三角學的應用
在第一層全班式教學中滲入資優教育三元素

九龍華仁書院
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CONTENT

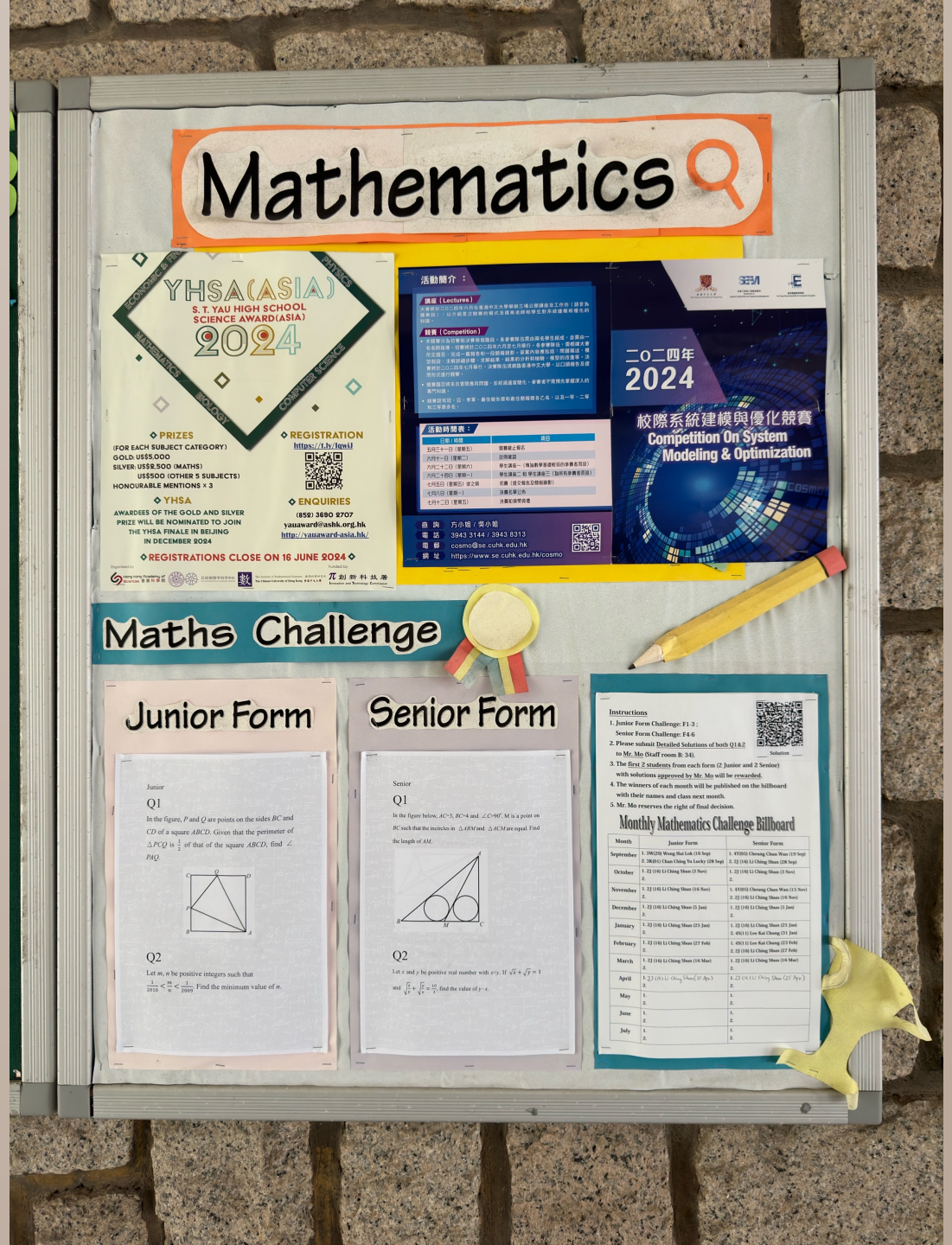
簡報內容

- 校本資料 Background
- 共同協作學與教資源 Teaching Resources
- 問答環節 Q&A

資優教育發展的情況

第一層：校本全班式教學	第二層：校本抽離式計劃	第三層：校外支援
主題早會	Scholar's Society - Mentorship Programme	校外課程： 香港資優教育學苑
正念與Examen	社會服務	校外活動： 本地大學舉辦的活動 / 比賽
學校價值觀 → 課程規劃 → 價值觀教育	天主教明愛賣物會	
	體驗式學習	

