

# 2017/18「第九屆香港中學數學創意解難比賽」

## 初賽題目（筆試）及參考答案

### 題 1 (2 分)

已知  $2015 \times 2016 \times 2017 \times 2018 + k$  為一完全平方數，如果  $k$  是一個正整數，求  $k$  的最小值。

### Question 1 (2 marks)

It is given that  $2015 \times 2016 \times 2017 \times 2018 + k$  is a perfect square. If  $k$  is a positive integer, find the least value of  $k$ .

建議題解：

設  $n$  為一正整數。

$$\begin{aligned} & n(n+1)(n+2)(n+3) + k \\ &= n(n+3)(n+1)(n+2) + k \\ &= (n^2 + 3n)(n^2 + 3n + 2) + k \\ &= (n^2 + 3n)^2 + 2(n^2 + 3n) + k \end{aligned}$$

如果  $k$  取最小值， $k = 1$  時上式將可以變為一個完全平方數。

### Suggested Solutions:

Let  $n$  be a positive integer.

$$\begin{aligned} & n(n+1)(n+2)(n+3) + k \\ &= n(n+3)(n+1)(n+2) + k \\ &= (n^2 + 3n)(n^2 + 3n + 2) + k \\ &= (n^2 + 3n)^2 + 2(n^2 + 3n) + k \end{aligned}$$

For the least value of  $k$ , if  $k = 1$ , a completed square could be formed.

**題 2 (2 分)**

定義  $n! = n \times (n-1) \times (n-2) \times \dots \times (1)$ ，其中  $n$  是正整數。

例如：

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

設  $x$  為  $2018!$ 。問  $x$  這數字的結尾有多少個「零」？

**Question 2 (2 marks)**

Define  $n! = n \times (n-1) \times (n-2) \times \dots \times (1)$ , when  $n$  is a positive integer.

For Example,

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Let  $x$  be  $2018!$ . How many zero digits are there at the end of  $x$ ?

**建議題解：**

結尾的 0 的數目與  $2018!$  中 10 的因數數目相同。在  $2018!$  中，先求出“5”的因數數目

$$\left\lfloor \frac{2018}{5} \right\rfloor = 403, \quad \left\lfloor \frac{2018}{25} \right\rfloor = 80, \quad \left\lfloor \frac{2018}{125} \right\rfloor = 16, \quad \left\lfloor \frac{2018}{625} \right\rfloor = 3$$

$$\text{因數 5 的總數目} = 403 + 80 + 16 + 3 = 502$$

∵ 因數 5 的總數目必定少於因數 2 的總數目，所以一定有足夠的因數 2 去與因數 5 配合成為因數 10。所以共有 502 個因數 10。

∴ 在  $2018!$  中，尾數共有 502 個零。

**Suggested Solutions:**

The number of zero digits is equal to the number of factor 10 in  $2018!$ .

We will find the number of factor 5 in  $2018!$  first.

$$\left\lfloor \frac{2018}{5} \right\rfloor = 403, \quad \left\lfloor \frac{2018}{25} \right\rfloor = 80, \quad \left\lfloor \frac{2018}{125} \right\rfloor = 16, \quad \left\lfloor \frac{2018}{625} \right\rfloor = 3$$

$$\text{Total no. of factor 5} = 403 + 80 + 16 + 3 = 502$$

∵ Total no. of factor 5 must be less than Total no. of factor 2. There must be enough 2s' to match the 5s' to form a 10. So there are 502 of factor 10.

∴ There are 502 zeros at the end of  $2018!$ .

**題 3 (2 分)**

吳媽媽有一子一女，阿忠和阿慈。她收入不多，但在每個月的月頭，都會把零用錢給阿忠或阿慈。為了公平，她設計了一個特別的輪換辦法。

第 1 至第 $n$ 個月	誰得零用錢
1	忠
1 - 2	忠 慈
1 - 4	忠 慈 慈 忠
1 - 8	忠 慈 慈 忠 慈 忠 忠 慈
1 - 16	忠 慈 慈 忠 慈 忠 忠 慈 慈 忠 忠 慈 忠 慈 慈 忠

在第四十和第四十一個月，誰人將會得到零用錢？

**Question 3 (2 marks)**

Mrs Ng has a son and a daughter, Bill and Amy ( $B$  &  $A$ ). She does not earn a lot but she will give pocket money either to Bill or Amy at the beginning of each month. She designs a fair way to give the pocket money.

1 <sup>st</sup> to $n^{\text{th}}$ month	Who gets the pocket money
1	$B$
1 – 2	$BA$
1 – 4	$BAAB$
1 – 8	$BAABABBA$
1 – 16	$BAABABBAABBABAAB$

Who will get the pocket money at the 40<sup>th</sup> and 41<sup>st</sup> months respectively?

建議題解：

第 40 和第 41 個月

⇔ (第 8 和第 9 個月) 的相反

⇔ (慈和慈) 的相反

⇔ 忠忠

### Suggested Solutions :

40<sup>th</sup> and 41<sup>st</sup>

$\Leftrightarrow$  negation of ( 8<sup>th</sup> and 9<sup>th</sup> )

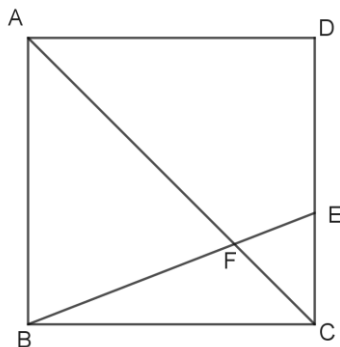
 $\Leftrightarrow$  negation of  $(A \text{ and } A)$  $\Leftrightarrow BB$

**題 4 (2 分)**

在圖中  $ABCD$  是一個邊長為 10 厘米的正方形，如果三角形  $ABF$  的面積是 36 平方厘米，求  $CE$  的長度。

**Question 4 (2 marks)**

In the figure,  $ABCD$  is a square with side 10 cm. If the area of  $\triangle ABF$  is  $36 \text{ cm}^2$ , find the length of the side  $CE$ .



建議題解：

$$\triangle ABC \text{ 面積} = \frac{10 \times 10}{2} = 50 \text{ cm}^2$$

$$\triangle BFC \text{ 面積} = 50 - 36 = 14 \text{ cm}^2$$

$$AF : FC = 36 : 14 = 18 : 7$$

$$\therefore \triangle ABF \sim \triangle CEF$$

$$\therefore AB : EC = AF : FC$$

$$\frac{10}{EC} = \frac{18}{7}$$

$$EC = \frac{35}{9} \text{ cm}$$

**Suggested Solutions:**

$$\text{Area of } \triangle ABC = \frac{10 \times 10}{2} = 50 \text{ cm}^2$$

$$\text{Area of } \triangle BFC = 50 - 36 = 14 \text{ cm}^2$$

$$AF : FC = 36 : 14 = 18 : 7$$

$$\therefore \triangle ABF \sim \triangle CEF$$

$$\therefore AB : EC = AF : FC$$

$$\frac{10}{EC} = \frac{18}{7}$$

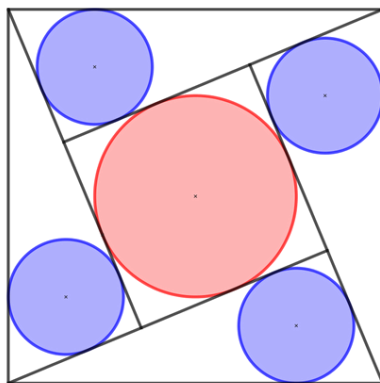
$$EC = \frac{35}{9} \text{ cm}$$

### 題 5 (2 分)

在圖中，正方形內有 4 個全等的直角三角形。如果大正方形的邊長為 13 厘米，而 4 個小圓形的半徑為 2 厘米，求大圓的半徑。

### Question 5 (2 marks)

In the figure, 4 congruent right-angled triangles are arranged inside a square as shown. If the larger square has side length 13 cm and the radii of small circles are 2 cm, find the radius of the larger circle.



