2015/16 The 7th HK Mathematics Creative Problem Solving Competition for Secondary School (Heat – Written)

CPS-ID:	Centre Code:	Session:	Seat No.:

Time allowed：50 minutes

Instructions：

1. The time allowed is 50 minutes.

2. The question paper consists of 12 pages. There are 19 questions in this paper.

3. A set of question paper will be given to each student in a team.
   Only ONE answer sheet (green) will be given to each team.
   All the questions should be discussed among team members. The agreed answers should be written onto the answer sheet. Only the answers on the answer sheet will be marked.

4. Participating teams should bring their own stationery and calculators. For the purpose of fairness, please use only scientific calculators on the "List of Approved Calculators" by the Hong Kong Examinations and Assessment Authority. Electronic dictionaries, computers, mobile phones and other communication devices are prohibited.

5. The blank space on each page of this question paper can be used for rough work. One rough work sheet will be distributed to each participant. Extra rough work paper will also be provided upon request.

6. The answer sheet, all question papers and rough work papers will be collected after the competition. Participants are not allowed to take away any of these papers or the team might risk disqualification.

2015/16 第七屆香港中學數學創意解難比賽
(初賽-筆試)

學校編號:	試場編號:	比賽場次:	座位編號:

比賽時間：**50 分鐘**

參加者須知：

1. 比賽時間共 50 分鐘。

2. 本問題卷共 12 頁，全卷共有 19 題。

3. 每位參賽同學獲派一份問題卷，每一隊參賽隊伍只會獲派一張(綠色)答題紙。題目須由各成員經過討論，然後將議定的答案寫於答題紙上。
   ** 只有寫於答題紙上的答案方可得到評分。

4. 參賽學生需自備文具及計算機。為公平考慮，比賽中只可使用香港考試及評核局「准用計算機型號名單」中的科學計算機 (Scientific Calculator)。本比賽中嚴禁使用電話、電子字典、電腦或其他有上網或通訊功能的工具。

5. 本試卷每頁空白位置可作為算草之用。每位參賽學生亦會獲派一張算草紙，如有需要，可要求額外算草用紙。

6. 在筆試完結後，各同學必須交回所有問題卷、答題紙及草稿紙。參賽學生不得取走任何於比賽中所派發之紙張文具，違規者全隊可被取消資格。

題 1

若 $S = 2\left(1-\dfrac{1}{2}\right) + 3\left(1-\dfrac{1}{3}\right) + \cdots + 2015\left(1-\dfrac{1}{2015}\right) + 2016\left(1-\dfrac{1}{2016}\right)$，求 S 的值。

(2 分)

答： $S = $ _____ 。

Question 1

If $S = 2\left(1-\dfrac{1}{2}\right) + 3\left(1-\dfrac{1}{3}\right) + \cdots + 2015\left(1-\dfrac{1}{2015}\right) + 2016\left(1-\dfrac{1}{2016}\right)$, find the value of S.

(2 marks)

Answer: $S = $ _____ .

題 2

已知 $y = -3(x+20)^2 + 16$。求 y 的最大可能值及 x 相應的取值。

(2 分)

答： 當 $x = $ _____ 時，$y = $ _____ 為最大可能值。

Question 2

It is given that $y = -3(x+20)^2 + 16$. Find the greatest possible value of y and the corresponding value of x.

(2 marks)

Answer: When $x = $ _____ , $y = $ _____ is the greatest possible value.

題 3

某校中四級共有 120 名學生，其中 70 名修讀地理科，80 名修讀經濟科，35 名同時修讀地理科及經濟科。

問沒有修讀地理科同時沒有修讀經濟科的學生有多少名？

(2 分)

答: 有 _____ 名學生沒有修讀地理科同時沒有修讀經濟科。

Question 3

There are 120 form four students in a school. 70 of these students take geography. 80 of them take economics. 35 of them take both geography and economics.

How many of these students take neither geography nor economics?
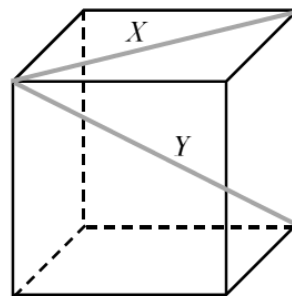
(2 marks)

Answer: There are _____ students who take neither geography nor economics.

