

2022/23 第十三屆香港中學數學創意解難比賽

25/2/2023 (星期六) 10:25-11:30

比賽時間：65 分鐘

參賽者須知：

1. 比賽時間：65 分鐘。建議在甲部用 50 分鐘作答，在乙部用 15 分鐘作答。
2. 本問題卷共 13 頁，甲部有 12 題數學題，乙部有 1 題創意解難題。
3. 每位參賽學生獲派一份問題卷及一份答題紙。
4. 比賽其間隊員可以討論題目，並於答題紙寫上議定的答案。
** 只有寫於隊長的答題紙上的答案方可得到評分。
5. 參賽隊伍需自備文具及計算機。為公平起見，比賽中只可使用非圖像計算機。本比賽中嚴禁使用電子字典、電腦、電話或其他有上網或通訊功能的工具。
6. 本試卷每頁的空白位置可作為草稿之用。每位參賽學生會獲派三張草稿紙，如有需要，可要求額外草稿紙。
7. 在筆試完結後，必須交回隊長的答題紙。

2022/23 The 13th Hong Kong Mathematics Creative Problem Solving Competition for Secondary Schools

25/2/2023 (Saturday) 10:25-11:30

Time allowed : 65 minutes

Instructions for participants :

1. **Time allowed: 65 minutes.** It is advised to spend 50 minutes in Section A and 15 minutes in Section B.
2. The question paper consists of 13 pages. There are 12 questions in Section A and 1 creative problem in Section B.
3. Each participant will get a set of question paper and a set of answer sheets.
4. Team members are allowed to discuss during the competition. The agreed answers should be written on the answer sheets.
**** Only the answers in the captain's answer sheet will be marked.**
5. Participating teams should bring their own stationery and calculators. For the purpose of fairness, please use only non-graphic calculators. Electronic dictionaries, computers, mobile phones and other online or communication devices are prohibited.
6. The blank space on each page of this question paper can be used for rough work. Each participant will get three rough work sheets. Extra rough work sheets will be provided upon request.
7. The captain's answer sheets will be collected after the competition.

甲部 (建議此部用 50 分鐘作答)

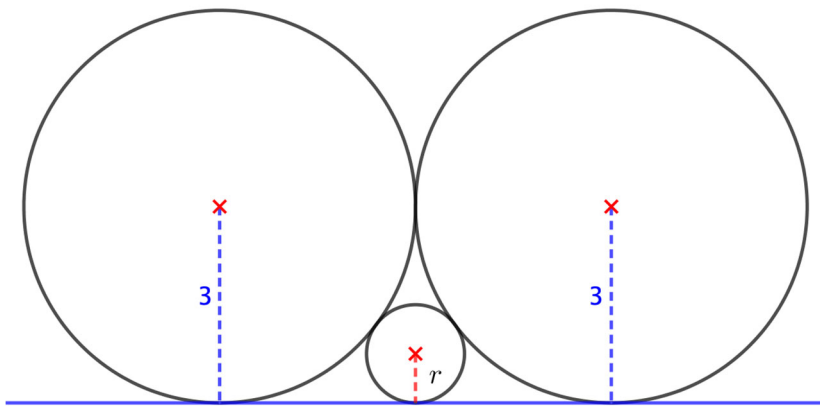
Section A (Suggested to use 50 minutes in this Section)

1. $\overline{a_1a_2 \cdots a_{n-1}3}$ is an n -digit number. If $\overline{3a_1a_2 \cdots a_{n-1}} = 2 \times \overline{a_1a_2 \cdots a_{n-1}3}$, find the least value of n .

$\overline{a_1a_2 \cdots a_{n-1}3}$ 是一個 n -位數。若 $\overline{3a_1a_2 \cdots a_{n-1}} = 2 \times \overline{a_1a_2 \cdots a_{n-1}3}$ ，求 n 的最小值。

2. Two circles with radius 3 touch each other and a straight line. A smaller circle touches the two circles and the line. Find the radius of the smaller circle.