### 建議題解 / Suggested Solutions :

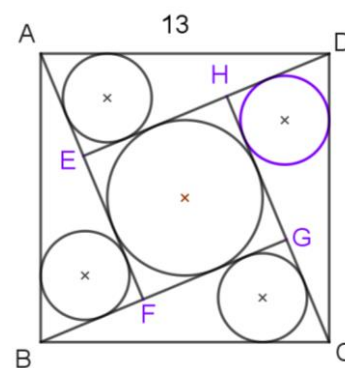
設  $AE = BF = CG = DH = a$  及  $AF = BG = CH = DE = b$

$$\begin{cases} ab = (2)(a + b + 13) & \dots (1) \\ a^2 + b^2 = 13^2 & \dots (2) \end{cases}$$

由 / From (1),

$$\begin{aligned} \frac{ab}{2} - 13 &= a + b \\ \frac{(ab)^2}{4} - 2\left(\frac{ab}{2}\right)(13) + 13^2 &= a^2 + 2ab + b^2 \\ \frac{(ab)^2}{4} - 13ab + 13^2 &= 13^2 + 2ab & (by (2)) \\ ab &= 60 \\ a^2 + b^2 - 2ab &= 13^2 - 2(60) \\ (b - a)^2 &= 49 \\ b - a &= 7 \end{aligned}$$

$\therefore$  大圓的半徑 / Radius of larger circle = 3.5 cm



**題 6 (2 分)**

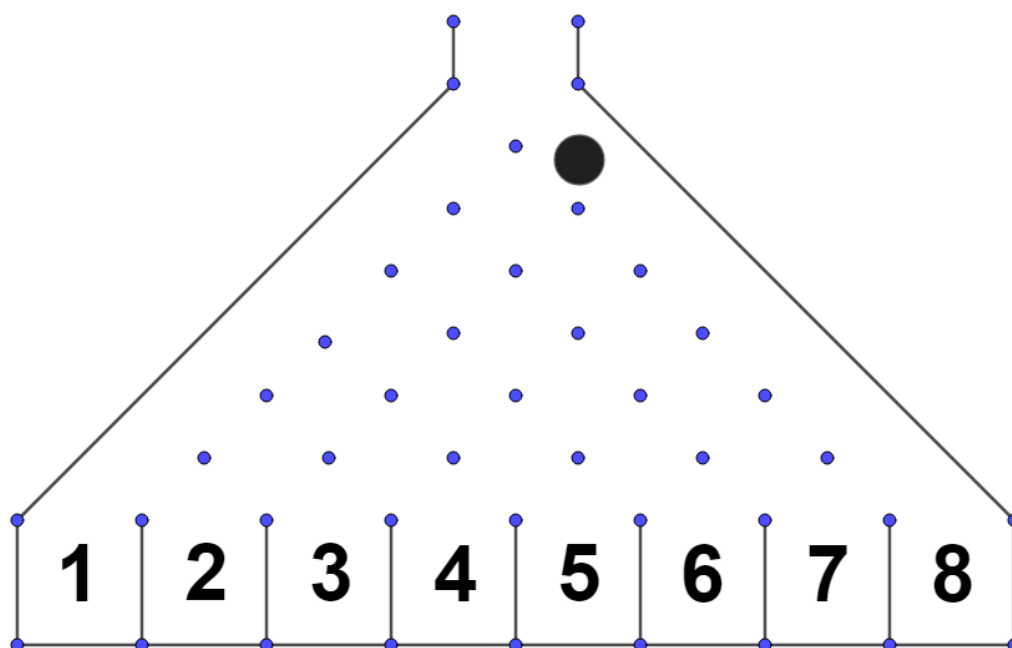
圖中的釘板是用作遊戲使用。你會放入一枚圖形的硬幣並讓它掉到 8 個位置中的其中一個。當硬幣碰到一口釘時，它彈至左或右的機會是均等的。

如果有一枚硬幣在圖中的位置掉下，它最有機會掉至那一個位置？

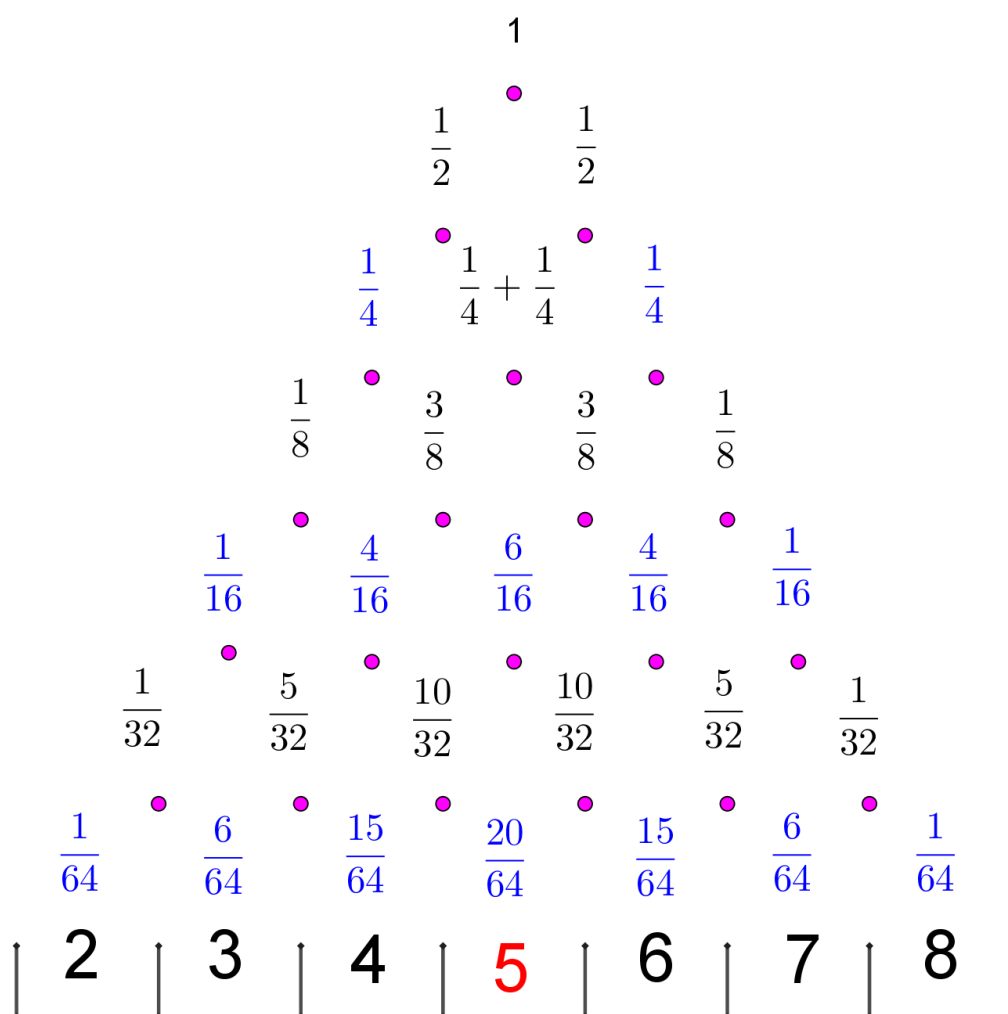
**Question 6 (2 marks)**

The figure shows a peg board used for a game show. Whenever the circular chip falls onto a peg, it has an equal chance to bounce left or right.

