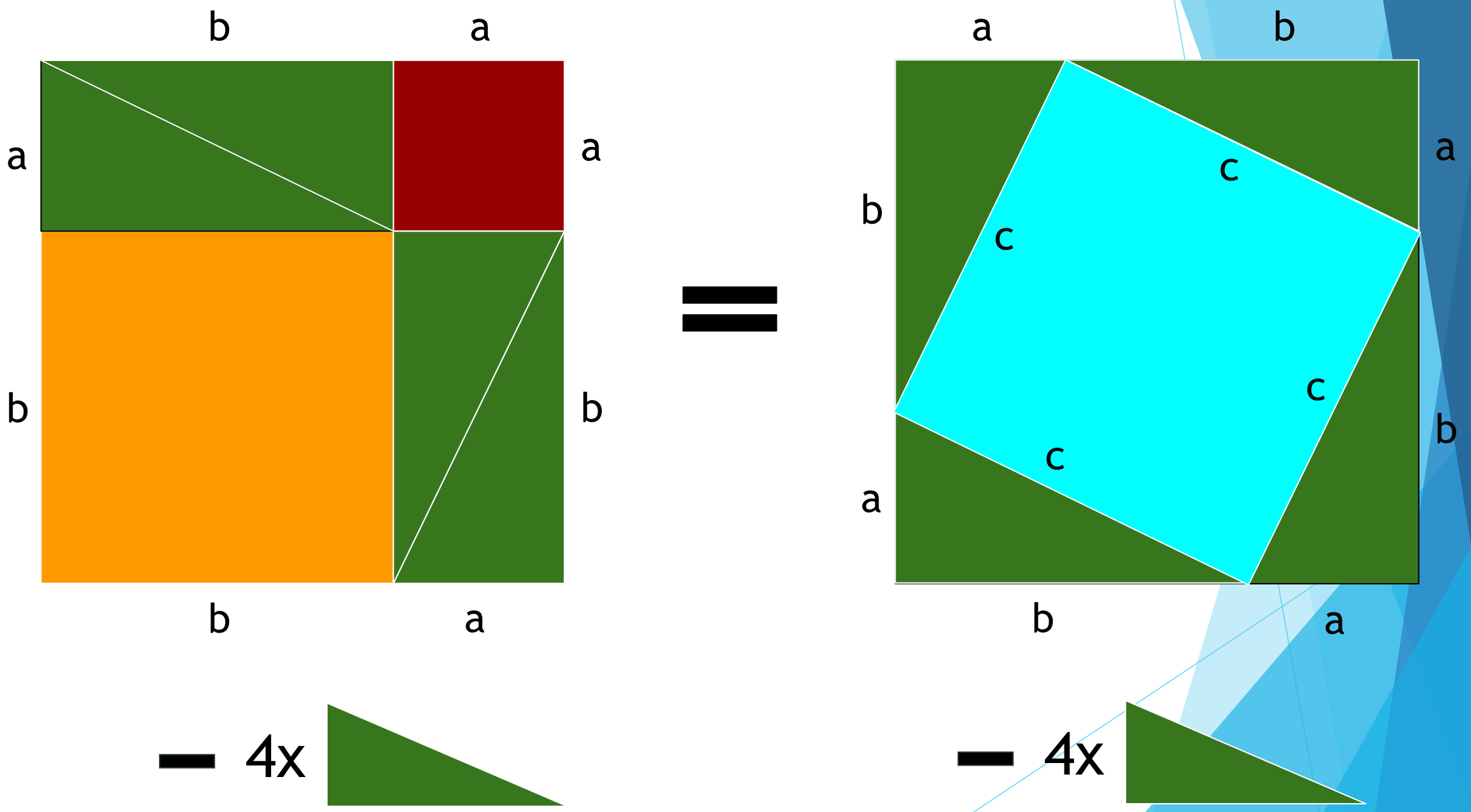
手動活動讓學生容易去了解發生什麼事



Area: (a+b) x (a+b)
= $(a+b)^2$



Area: (a+b) x (a+b)
= $(a+b)^2$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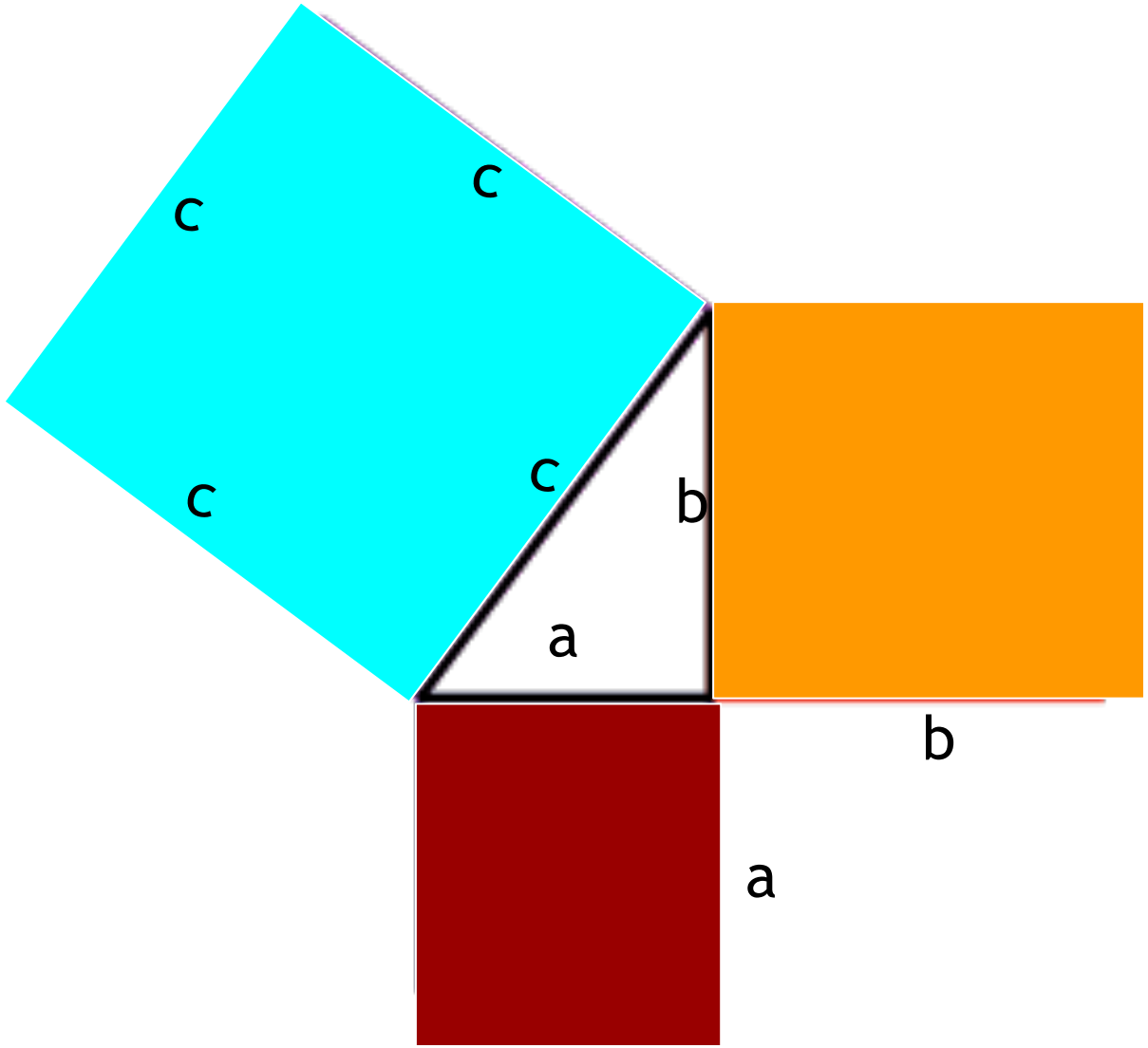
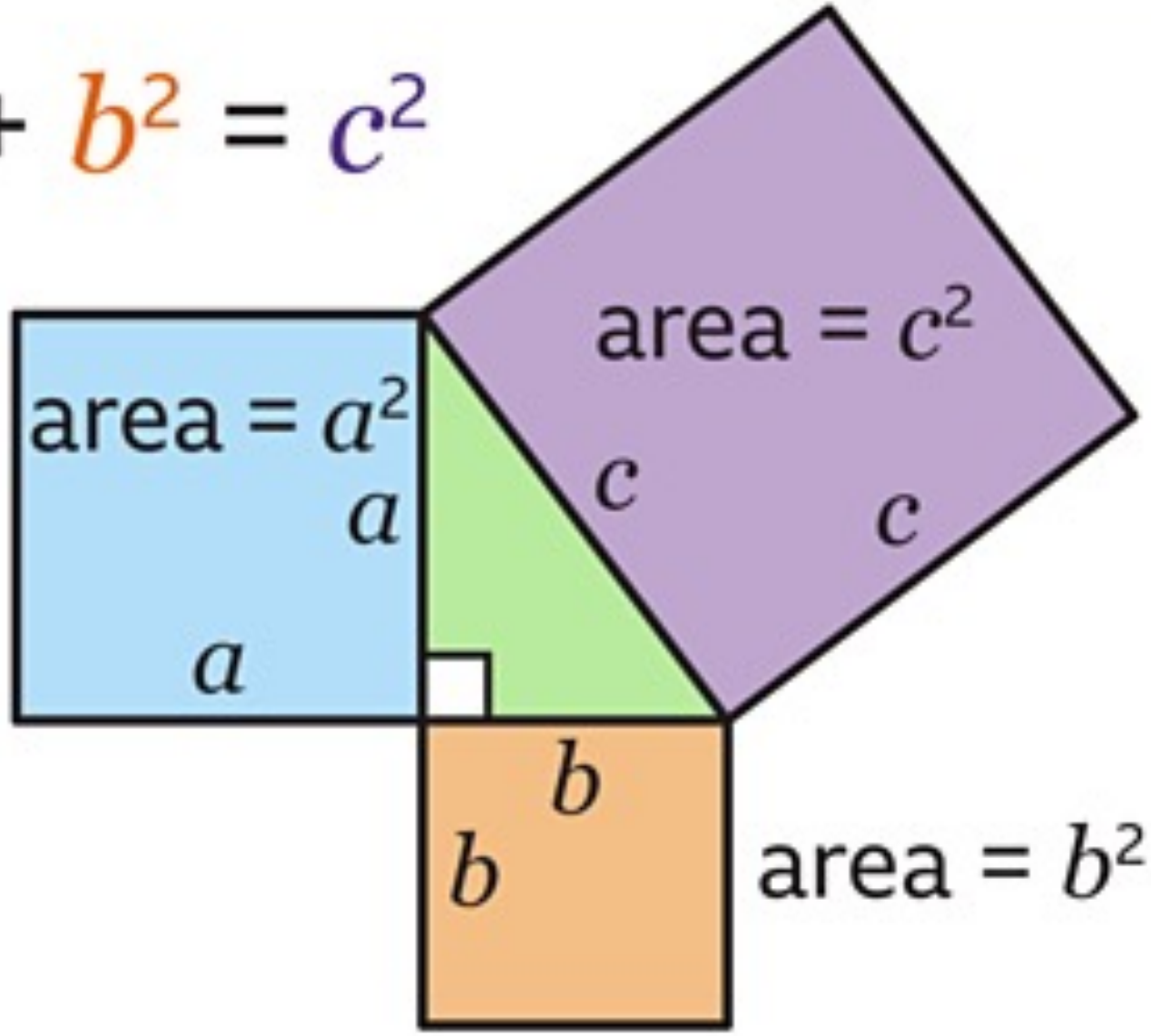
− 4x
− 4x



What can we say about the remaining area?

Pythagoras' Theorem

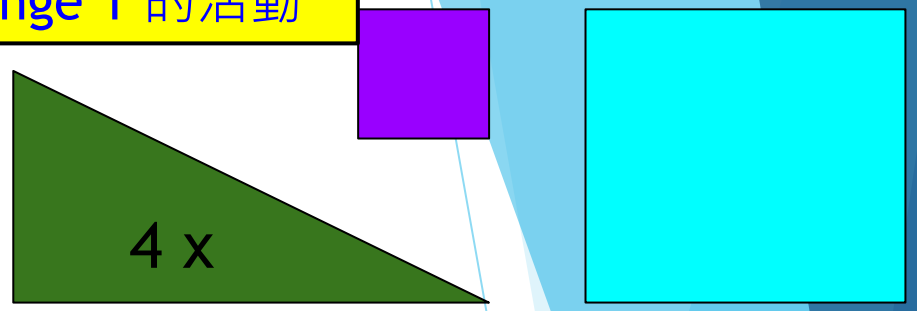
$$a^2 + b^2 = c^2$$



Challenge 2

You will be given 5 minutes, working in your groups to achieve the objective.