兩個半徑等於 3 的圓形彼此相切且和一直線相切。一個較小的圓和兩圓相切，也和直線相切。求小圓的半徑。

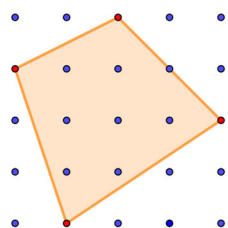


3. $2023 = a \times b \times c$ where a, b, c are integers and $a \leq b \leq c$. Find a, b, c such that they form the sides of a triangle.

$2023 = a \times b \times c$ 而 a, b, c 是整數並且 $a \leq b \leq c$ 。若 a, b, c 是三角形的邊長，求 a, b, c 。

4. Twenty-five points are arranged in a way such that the horizontal and vertical distances of adjacent points equal to 1 cm. If four points are selected such that they are all on different horizontal rows and on different vertical columns, a quadrilateral is then formed by using the selected points as the vertices. Find the greatest and smallest areas of the quadrilateral formed.

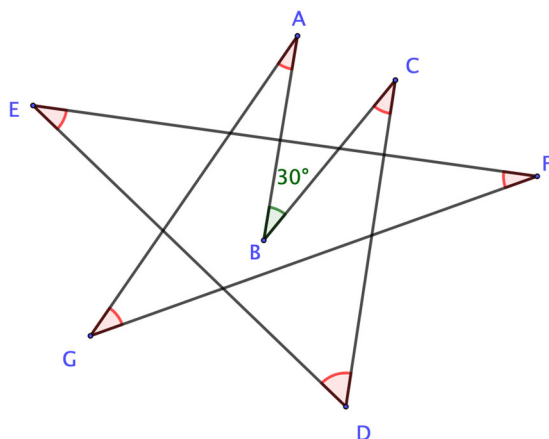
二十五點的排列方式使相鄰的點的垂直和橫向距離皆等於 1 cm。選出四點使得四點分佈在不同的橫行及不同的直列上，再以選出的四點為頂點作一個四邊形。求該四邊形的最大和最小面積。



5. Mr. Wong has $\$n$ in his wallet. It is known that $n \geq 200$. The price for one apple is $\$7$ while the price for one pear is $\$13$. If he uses his money to buy any many apples as possible, his will have $\$2$ left. If he uses his money to buy any many pears as possible, his will have $\$5$ left. What is the least possible value of n ?

黃先生的銀包有 $\$n$ ，已知 $n \geq 200$ 。一個蘋果的售價是 $\$7$ ，而一個梨的售價是 $\$13$ 。若他盡用銀包內的金錢全買蘋果，他會剩下 $\$2$ 。若他盡用銀包內的金錢全買梨，他會剩下 $\$5$ 。求 n 的最小可能值。

