

# 2014/15 第六屆香港中學數學創意解難比賽

## (決賽-解難實驗)

### Measuring Improvement in Learning 學習改善的量度

1. The pre-S1 placement mathematics test scores and the S1 mathematics final examination scores of the five S1 classes of a school are given in the Excel file provided. Calculate the mean score of the pre-S1 mathematics placement test for each class. Rank the classes accordingly (1 represents the best, 5 represents the worst).

在試算表檔案中已提供某校中一各班的入學分班試數學卷及中一期末數學考試的成績。試計算各班分班試數學卷的平均值，並按成績排列優次。(1 代表最佳，5 為最差。)

Class 班別	1A	1B	1C	1D	1E
Mean of pre-S1 placement test scores 分班試分數平均值	79.7 [1A]	66.3 [1A]	48.9 [1A]	43.7 [1A]	61.5 [1A]
Ranking 排名(1-5)	1	2	4	5	3

**\*\*\*\*Correct ranking [2A]**

2. After one academic year, the S1 classes attended the mathematics final examination. Calculate the mean score of the final mathematics examination for each class. Rank the classes accordingly (1 represents the best, 5 represents the worst).

到學期末，中一級學生進行期末數學考試。試計算各班期末考試的平均值，並按成績排列優次。(1 代表最佳，5 代表最差。)

Class 班別	1A	1B	1C	1D	1E
Mean of final examination scores 期末試分數平均值	73.4 [1A]	58.5 [1A]	53 [1A]	52 [1A]	58.9 [1A]
Ranking 排名(1-5)	1	3	4	5	2

**\*\*\*\*Correct ranking [2A]**

3. In the Excel worksheet and the Annex (a) to Annex (e), the scatter diagrams of the final mathematics examination scores against the pre-S1 placement mathematics test scores of the classes are given. On the diagram for class 1A, a *representative* straight line is drawn. It is expected that the equation of this straight line can best describe the relation between the two sets of scores.

Describe a way to draw such a *representative* straight line.

在 Excel 工作表中及本卷的附件(a)至附件(e)中的散點圖，顯示了各班的期末班試分數及分班試分數。其中 1A 班的散點圖中畫有一條有「代表性」的直線，此直線的方程應可用以代表兩組分數的關係。

試形容畫出這「代表性」直線的方法。

- Linear regression (Take this one as an example)
- no. of points above the line = no. of points below the line [3A for a reasonable answer]
- Other reasonable approaches

The equation of a straight line can be written as:  $y = mx + c$ ,  
where **m** is the **slope** and **c** is the **y-intercept**.

As indicated in the class 1A diagram,  
the y-intercept **c**, where the line cut the y-axis, is approximately -0.32.

The slope **m** can be calculated by using the y-intercept **c** and any other point  $(x_0, y_0)$  chosen on the straight line:

$$m = \frac{y_0 - c}{x_0}$$

For example, (80,74) is a point on the straight line in the diagram of Class 1A.

The value of **m** can be calculated as:

$$m = \frac{74 - c}{80} = \frac{74 - (-0.32)}{80} \approx 0.93$$

So, the equation of the representative straight line for scores of class 1A is  $y = 0.93x - 0.32$ .

一條直線的方程可寫成  $y = mx + c$ ，其中  $m$  為斜率、而  $c$  為  $y$  軸截距。

在 1A 班的圖上， $y$  軸截距  $c$  (即直線與  $y$ -軸相交之處) 約為  $-0.32$ 。

而斜率  $m$  則可從  $y$  軸截距  $c$  及直線上選取的任何一點  $(x_0, y_0)$  以算出：
$$m = \frac{y_0 - c}{x_0}$$

例如:  $(80, 74)$  為 1A 班的圖上的直線上的一點，斜率  $m$  的值可計算成:

$$m = \frac{74 - c}{80} = \frac{74 - (-0.32)}{80} \approx 0.93$$

因此，1A 班分數的代表直線的方程為  $y = 0.93x - 0.32$ 。

4. Using Q3 and above hints, write down the equation of the 'representative' straight line for each class.

運用題(3)及上述提示，寫出各班分數的「代表性」直線的方程。

Class 班別	Straight line equation 直線方程
1A	$y = 0.93x - 0.32$
1B	$y = 0.923x - 2.64$ [2A]
1C	$y = 0.893x + 9.34$ [2A]
1D	$y = 0.752x + 19.1$ [2A]
1E	$y = 0.996x - 2.32$ [2A]