If the chip is dropped from the location shown, which position would it most likely fall to?



建議題解/ Suggested Solutions :



它最有機會掉至 5 號位置。

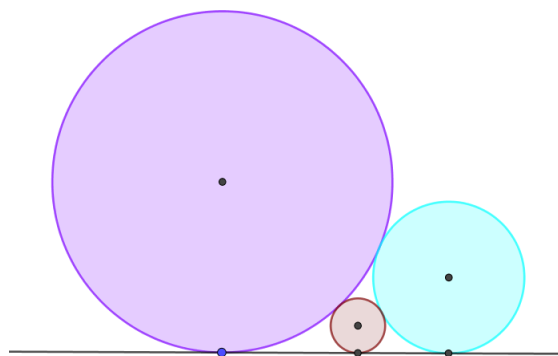
It most likely falls to position 5.

**題 7 (2 分)**

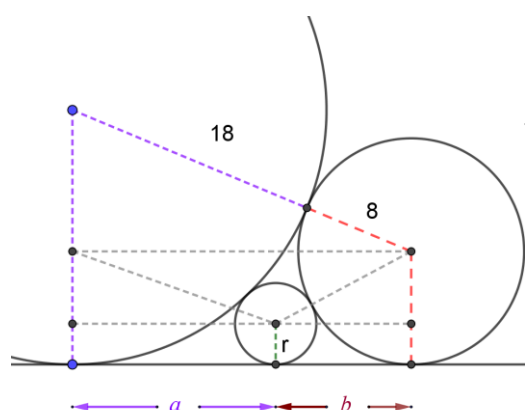
圖中有 3 個外接的圓，如果兩個較大的圓的半徑為 18 厘米及 8 厘米，求最小的圓的半徑。

**Question 7 (2 marks)**

The figure shows 3 touching circles. If the radii of the two larger circles are 18 cm and 8 cm respectively, find the radius of the smallest circle.



建議題解/ **Suggested Solutions :**



$$\begin{cases} a^2 &= (18 + r)^2 - (18 - r)^2 = 72r & \dots (*) \\ b^2 &= (8 + r)^2 - (8 - r)^2 = 32r & \dots (**) \\ (a + b)^2 &= (18 + 8)^2 - (18 - 8)^2 = 576 & \dots (***) \end{cases}$$

$$\sqrt{(*)(**)},$$

$$ab = \sqrt{(72r)(32r)} = 48r$$

From (\*\*\*),

$$\begin{aligned} a^2 + b^2 + 2ab &= 576 \\ 72r + 32r + 2(48r) &= 576 \\ r &= 2.88 \end{aligned}$$

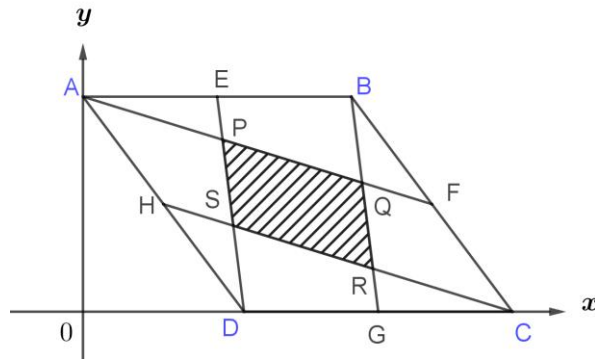


**題 8 (2 分)**

已知  $A$  點的坐標是  $(0, 4)$  而  $C, D$  都在  $x$  軸上， $E, F, G$  及  $H$  分別為  $AB, BC, CD$  及  $DA$  的中點。如果  $ABCD$  為一菱形及  $AB = 5$ ，求陰影部分  $PQRS$  的面積。

**Question 8 (2 marks)**

Given that the coordinate of  $A = (0, 4)$  and  $C, D$  lies on  $x$ -axis.  $E, F, G$  and  $H$  are mid-points of  $AB, BC, CD$  and  $DA$  respectively. If  $ABCD$  is a rhombus and  $AB = 5$ , find the area of the shaded region  $PQRS$ .

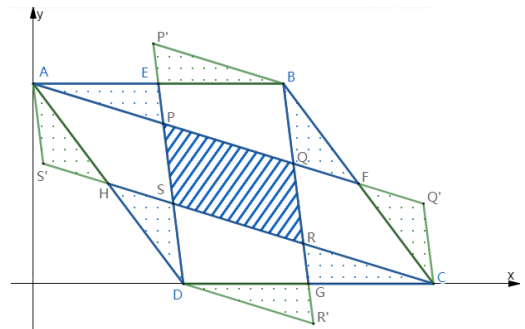


建議題解：

$\triangle SDH, \triangle PAE, \triangle QBF$  和  $\triangle RCG$  分別沿着  $H, E, F$  和  $G$  順時針方向旋轉  $180^\circ$ ，我們分別得出  $\triangle S'AH, \triangle P'BE, \triangle Q'CF$  和  $\triangle R'DG$ 。

得知平行四邊形  $PQRS, APSS', P'BQP, QQ'CR$  和  $SRR'D$  是全等。

$$\therefore PQRS \text{ 的面積} = \frac{1}{5}(5 \times 4) = 4 \text{ 平方單位}$$

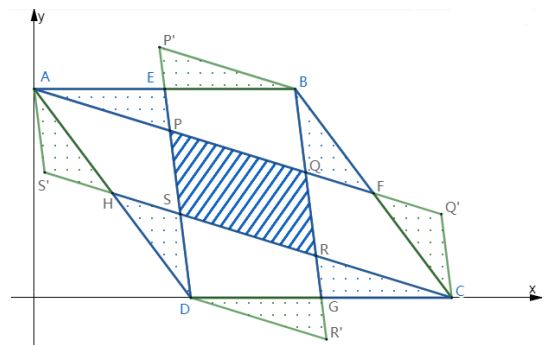


**Suggested Solutions:**

By rotating the triangles  $\triangle SDH, \triangle PAE, \triangle QBF$  and  $\triangle RCG$   $180^\circ$  clockwise about  $H, E, F$  and  $G$ , we obtain  $\triangle S'AH, \triangle P'BE, \triangle Q'CF$  and  $\triangle R'DG$  respectively.

