2016/17 The 8th HK Mathematics Creative Problem Solving Competition for Secondary School (Heat – Written)

School ID:

Seat No.:

Time allowed : 50 minutes

Instructions :

- 1. The time allowed is 50 minutes.
- 2. The question paper consists of 12 pages. There are 17 questions in this paper.
- A set of question paper will be given to each student in a team. Only <u>ONE</u> answer sheet (green) will be given to each team. All the questions should be discussed among team members. The agreed answers should be written onto the answer sheet. <u>Only the answers on the</u> <u>answer sheet will be marked</u>.
- 4. Participating teams should bring their own stationery and calculators. For the purpose of fairness, please use only scientific calculators on the "List of Approved Calculators" by the Hong Kong Examinations and Assessment Authority. Electronic dictionaries, computers, mobile phones and other communication devices are prohibited.
- 5. The blank space on each page of this question paper can be used for rough work. One rough work sheet will be distributed to each participant. Extra rough work paper will also be provided upon request.
- 6. The answer sheet, all question papers, appended sheets and rough work papers will be collected after the competition. Participants are not allowed to take away any of these papers or the team might risk disqualification.

2016/17 第八屆香港中學數學創意解難比賽

(初賽-筆試)

參賽學校編號:	
座位编號:	

比賽時間:50分鐘

參加者須知:

- 1. 比賽時間共 50 分鐘。
- 2. 本問題卷共12頁,全卷共有17題。
- 每位參賽同學獲派一份問題卷,每一隊參賽隊伍只會獲派一張(綠色)
 答題紙。題目須由各成員經過討論,然後將議定的答案寫於答題紙上。
 ** 只有寫於答題紙上的答案方可得到評分。
- 4. 參賽學生需自備文具及計算機。為公平考慮,比賽中只可使用香港考 試及評核局「准用計算機型號名單」中的科學計算機(Scientific Calculator)。本比賽中嚴禁使用電話、電子字典、電腦或其他有上網 或通訊功能的工具。
- 本試卷每頁空白位置可作為算草之用。每位參賽學生亦會獲派一 張算草紙,如有需要,可要求額外算草用紙。
- 在筆試完結後,各同學必須交回所有問題卷、答題紙、附頁及草稿紙。
 參賽學生不得取走任何於比賽中所派發之紙張文具,違規者全隊可被 取消資格。

<u>題1</u> (2分)

於圖1中的四個方格填入不同的整數,使它們成為正確的算式。



<u>Question 1</u> (2 marks)

In figure 1, fill in the four boxes with different integers to form two correct mathematical expressions.

<u>題2</u> (2分)

由 2017 至 20172017 , 求所有以 2017 結尾的整數之和。

答: 這些整數的和是 _____。

Question 2 (2 marks)

From 2017 to 20172017, find the sum of the all integers which end with 2017.

Answer: The sum of these integers is _____

<u>題3</u> (2分)

於 34³⁴ × 35³⁵ × 38³⁸ × 65⁶⁵ 的結果中,最後的數字會有一連串的 0。

求這一連串的0共有多少個0。

答: 共有一連串_____個0。

Question 3 (2 marks)

The result of $34^{34} \times 35^{35} \times 38^{38} \times 65^{65}$ ends in a chain of 0's.

Find the numbers of 0's in this chain of 0's.

Answer: There are _____ 0's in this chain.

<u>題4</u> (2分)

圖 4 中是一個一筆畫的七角星, a+b+c+d+e+f+g=?



Question 4 (2 marks)

Figure 4 shows a seven-pointed star that can be drawn with connected segments.

a + b + c + d + e + f + g = ?

Answer: $a + b + c + d + e + f + g = ____°$

<u>題5</u> (2分)

小明於早上九時正離家外出,他先走了一段平路,然後再往山上行,到達山頂後,稍為休息半小時,然後他便沿原路步行回家,並於上午十一時三十分返抵家中。已知他 平路步行的平均速度為4km/h,上山的平均速度為3km/h,下山的平均速度為6 km/h,請問他共走了多少km?

