

2022/23 第十三屆香港中學數學創意解難比賽

2022/23 The 13th Hong Kong Mathematics Creative Problem Solving Competition
for Secondary Schools

答題紙 Answer sheets

學校名稱 School Name :

得分 Score :
/31

甲部 Section A

答案 Answers	評分 Marks
1. Find the least value of n . 求 n 的最小值。	答/Ans: 18 /2
2. Find the radius of the smaller circle. 求小圓的半徑。	答/Ans: $\frac{3}{4}$ or 0.75 /2
3. Find a, b, c such that they form the sides of a triangle. 若 a, b, c 是三角形的邊長，求 a, b, c 。	答/Ans: $a = \underline{7}, b = \underline{17}, c = \underline{17}$ /3
4. Find the greatest and smallest areas of the quadrilateral formed. 求該四邊形的最大和最小面積。	答/Ans: 最大面積：Greatest area: 10 cm^2 (1 mark if unit is omitted) 最小面積：Smallest area: 1 cm^2 (1 mark if unit is omitted) /4
5. What is the least possible value of n ? 求 n 的最小可能值。	答/Ans: 226 /2
6. Find $\angle A + \angle C + \angle D + \angle E + \angle F + \angle G$. 求 $\angle A + \angle C + \angle D + \angle E + \angle F + \angle G$ 。	答/Ans: 210° (1 mark if unit is omitted) /2

<p>7. How many sets of solution are there? 以上方程共有多少個解?</p>	<p>答/Ans: 2043231</p> <p>/2</p>
<p>8. Find the least positive value of n. 求 n 的最小正數值。</p>	<p>答/Ans: 1</p> <p>/2</p>
<p>9. Find the value of n. 求 n 的值。</p>	<p>答/Ans: 314314</p> <p>/2</p>
<p>10. Find the area of the shaded region. 求圖中陰影部分的面積。</p>	<p>答/Ans: 1 cm² (1 mark if unit is omitted)</p> <p>/2</p>
<p>11. Find the minimum value of the sum of the area of the shaded regions. 求陰影面積之和的最小值。</p>	<p>答/Ans: 4 cm² (1 mark if unit is omitted)</p> <p>/2</p>
<p>12. (a) Find three possible integral value of n other than 2 such that $\sqrt{n+\sqrt{n+\sqrt{n+\dots}}}$ is a positive integer. 除了 2 以外，求 n 的另外三個整數值使得 $\sqrt{n+\sqrt{n+\sqrt{n+\dots}}}$ 為一個正整數。</p>	<p>答/Ans:</p> <p>(i) $n = 6$</p> <p>(ii) $n = 12$</p> <p>(iii) $n = 20$</p> <p>$n = k(k+1)$ for any positive integer k</p> <p>/3</p>
<p>(b) Find three possible integral value of m such that $\frac{1}{3}\sqrt{m+\frac{1}{3}\sqrt{m+\frac{1}{3}\sqrt{m+\dots}}}$ is a positive integer. 求 m 的三個整數值使得 $\frac{1}{3}\sqrt{m+\frac{1}{3}\sqrt{m+\frac{1}{3}\sqrt{m+\dots}}}$ 為一個正整數。</p>	<p>答/Ans:</p> <p>(i) $m = 8$</p> <p>(ii) $m = 34$</p> <p>(iii) $m = 78$</p> <p>$m = ab$ and $9a - b = 1$, where a and b are positive integers</p> <p>/3</p>

乙部 Section B

1.

- (a) Given that the triangle and the pentagon in Figure 2 have identical areas. Find the length of the dotted line in Figure 1.

已知圖二中的三角形和五邊形的面積相等。求圖一中的虛線長度。(2 marks)

$\sqrt{325} \text{ cm} / 5\sqrt{13} \text{ cm}$ (1 mark if unit is omitted)

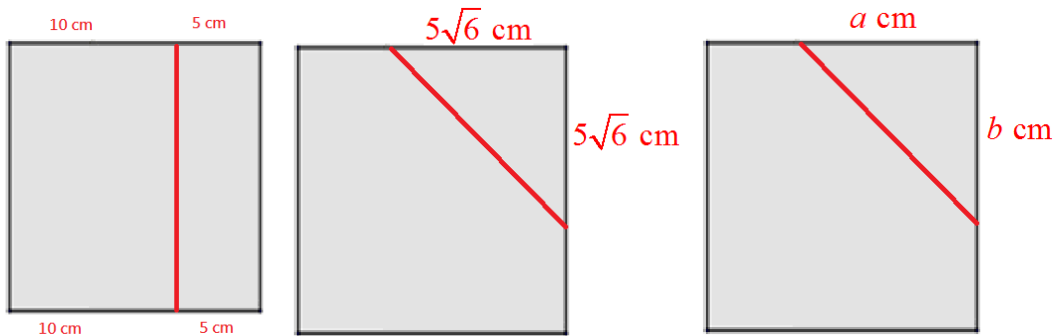
- (b) In Figure 3 below, draw a straight line which is different from the one in Figure 1, with the position clearly illustrated, such that when the square paper is folded along the straight line the two parts with different colours will have equal areas.

