2014/15 第十屆香港小學數學創意解難比賽

(初賽-筆試)

2014/15 The 10th Hong Kong Mathematics Creative Problem Solving Competition for Primary Schools

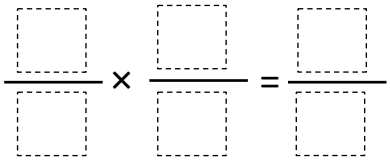
(Heat – Written Test)

題(1)

從 3, 4, 5, 6, 7, 8, 9, 10 中選出六個不同的數字，把每個數字填入圖(1) 的一個 方格中，使之組成正確的算式。 (2 分)

Question (1)

Choose six different numbers from 3, 4, 5, 6, 7, 8, 9 and 10. Fill each box in figure (1) with one of these numbers to show a correct arithmetic operation. (2 marks)



圖(1)

Figure (1)

題(2)

圖(2a) 是一個幾何板，板上有 20 個點排成兩行和兩列，行和列互相垂直且相鄰的點距離相等。

幾何板上的點可以直線連結成多邊形 (如圖(2b))，而圖(2c) 的正方形是這幾何板上可構成最小的正方形，這正方形的面積為 1 cm^2。

a. 於答題紙的圖(2d)上， 將一些點連結以畫出一個面積 2 cm^2 的正方形。
b. i. 於答題紙的圖(2e)上，畫出這幾何板上可構成面積最大的正方形。
   ii. 這正方形的面積是多少?

(3 分)

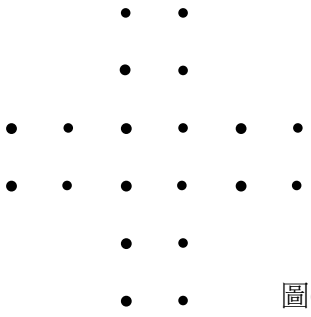
Question (2)

Figure (2a) shows a geoboard with 20 dots placed in two rows and two columns. The rows and columns are perpendicular to each other. All neighboring dots are equally spaced.

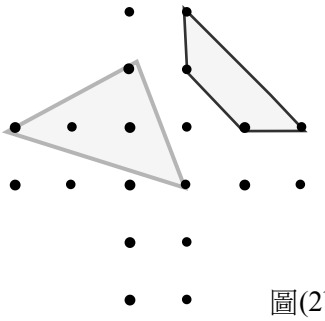
Polygons can be formed on the geoboard by connecting dots with straight lines, as shown in figure (2b). The square formed in figure (2c) is the smallest square that can be formed on this geoboard. The area of this square is 1 cm^2.

a. In figure (2d) in the answer sheet, draw a square of area 2 cm^2 by connecting the dots on the geoboard.
b. i. In figure (2e) in the answer sheet, draw a square of greatest possible area that can be formed on this geoboard.
   ii. What is the area of this square?

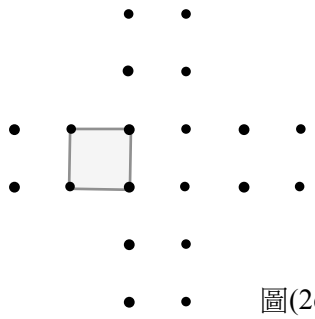
(3 marks)



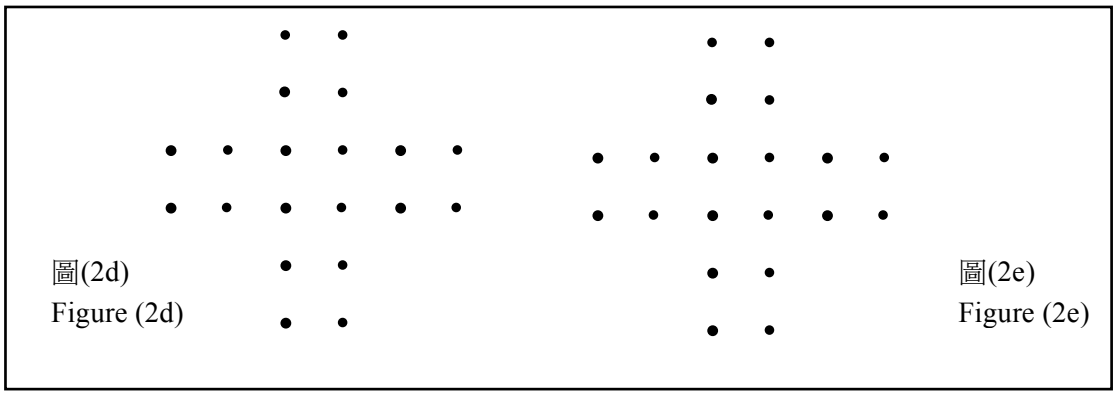
圖(2a)
Figure (2a)