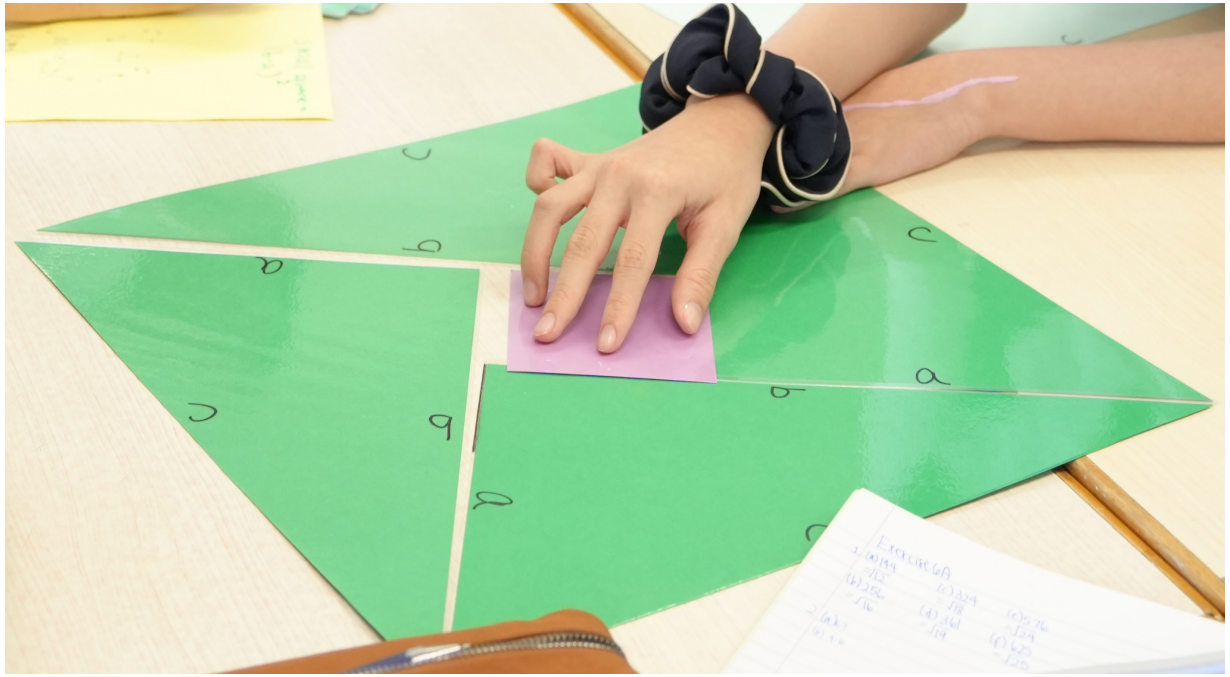
聯繫challenge 1 的活動



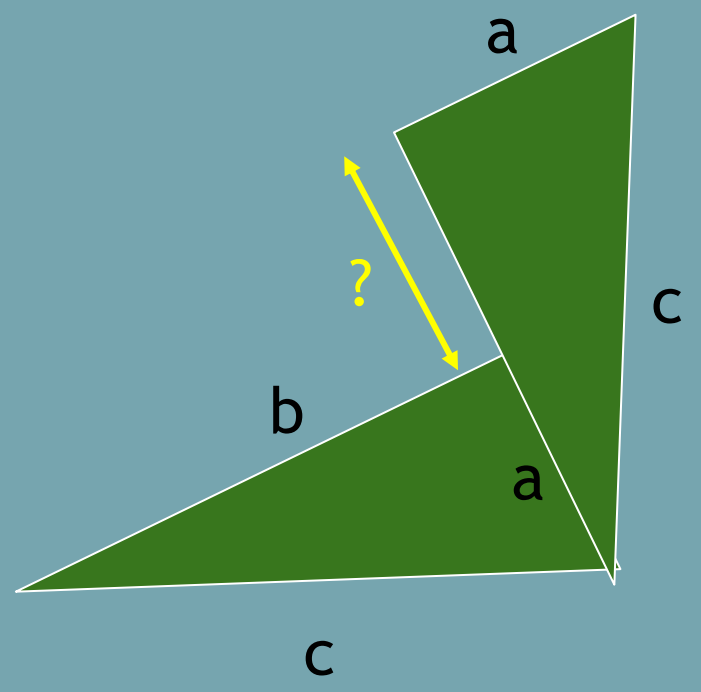
Your objective: 讓學生思考

Without any overlap, and by using ALL of the shapes provided, create 2 squares. Find the relationship between the area of the 2 squares.

Without any overlap, and by using ALL of the shapes provided, create 2 squares. Find the relationship between the area of the 2 squares.



Hint



學生實例:

挑戰學生學生去證明

$a^2+b^2=c^2$:3

Because in the first square we have 2 smaller squares. One with a length and width which is $a \times a = a^2$

