

# 2023/24 第十四屆香港中學數學創意解難比賽

2/3/2024 (星期六) 10:25-11:30

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## 比賽時間：65 分鐘

參賽者須知：

1. 比賽時間：65 分鐘。建議在甲部用 50 分鐘作答，在乙部用 15 分鐘作答。
2. 本問題卷共 9 頁、答題紙 7 頁，甲部有 10 題數學題，乙部有 1 題創意解難題。
3. 每位參賽學生獲派一份問題卷及一份答題紙。
4. 比賽期間隊員可以討論題目，並於答題紙寫上議定的答案。  
\*\* 只有寫於隊長的答題紙上的答案方可得到評分。
5. 參賽隊伍需自備文具及計算機。為公平起見，比賽中只可使用非圖像計算機。本比賽中嚴禁使用電子字典、電腦、電話或其他有上網或通訊功能的工具。
6. 本試卷每頁的空白位置可作為草稿之用。每位參賽學生會獲派三張草稿紙，如有需要，可要求額外草稿紙。
7. 在筆試完結後，必須交回隊長的答題紙。

# 2023/24 The 14<sup>th</sup> Hong Kong Mathematics Creative Problem Solving Competition for Secondary Schools

2/3/2024 (Saturday) 10:25-11:30

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**Time allowed : 65 minutes**

Instructions for participants :

1. **Time allowed: 65 minutes.** It is advised to spend 50 minutes in Section A and 15 minutes in Section B.
2. The question paper consists of 9 pages, the answer sheet consists of 7 pages. There are 10 questions in Section A and 1 creative problem in Section B.
3. Each participant will get a set of question paper and a set of answer sheets.
4. Team members are allowed to discuss during the competition. The agreed answers should be written on the answer sheets.  
**\*\* Only the answers in the captain's answer sheet will be marked.**
5. Participating teams should bring their own stationery and calculators. For the purpose of fairness, please use only non-graphic calculators. Electronic dictionaries, computers, mobile phones and other online or communication devices are prohibited.
6. The blank space on each page of this question paper can be used for rough work. Each participant will get three rough work sheets. Extra rough work sheets will be provided upon request.
7. The captain's answer sheets will be collected after the competition.

甲部 (建議此部用 50 分鐘作答)

Section A (Suggested to use 50 minutes in this Section)

1. 以 2, 3, 2, 4 這四個數字及任何運算符號計算出以下數值，算式中必需使用每個數字一次，數字可以任何次序出現：

Use the 4 digits 2, 3, 2, 4 and any math operations to compute the following values, where each digit must be used and be used once only, and the numbers can be arranged in any order:

26 = \_\_\_\_\_

27 = \_\_\_\_\_

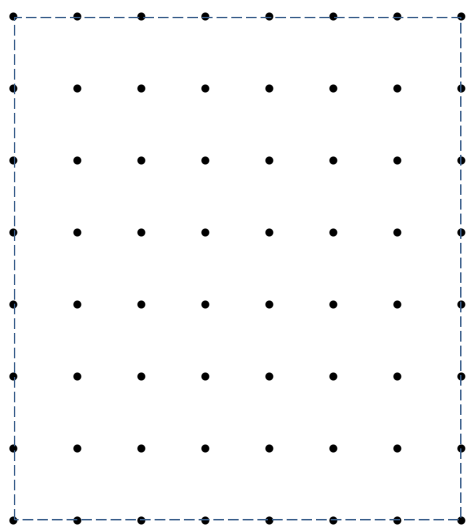
28 = \_\_\_\_\_

29 = \_\_\_\_\_

30 = \_\_\_\_\_

2. 下圖中直向及橫向每兩相鄰點的距離為 1 單位。試畫出三角形使其面積相等於虛線正方形的  
(a)  $\frac{1}{3}$ ; (b)  $\frac{1}{5}$ ; (c)  $\frac{1}{8}$ 。