答: 小明共走了 _____ km。

Question 5 (2 marks)

Sam left home for a walk at nine o'clock in the morning. He first walked along a level path. Then he walked up a path to the hill top and took a rest for half an hour. After the rest, he went back home along the same path. He reached home at half past eleven that morning. Sam's average speed for walking on a level track was 4 km/h. His average uphill speed was 3 km/h. His average downhill speed was 6 km/h. How many km in total had Sam walked?

Answer: Sam walked a total distance of _____ km.

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<u>題6</u> (2分)

已知 a 及 b 為正整數使得 (a + b + 2010) (a - b) = -2017。求 a² - b² 的值。

答: *a*² - *b*² = _____

<u>Question 6</u> (2 marks)

It is given that *a* and *b* are positive integers such that (a + b + 2010)(a - b) = -2017. Find the value of $a^2 - b^2$.

Answer: $a^2 - b^2 =$ _____

<u>題7</u> (2分)

圖 7 中,ACDE 為一平行四邊形,B 為 AC 的中點, AD 及 BE 相交於 F。若 ΔABF 的面積為 100 平方單位, 求 ΔBCD 的面積。

答: ΔBCD 的面積 = _____平方單位



<u>Question 7</u> (2 marks)

In figure 7, *ACDE* is a parallelogram. *B* is the mid-point of *AC*. *AD* and *BE* intersect at *F*. If the area of $\triangle ABF$ is 100 square units, find the area of $\triangle BCD$.

Answer: Area of $\triangle BCD =$ ______ square units

<u>題8</u> (2分)

房內有五個人,他們分別講了一句說話。

- A 說:「我們五個人中,有一個人說謊。」
- B 說:「我們五個人中,有兩個人說謊。」
- C 說:「我們五個人中,有三個人說謊。」
- D 說:「我們五個人中,有四個人說謊。」
- E 說:「我們五個人中,全都說謊。」
- (a) 有多少人說了真話?
- (b) 哪(些)人說了真話?
- 答: a. 有_____個人說了真話。
 - b. ______說了真話。

Question 8 (2 marks)

In a room, there are five persons. Each of them has said something.

A said, "Among the five of us, one is lying."

B said, "Among the five of us, two are lying."

C said, "Among the five of us, three are lying."

D said, "Among the five of us, four are lying."

E said, "Among the five of us, all are lying."

- (a) How many of them told the truth?
- (b) Who told the truth?

Answer: a. _____ of them told the truth.

b. _____ told the truth.

<u>題9</u> (3分)

圖 9 中有一正方形紙張 ABCD, E 和 F 分別為 AD 和 BC 的中點。P 和 Q 為 AD 上的兩點, 若將這紙張沿 BP 及 CQ 摺合, 原來的頂點 A 和 D 會於 EF 上的一點 G 重合。(如圖)

求圖中的角x。(圖形並不依比例繪畫)

答: x = _____°



Question 9 (3 marks)

In figure 9, *ABCD* is a piece of paper in the shape of a square. *E* and *F* are respectively the midpoints of *AD* and *BC*.

P and *Q* are two points on *AD*. When the paper is folded along *BP* and *CQ*, the original vertices at *A* and *D* will meet at a point *G* on *EF*, as shown.

Find the angle *x* in the figure. (The figure is not drawn to scale.)

Answer: $x = ___^\circ$

<u>題10</u>(3分)

圖 10a 中, ABCDEFGH 為一個正方體, 按圖示切割就可切割出一個正六邊形切面。 圖 10b 為這正方體的展開圖, 請在圖 10b 中畫出這個六邊形切面所有的邊。



Question 10 (3 marks)

In figure 10a, *ABCDEFGH* is a cube. A cross section in the shape of a regular hexagon can be formed by cutting the cube as shown in the figure. Figure 10b shows a net of the cube. Draw on figure 10b all the edges of the hexagonal cross section.