For the second ~~square~~ square is b height and b width so is b^2 is the area

so ~~since of~~

Given that $a^2+b^2=c^2$ = we can proof that both squares have the same area

$L.H.S. = c \times c = c^2$

$R.H.S. = c \times c = c^2$

$\therefore L.H.S. = R.H.S. = c^2$

$L.H.S. = c^2$

$= 4\left(\frac{ab}{2}\right) + (b-a)^2$

$= 2ab + b^2 - 2ab + a^2$

$= a^2 + b^2$

$\because L.H.S. = R.H.S. = c^2$

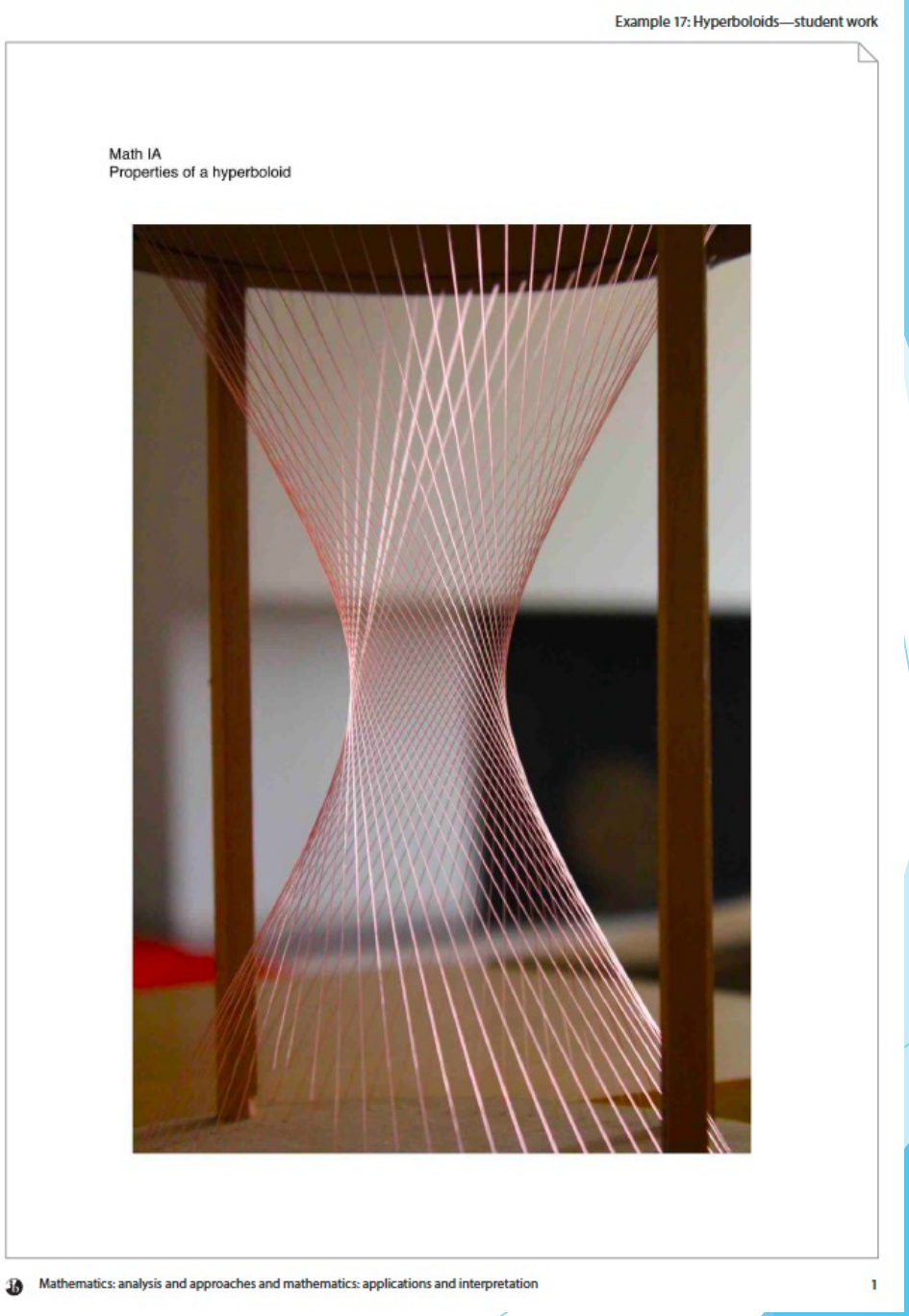
$\therefore c^2 = a^2 + b^2$

挑戰學生去思考其他證明方法

鼓勵數學能力高的學生自選主題去研究

IB Internal Assessment

- IB公開試數學的內部評估
- 學生利用所學的數學知識來研究一個主題



Example 17: Hyperboloids—student work

Math IA
Properties of a hyperboloid

Mathematics: analysis and approaches and mathematics: applications and interpretation 1

(Adapted from: IBO Maths IA sample)

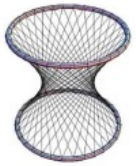
Introduction

I chose the topic for my exploration in a rather coincidental manner. Being slightly distracted during my literature class, I began doodling, and found myself drawing a circular cylinder with its vertical lengths rotated relative to their corresponding circumference. Doing this, I found that by increasing the rotation, the cross sectional circumference in its center would be reduced. With this in mind, I began wondering about the possible patterns related to the properties of this shape, known mathematically as a hyperboloid, and mathematical approaches to model them.

As a prospective design student, I have always been interested in the shape and structure of visually compelling objects, and how the most abstract forms are usually linked to some sharp functionality. Besides this, by being interested in the power of mathematics of modeling the most abstract shapes found, and often unfound, around us, I chose this topic for my exploration, feeling like it would be an interesting topic in which to engage.

In this exploration, I will try to find expressions relating the rotation of one face in a hyperboloid, its central radius and volume, through different methods such as physical modeling, data regression, and deduction of functions. Furthermore, I will also evaluate the validity of this expressions, and its possible applications in real life scenarios.

Hyperboloid

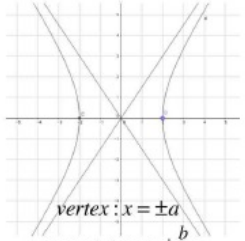


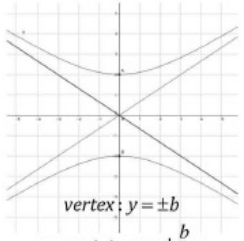
The shape studied is referred mathematically as a one-sheet hyperboloid, which has a continuous surface made up as "a surface of revolution obtained by rotating a hyperbola about the perpendicular bisector to the line between the foci" (Wolfram Alpha, 2016).

A hyperbola is a conic section which can be defined in a Cartesian plane with the general formula:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad \text{or} \qquad \frac{y^2}{b^2}-\frac{x^2}{a^2}=1$$

where a and b are real constants. Here, the negative coefficient belongs to x or y depending on the axis into which the hyperbola opens (Wolfram Alpha, 2016).