Notice the parallelograms  $PQRS, APSS', P'BQP, QQ'CR$  and  $SRR'D$  are congruent.

$$\therefore \text{Area of } PQRS = \frac{1}{5}(5 \times 4) = 4 \text{ sq. units}$$



**題 9 (2 分)**

我們將定義以下的運算方法:

$$(a, b) * (c, d) = (ad + bc, bd)$$

$$(ka, kb) = (a, b)$$

$$(a, b) \Delta (c, d) = (ac, bd)$$

其中  $a, b, c, d, k$  為實數，並且  $b, d$  和  $k$  不等於 0。

(a) 如果  $(1, 2) * (x, y) = (5, 6)$ ，求  $(x, y) \Delta (1, 6) = ?$

(b)  $(1, 2) \Delta (2, 3) \Delta (3, 4) \Delta \dots \Delta (2017, 2018) = ?$

**Question 9 (2 marks)**

We are going to define the following operations:

$$(a, b) * (c, d) = (ad + bc, bd)$$

$$(ka, kb) = (a, b)$$

$$(a, b) \Delta (c, d) = (ac, bd)$$

Where  $a, b, c, d, k$  are all real numbers with  $b, d$  and  $k$  are not equal to 0.

(a) Given  $(1, 2) * (x, y) = (5, 6)$ , then  $(x, y) \Delta (1, 6) = ?$

(b)  $(1, 2) \Delta (2, 3) \Delta (3, 4) \Delta \dots \Delta (2017, 2018) = ?$

**建議題解/ Suggested Solutions :**

(a)

$$(1, 2) * (x, y) = (5, 6)$$

$$(y + 2x, 2y) = (5, 6)$$

$$\therefore y = 3k, x = k$$

$$(x, y) \Delta (1, 6)$$

$$= (x, 6y)$$

$$= (k, 18k)$$

$$= (1, 18)$$

(b)

$$(1, 2) \Delta (2, 3) \Delta (3, 4) \Delta \dots \Delta (2017, 2018)$$

$$= (1 \cdot 2 \cdot 3 \dots \cdot 2017, 2 \cdot 3 \cdot 4 \dots \cdot 2018)$$

$$= (1, 2018)$$

**題 10 (2 分)**

在一個正方形內有 15 點，它們與正方形的 4 個頂點均被一些不相交的線段連起，使到整個正方形完全被分割成很多個三角形。請問總共可以分割成多少個三角形呢？

**Question 10 (2 marks)**

There are 15 points inside a square. They are all connected by non-intersecting line segments with each other and with the vertices of the square, in such a way that the square is dissected into triangles. How many triangles will we have?

**建議題解：**

開始時，我們有正方形的 4 個頂點，

在正方形內任意加上一點，我們得 4 個三角形。

在其中一個部份加上第二點，我們再得 2 個三角形。

在其中一個部份加上第三點，我們再得 2 個三角形。

....

因此，

共有三角形 =  $4 + 2 + 2 + \dots 2 = 4 + 2 (14) = 32$  個。

} 14 次

**Suggested Solutions :**

Initially, we have a square with 4 vertices only,

After adding the 1st point inside the square we have 4 triangles.

Adding the 2nd point inside any region we have 2 more triangles.

Adding the 3rd point inside any region we have 2 more triangles.

....

Therefore,

Total number of triangles =  $4 + 2 + 2 + \dots 2 = 4 + 2 (14) = 32$ .

} 14 times

**題 11 (2 分)**

小明在用晚餐前看了手錶，發現時針與分針成  $121^\circ$ ；用餐後他再看手錶，發現時針與分針又再次成  $121^\circ$ 。如果已知他是在晚上 6 時至 7 時用餐，求他的用餐時間。

**Question 11 (2 marks)**

Ming watched his watch before his dinner. He found that the angle formed between the hour hand and the minute hand was  $121^\circ$ . After the dinner, he watched his watch again. The angle formed between the hour hand and the minute hand was also  $121^\circ$ . If he had his dinner between 6 p.m. to 7 p.m., find the time spent on his dinner.

**建議題解：** 時針的速度是：

$$\frac{360^\circ}{12} \text{ 每 } 60 \text{ 分}$$

$$0.5^\circ \text{ 每分}$$

分針的速度是：  $6^\circ \text{ 每分}$

$$\text{晚餐是由 } 6:x \text{ 至 } 6:y \quad 180^\circ + 0.5^\circ x - 6^\circ x = 121^\circ$$

$$59^\circ = 5.5^\circ x$$

$$x = \frac{118}{11}$$

$$6^\circ y - (180^\circ + 0.5^\circ y) = 121^\circ$$

$$5.5^\circ y = 301^\circ$$

$$y = \frac{602}{11}$$

$$\text{用餐時間是} = \frac{602}{11} - \frac{118}{11} = 44 \text{ 分鐘}$$

**Suggested Solutions:** Speed of hour arm is

$$\frac{360^\circ}{12} \text{ per } 60 \text{ min} = 0.5^\circ \text{ per min}$$

Speed of minutes arm:  $6^\circ \text{ per min}$

$$\text{Dinner is from } 6:x \text{ to } 6:y \quad 180^\circ + 0.5^\circ x - 6^\circ x = 121^\circ$$

$$59^\circ = 5.5^\circ x$$

$$x = \frac{118}{11}$$

$$6^\circ y - (180^\circ + 0.5^\circ y) = 121^\circ$$

$$5.5^\circ y = 301^\circ$$

$$y = \frac{602}{11}$$

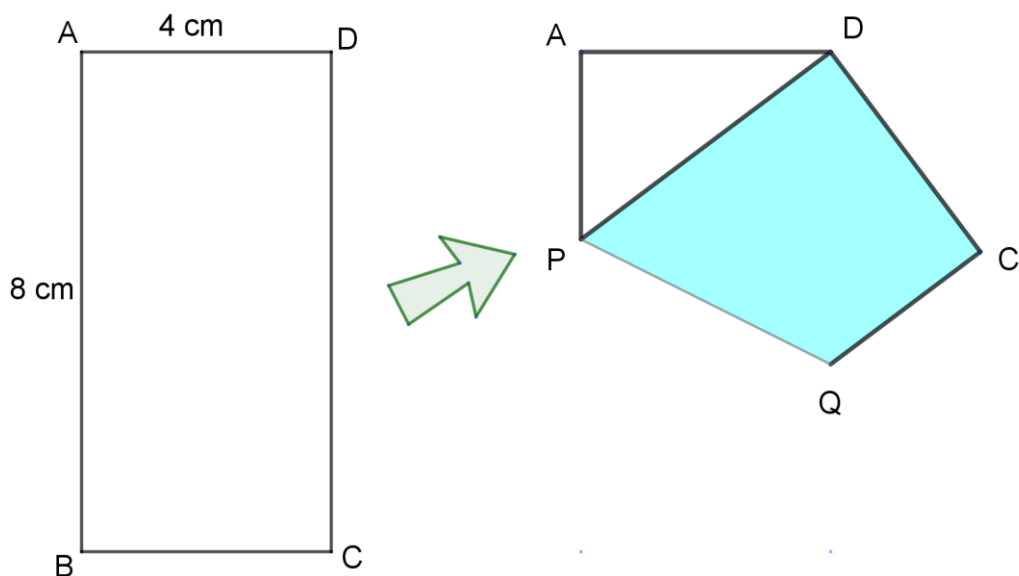
$$\text{Time spent} = \frac{602}{11} - \frac{118}{11} = 44 \text{ min.}$$

**題 12 (2 分)**

把一個  $4\text{ cm} \times 8\text{ cm}$  的長方形，如圖摺起使  $B$  點和  $D$  點重疊成五邊形  $APQCD$ 。求五邊形  $APQCD$  的面積。

**Question 12 (2 marks)**

A rectangle  $4\text{ cm} \times 8\text{ cm}$  in dimension is folded as shown in the figure so that point  $B$  and point  $D$  are overlapped to form a pentagon  $APQCD$ . Find the area of  $APQCD$ .