圖(2b)
Figure (2b)



圖(2c)
Figure (2c)



圖(2d)
Figure (2d)

圖(2e)
Figure (2e)

題(3)

圖(3a)中，一個等邊三角形紙片按所示的步驟折疊三次，得到一個較小的三角形。

現於折疊的三角形上通過兩邊的中點剪去一角(如圖(3b))，再將剩下的紙片展開放平。

於答題紙的圖(3c)中，畫出該剩下紙片的圖形。 (2 分)

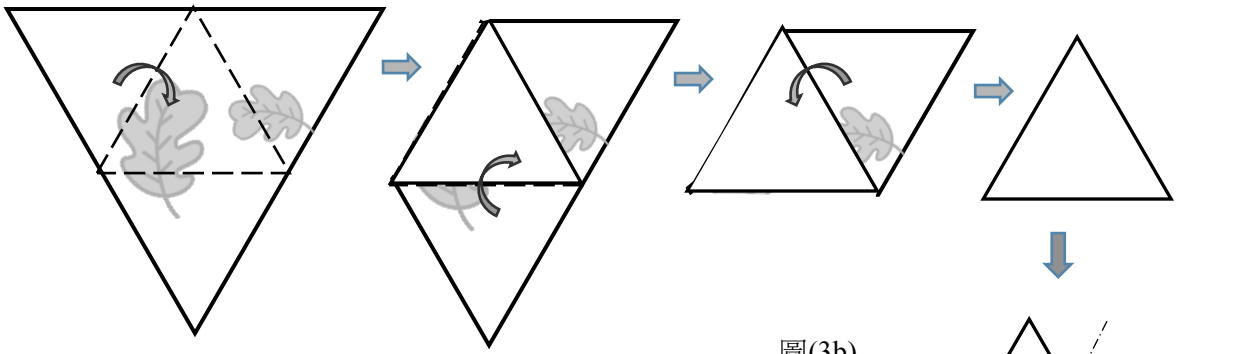
Question (3)

As shown in figure (3a), a piece of paper in the shape of an equilateral triangle is folded three times to form a smaller triangle.

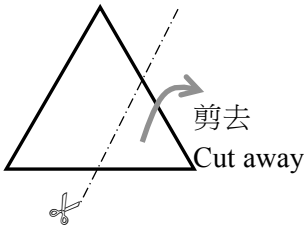
A cut is made across the midpoints of two sides of the folded triangle to cut away a corner (Figure (3b)). The remaining piece is then unfolded.

In figure (3c) in the answer sheet, draw the shape of the remaining piece. (2 marks)

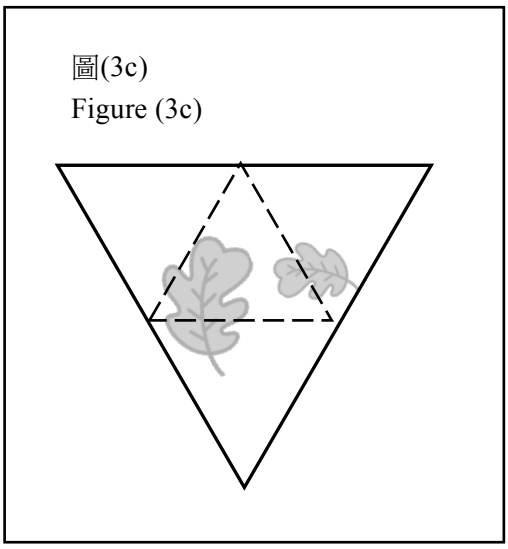
圖(3a)
Figure (3a)



圖(3b)
Figure (3b)



圖(3c)
Figure (3c)