數學科


Mathematics
YHSA(ASIA)
S. T. YAU HIGH SCHOOL SCIENCE AWARD(ASIA)
2024
PRIZES
(FOR EACH SUBJECT CATEGORY)
GOLD: US$5,000
SILVER: US$2,500 (MATHS)
US$500 (OTHER 5 SUBJECTS)
HONOURABLE MENTIONS × 3
YHSA
AWARDEES OF THE GOLD AND SILVER PRIZE WILL BE NOMINATED TO JOIN THE YHSA FINALE IN BEIJING IN DECEMBER 2024
REGISTRATION
https://t.ly/lqwiJ
ENQUIRIES
(852) 3680 2707
yauaward@ashk.org.hk
http://yauaward-asia.hk/
REGISTRATIONS CLOSE ON 16 JUNE 2024
活動簡介：
講座（Lectures）
競賽（Competition）
活動時間表：
二〇二四年
2024
校際系統建模與優化競賽
Competition On System Modeling & Optimization
查詢 方小姐 / 樊小姐
電話 3943 3144 / 3943 8313
電郵 cosmo@se.cuhk.edu.hk
網址 https://www.se.cuhk.edu.hk/cosmo
Maths Challenge
Junior Form
Junior
Q1
In the figure, P and Q are points on the sides BC and CD of a square ABCD. Given that the perimeter of △PCQ is 1/2 of that of the square ABCD, find ∠PAQ.
Q2
Let m, n be positive integers such that 1/2010 < m/n < 1/2009. Find the minimum value of n.
Senior Form
Senior
Q1
In the figure below, AC=5, BC=4 and ∠C=90°. M is a point on BC such that the incircles in △ABM and △ACM are equal. Find the length of AM.
Q2
Let x and y be positive real number with x>y. If √x + √y = 1 and √(1/x) + √(1/y) = √(15/7), find the value of y−x.
Instructions
1. Junior Form Challenge: F1-3 ;
Senior Form Challenge: F4-6
2. Please submit Detailed Solutions of both Q1&2 to Mr. Mo (Staff room B: 34).
3. The first 2 students from each form (2 Junior and 2 Senior) with solutions approved by Mr. Mo will be rewarded.
4. The winners of each month will be published on the billboard with their names and class next month.
5. Mr. Mo reserves the right of final decision.
Solution
Monthly Mathematics Challenge Billboard
Month
Junior Form
Senior Form
September
October
November
December
January
February
March
April
May
June
July

每月
挑戰

課外比賽

The Hong Kong Mathematical High Achievers Selection Contest 2023-2024
團體賽首十名最佳成績

2024 年香港華羅庚金杯少年數學邀請賽（決賽）
中學組團體冠軍

Mathematics Project Competition for Secondary Schools 2023/24
Champion

Creative Infographic Design Competition on Applications of Mathematics for Primary and Secondary Schools (2023/24) – Gold Award