建議題解：

設  $AP$  為  $x\text{ cm}$

$$AP^2 + AD^2 = PD^2 \quad (\text{畢氏定理})$$

$$x^2 + 4^2 = (8 - x)^2$$

$$x = 3$$

$$\therefore AP = 3\text{ cm}, PD = 5\text{ cm}$$

運用對稱性,  $QC = 3\text{ cm}$

$$\begin{aligned} \text{Area of } APQCD &= \frac{(5 + 3) \times 4}{2} + \frac{3 \times 4}{2} \\ &= 22\text{ cm}^2 \end{aligned}$$

**Suggested Solutions:**

Let  $AP$  be  $x\text{ cm}$

$$AP^2 + AD^2 = PD^2 \quad (\text{Pyth. Thm.})$$

$$x^2 + 4^2 = (8 - x)^2$$

$$x = 3$$

$$\therefore AP = 3\text{ cm}, PD = 5\text{ cm}$$

By symmetry,  $QC = 3\text{ cm}$

$$\text{Area of } APQCD = \frac{(5 + 3) \times 4}{2} + \frac{3 \times 4}{2} = 22\text{ cm}^2$$

**題 13 (2 分)**

設  $\lceil x \rceil$  和  $\lfloor x \rfloor$  分別代表向上和向下捨入的函數。

(例如  $\lceil 4.2 \rceil = \lceil 4.8 \rceil = 5$  而  $\lfloor 3.46 \rfloor = \lfloor 3.99 \rfloor = 3$ )

試計算下列算式

$$\lceil \sqrt{1} \rceil + \lceil \sqrt{2} \rceil + \lceil \sqrt{3} \rceil + \cdots + \lceil \sqrt{100} \rceil - (\lfloor \sqrt{1} \rfloor + \lfloor \sqrt{2} \rfloor + \lfloor \sqrt{3} \rfloor + \cdots + \lfloor \sqrt{100} \rfloor)$$

的值。

**Question 13 (2 marks)**

Let  $\lceil x \rceil$  and  $\lfloor x \rfloor$  represents the round up and round down functions respectively.

(For example,  $\lceil 4.2 \rceil = \lceil 4.8 \rceil = 5$  while  $\lfloor 3.46 \rfloor = \lfloor 3.99 \rfloor = 3$ )

Find the value of the following expression,

$$\lceil \sqrt{1} \rceil + \lceil \sqrt{2} \rceil + \lceil \sqrt{3} \rceil + \cdots + \lceil \sqrt{100} \rceil - (\lfloor \sqrt{1} \rfloor + \lfloor \sqrt{2} \rfloor + \lfloor \sqrt{3} \rfloor + \cdots + \lfloor \sqrt{100} \rfloor)$$

建議題解：

$$\lceil \sqrt{x} \rceil - \lfloor \sqrt{x} \rfloor = \begin{cases} 0 & \text{當 } x \text{ 為正方形數.} \\ 1 & \text{其他情況} \end{cases}$$

$\therefore$  1 至 100 內有 10 個正方形數

$\therefore$  要求的值 =  $100 - 10 = 90$

**Suggested Solutions:**

$$\lceil \sqrt{x} \rceil - \lfloor \sqrt{x} \rfloor = \begin{cases} 0 & \text{when } x \text{ is a square no.} \\ 1 & \text{otherwise} \end{cases}$$

$\therefore$  There are 10 square no. from 1 to 100 inclusively

$\therefore$  Required value =  $100 - 10 = 90$

**題 14 (2 分)**

如果  $k$  個連續整數之和是 2018 並且  $1 < k < 2018$ ，求這  $k$  個連續整數中最小的數。

**Question 14 (2 marks)**

If the sum of  $k$  consecutive integers is 2018 and  $1 < k < 2018$ . Find the smallest number in these  $k$  integers.

建議題解：

$$\begin{aligned}\frac{k}{2}(2a + k - 1) &= 2018 \\ k(2a + k - 1) &= 4036\end{aligned}$$

把 4036 因式分解， $k$  的可能值為 1009 或 2018。考慮到  $a$  為整數， $k$  只可以為 1009，因此  $a = -502$ 。

即首項為 -502，共有 1009 連續整數。

**Suggested Solutions:**

$$\begin{aligned}\frac{k}{2}(2a + k - 1) &= 2018 \\ k(2a + k - 1) &= 4036\end{aligned}$$

By factorizing 4036, possible values of  $k = 1009$  or 2018. By considering  $a$  is also an integer, the only possible value of  $k$  is 1009, thus returning  $a = -502$ .

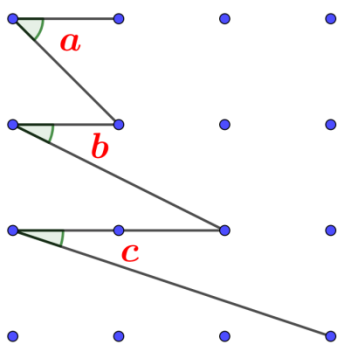
i.e. the 1st term is -502 and there are 1009 consecutive numbers.

**題 15 (2 分)**

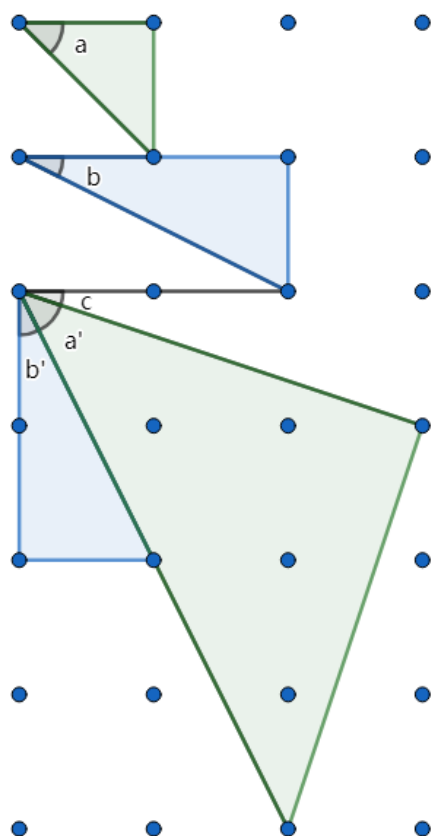
圖中為一等距釘板，求圖中角  $a, b$  及  $c$  之和。

**Question15 (2 marks)**

The figure shows an equidistant geoboard. Find the sum of angles  $a, b$  and  $c$  shown in the figure.