1

Furthermore, we can even consider the formula for an unknown height h, and a base circumference radius r, as done before with the radius. With this, the solution for a, would have constant values as to adjust the hyperbola to the known points:

$$\frac{y^2}{\left(\left|\frac{r\sqrt{1+\cos\theta}}{2}\right|\sqrt{2}\right)^2}-\frac{x^2}{\left(\frac{h^2}{\left(\frac{r^2}{\left(\left|\frac{r\sqrt{1+\cos\theta}}{2}\right|\sqrt{2}\right)^2}-1\right)}\right)}=1 \qquad r,h\in\mathbb{R}^+ \quad \theta\in\left]-180°,180°\right[$$

This expression would be of great power in calculating the form of a hyperbola from reestablished specifications, assuming the expressions above are all correct. However, it does have the limitation of being a hyperbola which opens along the y-axis, and considering that realistically a hyperbola is most likely to be used when opening along the x-axis, it would need to be adjusted to be fully valid and useful in a real life scenario. Luckily, the change needed would only be a 90° rotation along the origin, which can be done by switching the x and y components in the above expression.

Volume

To find the volume, I first have to transform the hyperbolic expression described above into a function, which can then be rotated 360° about the x-axis. To do this, I rearranged the expression to solve for y:

$$\frac{y^2}{s^2}-\frac{x^2}{\left(\frac{11^2}{\left(\frac{7^2}{s^2}-1\right)}\right)}=1\Rightarrow\frac{y^2}{s^2}=1+\frac{x^2}{\left(\frac{11^2}{\left(\frac{7^2}{s^2}-1\right)}\right)}\Rightarrow y=\pm\sqrt{s^2+\frac{s^2x^2}{\left(\frac{11^2}{\left(\frac{7^2}{s^2}-1\right)}\right)}}$$

Because the expression needs to be transformed to a function, the $\pm$ needs to be ignored, yet when rotated the shape will remain the same. Additionally, because s, is the central radius, we can consider it a constant for each rotation in the hyperboloid, so we will replace it with the rotation function later on.

Having the resulting function of:

$$f(x)=\sqrt{s^2+\frac{s^2x^2}{\left(\frac{11^2}{\left(\frac{7^2}{s^2}-1\right)}\right)}}$$

We then consider it's 360° solid of revolution about the x-axis, by using the formula:

$$V=\pi\int_a^b(f(x))^2\,dx$$

where a, and b, are the x-values for which it is bound.

12

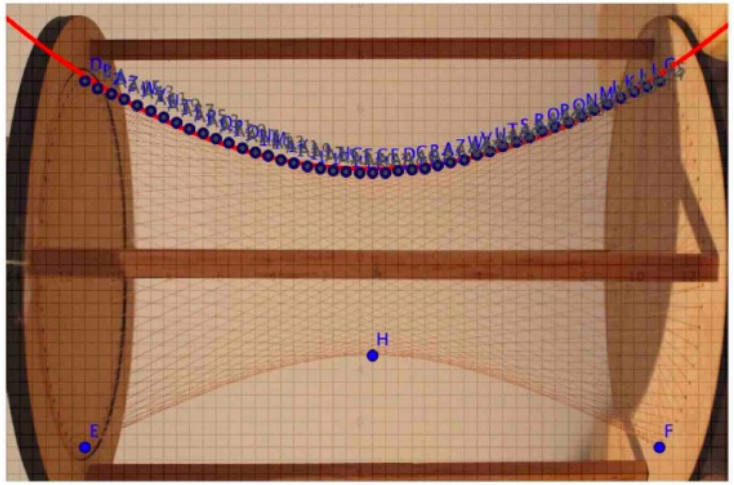
Considering that the model has a height of 22cm, and is divided symmetrically by the y-axis, it would be bound at ± 11:

$$V=\pi\int_{-11}^{11}\left(\sqrt{s^2+\frac{s^2x^2}{\left(\frac{11^2}{\left(\frac{7^2}{s^2}-1\right)}\right)}}\right)^2 dx \Rightarrow \pi\int_{-11}^{11} s^2+\frac{s^2}{\left(\frac{11^2}{\left(\frac{7^2}{s^2}-1\right)}\right)}x^2\,dx \Rightarrow \pi\left[s^2x+\frac{s^2}{3\left(\frac{11^2}{\left(\frac{7^2}{s^2}-1\right)}\right)}x^3\right]_{-11}^{11}$$

Considering this, we can then solve the volume of revolution of a hyperbola with a known central radius; $s(120°)=3.5$

$$V=\pi\int_{-11}^{11}\left[3.5^2x+\frac{3.5^2}{3\left(\frac{11^2}{\left(\frac{7^2}{3.5^2}-1\right)}\right)}x^3\right]_{-11}^{11} \Rightarrow V=\pi\left(3.5^2(-11)+\frac{3.5^2}{3\left(\frac{11^2}{\left(\frac{7^2}{3.5^2}-1\right)}\right)}(-11^3)\right)-\left(3.5^2(11)+\frac{3.5^2}{3\left(\frac{11^2}{\left(\frac{7^2}{3.5^2}-1\right)}\right)}(11^3)\right) \Rightarrow V=1693{,}3cm^3$$

Lastly, we can check the accuracy of this result, by using a different method to get the function with which to do a solid of revolution. For this, we can use Geogebra to plot a set of points in a picture of the model I made, adjusted to fit the coordinates of the real dimensions in a Cartesian plane, and then use curve fitting to find a quadratic equation that best fits the image.



Having done this when the model was at a rotation of 120°, we can then compare the values given from the expected volume according solid of revolution of the deduced function, and the volume according to the solid of revolution resulting from Geogebras line fitting.

Angle (degrees)	Formula-based volume (cm_3)	Regression based volume (cm_3)	Difference
120	1693,3	1713,4	20,1

Here, we can see how both values for volume when rotated 120° are very close, which further confirms the accuracy of the deduced function.

Conclusion

In my exploration, I investigated the properties of a hyperboloid model I made, and found functions and expressions relating the rotation of one face and the central radius, as well as the shape and volume of the hyperboloid. This findings were confirmed by comparing with values of the actual model, and were mostly seen to be true, at least when the rotation was in between -180 and 180 degrees. With this in mind, we can see how my functions and expressions are useful in real life scenarios, as not only they apply for my model, but can calculate the wanted properties in a hyperboloid of any height and base circumference radius.