「華夏盃」全國數學奧林匹克邀請賽 2024 全國總決賽
一等獎

Canadian Senior and Intermediate Mathematics Contests 2023
Distinction

Thailand International Mathematical Olympiad (TIMO)
Gold Award

Trigonometric Ratios

中二：三角學簡介

課堂背景

29名學生

有非華語學生

傳統教學、小組形式

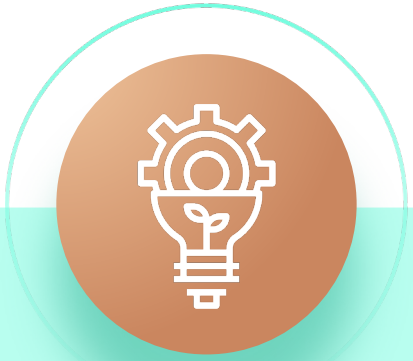
有部分學生在數學上
有過人天賦

設計資優課程的策略

建立高階思維
develop high-order thinking

開放性問題

小組討論（拼圖法、結隊）



培養創意
cultivate creativity

解決現實生活中的問題

跳出傳統思維框框

鍛鍊個人及社交能力
working on personal-social competence

小組討論

尊重他人意見

學習目標

一般能力

All Students

已有知識：畢氏定理、正弦（sin）、餘弦（cos）、正切（tan）

- 製作圓規
- 使用圓規量度與水平面的角度
- 以正切算出以三角形的角
- 以正切應用在三角學的應用題上

資優 / 高能力

Gifted / abled Students

- 運用創意，結合所學求出物體與水平線的角度

- 能在複雜的應用題上求出相關角度

教學流程（一）

隨機分組

隨機將能力不一的學生
分入6小組內

分配物體

安排不同小組分別量度學校中
的不同物體

量度限制

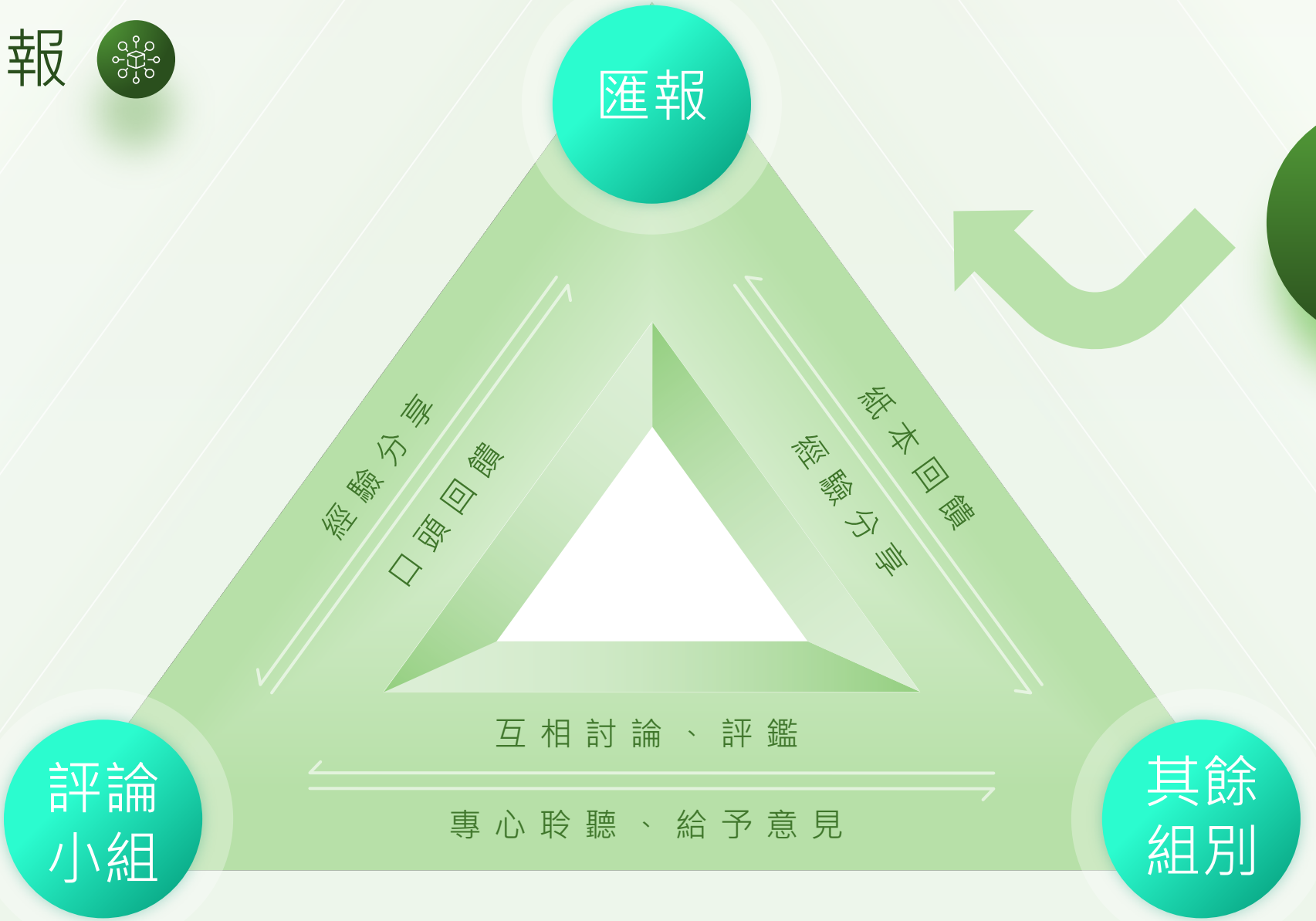
同學不能使用應用程式
或超過一米長的捲尺幫
助量度

引發討論

鼓勵同學討論量度長度的方法與
量度自己有興趣的物體


WAH YAN
KOWLOO

分組彙報



分組彙報問題舉隅

Reflect on the process of measuring the height of different objects using trigonometry. What challenges did you encounter? How did you overcome them?

Evaluate your current design. How can you create an improved version to increase the accuracy of your measurement and calculation?

Consider the limitations of using trigonometry to measure height. Are there any situations or objects where this method might not be applicable? Explain your reasoning.