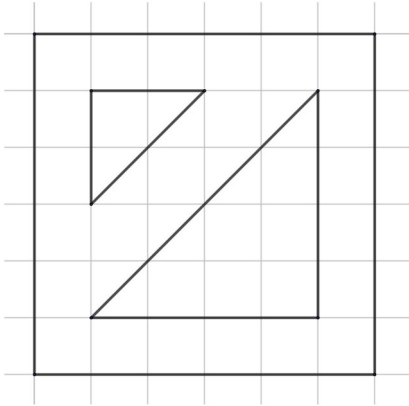
In the following figure, the vertical and horizontal distance between adjacent dots is 1 unit. Try to make triangles whose areas equal to (a)  $\frac{1}{3}$ ; (b)  $\frac{1}{5}$ ; (c)  $\frac{1}{8}$  of the square with dotted lines.



3. 下圖顯示兩個直角三角形。畫出第三個三角形使其與已知的兩個三角形相似，並使三個三角形的面積成：

In the figure, there are two right-angled triangles. Draw a third triangle similar to the given triangles inside the square such that the areas of the three triangles:

- (a) 一個等差數列；  
Form an arithmetic sequence;
- (b) 一個等比數列。  
Form a geometric sequence.



4. 考慮  $P = 20 \square 2 \square 4 \square 3 \square 2$ ，當中可在任何  $\square$  內填上乘號使其成為一條算式。  
例如， $P = 20 \times 2 \square 4 \square 3 \times 2$  的意思為  $P = 20 \times 243 \times 2 = 9720$ 。求  $P$  的值使其有  
Consider  $P = 20 \square 2 \square 4 \square 3 \square 2$ , Multiplication signs can be filled to any  $\square$  to form an expression.  
For example,  $P = 20 \times 2 \square 4 \square 3 \times 2$  means  $P = 20 \times 243 \times 2 = 9720$ . Find the values of  $P$  such that it has
- (a) 最多的正因數；  
the most number of positive factors;
- (b) 最少的正因數。  
the least number of positive factors.

5. 有一個長方形框架  $ABCD$ ，當中  $BC = 16$  cm。從  $A$  點發射一顆細小的彈珠至  $BC$  上的  $E$  點，當中  $BE = 6$  cm。當彈珠擊中框架的一邊時會反射；當彈珠擊中框架的一角時會沿原來的路徑反彈。  
當彈珠擊中框架  $n-1$  次後，它會在第  $n$  次返回  $A$  點。求  $n$  的值。  
In a rectangular frame  $ABCD$ ,  $BC = 16$  cm. A tiny marble shoots from  $A$  to a point  $E$  on  $BC$  such that  $BE = 6$  cm. When the marble hits the side of the frame, it rebounds. When the marble hits the corner of the frame, it returns back to its original path. After hitting the frame  $n-1$  times, it first goes back to  $A$  at the  $n$ th time. Find  $n$ .

6. 現有長方形  $ABCD$ ， $E$ 、 $F$ 、 $G$  及  $H$  分別為  $AB$ 、 $BC$ 、 $CD$  及  $AD$  上的點使得  $AE:EB = CG:GD = 3:2$  及  $BF:FC = DH:HA = 4:1$ 。  $P$  為  $ABCD$  內的一點使得  $BEPF$ 、 $CFPG$  及  $DGPH$  的面積分別為 5、11 及 19。求  $AHPE$  的面積。

In a rectangle  $ABCD$ ,  $E$ ,  $F$ ,  $G$  and  $H$  are points on  $AB$ ,  $BC$ ,  $CD$  and  $AD$  respectively such that  $AE:EB = CG:GD = 3:2$  and  $BF:FC = DH:HA = 4:1$ .  $P$  is a point in  $ABCD$  such that the areas of  $BEPF$ ,  $CFPG$  and  $DGPH$  are 5, 11 and 19 respectively. Find the area of  $AHPE$ .

7. 在一個  $4 \times 5$  的數字方格，以一個  $2 \times 2$  的正方形框架圍出四個數字。

In a  $4 \times 5$  number grid, four numbers are enclosed by a  $2 \times 2$  square mesh.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

框架內把右上及左下數字的乘積設為  $p$ ，把左上及右下數字的乘積設為  $q$ 。

The product of the numbers in the top right corner and in the bottom left corner of the mesh is denoted by  $p$  and product of the numbers in the top left corner and in the bottom right corner of the mesh is denoted by  $q$ .