With this in mind, we can see how this has implications in fields like architecture and civil engineering, were building in the shape of hyperboloids, such as nuclear plant chimneys, have to be precisely modeled, with reestablished dimensions. However, it is important to remember my model is based on threads, and in real life, the walls of a building would change the results quite significantly. Regardless of this, my models and expressions are some quite good approximations, with which I am quite happy.

Besides enjoying the process thoroughly, I am happy to say I have learned quite a lot in this exploration, and it very satisfying to know that I might be able to use my findings if I decide to study architecture or engineering. For this, I´d like to thank everyone involved in the making of this exploration, whose endless support and motivation were key to achieving the final result.

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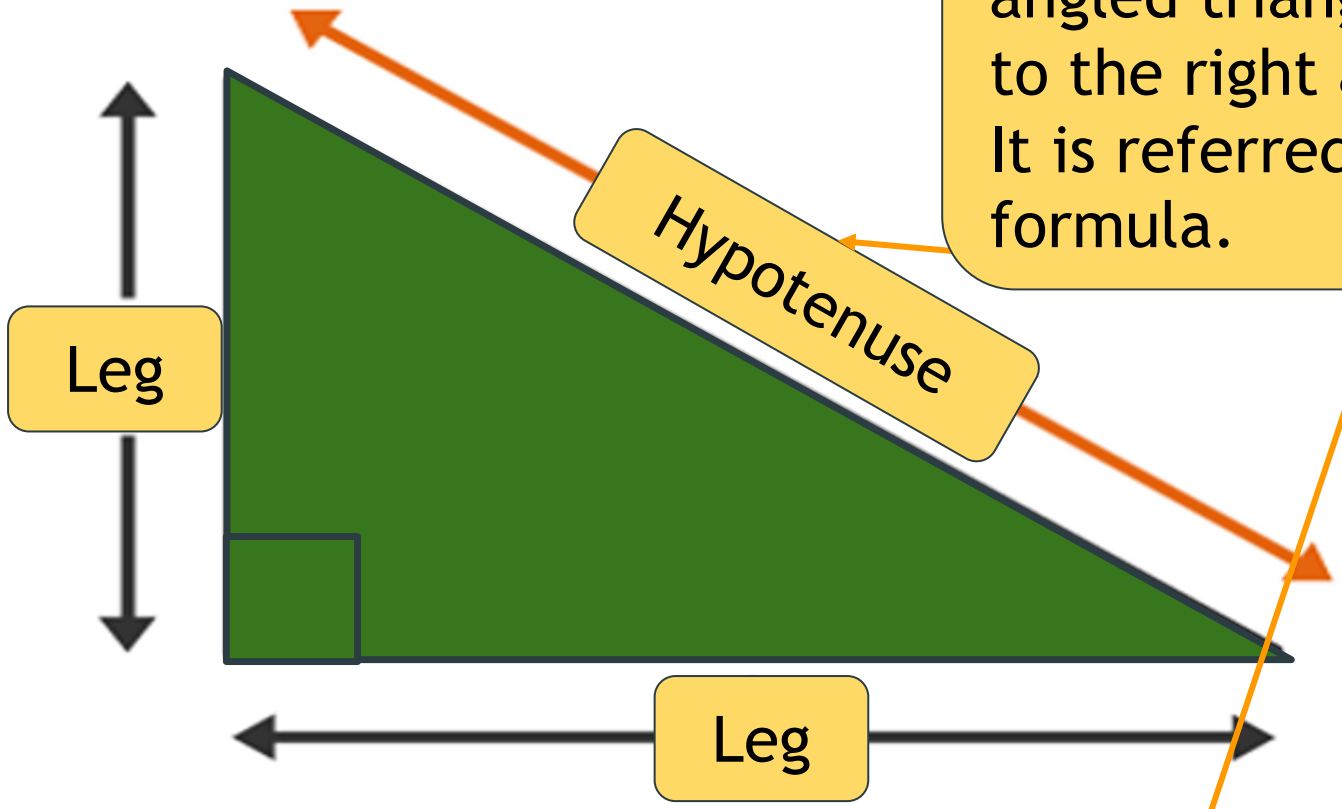
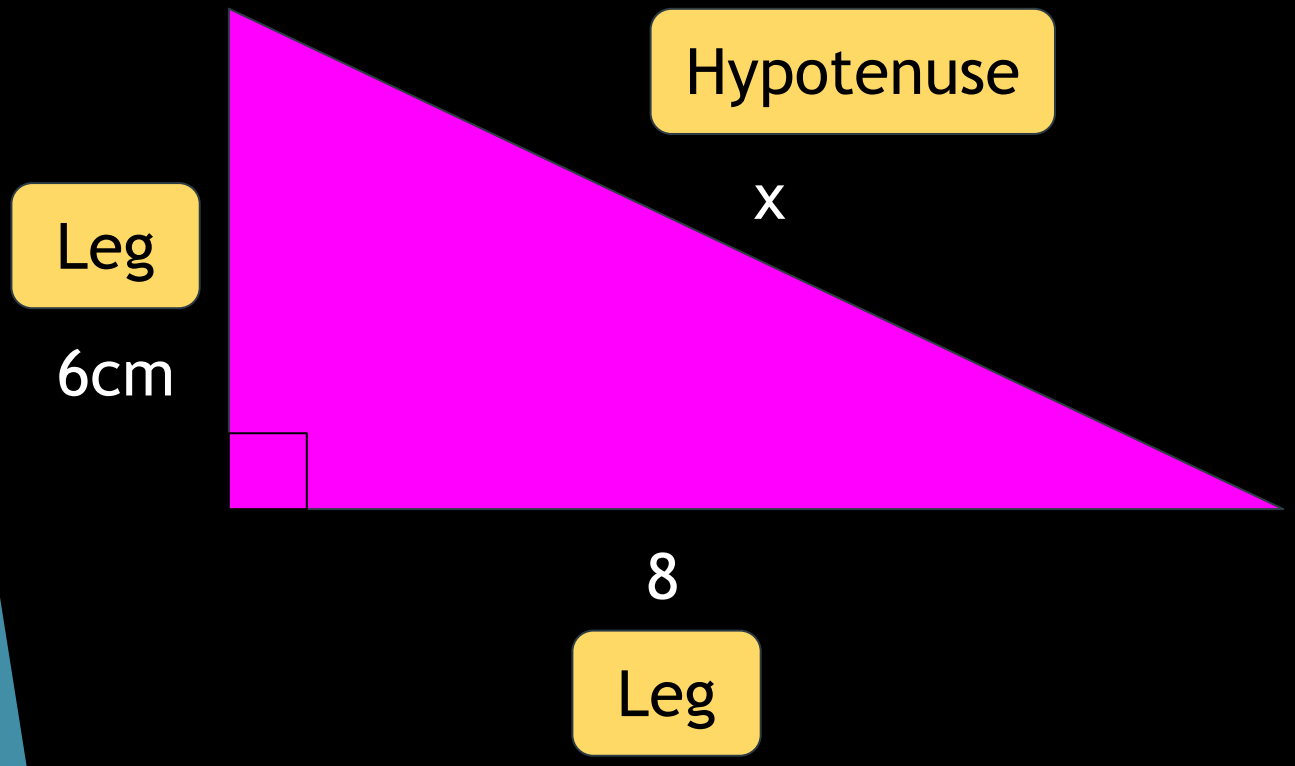
The longest side in a right-angled triangle, opposite to the right angle.
It is referred to as c in the formula.