Reflect on the **accuracy** of your measurements and calculations. What factors could have contributed to any errors or inaccuracies?

Calculate the values of the following: tan 1 / tan 2 / tan 88 / tan 89

What can you conclude from the above results?

應用題工作紙

分層設計

老師介紹情境

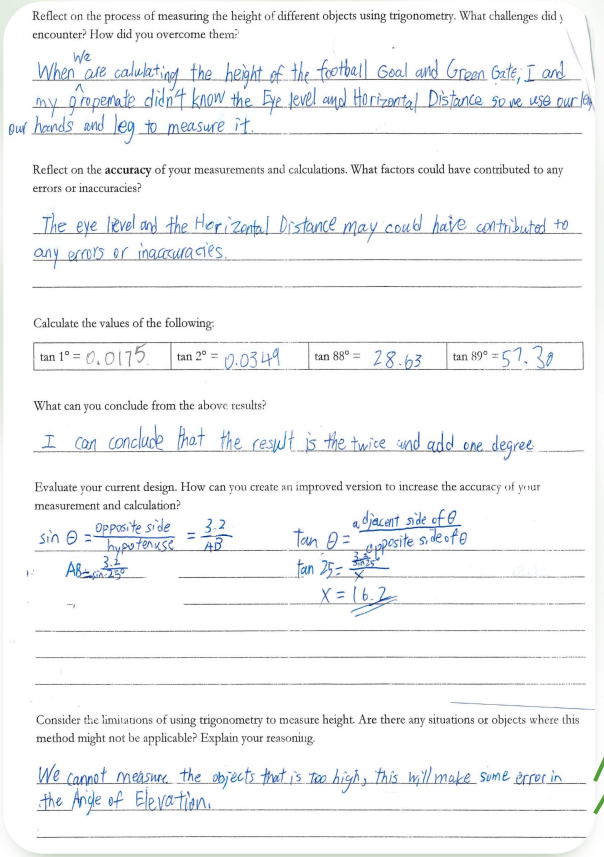
引導同學做嘗試

資優 / 高能力

闡釋如何計算

自行解出答案

同學功課舉隅



Reflect on the process of measuring the height of different objects using trigonometry. What challenges did you encounter? How did you overcome them?

When we are calulating the height of the football Goal and Green Gate, I and my groperate didn't know the Eye level and Horizontal Distance so we use our hands and leg to measure it.

Reflect on the **accuracy** of your measurements and calculations. What factors could have contributed to any errors or inaccuracies?

The eye level and the Horizontal Distance may could have contributed to any errors or inaccuracies.

Calculate the values of the following:

tan 1° = 0.0175	tan 2° = 0.0349	tan 88° = 28.63	tan 89° = 57.30

What can you conclude from the above results?

I can conclude that the result is the twice and add one degree

Evaluate your current design. How can you create an improved version to increase the accuracy of your measurement and calculation?