例如如上圖圍出四個數字 8, 9, 13 及 14。

For example, enclose the 4 numbers 8, 9, 13 and 14 as shown in the above figure.

$$q - p = (8)(14) - (9)(13) = -5$$

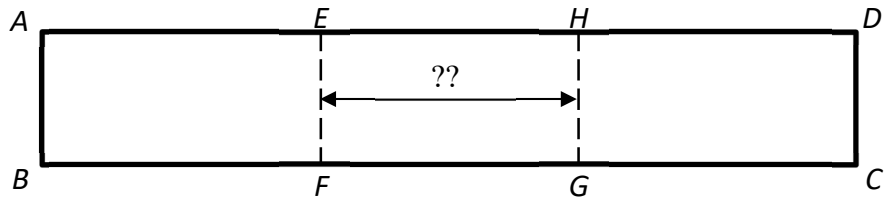
下圖顯示一個  $2023 \times 2024$  的數字方格，現把一個  $2 \times 2$  的正方形框架隨機放在數字方格以圍出四個數字。 $p$  與  $q$  的定義同上，求  $q - p$  的值。

In a  $2023 \times 2024$  number grid shown below, a  $2 \times 2$  square mesh is randomly placed on the grid to enclose four numbers.  $p$  and  $q$  are defined as above. Find the value of  $q - p$ .

		2024 列 columns						
2023 行 rows		1	2	3	...	2022	2023	2024
		2025	2026	2027	...	...	...	...
		...	...	...	...	...	...	...
		...	...	...	...	...	...	...
		...	...	...	...	...	...	...
		...	...	...	...	...	...	...

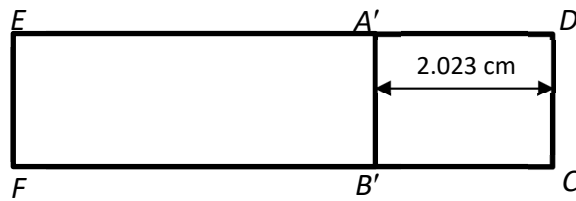
8. 下圖顯示一個長方形  $ABCD$ 。

The figure below shows a rectangle paper  $ABCD$ .



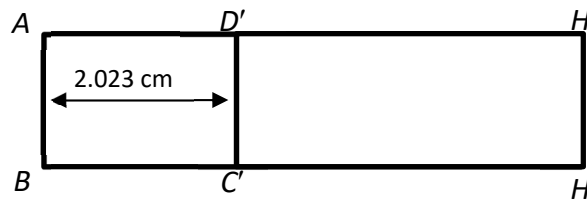
當該長方形沿  $EF$  摺疊， $AB$  會被摺到  $A'B'$ ， $A'B'$  及  $CD$  間的距離為  $2.023\text{cm}$ 。

When it is folded along  $EF$ ,  $AB$  land on  $A'B'$ . The distance between  $A'B'$  and  $CD$  is  $2.023\text{ cm}$ .



當長方形沿  $GH$  摺疊， $CD$  會被摺到  $C'D'$ ， $C'D'$  及  $AB$  間的距離亦為  $2.023\text{cm}$ 。

When it is folded along  $GH$ ,  $CD$  land on  $C'D'$ . The distance between  $C'D'$  and  $AB$  is also  $2.023\text{ cm}$ .

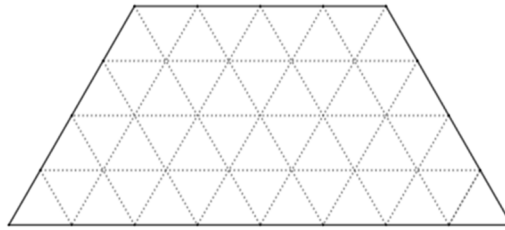


求  $EF$  及  $GH$  間的距離。

Find the distance between  $EF$  and  $GH$ .