題(4)

計算 $2015.02\dot{0}\dot{7} + 0.\dot{1}2\dot{3}$ 。 (2 分)

註:

循環小數是一個無限長的小數，這小數中有一個不斷重覆的部分。表達循環小數時通常會在它的一個或兩個數位上加上一點以表示重覆的部分。例如

$1.23\dot{4}$ 代表循環小數 1.23444…. (其中 '4' 不斷重覆)。

$1.2\dot{3}\dot{4}$ 代表循環小數 1.234343434…. (其中 '34' 不斷重覆)。

$42.\dot{1}00\dot{3}$ 代表循環小數 42.100310031003… (其中'1003'不斷重覆)。

Question (4)

Evaluate $2015.02\dot{0}\dot{7} + 0.\dot{1}2\dot{3}$. (2 marks)

Note :

A repeating decimal is an infinitely long decimal with one or two dot(s) above the numerals to indicate the repeating part. For example:

$1.23\dot{4}$ represents the repeating decimal 1.23444…. (with '4' repeating indefinitely).

$1.2\dot{3}\dot{4}$ represents the repeating decimal 1.234343434…. (with '34' repeating indefinitely).

$42.\dot{1}00\dot{3}$ represents the repeating decimal 42.100310031003… (with '1003' repeating indefinitely).

題(5)

圖(5)中，一個長方形被分割成 9 個不同大小的正方形。

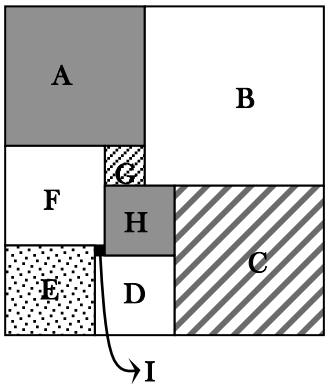
已知正方形 A、C 及 I 的邊長分別是 14、15 及 1，求正方形 B 的邊長。 (2 分)

Question (5)

In the figure (5), a rectangle is cut into 9 squares of different sizes.

It is known that the squares A, C and I are of sides 14, 15 and 1 respectively.

Find the length of side of the square B. (2 marks)



圖(5)
Figure (5)

題(6)

圖(6)中，ABCD 及 DEFG 為正方形，邊長分別為 16 及 6。ADE 與 GDC 皆為直線。

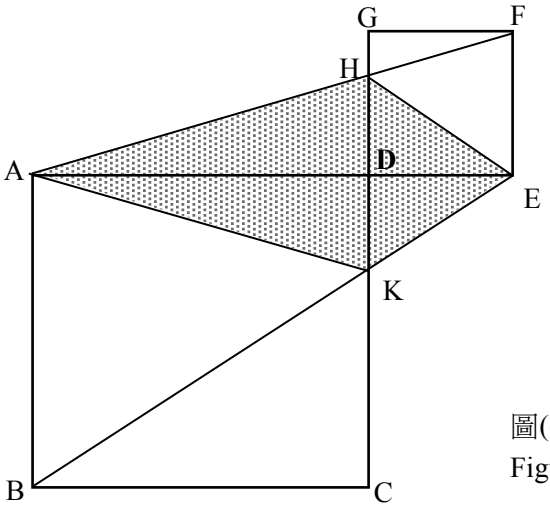
AF 及 GD 相交於 H，BE 及 DC 相交於 K。

求四邊形 AKEH 的面積。 (2 分)

Question (6)

In figure (6), ABCD and DEFG are squares of sides 16 and 6 respectively. ADE and GDC are straight lines. AF intersects GD at H. BE intersects DC at K.

Find the area of AKEH. (2 marks)



圖(6)
Figure (6)

題(7)

A、 B、 C、 D 四人共賽跑四次。

A 的成績很好，得了兩次第一名，且從未排最後。

B 的表現很平均，四次比賽中取得四個不同的名次。

C 最愛跟 A 較量。綜合四次結果，他倆勝過對方的場數相同。

D 每次都輸給 C，但 D 卻能在其中一場比賽中贏了 A 和 B。

第一場比賽結果由第一至第四名依次為 A、B、C、D。試列出其他三場比賽的結果。

(3 分)

Question (7)

A, **B**, **C** and **D** competed in four races.