5. What is the meaning of the slope in Q4? What is the meaning of the mean of all slopes?

Q4 的斜率有什麼意義？所有斜率的平均值有何意義？

- Slope: The final examination score improvement per score of pre-S1 placement test for a particular class. [2A]
- Average of all slopes: The average of slope over individual class. [2A]

6. Calculate the mean of all slopes and the mean of all y-intercepts in Q4.

計算所有 Q4 斜率的平均值及所有 Y-軸截距的平均值。

<b>M</b> = Mean of all slopes <b>M</b> = 斜率平均值	0.898	[1A]
<b>C</b> = Mean of all y-intercepts <b>C</b> = Y-軸截距平均值	4.64	[1A]

7. A new straight line:  $y = \mathbf{M}x + \mathbf{C}$  is constructed by using the two answers of Q6 as **M** and **C** respectively. For a particular  $x$ , what is the meaning of the  $y$  calculated?

以 Q6 所計算出的兩個答分別作為 **M** 及 **C**，定義出一條新的直線： $y = \mathbf{M}x + \mathbf{C}$ 。對應於某個  $x$  值所計算出來的  $y$  有何意義？

A student of having pre-S1 placement test score  $x$  would on average obtain score  $y$  in the final examination. [2A]

8. Create a new index  $V$  and state clearly how to use it to measure the "improvement" in learning.

創作一個新的指數  $V$ ，並說明它如何量度學習的「改善」。

$$V = F_i - (MS_i + C)$$

*Note:  $V$  is in fact the Value addedness in literacy*

Where

$i = A, B, C, D$  and  $E$

$F_i$  is the average of final examination score of class  $i$

$S_i$  is the average of pre-S1 test score of class  $i$

[5M]

9. According to your answer in Q8, calculate the value of  $V$  for each class. Rank their learning improvement accordingly. (1 represents the best, 5 represents the worst).

根據你們在 Q8 的描述，計算各班的  $V$  值，並排列各班學習改善情況的優次。(1 代表最佳，5 為最差。)

Class 班別	1A	1B	1C	1D	1E
$V$	-2.73 [1A]	-5.63 [1A]	4.46 [1A]	8.13 [1A]	-0.95 [1A]
Ranking 排序(1-5)	4	5	2	1	3

\*\*\*\***Correct ranking [2A]**

10. According to Q9, the Principal wants to award the mathematics teacher of the class who got the highest "improvement" in learning. What are the limitations of using the index  $V$  to measure the "improvement"?

據 Q8 校長有意嘉許有最佳學習「改善」的一班的數學科老師。然而以這個指數  $V$  作為量度學習的「改善」的方法，有什麼局限？

- $V$  is an overall "improvement". This improvement may consist of teacher effort ( $V_t$ ), students effort ( $V_s$ ), private tutor effort ( $V_{pt}$ ), parents effort ( $V_p$ ) etc. i.e.  $V = V_t + V_s + V_{pt} + V_p$
- If the class size is small, the  $V$  calculated is not reliable. [2A for a reasonable answer]
- It is a linear model.

11. That teacher mentioned in Q10 was very humble. He said he got the highest  $V$  because his students studied very hard in the past academic year. Do you think his comment is justified mathematically?

Q10 中的老師為人謙虛，他說能有如此佳績，是學生過去一年的努力成果。你認為他的說法在數學上是否合理？

Yes, he is right. Referring to Q15's answer. i.e.  $V = V_t + V_s$  [2A]

12. Which teacher is more capable of helping students of lower mathematics ability to improve? Why?

哪一班的老師較能有效提升數學能力弱的學生？為什麼？

1D's teacher because he has the highest  $V$  and the slope is smallest. Smaller slope means the teacher is more able to improve low achievers. [3A]

13. Suppose you are good at mathematics. Which teacher do you prefer to be your mathematics teacher? Why?

假設你的數學成績理想，你會選哪一班的老師做你的數學老師？為什麼？

1E's teacher because he has high  $V$  and the slope is greatest. Greater slope means the teacher is more able to improve high achievers. [3A]

[End of Paper 全卷完]