在圖三畫一條與圖一不同的直線，並清楚描述直線的位置，使得當方形咭紙沿直線摺紙時，不同顏色的兩部分的面積相同。(2 marks)



Figure 3

圖三



$a \times b = 150$

$a \neq 15 \text{ and } b \neq 15$

Marking guideline:

Grease line only: 0 mark

Grease line with description which is not precise enough: 1 mark

Grease line with precise description: 2 marks

- (a) Suppose the number of folds is not restricted. Draw the grease lines on Figure 4, with the position clearly illustrated, such that when the square paper is folded along the grease lines, it will obtain a new plane figure with a smaller square inside a bigger square of different colours such that $\frac{\text{the area of smaller square}}{\text{the area of larger square}} = \frac{m}{n}$, where m and n are positive integers and are relatively prime.

Write down the values of m and n . (4 marks)

假設摺紙的次數不受限制。在圖四上畫出摺痕，並清楚描述摺痕的位置，使得當正方形咭紙沿著摺痕摺疊後，會出現一個小正方形在大正方形內，兩個正方形的顏色不同，並且

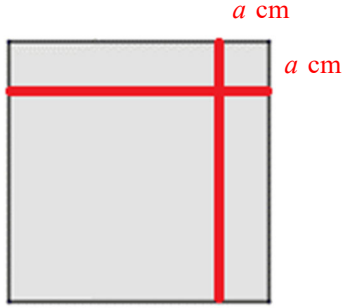
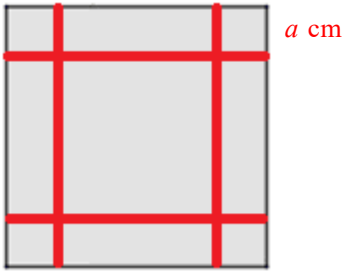
同，並且 $\frac{\text{小正方形面積}}{\text{大正方形面積}} = \frac{m}{n}$ ，其中 m 及 n 為互質的正整數。

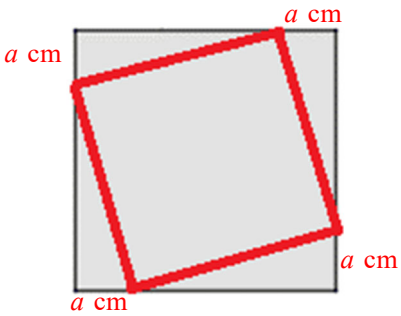
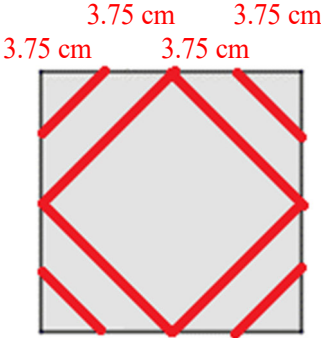
寫出 m 及 n 的值。



Figure 4

圖四

 <p>$a < 7.5$</p>	<p>Area of larger square $= (15 - a)^2 \text{ cm}^2$ Area of smaller square $= (15 - 2a)^2 \text{ cm}^2$</p>	$\frac{m}{n} = \frac{(15 - 2a)^2}{(15 - a)^2}$
 <p>$a < 3.75$</p>	<p>Area of larger square $= (15 - 2a)^2 \text{ cm}^2$ Area of smaller square $= (15 - 4a)^2 \text{ cm}^2$</p>	$\frac{m}{n} = \frac{(15 - 4a)^2}{(15 - 2a)^2}$

 <p>$a < 7.5$</p>	<p>Area of larger square $= [a^2 + (15 - a)^2] \text{ cm}^2$</p> <p>Area of smaller square $= (15 - 2a)^2 \text{ cm}^2$</p>	$\frac{m}{n} = \frac{(15 - 2a)^2}{a^2 + (15 - a)^2}$
	<p>Area of larger square $= 7.5^2 + 7.5^2$ $= 112.5 \text{ cm}^2$</p> <p>Area of smaller square $= 3.75^2 + 3.75^2$ $= 28.125 \text{ cm}^2$</p>	$\frac{m}{n} = \frac{28.125}{112.5} = \frac{1}{4}$

Marking guideline:

Grease line only:

0 mark

Grease line with description which is not precise enough:

1 mark

Grease line with precise description:

2 marks

Correct Grease line with precise description and correct values of m and n : 4 marks

全卷完

End of Paper