A did excellently well. He came first in two of the races and did not come last in any one race.

B did fairly well. He got all the four different ranks in the four races.

C always wanted to compare with **A.** Over the four races, they beat one another in equal number of races.

D was behind **C** in all races. However, **D** was able to beat both **A** and **B** in one of the races.

In the first race, the first to the fourth places in order were **A**, **B**, **C** and **D**. Write down the results of the other three races?

(3 marks)

題(8)

志強跑步的路徑是一段單車與跑手共用的路段，這路段從起點 A 至終點 B 全長 8 km。志強以 9 km/h 的均速在這路上從 A 跑到 B。

一隊少年單車隊的 45 個成員亦在這時在這路段上練習，每個單車隊員均以 15 km/h 的速度由 A 騎行至 B。第一個單車隊員在志強出發後 10 分鐘開動，其後每隔 2 分鐘便有一位隊員出發。

a. 志強從起點 A 跑到終點 B 期間，共有多少部單車隊的單車追上他?

b. 若志強到終點 B 後，立即掉頭以 6 km/h 的速度跑回起點 A，這回頭路上他又會遇上多少部單車隊的單車?

(3 分)

Question (8)

Johnny jogs along a road that cyclists and joggers run side by side. The distance between the starting point A and end point B is 8 km. Johnny jogs from A to B with a constant speed of 9km/h.

45 cyclists of a junior cycling team also practice cycling along the same road. All cyclists cycle from A to B with the speed of 15 km/h. The first cyclist starts 10 minutes after Johnny. Then the other cyclists start one by one at 2 minutes' interval.

a. When Johnny jogs from the starting point A to the end point B, how many cyclists of the team will overtake him?

b. Johnny turns back immediately after reaching the end point B and jogs back to A with a speed of 6 km/h. How many more cyclists of the team will he meet on his way back?

(3 marks)

題(9)

將 2015 寫成連續數的和，共有好幾種方法。

例如: 2015 = 1007 + 1008 或 2015 = 401 + 402 + 403 + 404 + 405 。

寫出其中用上最多個連續數相加的方法。

2015 = _____ + _____ + _____ + …… + _____

(3 分)

Question (9)

There are several ways to express 2015 as the sum of consecutive numbers.

For example: 2015 = 1007 + 1008 or 2015 = 401 + 402 + 403 + 404 + 405.

Write down the way that uses the greatest number of consecutive numbers in the sum.

2015 = _____ + _____ + _____ + …… + _____

(3 marks)

題 (10)

志文、志强和志玲有許多綠色和紅色的螢光棒，全部同一長度。他們要將其中 8 條組成如圖 (10) 的一個裝飾。在這裝飾中，2 條螢光棒被屈曲成圓形，另外的 6 條則連結著兩個圓形並作相等距離的排列。

a. 志文取了 2 條綠色螢光棒和 6 條紅色螢光棒，他有多少種不同的方法組成這個裝飾?

b. 志强取了 4 條綠色螢光棒和 4 條紅色螢光棒，他有多少種不同的方法組成這個裝飾?

c. 志玲只想她的裝飾中，有不同數目的綠色和紅色螢光棒，她有多少種不同的方法組成這個裝飾?

(4 分)

註:

裝飾可自由以各方向轉動或翻動，兩個裝飾若經這些轉動或翻動後變成相同，它們只算作以同一種方法組成。

Question (10)

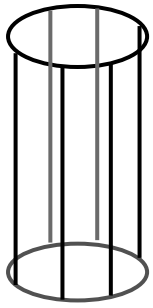
Peter, Paul and Mary have a lot of green and red glow sticks. All sticks are of the same length. They want to put 8 glow sticks together to make a decoration as shown in figure (10). In this decoration, 2 glow sticks are bent into circles. The other 6 glow sticks are spaced equally to connect the circles.

a. Peter takes 2 green sticks and 6 red sticks. In how many different ways can he put them together to make the decoration?
b. Paul takes 4 green sticks and 4 red sticks. In how many different ways can he put them together to make the decoration?
c. Mary wants her decoration to have different number of green and red sticks. In how many different ways can she put them together to make the decoration?