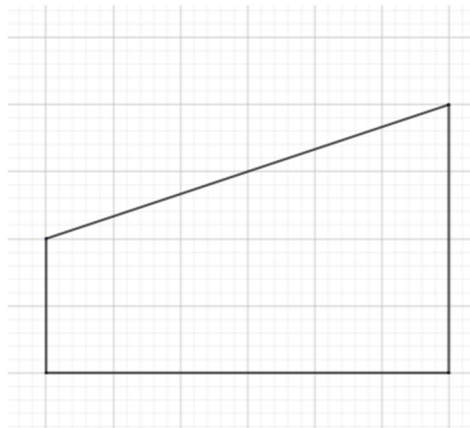
9. (a) 以兩條直線把以下圖案分割成三份使其面積比為 1:2:3。

Dissect the shape in the figure by two straight lines into three parts with area ratio 1:2:3.



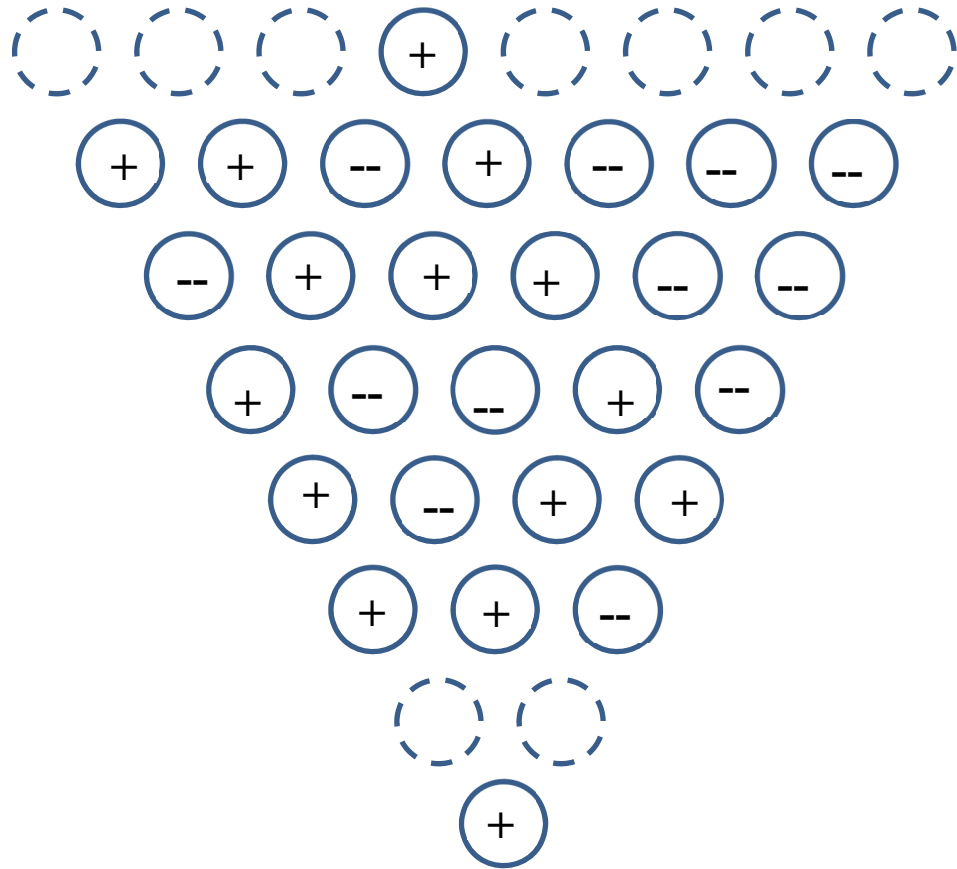
- (b) 以兩條直線把以下圖案分割成三份使其面積比為 1:2:3。

Dissect the shape in the figure by two straight lines into three parts with area ratio 1:2:3.



10. 以下圓形根據某些定律填上‘+’或‘--’符號，試定出虛線圓形內的符號。

The circles below are filled with either ‘+’ or ‘--’ signs according to a certain rule. Determine the signs of the dotted line circles.