<u>題11</u> (3分)

以下的表格第二行的每一個格子內的數字都恰好是第一行它上面的那個數字在第二行 出現的次數。如:在第二行中,「0」出現1次、「1」出現2次、…

第一行	0	1	2	3
第二行	1	2	1	0

試在以下的表格第二行的每一個空格內,填入一個數字,使得該數字恰好是第一行它 上面的那個數字在第二行出現的次數。

 (a)
 第一行
 0
 1
 2
 3
 4

 第二行

(b)	第一行	0	1	2	3	4	5	6	7	8	9
	第二行										

Question 11 (3 marks)

The table below has two rows of numbers. Each value in row 2 is the number of times the value above it (in row 1) appearing in row 2.

For examples: In row 2, "0" appears for 1 time and "1" appears for 2 times and

Row 1	0	1	2	3
Row 2	1	2	1	0

In the tables below, fill in the cells in row 2 such that each value in row 2 is the number of times the value above it (in row 1) appearing in row 2.

(a)	Row 1	0	1	2	3	4
	Row 2					

(b)	Row 1	0	1	2	3	4	5	6	7	8	9
	Row 2										

<u>題12</u>(3分)

100 個學生圍成一圈 (如圖 12),並以順時針方向依次編為 1 至 100 號。從第 1 號的同 學開始,圍圈的同學順時針以 1、2、1、2、...的形式不停報數,凡所報之數為 "1" 的學生便需退出圈子,若這樣循環進行到剩下最後一個學生為止,問最後剩下的這個 學生的編號是甚麼?

答: 最後剩下的這個學生的編號是 _____。



圖 12

Figure 12

Question 12 (3 marks)

100 students stand in a circle as in figure 12. The students are numbered 1 to 100 in a clockwise direction. Beginning with student number 1, students standing in this circle keep counting off by twos as "1", "2", "1", "2", …, , in the clockwise direction. All students counting out "1" will leave the circle. The counting off keeps on until there is only one student left. What is student number of the last student left?

Answer: The student number of the last student left is _____.

<u>題13</u> (3分)

某青年中心共有 50 個會員,會員必須參加一或兩球隊。,若其中 40 人參加籃球隊, 30 人參加排球隊,20 人參加足球隊,10 人同時參加籃球隊及足球隊,5 人同時參加排 球隊及足球隊。有多少人同時參加籃球隊及排球隊?

答: 有_____同時參加籃球隊及排球隊。

Question 13 (3 marks)

There are 50 members in a youth centre. Every member has to join one or two of the ball teams. 40 members join the basketball team. 30 members join the volleyball team. 20 members join the football team. 10 members join both the basketball team and the football team. 5 members join both the volleyball team and the football team.

How many members join both the basketball team and the volleyball team?

Answer: ______ members join both the basketball team and the volleyball team.

<u>題14</u> (4分)

圖 14 中有一個正立方體有蓋箱子,邊長 1 m。箱子內放了兩個大小相同的球體並蓋上 蓋子。求每個球體半徑的最大可能值。

(答案須為真確值或準確至最接近的 0.001 m。)



Question 14 (4 marks)

Figure 14 shows a cubical box of side 1 m. Two identical spheres are put into the box with the lid covered. Find the greatest possible value of the radius of each sphere.

(Answer should be exact or correct to the nearest 0.001 m.)

Answer: The greatest possible value of the radius of the sphere is ______ m.

<u>題15</u> (4分)

圖 15 中的圓柱體高 6cm,半徑 1 cm。有一繩子從底至頂將這圓柱緊緊環繞了剛好四 圈,每圈的距離相等。

求這繩子的長度。答案準確至最接近的 0.1 cm。

[註: 圓形周界為2×半徑×π, 其中π=3.1416。]

答: 繩子的長度是 _____ cm。



Question 15 (4 marks)

Figure 15 shows a cylinder of radius 1 cm and height 6 cm.

A piece of string, going from the bottom of the cylinder to its top, winds around the cylinder tightly for exactly four rounds. Each round is equally spaced. Find the length of this piece of string. Give the answer correct to the nearest 0.1cm.