(4 marks)

Remark:

The decoration can be turned or flipped in any directions. Two decorations that appear the same after these turns or flips are counted as the same design.



圖(10)

Figure (10)

題 (11)

邦邦正學習化簡分數和比較它們的大小。他每天都寫出一個系列的數字，並將它們按大小排序。

第 1 天，他寫出兩個數: $\frac{0}{1},\frac{1}{1}$

第 2 天，他再加入一個以 **2** 為分母的最簡真分數: $\frac{0}{1},\frac{1}{2},\frac{1}{1}$

第 3 天，他再加入兩個以 **3** 為分母的最簡真分數: $\frac{0}{1},\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{1}{1}$

第 4 天，他再加入以 **4** 為分母的最簡真分數: $\frac{0}{1},\frac{1}{4},\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{3}{4},\frac{1}{1}$

……

他每天如是加入新的最簡真分數，新增分數的分母都比前一天增添的增加 1，增添分數後再將所有數按大小排序。

a. 如是到了第 26 日，他的數列上共有 213 個數，這數列是 " $\frac{0}{1},\frac{1}{26},\frac{1}{25},\ldots\ldots,\frac{1}{1}$"。

   i. 在這數列中，最後的三個數是甚麼?

   ii. 邦邦觀察所得，從第 2 天的數列開始，$\frac{1}{2}$ 這分數總是在數列的中央。在這天的數列中，在 $\frac{1}{2}$ 之前的一個和之後的一個分數是甚麼?

   iii. 在這天的數列中，邦邦觀察到從 $\frac{1}{26}$ 開始，接著一連串的分數的分子都是 1。這一連串的分數共有多少個?

b. 如果邦邦每天繼續這個練習直到第 29 天，那天他的數列裏共有多少個數?

(6 分)

Question (11)

Bobby was learning to simplify fractions and to compare their sizes. He made a new list of numbers every day and arranged them in order of their sizes.

On day 1, he listed two numbers: $\frac{0}{1}, \frac{1}{1}$.

On day 2, he added another simple proper fraction with denominator **2**: $\frac{0}{1}, \frac{1}{2}, \frac{1}{1}$.

On day 3, he added another two simple proper fractions with denominator **3**: $\frac{0}{1}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{1}$.

On day 4, he added in new simple proper fractions with denominator **4**: $\frac{0}{1}, \frac{1}{4}, \frac{1}{3}\ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{1}{1}$.

……

Every day, he added in new simple proper fractions with denominator greater than the previous day's by 1. The new fractions are inserted in places such that all numbers are ranked in order of their sizes. He kept doing the exercise every day.

a. On day 26, he had a list with 213 numbers. The list went as " $\frac{0}{1}, \frac{1}{26}, \frac{1}{25}, \ldots\ldots, \frac{1}{1}$ "

   i. What were the last three numbers in this list?

   ii. Bobby observed that, starting from day 2, the fraction $\frac{1}{2}$ was always in the middle of the list. In this list, what were the fraction immediately before $\frac{1}{2}$ and the fraction immediately after $\frac{1}{2}$?

   iii. Bobby also observed that, starting with the term $\frac{1}{26}$ in this list, there was an unbroken chain of fractions with numerator 1. How many fractions were there in this unbroken chain?

b. If Bobby kept working with this exercise every day until day 29, how many numbers would there be in the list on that day?

(6 marks)

題 (12)

圖(12a)中，三角形 ABC 的三邊各自向外伸延一倍的長度，即 AP = 2 AB、 BQ = 2 BC 及 CR = 2 CA，再以延伸線段的端點擴展成較大的三角形 PQR。

在圖(12b)中，以上述的ΔABC 和ΔPQR 作為圖案 1 及圖案 2，將圖形繼續發展成更大的三角形 XYZ (即圖案 3，其中 PX = 2 PR、QY = 2 QP 及 RZ = 2 RQ)，並依循此方式擴展出愈來愈大的三角形。

a. 若ΔABC 的面積為 1 cm^2，圖(12b)中 ΔXYZ 的面積是多少?
b. 若以這個方式不斷擴展並將每一次新增的部分反覆填上白色和灰色 (如圖(12b))，在哪一個圖案中灰色部分的面積會首次超越 9000 cm^2?