$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{3.2}{AB}$

$AB = \frac{3.2}{\sin 25^\circ}$

$\tan\theta = \frac{\text{adjacent side of } \theta}{\text{opposite side of } \theta}$

$\tan 25 = \frac{\sin 25^\circ}{x}$

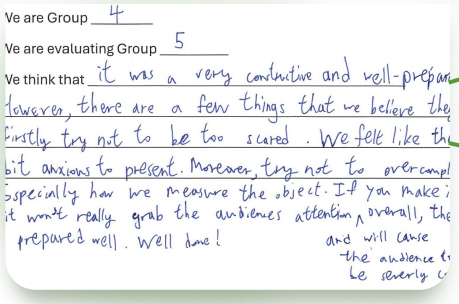
$x = 16.2$

Consider the limitations of using trigonometry to measure height. Are there any situations or objects where this method might not be applicable? Explain your reasoning.

We cannot measure the objects that is too high, this will make some error in the Angle of Elevation.

We cannot measure the objects that is too high, this will make some error in the Angle of Elevation.

同學功課舉隅


We are Group 4

We are evaluating Group 5

We think that it was a very conductive and well-prepar

however, there are a few things that we believe the

Firstly try not to be too scared. We felt like th

bit anxious to present. Moreover, try not to overcompl

Especially how we measure the object. If you make

it won't really grab the audiences attention, overall, the

prepared well. Well done!

and will cause

the audience t

be severly c

We think that it was a very constructive and well-prepared presentation. However, there are a few things that we believe they can improve firstly try not to be too scared. We felt like they were a bit anxious to present. Moreover, try not to overcomplicate things. Especially how we measure the object. If you make it overcomplicated, it won't really grab the audiences attention and will cause the audience to be severly confused. Overall, they really prepared well. Well done!

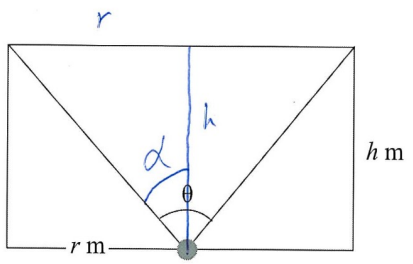
同學功課舉隅

Extended Questions

The frog is standing in the middle of the well. $r = 12 - h$

1. Let $2h + 2r = 24$, $\rightarrow h + r = 12$ $h = 12 - r$

   Find r and h so that the viewing angle of the frog is 40°.



$\tan 20° = \frac{r}{h} = \frac{r}{12-r}$ $\tan 20° = \frac{12-h}{h}$

$12\tan 20° - \tan 20° \cdot r = r$ $h \cdot \tan 20° = 12 - h$

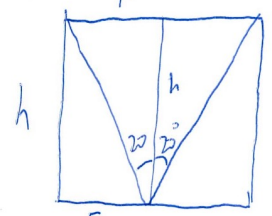
$r = \frac{12\tan 20°}{1 + \tan 20°}$ $h = \frac{12}{1 + \tan 20°}$

$= 3.20$ (3sf) $= 8.80$ (3sf)

2. The volume of a cylinder is given by $\pi r^2 h$. If the volume of the well must be 120m^3, with the frog at the bottom of the well, find h and r if the viewing angle to be 40°.

$\pi r^2 h = 120$

$h = \frac{120}{\pi r^2}$



$\tan 20° = \frac{r}{h}$

$\tan 20° = r \cdot \frac{\pi r^2}{120}$

$\tan 20° = \frac{\pi}{120} \cdot r^3$

$r^3 = \frac{120 \tan 20°}{\pi}$

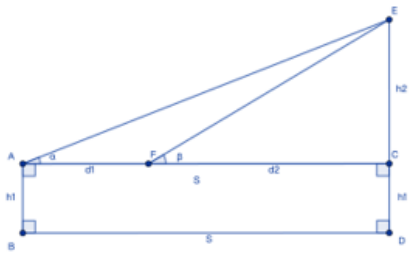
$r = \sqrt[3]{\frac{120 \tan 20°}{\pi}} = 2.4045$ (5sf) $= 2.40$ (3s.f.)

$h = \frac{120}{\pi} \cdot \frac{1}{r^2} = 6.6064$ (5sf) $= 6.61$ (3s.f.)