**乙部 (建議此部用 15 分鐘作答)**

**Section B (Suggested to use 15 minutes in this Section)**

1. 彼德從父親處獲得一把長度為  $m$  的舊間尺，當中部分刻度已完全褪色。他發現該間尺仍可量度  $1, 2, \dots, (m-1)$  及  $m$  的長度。他把該間尺命名為「懶惰尺」。

Peter has an old ruler of length  $m$  given by his father, in which some markings on the ruler are missing. He finds that the ruler can still be used to measure all the lengths in  $1, 2, \dots, (m-1)$  and  $m$ . He names the rulers with this property as 'Lazy Ruler'.

例如，一把長度為 5 的「懶惰尺」只有  $\{0, 1, 3, 5\}$  這四個刻度。我們有  $1-0=1, 3-1=2, 3-0=3, 5-1=4$  及  $5-0=5$ ，所以  $0, 1, 2, 3, 4$  及  $5$  中所有長度都可被量度出。

For example, a 'Lazy Ruler' of length 5 with 4 markings can contain the markings  $\{0, 1, 3, 5\}$ . We have  $1-0=1, 3-1=2, 3-0=3, 5-1=4$  and  $5-0=5$ . So, all lengths in  $0, 1, 2, 3, 4$  and  $5$  can be measured.

- (a) 構作一個長度為 6 而只有 4 個刻度的「懶惰尺」。

Give an example of a 'Lazy Ruler' of length 6 with 4 markings.

- (b) 保羅聲稱他可以構作一個長度為 11 而只有 5 個刻度的「懶惰尺」。是否可行？試解釋你的判斷。

Paul claims that there is a 'Lazy Ruler' of length 11 with 5 markings only. Is that possible?

Explain your judgement.

- (c) 一把長度為 11 的間尺最少需有多少個刻度才能成為「懶惰尺」呢？試舉例並解釋你的答案。

What is minimum number of markings that a 'Lazy Ruler' of length 11 can have? Justify your answer and give an example.

- (d) 另一把「懶惰尺」上刻度的數目和(c)題的一把相同，這把「懶惰尺」的長度最長為多少？Another 'Lazy Ruler' has the same number of markings as that in (c). What is the greatest length of this 'Lazy Ruler'?

全卷完

End of Paper

2023/24 第十四屆香港中學數學創意解難比賽

2023/24 The 14<sup>th</sup> Hong Kong Mathematics Creative Problem Solving Competition for  
Secondary Schools

答題紙 Answer sheets

學校編號 School Code : S\_\_\_\_\_

學校名稱 School Name :

得分 Score :

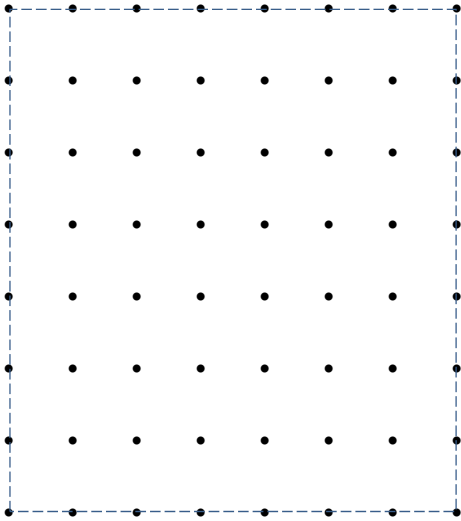
/40

甲部 Section A

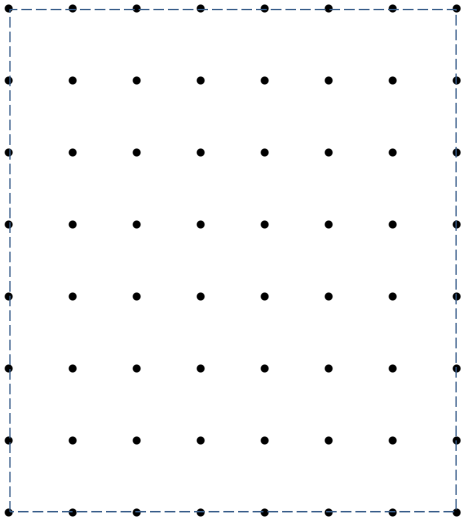
答案 Answers		評分 Marks
1.	26 = _____ 27 = _____ 28 = _____ 29 = _____ 30 = _____	/5

2.