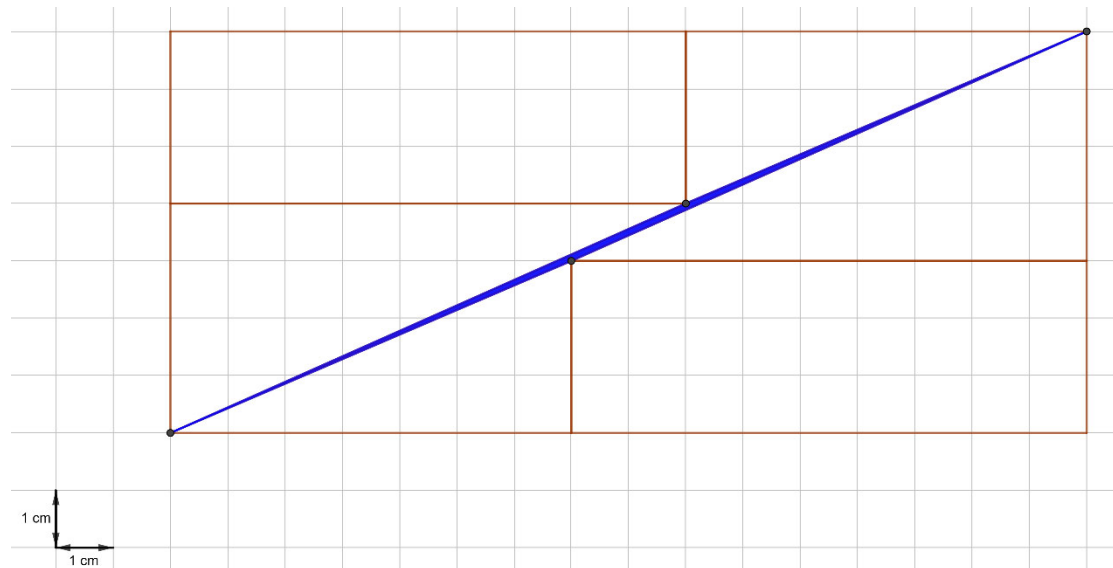
6. In the diagram below, it is given that $\angle B = 30^\circ$. Find $\angle A + \angle C + \angle D + \angle E + \angle F + \angle G$.
 從下圖得知 $\angle B = 30^\circ$ 。求 $\angle A + \angle C + \angle D + \angle E + \angle F + \angle G$ 。



7. If a, b, c are positive integers such that $a + b + c = 2023$, how many sets of solution are there?
 若 a, b, c 為正整數使得 $a + b + c = 2023$ ，以上方程共有多少個解？
8. Given that $11a + 13b = 1$, where a, b are integers and $n = a + b$. Find the least positive value of n .
 已知 $11a + 13b = 1$ ，當中 a, b 為整數及 $n = a + b$ 。求 n 的最小正數值。

9. Mr. Chan want to share n apples to his 6 sisters, 10 brothers and 12 friends. He first divides the apples into 7 equal portions, 6 of the portions is given to his sisters. The portion left is then divided into 11 portions, 10 of which is given to his brothers. The remaining portion is further divided into 13 portions and 12 of them is given to his friends. Finally, there are 314 apples left. Find the value of n .
 陳先生想把 n 個水果分給 6 個姐姐，10 個弟弟及 12 個朋友。他先將水果分為 7 等份，6 份給姐姐們。餘下的再分為 11 份，10 份給弟弟們。再將剩餘的分 13 份，其中 12 份給朋友們。最後有 314 個水果留給自己。求 n 的值。

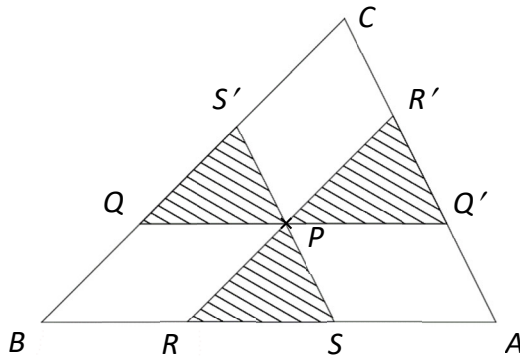
10.



In the diagram, the large rectangle is formed by two small rectangles, four right-angled triangles and the shaded region. Find the area of the shaded region.

圖中的大長方形由兩個小長方形、四個直角三角形及陰影部分組成。求圖中陰影部分的面積。

11.



P is a point inside the $\triangle ABC$, Q, Q', S, S', R and R' are points on the sides such that $QQ' \parallel BA$, $SS' \parallel AC$ and $RR' \parallel BC$.

If the area of $\triangle ABC$ is 12 cm^2 , find the minimum value of the sum of the area of the shaded regions.