題 4

圖(4)為一個正立方體，連接兩個不相鄰頂點的線段稱為「對角線」，例如，圖中 *X* 及 *Y* 均為「對角線」。問一個正立方體共有多少條「對角線」？

(2 分)

答: 正立方體共有 _____條「對角線」。

圖(4)
Figure (4)

Question 4

Figure (4) shows a cube. A 'diagonal' is a line segment connecting two non-adjacent vertices.

For example, *X* and *Y* in the figure are both 'diagonals'. How many diagonals does a cube have?

(2 marks)

Answer: A cube has _____ diagonals.

若在圖(5)的九宮格中填入 1、2 或 3 使得每一行、每一列都有 1、2、3 這三個數字，求 $A+B$ 的值。(其中兩格已填入數字。)

(2 分)

答： $A + B =$ _____ 。

圖(5)

Figure (5)

1		
	2	← A
		← B

Question 5

If the 3×3 grid diagram in figure (5) is filled such that each of the three numbers 1, 2 and 3 appear in every row and every column. Find the value of $A+B$. (Two of the squares are filled.)

(2 marks)

Answer: $A + B =$ _____ .

題 **6**

已知 $\dfrac{1}{a-2013} = \dfrac{1}{b+2014} = \dfrac{1}{c-2015} = \dfrac{1}{d+2016}$，請把 a、b、c 及 d 由小到大排列。

(2 分)

答： _____ < _____ < _____ < _____

Question 6

It is given that $\dfrac{1}{a-2013} = \dfrac{1}{b+2014} = \dfrac{1}{c-2015} = \dfrac{1}{d+2016}$, arrange a, b, c and d in ascending order of magnitude.

(2 marks)

Answer: _____ < _____ < _____ < _____

題 7

已知 x、y、$\sqrt{2016x}$ 及 $\sqrt{2016+y}$ 均為正整數。求 x 和 y 最小值。

(3 分)

答: x 的最小值為 ＿＿＿＿＿。y 的最小值為 ＿＿＿＿＿。

Question 7

It is given that x, y, $\sqrt{2016x}$ and $\sqrt{2016+y}$ are all positive integers.

Find the smallest possible values of x and y.

(3 marks)

Answer:

The smallest possible value of x is ＿＿＿＿＿. The smallest possible value of y is ＿＿＿＿＿.

題 8

媽媽對女兒說：「當我在你現在的年齡時，你只是 7 歲。但當你在我現在的年齡時，我已是 88 歲了。」請問媽媽及女兒現在多少歲？

(3 分)

答: 媽媽現在＿＿＿＿歲。女兒現在＿＿＿＿歲。

Question 8

A mother says to her daughter, "When I was your age, you were only 7. But, when you become my age, I will be 88 years old." What are the present ages of the mother and that of the daughter?

(3 marks)

Answer : The mother is now ＿＿＿＿ years old. The daughter is now ＿＿＿＿ years old.

題 9

圖(9)為一個邊長為 8 單位的正八邊形 *ABCDEFGH*。求 *AF* 的長度。

(3 分)

答:　*AF* 的長度是＿＿＿＿＿＿＿。

圖(9)
Figure (9)

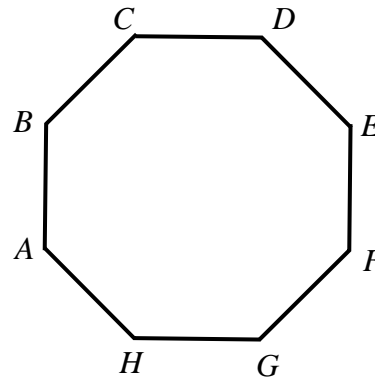## Question 9

Figure (9) shows a regular octagon *ABCDEFGH* of side 8. Find the length of *AF*.

(3 marks)

Answer: The length of *AF* is ＿＿＿＿＿＿＿.

題 10

已知 $x + y = 1$，$x + z = 2$ 及 $y + z = 3$，求 $x^2 + y^2 + z^2 + xy + yz + zx$ 的值。

(3 分)

答:　　$x^2 + y^2 + z^2 + xy + xz + yz = $ ＿＿＿＿＿＿＿＿＿＿。

Question 10

Is is given that　$x + y = 1$, $x + z = 2$ and　$y + z = 3$. Find the value of　$x^2 + y^2 + z^2 + xy + yz + zx$.