$$a^2 + b^2 = c^2$$

Example 1 - Find x



$$a^2 + b^2 = c^2$$

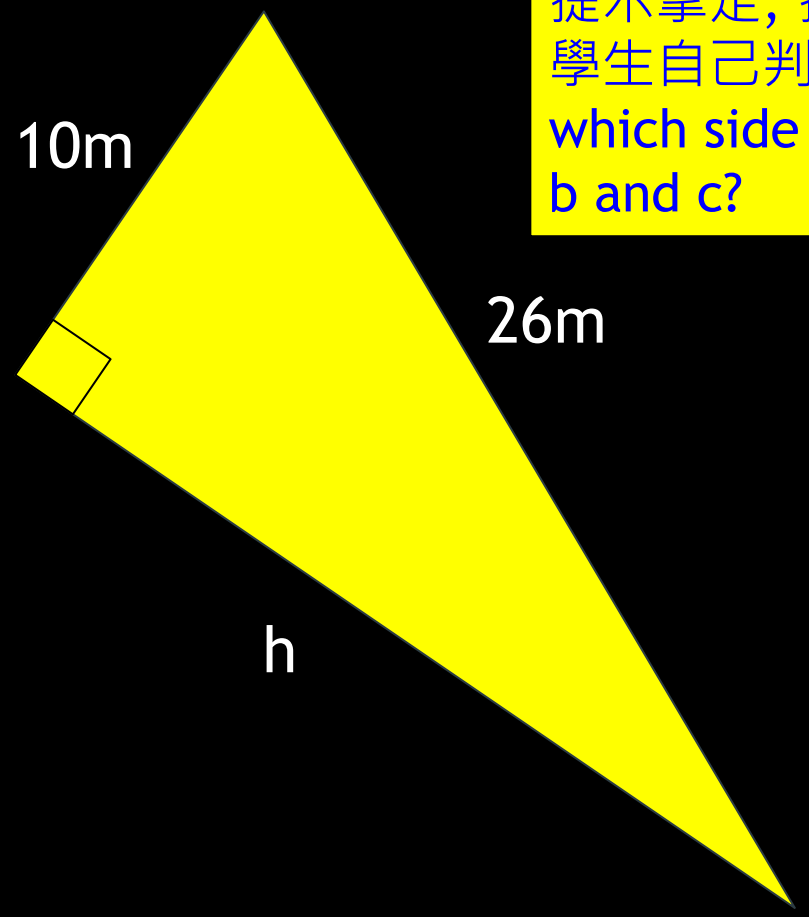
$$8^2 + 6^2 = x^2 \ (pyth.theorem)$$

$$64 + 36 = x^2$$

$$100 = x^2$$

$$x = 10cm$$

Example 2 - Find h



提示拿走，挑戰學生自己判斷 which side is a, b and c?