(4 分)

Question (12)

In figure (12a), the triangle ABC grows by extending its three sides outward by doubling their lengths, i.e. AP = 2 AB, BQ = 2 BC and CR = 2 CA. The ends of the extension then form a bigger triangle PQR.

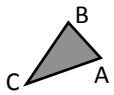
In figure (12b), with ΔABC as Pattern 1 and ΔPQR as described above as Pattern 2, the pattern further expands in the same way to form an even bigger triangle XYZ (i.e. as in Pattern 3, where PX = 2 PR, QY = 2 QP and RZ = 2 RQ). It then 'expands' in the same way to form bigger and bigger triangles.

a. If the area of ΔABC is 1 cm^2, what is the area of the ΔXYZ in figure (12b)?

b. If the above expansion goes on with the newly added parts colored alternately in white and in grey, as shown in figure (12b), in which pattern will the total area of the grey regions first exceed 9000 cm^2?

(4 marks)

圖(12a)
Figure (12a)



圖(12b)
Figure (12b)

圖案 1
Pattern 1

圖案 2
Pattern 2

圖案 3
Pattern 3

圖案 4
Pattern 4

……

題 (13)

A、B、C、D 四人想要知道自己的體重。

他們只找到一個怪磅，這個磅只能準確地顯示 150 kg 至 200 kg 之間的整數個 kg 的重量。

這四人估計他們每三個人的體量的和應該在這範圍，所以安排每三個一同在怪磅上量重一次，從而再計算各自的體重

結果如下: A、B 和 C 的重量合計為 193 kg；

B、C 和 D 的重量合計為 155 kg；

C、D 和 A 的重量合計為 199 kg 。

但當 D、A 和 B 一同量重時，怪磅只能顯示 14♣ kg，最後一個數位未能清楚顯示。

a. 從量得的結果計算 C 的最小可能體重。

b. 只從以上結果，根本無從得知四人各自體重。但他們認為只需再用上這怪磅一次，便有方法計算各人體重。你覺得有可能嗎? 若有可能，這一次該量甚麼？

(4 分)

Question (13)

Four persons A, B, C and D wanted to find out their own weights.

They found a strange weighing scale that could only show the accurate weight between 150kg and 200kg in integral number of kg.

They estimated that the total weight of any three of them would be in this range. So, they put three of them on the weighing scale each time. Then, they could calculate their individual weights from the results.

They found that the total weight of A, B and C was 193 kg.

The total weight of B, C and D was 155 kg.

The total weight of C, D and A was 199 kg.

When D, A and B were put on the weighing scale, the weight was displayed as 14♣ kg. The last digit cannot be shown clearly.

a. Find the least possible weight of C from the results.

b. With the above results, the individual weights of the four persons could not be found. They thought they could certainly find their weights if they could use this weighing scale one more time. Do you think that it is possible? If yes, what is/are to be weighed?

(4 marks)

題 (14) : 動手題

在桌上有 4 個預先摺成的組件，形狀如圖(14)。請將組件拼合成一個具備以下特性的立體:

(1) 具對稱特質；
(2) 有 5 個面；
(3) 有 8 條稜邊。

組件必須穩固地拼合成立體，且過程中不可使用剪刀、膠水或膠紙等工具。

完成後，請先將成品以紙杯覆蓋，並舉手示意請監考員到桌前檢查完成品及評分。

(3 分)

Question (14) : Hands-on Question

4 folded components, in the shape as shown in figure (14), are distributed on your table. Put the components together to form a 3-D shape with the following properties:

(1) It is symmetrical.
(2) It has 5 faces.
(3) It has 8 edges.

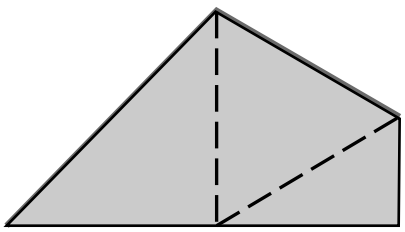
The components must be securely put together to form the 3-D shape. No scissors, glue, adhesive tape nor similar tools can be used in the process.

When the task is done, cover the product with the paper cup. Raise your hand to invite the invigilator to come to your desk to examine and grade the product.

(3 marks)

圖(14)

Figure (14)



[End of Paper]