(3 marks)

Answer :　　$x^2 + y^2 + z^2 + xy + xz + yz = $ ＿＿＿＿＿＿＿＿＿＿.

題 **11**

袋中有紅色、白色及黃色襪子各有 2016 隻，若隨意從該袋中取出襪子，問最少要取出多少隻襪子，才能肯定當中最少有兩對不同顏色的襪子？

(3 分)

答：　須取出＿＿＿＿＿隻襪子。

Question 11

A bag contains red, white and yellow socks. There are 2016 socks for each colour. If the socks are taken from the bag at random, at least how many socks are to be taken to make sure that there are two pairs of socks of different colours?
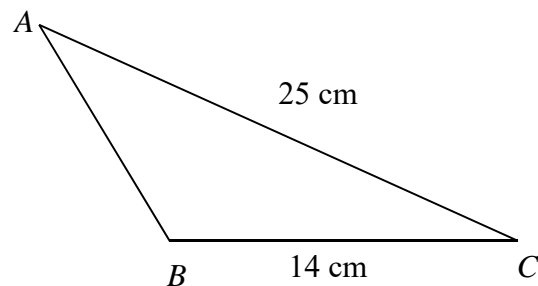
(3 marks)

Answer: _____ socks have to be taken out.

題 **12**

圖(12)中，ΔABC 為一面積為 49 cm^2 的三角形，其中 $BC = 14$ cm 及 $AC = 25$ cm。

若 $AB = \sqrt{x}$ cm，求 x 的值。

(3 分)

答：　$x =$ ＿＿＿＿＿。

圖(12)
Figure (12)

Question 12

In figure (12), ΔABC is a triangle of area 49 cm^2. $BC = 14$ cm and $AC = 25$ cm.

If $AB = \sqrt{x}$ cm, find the value of x.

(3 marks)

Answer: $x =$ _____.

在一集會中，每一個人都與其他每一個人握一次手。如果集會中握手的總次數為 2016 次，那麼該集會中共有多少人？ (註：A 和 B 握手、B 和 A 握手只算同一次)

(3 分)

答：　集會中共有 ＿＿＿＿＿＿ 人。

Question 13

In a party, every person shakes hand once with each of the other persons. If there are altogether 2016 handshakes in this party, how many persons are there in this party?(Note : A shaking hand with B and B shaking hand with A will be counted as ONE handshake only.)

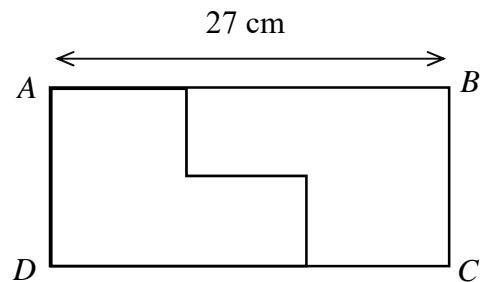(3 marks)

Answer:　There are ＿＿＿＿＿＿ persons in the party.

圖(14)中，*ABCD* 為一長方形，*AB* = 27 cm。若將該長方形 *ABCD* 如圖所示分割成兩個六邊形，然後再將該兩個六邊形併合，可成一個正方形。求這正方形的周界。

(3 分)

答：　正方形的周界為 ＿＿＿＿＿ cm。

圖(14)
Figure(14)

Question 14

Figure(14) shows a rectangle *ABCD* with *AB* = 27 cm. If *ABCD* is be cut into two hexagons as shown in the figure, the two hexagons can then be fitted together to form a square. Find the perimeter of this square.

(3 marks)

Answer: The perimeter of the square is ＿＿＿＿ cm.
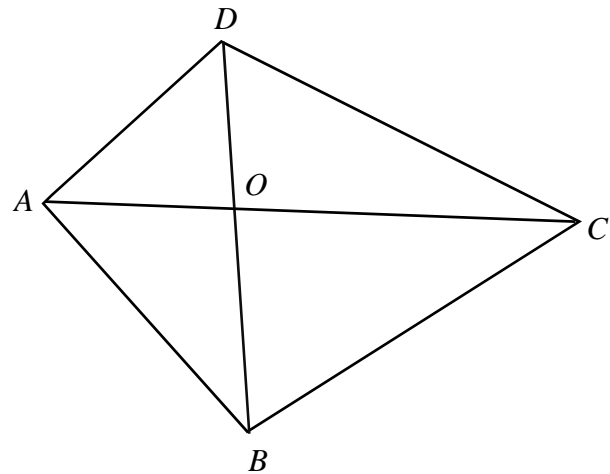
圖(15)中，*ABCD* 為一個四邊形，對角線 *AC* 及 *BD* 相交於點 *O*。