建議題解 / Suggested Solutions :



$$a' = a, b' = b$$

$$\therefore a + b + c = a' + b' + c = 90^\circ$$



**題 16 (4 分)**

在方程

$$(x^2 - 5x + 5)^{(x^2 - 11x + 30)} = 1$$

中，求所有的實數解之和。

**Question 16 (4 marks)**

Find the sum of all real roots in this equation,

$$(x^2 - 5x + 5)^{(x^2 - 11x + 30)} = 1.$$

**建議題解：**

$$(x^2 - 5x + 5)^{(x^2 - 11x + 30)} = 1.$$

$$x^2 - 11x + 30 = (x - 5)(x - 6)$$

情況一：  $x^2 - 5x + 5 = 1$  和  $x^2 - 11x + 30 = K$ ， $K$  為非零整數

兩實根之和 = 5

情況二：  $x^2 - 5x + 5 \neq 0$  和  $x^2 - 11x + 30 = 0$

兩實根之和 = 11

情況三：  $x^2 - 5x + 5 = -1$  和  $x^2 - 11x + 30 = 2K$ ， $K$  為非零整數

兩實根之和 = 5

∴ 所有的實數解之和 = 5 + 5 + 11 = 21

**Suggested Solutions:**

$$(x^2 - 5x + 5)^{(x^2 - 11x + 30)} = 1.$$

$$x^2 - 11x + 30 = (x - 5)(x - 6)$$

Case I:  $x^2 - 5x + 5 = 1$  and  $x^2 - 11x + 30 = K$ , where  $K$  is a non-zero integer

Sum of 2 real root = 5

Case II:  $x^2 - 5x + 5 \neq 0$  and  $x^2 - 11x + 30 = 0$

Sum of 2 real root = 11

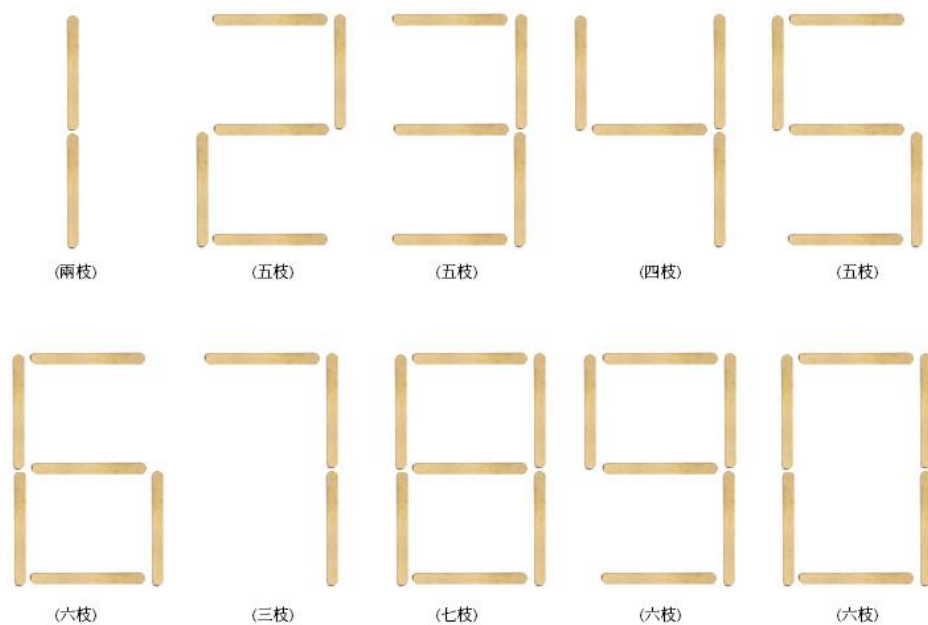
Case III:  $x^2 - 5x + 5 = -1$  and  $x^2 - 11x + 30 = 2K$ , where  $K$  is a non-zero integer

Sum of 2 real root = 5

∴ sum of all real roots = 5 + 5 + 11 = 21

**題 17 (3 分)**

利用雪條棒可以砌出以下的數字。

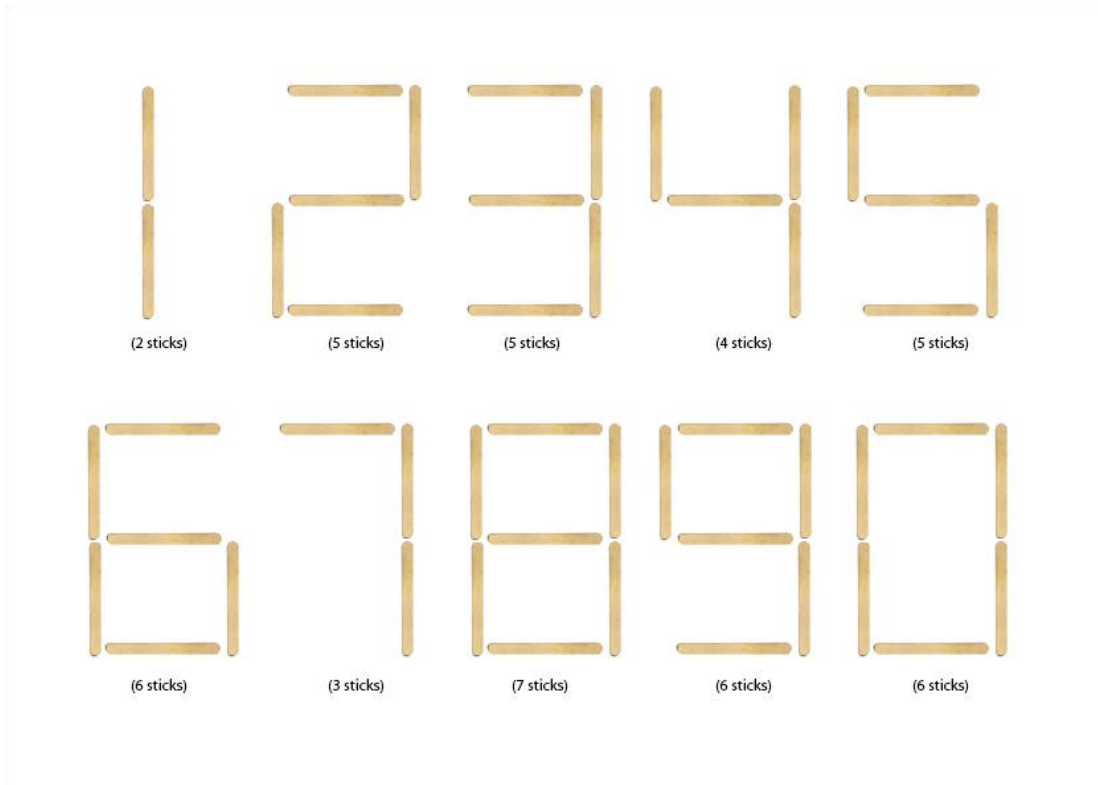


現有 30 枝雪條棒，用盡 30 枝雪條棒砌出下列要求的數字。

- (a) 最大的雙數;
- (b) 最小 5 的倍數;
- (c) 最大的 6 位 3 的倍數。

**Question 17 (3 marks)**

The following digits can be formed by using popsicle sticks.