同學功課舉隅

Math Project: Finding Building Height

By [redacted]



(Fig 1.1)

AB(h1) is the height of our eyes. DE(h1+h2) is the building's height. α is the _____ angle. β is the _____ angle when we walked to point F where d1 is the distance between A and F. Given h1, d1, α and β, we are going to calculate DE.

First, we can use sine law to find AE and EF.

In ΔAEF,

$$\frac{EF}{\sin\alpha} = \frac{AF}{\sin(\beta-\alpha)} \dots\dots\dots\dots \text{(sine law)}$$

Which can be simplified to

$$\frac{EF}{\sin\alpha} = \frac{d1}{\sin(\beta-\alpha)} \dots\dots\dots\dots\dots\dots (1)$$

From (1), we get $EF = \frac{d1\times\sin\alpha}{\sin(\beta-\alpha)} \dots\dots\dots\dots\dots (2)$

In ΔFEC, we have

$$h2 = EF \times \sin\beta \dots\dots\dots\dots\dots (3)$$

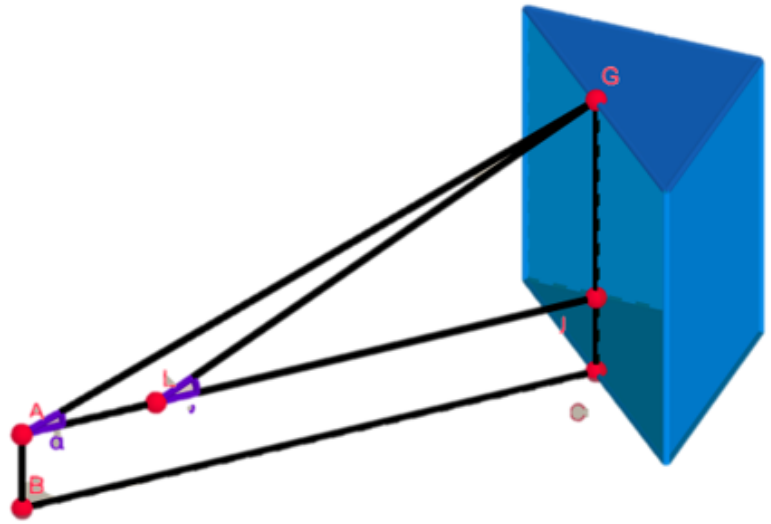
Put (2) into (3).

$$h2 = \sin\beta \times \frac{d1 \times \sin\alpha}{\sin(\beta-\alpha)}$$

$$= \frac{d1 \times \sin\alpha \sin\beta}{\sin(\beta-\alpha)}$$

Therefore, the height of the building$= \frac{d1\times\sin\alpha\sin\beta}{\sin(\beta-\alpha)} + h1$

Here is how to find the actual height in the 3D model.



(Fig 1.2)

This figure is like Fig 1.1.

Noted that ALJ is one line and $AJ \perp GC$.

總結

第一層全班式教學

分組 / 獨立專題研究

- 以能力 / 隨機分組
- 讓同學主導學習
- 引導學生探討、深究、表達所學，**鍛鍊社交能力**

深度與複雜度

- 結合布盧姆分類學，將**高階思維**融合教學
- 提供增潤活動與題目
- 鼓勵同學自主探索

調節步伐

- 預先了解同學進度
- 調適教學進度
- 提供平台與機會讓同學用**多元化**的學習模式，激發**創意**

如何改良教學以融入資優教育？

	傳統教學	改良方向
 步伐	部分學生完成作業與測驗的速度不一，資優學生往往比普通同學快接近一倍的時間完成。很多時候他們會在完成課堂工作後容易分心。	提供機會讓資優同學多完成一些題目，在不影響分數的情況下增添試卷的難度與題目的數量。
 深度	資優同學大多覺得傳統教學內容沉悶、缺乏挑戰性。	在課間工作紙、測驗、考試當中增添LV3題目，提供挑戰與趣味性給予資優同學，
 進度	不同同學的學習進度不一，難以為部分同學（資優 / 水平稍遜）作調適。	考慮在課程設計當中加入分級元素，如在課堂上使用大部分時間教導一般課程，但可在課堂完結前加入少許延伸部分。

Q&A Q&A Q&A Q&A

Trigonometric Ratios
中二：三角學簡介

問答環節

Questions?