其中 ΔOAD 的面積 $= 20$ ， ΔOBC 的面積 $= 45$。

若 *ABCD* 的面積為 *S*，求 *S* 的最小值。

(3 分)

答: *S* 的最小值是 _____。

圖(15)
Figure(15)

Question 15

Figure(15) shows a quadrilateral *ABCD*. The diagonals *AC* and *BD* intersect at *O*.

Area of ΔOAD = 20. Area of ΔOBC = 45.

If the area of *ABCD* is *S*, find the smallest possible value of *S*.

(3 marks)

Answer: The smallest possible value of *S* is _____.

題 16

在某次中一級班際 100 米接力比賽中，1A、1B、1C、1D 當中三班得冠、亞、季軍。小強、小明和小華在賽前分別預測了四班的成績，情況如下：

	1A	1B	1C	1D
小強的預測	季軍	沒得獎	亞軍	冠軍
小明的預測	沒得獎	冠軍	亞軍	季軍
小華的預測	季軍	亞軍	沒得獎	冠軍

就這三個作了預測的，其中一人於四班的成績都全預測錯誤，其中一人對於其中兩班的成績預測正確，剩下的一個預測得比這兩個都好。

比賽的冠、亞、季軍分別是哪三班?

(3 分)

答: 冠軍是 _____，亞軍是 _____，季軍是 _____。

Question 16

In a S1 inter-class 100m relay competition, three classes among 1A, 1B, 1C and 1D got the first, second and third places. John, Michael and William made predictions about the prizes for the four classes. Their predictions were as follows:

	1A	1B	1C	1D
John's predictions	Third	No prize	Second	First
Michael's predictions	No prize	First	Second	Third
William's predictions	Third	Second	No prize	First

Among the three who predicted the results, one had all the predictions wrong. One predicted the correct results for two of the classes. The remaining one made better predictions than the other two.

Which classes got the first, second and third places respectively in the race?

(3 marks)

Answer: _____ got the first place. _____ got the second place. _____ got the third place.

題 17

圖(17)中有四邊形 *PQRS*，其中 *PQ* ⊥ *QR*、*RS* ⊥*SP*，*PS* = 8 cm、*QR* = 6 cm。求 *RS* 的長度。(提示: 可考慮一對合適的直角三角形。)

(3 分)

答: *RS* = _____ cm。

Q

6 cm

圖(17)
Figure (17)

R

60°

P

S

8 cm
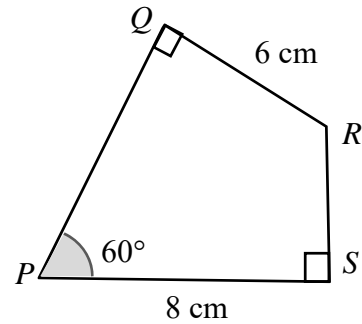
Question 17

Figure (17) shows a quadrilateral *PQRS*. *PQ* ⊥ *QR* and *RS* ⊥ *SP*. *PS* = 8 cm and *QR* = 6 cm. Find the length of *RS*. (Hint: Consider a pair of right-angled triangles.)

(3 marks)

Answer: *RS* = _____ cm.

題 18

設 a_1、a_2、 …、a_{2015}、a_{2016} 為 2016 個正整數，其中 $a_1 < a_2 < … < a_{2016}$。若隨意從這 2016 個正整數選 2015 個數，並計算其平均值，不同數字的選取所得平均值全不相同，且設為 x_1、x_2、…、x_{2015} 及 x_{2016} ，其中 $x_1 < x_2 < … < x_{2016}$。

求 $\dfrac{a_1 + a_2 + \cdots + a_{2016}}{x_1 + x_2 + \cdots + x_{2016}}$ 的值。

(4 分)

答: $\dfrac{a_1 + a_2 + \cdots + a_{2016}}{x_1 + x_2 + \cdots + x_{2016}} = $ _____ 。

Question 18

Let a_1, a_2, …, a_{2015}, a_{2016} be 2016 positive integers, where $a_1 < a_2 < … < a_{2016}$.