Now there are 30 popsicle sticks, by using all 30 sticks, form the required numbers to the following conditions.

- (a) The greatest even number;
- (b) The smallest number which is a multiple of 5;
- (c) The greatest 6-digit number which is also a multiple of 3.

**建議題解/ Suggested Solutions :**

- |                           |                            |
|---------------------------|----------------------------|
| (a) 11,111, 111, 111, 114 | $(2 \times 13 + 4)$        |
| (b) 20080                 | $(5 + 6 \times 2 + 7 + 6)$ |
| (c) 999531                | $(6 \times 3 + 5 + 5 + 2)$ |

**題 18 (4 分)**

每隊參賽隊伍請提供一個 0 至 100 之間的數字（包括 0 和 100，也可以是小數）。計算所有收集的數字的平均數，再把答案乘以 0.8，以得到一個數字  $C$ 。

所提供的數字最接近  $C$  的 5 隊會各得 4 分。

不提供數字的隊伍和所提供的數字最不接近  $C$  的 10 隊會各得 0 分，其餘隊伍各得 1 分

**Question 18 (4 marks)**

Every team is required to provide a number between 0 and 100 (including 0 and 100, decimal number is also allowed). The average of all the numbers collected will be calculated and then be multiplied by 0.8 to get a number  $C$ .

The 5 teams that provide numbers closest to  $C$  will get 4 marks.

The 10 teams that provide numbers furthest away from  $C$  and the teams have not provided any numbers will get 0 marks. All the other teams will get 1 mark.

**建議題解/ Suggested Solutions :**

(不適用/NA)

**題 19 (4 分)**

只用直尺，完成以下任務：

註：你只可以連接圖中已有的點（包括 線/線段 與 線/線段 的新交點）來作出新的 線/線段。

- (a) 在此矩形內圍出一塊面積等如此矩形面積五分之一的四邊形。
- (b) 在此正方形內圍出一塊面積等如此正方形面積三分之一的三角形。
- (c) 用一條直線把梯形分為面積相等的兩部分。
- (d) 在此三角形內圍出面積等如此三角形面積四分之一的另一個三角形。

**Question 19 (4 marks)**

Using straightedge only to complete the following tasks:

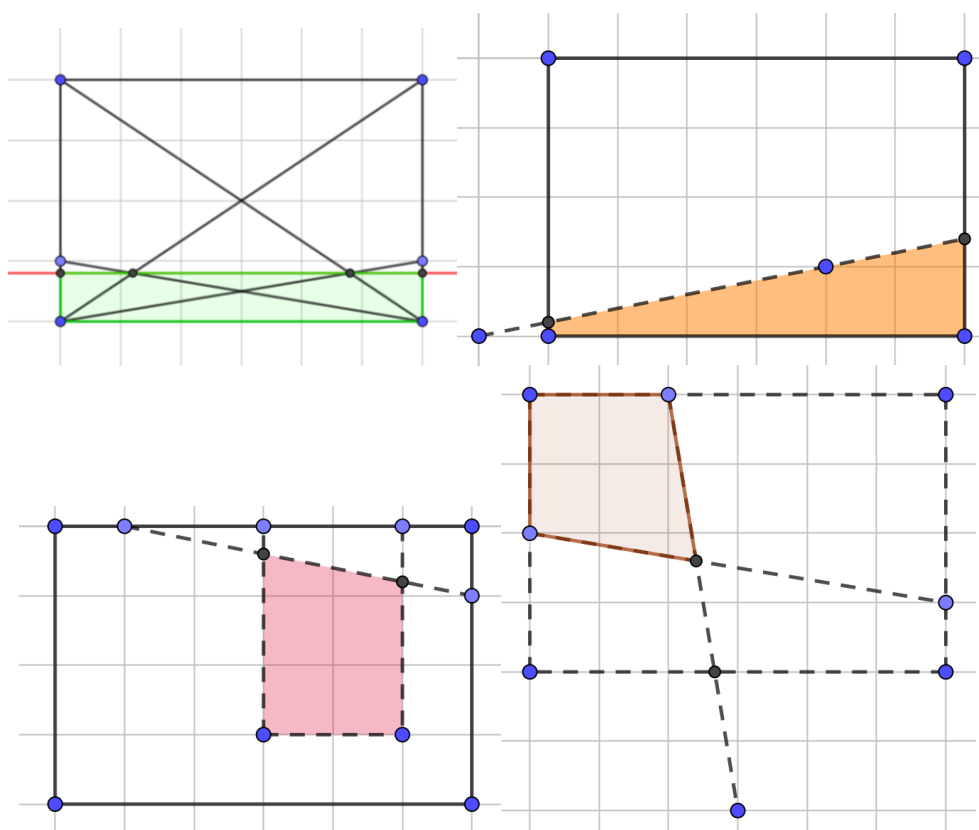
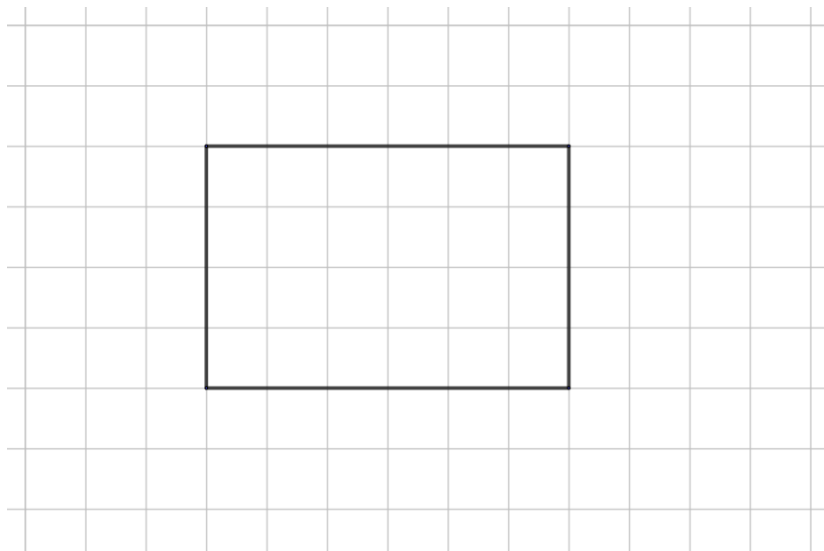
Remark: You are only allowed to join points (including new intersections of lines / line segments) to form new lines / line segments.

- (a) Form a quadrilateral inside the rectangle which has one fifth of its area.
- (b) Form a triangle inside the square which has one third of its area.
- (c) Use a line to divide the trapezium into two parts having equal area.
- (d) Form a triangle inside the given triangle which has one fourth of its area.