$$a^2 + b^2 = c^2$$

$$h^2 + 10^2 = 26^2 \ (pyth.theorem)$$

$$h^2 = 26^2 - 10^2$$

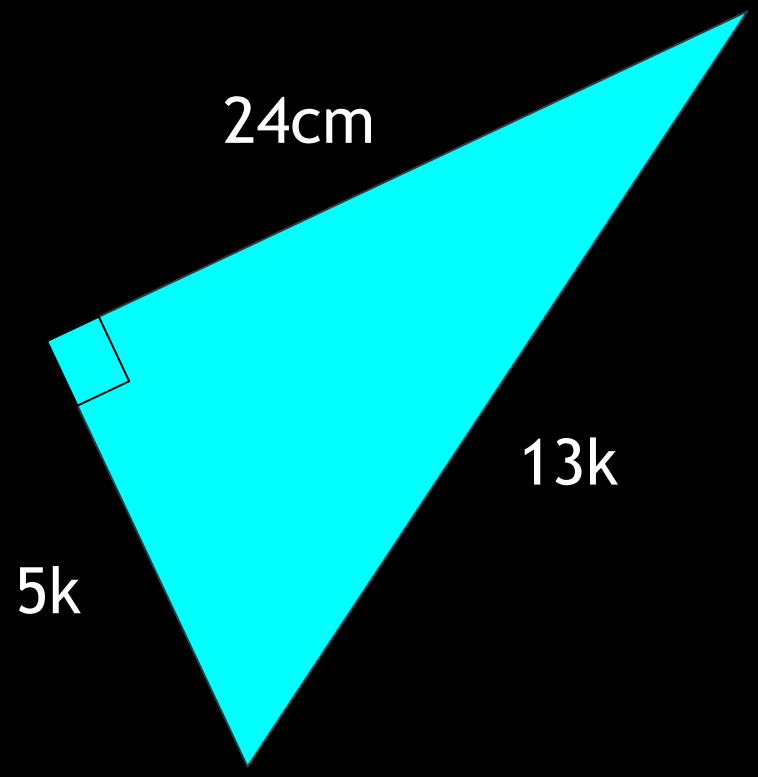
挑戰學生 rearrange the subject

$$h^2 = 576$$

$$h = 24m$$

Challenge 3 - Find k

加入代數,挑戰數學能力高的學生



$$a^2 + b^2 = c^2$$

$$(5k)^2 + 24^2 = (13k)^2 \ (pyth.theorem)$$

$$24^2 = (13k)^2 - (5k)^2$$

$$24^2 = 169k^2 - 25k^2$$

$$576 = 144k^2$$

$$4 = k^2$$

挑戰數學能力高的學生，考慮捨去答案

$$k = 2cm \ or - 2cm$$

$$k = 2cm$$

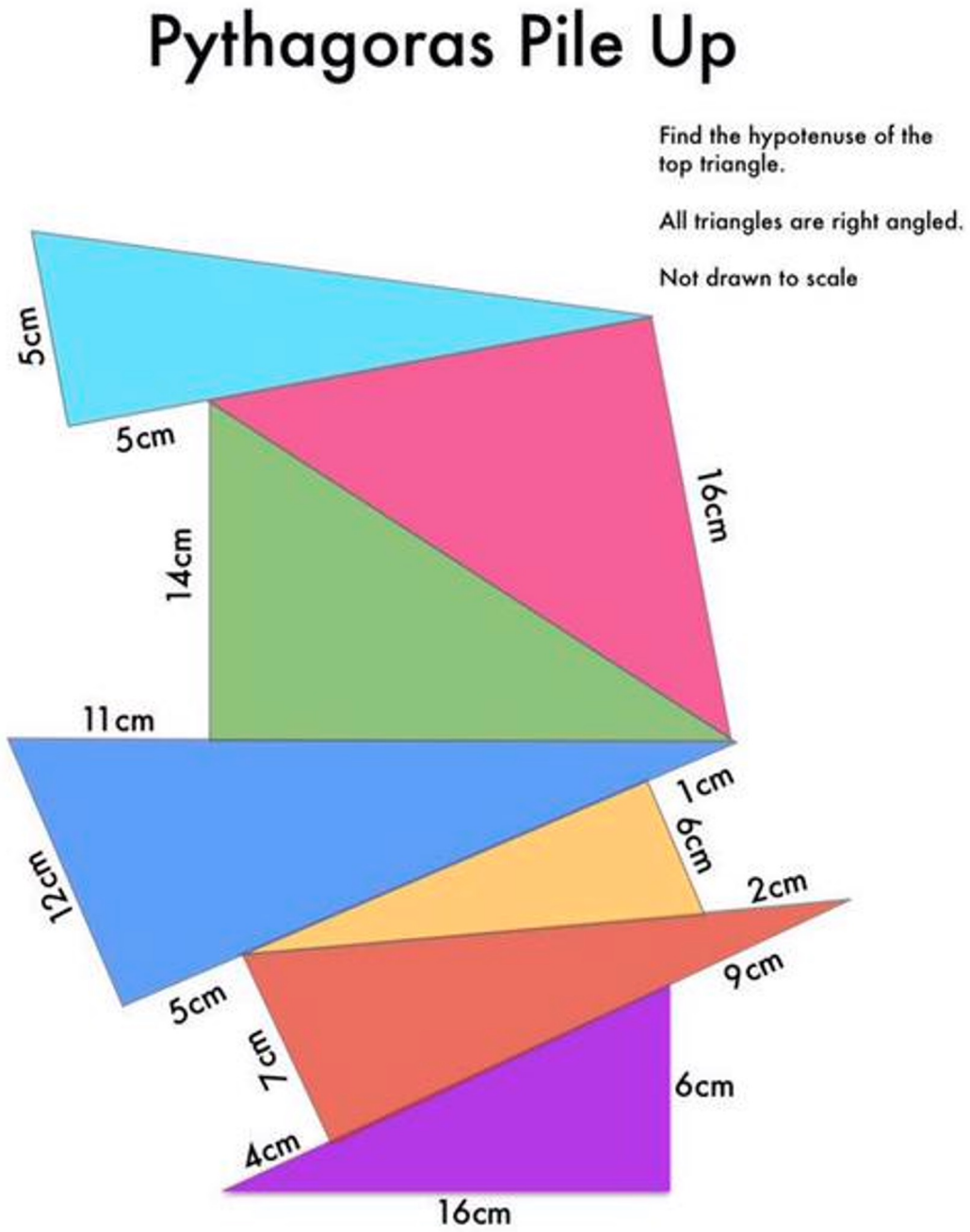
利用比賽評估學生是否理解
(分工合作)

Working around the table, with each student completing 1 question at a time (each member must complete at least 1 question), find the hypotenuse of the triangle at the top (light blue).

No steps required!

Hint: Use 1 single calculator and the ANS function.

Fastest team wins!



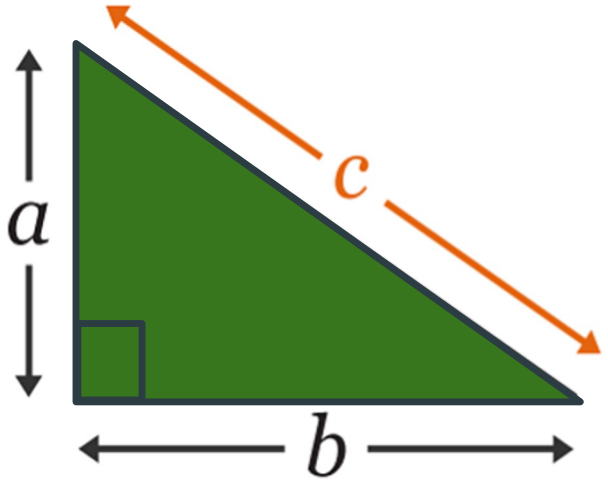
Adapted from: https://mrallanmaths.wordpress.com/2015/08/07/pythagoras-theorem-pile-up/

Something to think about…

挑戰數學能力高的學生，學生為下一課做準備

All the triangles we have considered so far are right-angled triangles.

1. What happens if the triangle is not right-angled?
2. How can I prove if a triangle is right-angled?



$$a^2 + b^2 = c^2$$

寄語:

學生有無限的創意，

說數是不可缺少的元素，

學校的氛圍要做好!

- 李澤林老師
- Email: forrest_li@cky.edu.hk