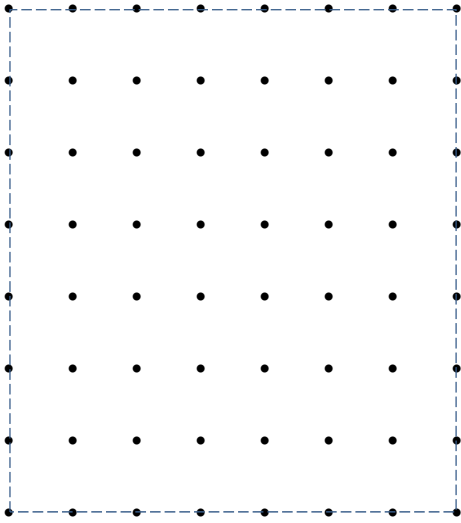
(a)



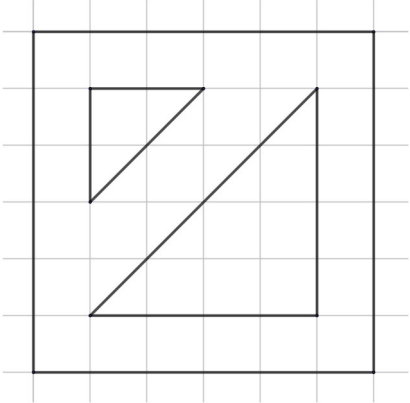
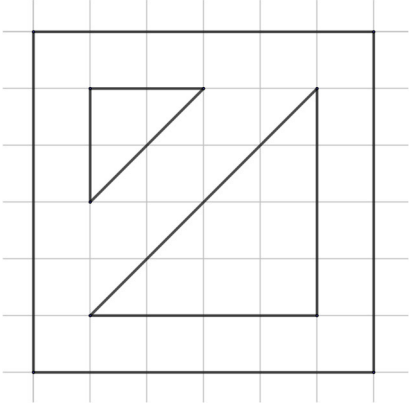
(b)

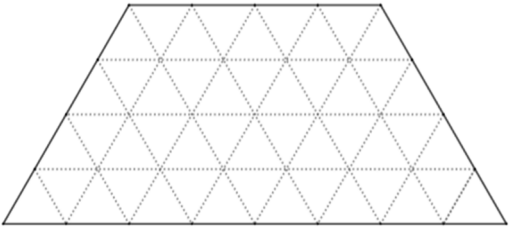
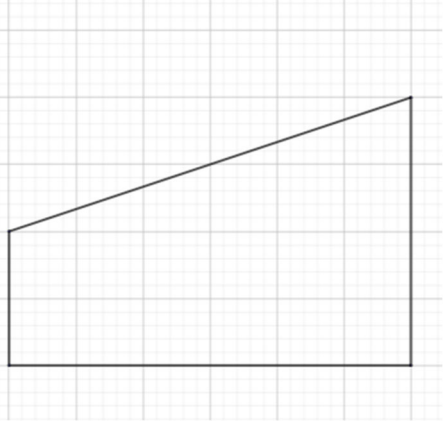


(c)