[Note: circumference of the a circle is $2 \times \text{radius} \times \pi$, where $\pi = 3.1416$.]

Answer: The length of the string is _____ cm.

<u>題16</u> (5分)

當志明疲累的時候,他會做一個「方格填色」作休息放鬆。他會以紅、綠、藍三種顏 色筆為一排的方格填色,他每次用紅色都填一個方格、每次用綠色都填兩個連續的方 格、每次用藍色都填連續3個方格。如圖16a所見,志明有4種不同的方法為一排三 個方格填色。如圖16b所見,志明有7種不同的方法為一排四個方格填色。

志明有多少種不同方法為下列方格填色?

- a. 一排5 個方格;
- b. 一排9個方格。
- 答: a. 志明有_____種不同方法為一排5個方格填色。
 - b. 志明有_____種不同方法為一排9個方格填色。

R: Red (紅色), G: Green(綠色), B:Blue (藍色)

\odot	R	R	R	ð
\odot	R	G	G	ð
\odot	G	G	R	ð
\odot	B	B	B	R

\odot	R	R	R	R	Ŗ	\odot	G	G	G	G	ð
\odot	R	R	G	G	R	\odot	R	В	В	В	Ŗ
\odot	R	G	G	R	Ŗ	\odot	В	В	В	R	g
\odot	G	G	R	R	R						

Figure 16a (圖 16a)

Figure 16b (圖 16b)

Question 16 (5 marks)

When Peter is tired, he will relax himself by a "colouring boxes" routine. He will use three colour pencils, red, green or blue, to fill up boxes arranged in a row. He will use the red pencil to fill one box at a time, the green pencil to fill two consecutive boxes at a time and the blue pencil to fill three consecutive boxes at a time. As shown in figure 16a, there are 4 different ways for Peter to fill a row of three boxes. As shown in figure 16b, there are 7 different ways for Peter to fill a row of four boxes.

In how many different ways can Peter fill the following rows?

- a. A row of 5 boxes;
- b. A row of 9 boxes.

Answer: a. Peter has ______ different ways to fill a row of 5 boxes.

b. Peter has ______ different ways to fill a row of 9 boxes.

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<u>題 17</u> (動手題) (5分)

有一種四巧板 (或「T-字板」)的拼圖遊戲據說源自中國古代。它是由四塊不同的形狀 的圖形板塊組成:一個三角形、一個短直角梯形、一個長直角梯形及一個不規則五邊 形。(如圖 17)

這四塊圖形可拼合並剛好放於一個長方形盒子內,亦可拼合成一個英文大楷「T」形狀的圖形(即:一橫向矩形下加一直向矩形)。

試將四巧板的四塊圖形板塊以兩種方法拼合,分別拼造出一個長方形及一個"**T**"形, 並將拼合的方法分別畫於圖 17(I) 和 17(II) 中.

你可以剪下附件1的圖形板塊作試驗,答案以畫於答題紙上為準。



Question 17 (Hands-on Question) (5 marks)

The four-piece-tangram (or "**T**-puzzle") is said to have a Chinese origin. The puzzle consists of four pieces of different shapes: one triangle, one small right trapezoid, one long right trapezoid and one irregular pentagon. (as shown in figure 17).

The four pieces can be just fitted into a rectangular box. They can also be fitted together to form the shape of a capital letter **T**, i.e. a horizontal rectangle on top of a vertical rectangle.

Fit the four pieces in two different ways: one to form a rectangle and another to form a **T**-shape. Draw, in each case, how the pieces are fitted together in figure 17(I) and figure 17(II) respectively.

The shapes printed on appendix 1 can be cut out for experiment. Only the drawings on the answer sheet will be marked.