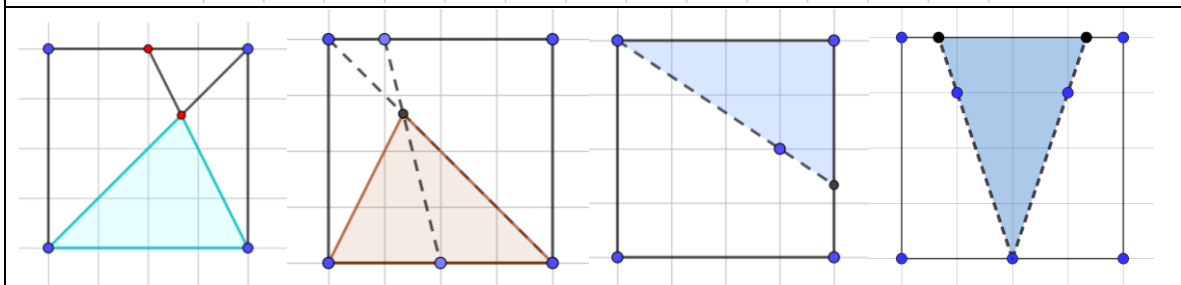
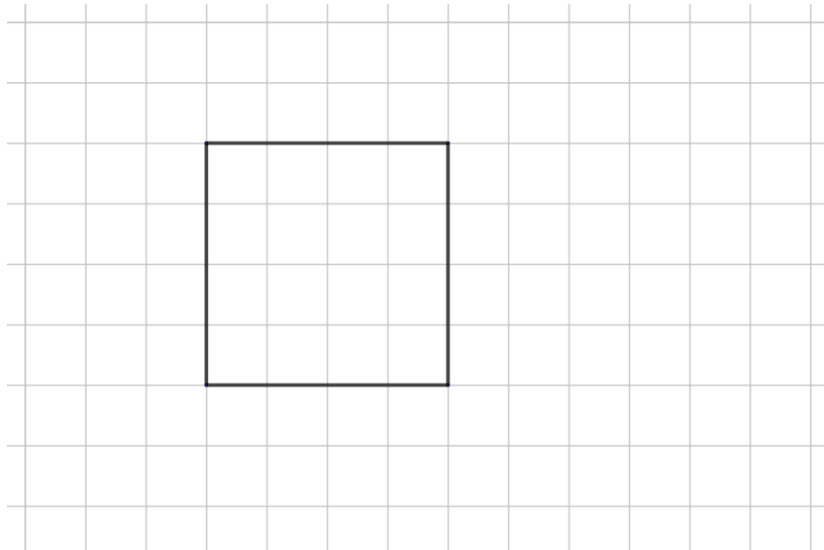
建議題解/ Suggested Solutions :

(接受其他合理答案/Accept any reasonable answers)

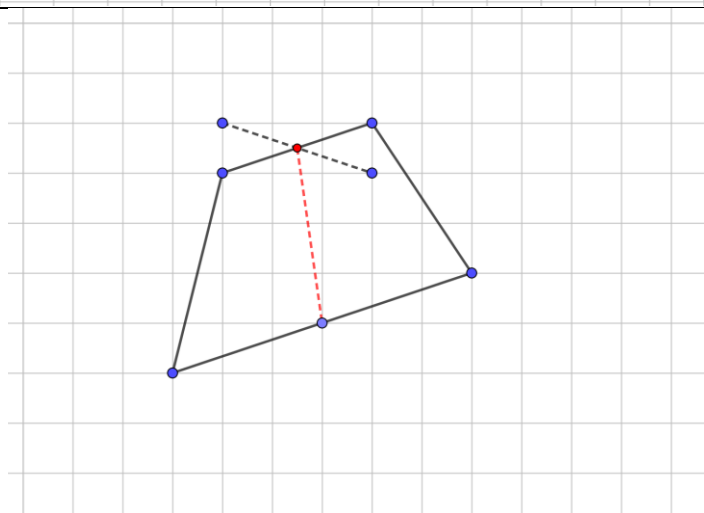
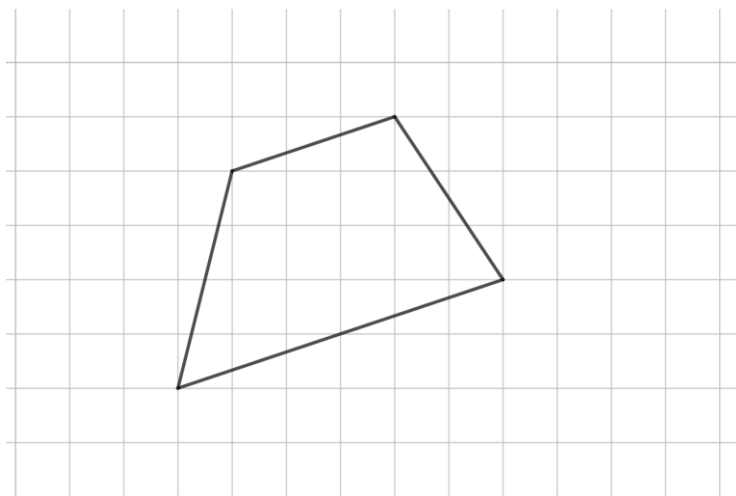
(a)



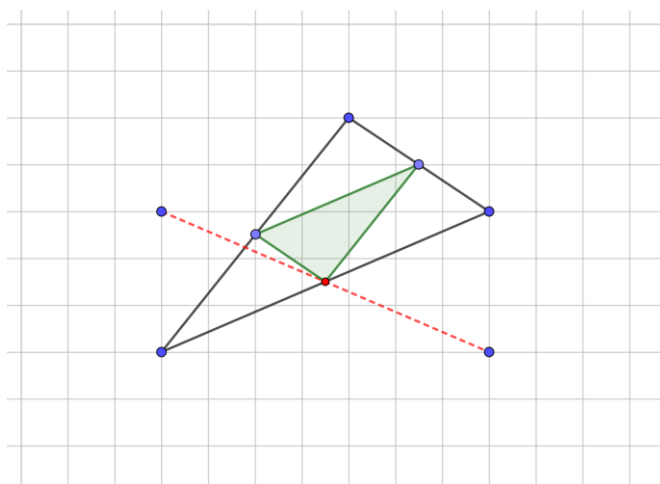
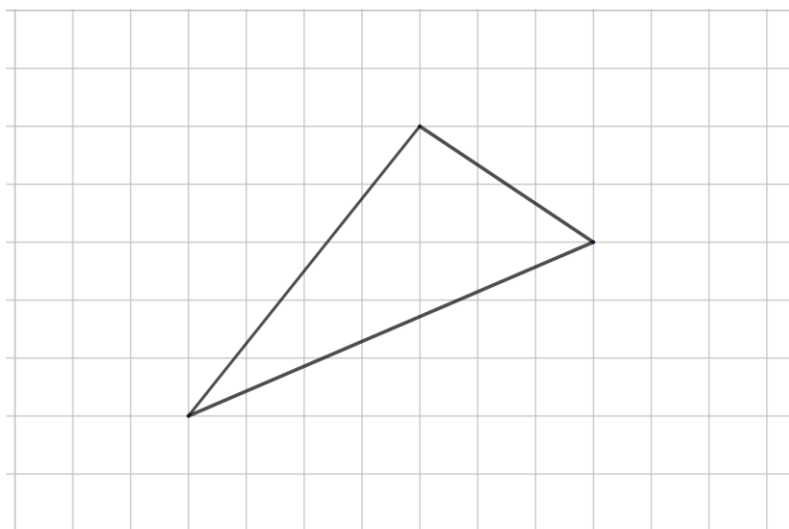
(b)



(c)



(d)



z

全卷完  
End of Paper