If we choose any 2015 integers from these 2016 integers and calculate their average, a different average value will be obtained from a different choice of numbers. Let these averages be x_1, x_2, …, x_{2015} and x_{2016} where $x_1 < x_2 < … < x_{2016}$.

Find the value of $\dfrac{a_1 + a_2 + \cdots + a_{2016}}{x_1 + x_2 + \cdots + x_{2016}}$.

(4 marks)

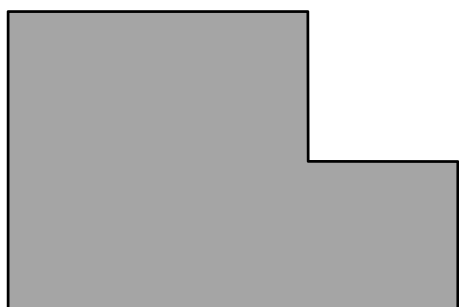Answer: $\dfrac{a_1 + a_2 + \cdots + a_{2016}}{x_1 + x_2 + \cdots + x_{2016}} = $ _____.

題 19 (動手題)

圖(19a) 為一個 L-形的圖形。這圖形可分割成若干部分再重併成一個正方形。

a. 於答題紙的圖(19b)方格中畫出一個與這 L-形面積相等的正方形。

b. 你們桌上有一顏色紙，形狀與圖(19a) L-形圖形全等，將顏這色紙圖形剪成不多於 5 部分，再將各部併合於所(a)部所畫出的正方形中，並以膠水或膠紙固定。

(桌上另有白色全等圖形的紙樣可作練習之用。)

(4 分)

圖(19a)
Figure(19a)

圖(19b)
Figure(19b)

Question 19 (Hands-on Question)

The L-shape figure shown in figure (19a) can be cut into several parts which can be rearranged to form a square.

a. On your answer sheet, draw a square in the box in figure (19b) which is of the same area as the L-shape.

b. On your desk, there is a piece of colour paper whose shape is congruent to L-shape shown in figure(19a). Cut the colour figure into at most 5 pieces. Rearrange all pieces to fit into the square drawn in part (a). Fix the pieces with glue or adhesive tapes.

(On your desk are the congruent L-shape in white papers for your practice.)
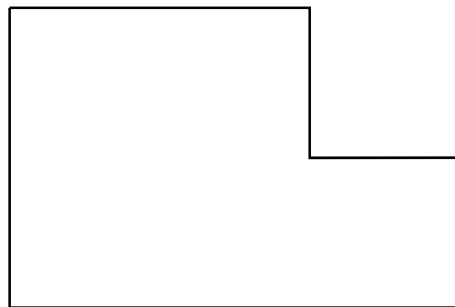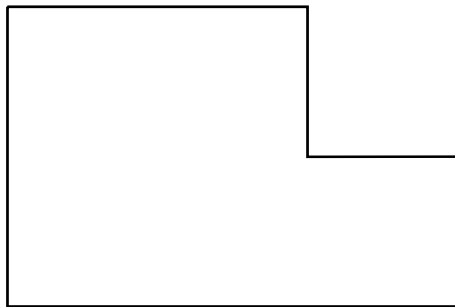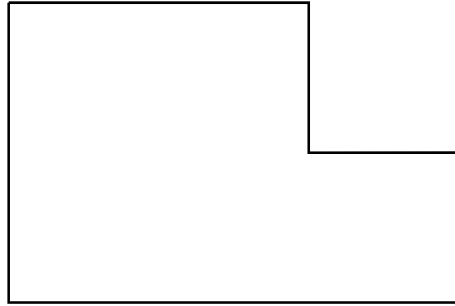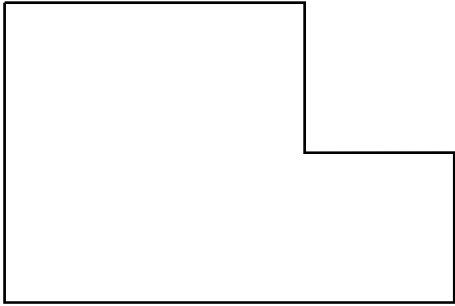
(4 marks)

全卷完

End of Paper

Question 19 – Appendix

The following figures can be cut out for practice. Only the cut out pieces fitted on the answer sheet will be marked as answer. Only one red figure will be given to each team.