End of Paper

Appendix 1



Appendix 1



「第八屆香港中學數學創意解難比賽」

初賽題目(筆試)

参考答案

題1

2017 - 218 = 1799

算式中的除數可以整除 1799

 $1799 = 7 \times 257$

1799 可整除 7、257 或 1799;但當除以7時,餘數不會是 218。

所以	2017 ÷	257	=	7	 218
	2017 ÷	1799	=	1	 218

題2

$$2017 + 12017 + 22017 + \dots + 20172017$$

= (2017)(2018) + 10000 + 20000 + \dots + 20170000
= (2017)(2018) + (10000)(1 + 2 + \dots + 2017)
= (2017)(2018) + (10000)\frac{(2017)(2018)}{2}
= (2017)(2018)(5001) = 20355600306

 $34^{34} \times 35^{35} \times 38^{38} \times 65^{65}$

- = 2³⁴ × 5³⁵ × 2³⁸ × 5⁶⁵ × 其餘的數均不包含 2 或 5 為因子
- = 2⁷² x 5¹⁰⁰ x 其餘的數均不包含 2 或 5 為因子
- = 10⁷² x其餘的數均不包含 10 為因子
- :: 共有一連串 72 個 0。

題4

方法 1 In ACEV, $a + c + e + x = 360^{\circ}$ In BDUG, $b + d + y + g = 360^{\circ}$ In ΔVFU , $f + (180^{\circ} - x) + (180^{\circ} - y) = 180^{\circ}$



三個方程相加: $a + b + c + d + e + f + g + x + y + (180^{\circ} - x) + (180^{\circ} - y)$ $= 360^{\circ} + 360^{\circ} + 180^{\circ}$ $a + b + c + d + e + f + g = 360^{\circ} + 180^{\circ} = 540^{\circ}$

<u>方法 2</u>

七個三個形的內角總和 = 7 × 180° 以"Δ"標示的各角的總和 = 以"•"標示的各角的總和 = 多邊形的外角和= 360°

a + b + c + d + e + f + g= 7 × 180° - 2 × 360° = <u>540°</u>



設平路有 x km,山路有 y km。

$$\frac{2x}{4} + \frac{y}{3} + \frac{y}{6} = 2$$
$$\frac{6x + 4y + 2y}{12} = 2$$
$$x + y = 4$$

小明共走了<u>8 km</u>。

題6

2017 是一個質數。

a - *b* = - 1

a + b + 2010 = 2017, Bp a + b = 7.

a = 3 and b = 4 $rightarrow a^2 - b^2 = (a + b)(a - b) = -7$ $a^2 - b^2 = -7$

題 7

$$AB: ED = 1:2, \quad \therefore AF: FD = 1:2$$
$$\Delta ABF = 100, \quad \therefore \Delta BDF = 200$$
$$\Delta BCD = \Delta ABD = \Delta ABF + \Delta BDF = 100 + 200 = 300$$

(a) <u>1</u>, (b) <u></u>**D**

這五個人各人說的話都不同,若有一人說真話,其他的都不可能是真話。說真 話的人最多是1個。

若五人中沒有人說真話,則E的話正確,生了矛盾。所以有最少一人說真話。 這人必定是D。

題9

BG = GC = BC $\therefore \Delta GBF 為等邊三角形, \angle GBF = 60^{\circ} \circ$ $\Delta ABP \cong \Delta GBP$ $\therefore \angle APB = \angle GPB = (90^{\circ} - 60^{\circ}) \div 2 = 15^{\circ}$ $x = 180^{\circ} - 90^{\circ} - 15^{\circ} = \frac{75^{\circ}}{2}$



題 10

可先考慮摺立方體於摺紙圖形上的對應位置。



a. 第二行的五個數字為0至4的整數,它們的和是5°先考慮這5個數的組合。

其中 最大數字為 4: (4,1,0,0,0)

最大數字為 3: (3,2,0,0,0), (3,1,1,0,0)均不可能是答案。 考慮 (2,2,1,0,0) 或 (2,1,1,1,0).