$\triangle ABC$ 內有一點 P ，三邊上有 Q, Q', S, S', R 及 R' 使得 $QQ' \parallel BA$, $SS' \parallel AC$ and $RR' \parallel BC$ 。

若 $\triangle ABC$ 的面積為 12 cm^2 ，求陰影面積之和的最小值。

12. It is given that $\sqrt{2+\sqrt{2+\sqrt{2+\dots}}}$ is a positive integer.

已知 $\sqrt{2+\sqrt{2+\sqrt{2+\dots}}}$ 是一個正整數。

(a) Find three possible integral value of n other than 2 such that

$\sqrt{n+\sqrt{n+\sqrt{n+\dots}}}$ is a positive integer.

除了 2 以外，求 n 的另外三個整數值使得 $\sqrt{n+\sqrt{n+\sqrt{n+\dots}}}$ 為一個正整數。

(b) Find three possible integral value of m such that

$\frac{1}{3}\sqrt{m+\frac{1}{3}\sqrt{m+\frac{1}{3}\sqrt{m+\dots}}}$ is a positive integer.

求 m 的三個整數值使得 $\frac{1}{3}\sqrt{m+\frac{1}{3}\sqrt{m+\frac{1}{3}\sqrt{m+\dots}}}$ 為一個正整數。

乙部 (建議此部用 15 分鐘作答)

Section B (Suggested to use 15 minutes in this Section)

1. Figure 1 shows a piece of square paper (15 cm x 15 cm) with different colours on two faces. The paper is then folded to form a new plane figure with two colours as shown in Figure 2.

圖一為一張底面顏色不相同且邊長為15 cm的正方形咭紙。把咭紙沿著虛線摺疊得出圖二中有兩種顏色的新形狀。

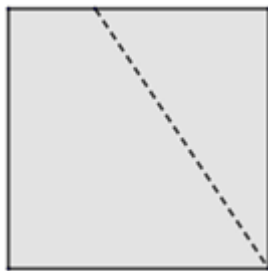


Figure 1

圖一

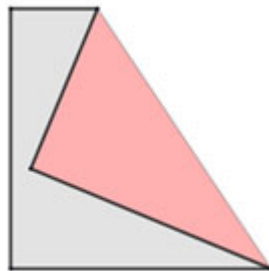


Figure 2

圖二

- (a) Given that the triangle and the pentagon in Figure 2 have identical areas. Find the length of the dotted line in Figure 1.

已知圖二中的三角形和五邊形的面積相等。求圖一中的虛線長度。

- (b) In Figure 3 below, draw a straight line which is different from the one in Figure 1, with the position clearly illustrated, such that when the square paper is folded along the straight line the two parts with different colours will have equal areas.

在圖三畫一條與圖一不同的直線，並清楚描述直線的位置，使得當方形咭紙沿直線摺紙時，不同顏色的兩部分的面積相同。



Figure 3

圖三

- (c) Suppose the number of folds is not restricted. Draw the grease lines on Figure 4, with the position clearly illustrated, such that when the square paper is folded along the grease lines, it will obtain a new plane figure with a smaller square inside a bigger square of different colours such that $\frac{\text{the area of smaller square}}{\text{the area of larger square}} = \frac{m}{n}$, where m and n are positive integers.

Write down the values of m and n .

假設摺紙的次數不受限制。在圖四上畫出摺痕，並清楚描述摺痕的位置，使得當正方形咭紙沿著摺痕摺疊後，會出現一個小正方形在大正方形內，兩個正方形的顏色不同，並且 $\frac{\text{小正方形面積}}{\text{大正方形面積}} = \frac{m}{n}$ ，其中 m 及 n 為正

整數。

寫出 m 及 n 的值。



Figure 4

圖四

全卷完

End of Paper

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2022/23 The 13th Hong Kong Mathematics Creative Problem Solving Competition for
Secondary Schools

答題紙 Answer sheets

學校名稱 School Name :

得分 Score :