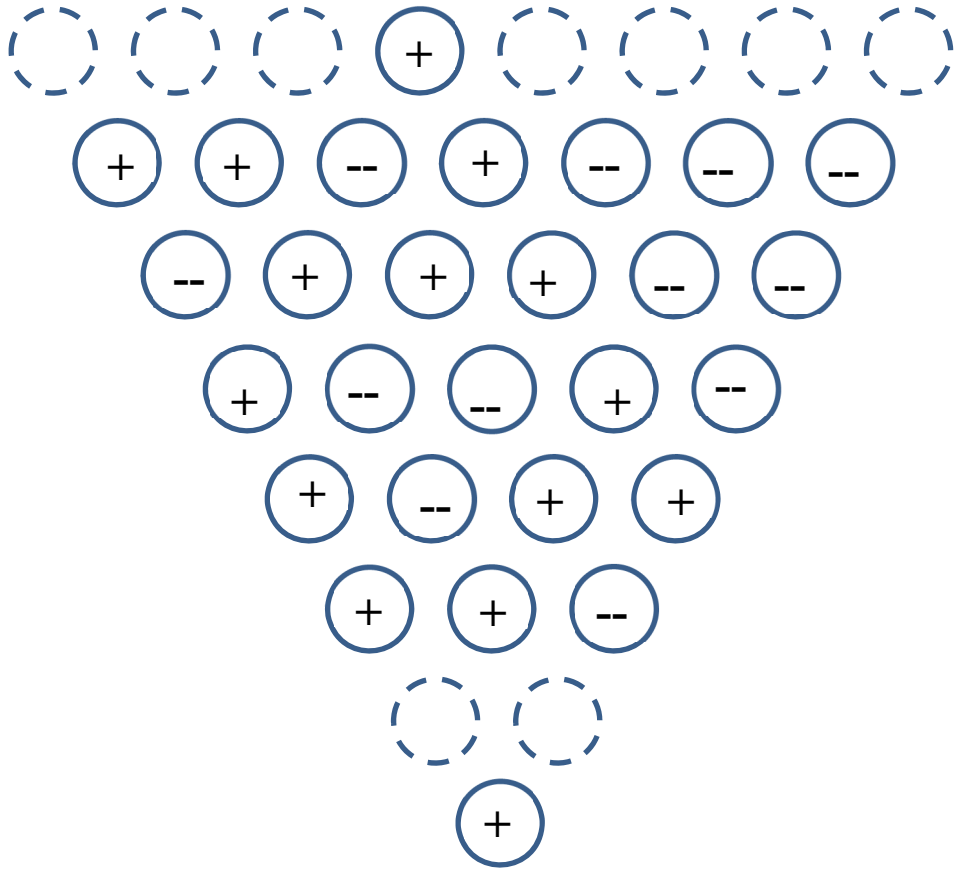


/3

3.	<p>(a)</p>  <p>(b)</p> 	/4
4.	<p>(a) <math>P =</math> _____</p> <p>(b) <math>P =</math> _____</p>	/4
5.	<p>Find <math>n</math>.</p> <p>求 <math>n</math> 的值。</p> <p>答：</p> <p>Answer: _____</p>	/2
6.	<p>求 <math>AHPE</math> 的面積。</p> <p>Find the area of <math>AHPE</math>.</p> <p>答：</p> <p>Answer: _____</p>	/2
7.	<p>求 <math>q - p</math> 的值。</p> <p>Find the value of <math>q - p</math>.</p> <p>答：</p> <p>Answer: _____</p>	/2

8.	<p>求 <math>EF</math> 及 <math>GH</math> 間的距離。</p> <p>Find the distance between <math>EF</math> and <math>GH</math>.</p> <p>答： Answer: _____</p>	/2
9.	<p>(a)</p>  <p>(b)</p> 	/4

10.



/2

**乙部 Section B**

1. (a) 構作一個長度為 6 而只有 4 個刻度的「懶惰尺」。(1 分)

Give an example of a 'Lazy Ruler' of length 6 with 4 markings. (1 mark)

(b) 保羅聲稱他可以構作一個長度為 11 而只有 5 個刻度的「懶惰尺」。是否可行？試解釋你的判斷。(2 分)

Paul claims that there is a 'Lazy Ruler' of length 11 with 5 markings only. Is that possible?

Explain your judgement. (2 marks)

- (c) 一把長度為 11 的間尺最少需有多少個刻度才能成為「懶惰尺」呢？試舉例並解釋你的答案。(3 分)

What is minimum number of markings that a 'Lazy Ruler' of length 11 can have? Justify your answer and give an example. (3 marks)

- (d) 另一把「懶惰尺」上刻度的數目和(c)題的一把相同，這把「懶惰尺」的長度最長為多少？(4 分)

Another 'Lazy Ruler' has the same number of markings as that in (c). What is the greatest length of this 'Lazy Ruler'? (4 marks)

全卷完  
End of Paper