其中 (2,2,1,0,0)中, "0"和 "2"出現 2 次, "1"出現一次。可作以下答案: 0 1 2 3 4 2 1 2 0 0

b. 第二行的十個數字為0至9的整數,它們的和是10。

考慮最大數字為9、8或7的組合:

(9, 1, 0,....0), (8, 2, 0,0), (8, 1, 1, 0, ...,0), (7, 3, 0, ...0), (7, 2, 1, 0, ...,0),

(7,1,1,1,0,..,0) 等顯然不是正確的組合。

考慮最大數字為"6"的可能: 若6之下是1,0之下必定是6。

0	1	2	3	4	5	6	7	8	9
6	а	b	С	d	е	1	0	0	0

其中,a+b+c+d+e=3,

因為第二行應有6個"0",a、b、c、d、e當中有3個數是0。

這10個數的組合是 (6,2,1,1,0, 0,0,0,0,0),合題意。

0	1	2	3	4	5	6	7	8	9
6	2	1	0	0	0	1	0	0	0

循環	第一個和最後報數的學生編號 (報出的數)	留下次輪報數的學生編號 (學生數目)	
原先		1, 2, 3, 4, 5, 6 , 100	(100)
第一	1 (1), 100(2)	2, 4, 6, 8,,98, 100	(50)
第二	2 (1), 100(2)	4, 8, 12,,96, 100	(25)
第三	4 (1), 100(1)	8,16,24,,88, 96	(12)
第四	8 (2), 96(1)	8, 24, 40, 56, 72, 88	(6)
第五	8 (2), 88(1)	8, 40, 72	(3)
第六	8 (2), 72(2)	8, 72	(2)
第七	8 (1), 72(2)	72	(1)

因此,最後剩下的學生的編號是 72。

題 13

設同時參加籃球隊及排球隊的有 X 人。

則只參加籃球隊及只參加排球隊的人數分別為 (40-10-x)及(30-5-x)。(如圖)

不參加足球隊的人數= x + (40-10-x) + (30-5-x)

 $\therefore x + (40 - 10 - x) + (30 - 5 - x) + 20 = 50$ 75 - x = 50 x = 25 籃(40) 40-10-x 10 20-10-5=5足 (20)

即共有 25 人同時參加籃球隊及排球隊。



當兩個球體為最大可能,圖中應為一對稱設置,球體各自接觸盒子的三個面,兩個球體亦互相接觸。

考慮圖中所設置的三個立方體,三個立方體均與盒子的方向相同。

中間一個以兩個球體的中心為它兩個最遠的頂點。(紅色部分)

另外兩個以一個球體的中心及盒子的最近的一個頂點為它兩個最遠的頂點。

如圖,紅色立方體的邊長為 (1 - 2r)m。

用畢氏定理,這立方體一個面的對角邊長 $\sqrt{2}(1 - 2r)$ m。

立方體的對角線長 $\sqrt{3}(1 - 2r)$ m。

 $\sqrt{3}(1 - 2r) = 2r$ r = $\frac{\sqrt{3}}{2(1+\sqrt{3})}$ (= 0.317)

以一過A點並垂直於圓柱體底部的直線剪開圓柱體的曲面,並將它展開成一長 方形。(如圖)

長方形的闊度 = $2\pi \times 1$ cm

長方形的高度 = 6 cm

繩子的長度 = 4 × $\sqrt{(2\pi)^2 + (6 \div 4)^2}$ = $\frac{25.8 \text{ cm}}{25.8 \text{ cm}}$ (準確至最接近的 0.1 cm)



a. 一排兩個方格,只有兩種方法填色: RR, GG.

.: 兩格有2種方法,三格有4種方法,四格有7種方法。

若要為一排5格填色:

若第一格填了紅色,餘下四格有7種方法填色。 若第一二格填了綠色,餘下三格有4種方法填色。 若第一二三格填了綠色,餘下兩格有2種方法填色。 2+4+7=13。共有13種方法。

b. 以(a)部相類的推論:

格子的數目	填色的方法	總數
2	2	2
3	4	4
4	7	7
5	2+4+7	13
6	4+7+13	24
7	7+13+24	44
8	13+24+44	81
9	24+44+81	149

共有149種方法為9個方格填色。