/31

甲部 Section A

答案 Answers	評分 Marks
1. Find the least value of n . 求 n 的最小值。	答/Ans: _____ /2
2. Find the radius of the smaller circle. 求小圓的半徑。	答/Ans: _____ /2
3. Find a , b , c such that they form the sides of a triangle. 若 a 、 b 、 c 是三角形的邊長，求 a 、 b 、 c 。	答/Ans: $a = \underline{\quad}$, $b = \underline{\quad}$, $c = \underline{\quad}$ /3
4. Find the greatest and smallest areas of the quadrilateral formed. 求該四邊形的最大和最小面積。	答/Ans: 最大面積：Greatest area: _____ 最小面積：Smallest area: _____ /4
5. What is the least possible value of n ? 求 n 的最小可能值。	答/Ans: _____ /2
6. Find $\angle A + \angle C + \angle D + \angle E + \angle F + \angle G$. 求 $\angle A + \angle C + \angle D + \angle E + \angle F + \angle G$ 。	答/Ans: _____ /2

<p>7. How many sets of solution are there? 以上方程共有多少個解?</p>	<p>答/Ans: _____</p>	<p>/2</p>
<p>8. Find the least positive value of n. 求 n 的最小正數值。</p>	<p>答/Ans: _____</p>	<p>/2</p>
<p>9. Find the value of n. 求 n 的值。</p>	<p>答/Ans: _____</p>	<p>/2</p>
<p>10. Find the area of the shaded region. 求圖中陰影部分的面積。</p>	<p>答/Ans: _____</p>	<p>/2</p>
<p>11. Find the minimum value of the sum of the area of the shaded regions. 求陰影面積之和的最小值。</p>	<p>答/Ans: _____</p>	<p>/2</p>
<p>12. (a) Find three possible integral value of n other than 2 such that $\sqrt{n+\sqrt{n+\sqrt{n+\dots}}}$ is a positive integer. 除了 2 以外，求 n 的另外三個整數值使得 $\sqrt{n+\sqrt{n+\sqrt{n+\dots}}}$ 為一個正整數。</p>	<p>答/Ans: (i) $n =$ _____ (ii) $n =$ _____ (iii) $n =$ _____</p>	<p>/3</p>
<p>(b) Find three possible integral value of m such that $\frac{1}{3}\sqrt{m+\frac{1}{3}\sqrt{m+\frac{1}{3}\sqrt{m+\dots}}}$ is a positive integer. 求 m 的三個整數值使得 $\frac{1}{3}\sqrt{m+\frac{1}{3}\sqrt{m+\frac{1}{3}\sqrt{m+\dots}}}$ 為一個正整數。</p>	<p>答/Ans: (i) $m =$ _____ (ii) $m =$ _____ (iii) $m =$ _____</p>	<p>/3</p>

乙部 Section B

- (a) Given that the triangle and the pentagon in Figure 2 have identical areas. Find the length of the dotted line in Figure 1.

已知圖二中的三角形和五邊形的面積相等。求圖一中的虛線長度。

- (b) In Figure 3 below, draw a straight line which is different from the one in Figure 1, with the position clearly illustrated, such that when the square paper is folded along the straight line the two parts with different colours will have equal areas.

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Figure 3

圖三

- (c) Suppose the number of folds is not restricted. Draw the grease lines on Figure 4, with the position clearly illustrated, such that when the square paper is folded along the grease lines, it will obtain a new plane figure with a smaller square inside a bigger square of different colours such that $\frac{\text{the area of smaller square}}{\text{the area of larger square}} = \frac{m}{n}$, where m and n are positive integers and are relatively prime.

Write down the values of m and n .

假設摺紙的次數不受限制。在圖四上畫出摺痕，並清楚描述摺痕的位置，使得當正方形咭紙沿著摺痕摺疊後，會出現一個小正方形在大正方形內，兩個正方形的顏色不同，並且

$\frac{\text{小正方形面積}}{\text{大正方形面積}} = \frac{m}{n}$ ，其中 m 及 n 為互質的正整數。

寫出 m 及 n 的值。



Figure 4

圖四

全卷完

End of Paper