

Suggested Solution

學校名稱：School Name:

第十三屆香港中學數學創意解難比賽

The 13th Hong Kong Mathematics Creative Problem Solving Competition for Secondary Schools

決賽

Final Event

Suggested Solution

問題一：(20 分)

Question 1: (20 marks)

甲部：製作容積量度工具

(你們可以用到這量度工具協助解題)

Part A – Making volume-measuring tools

(You will use these measuring tools in solving problems)

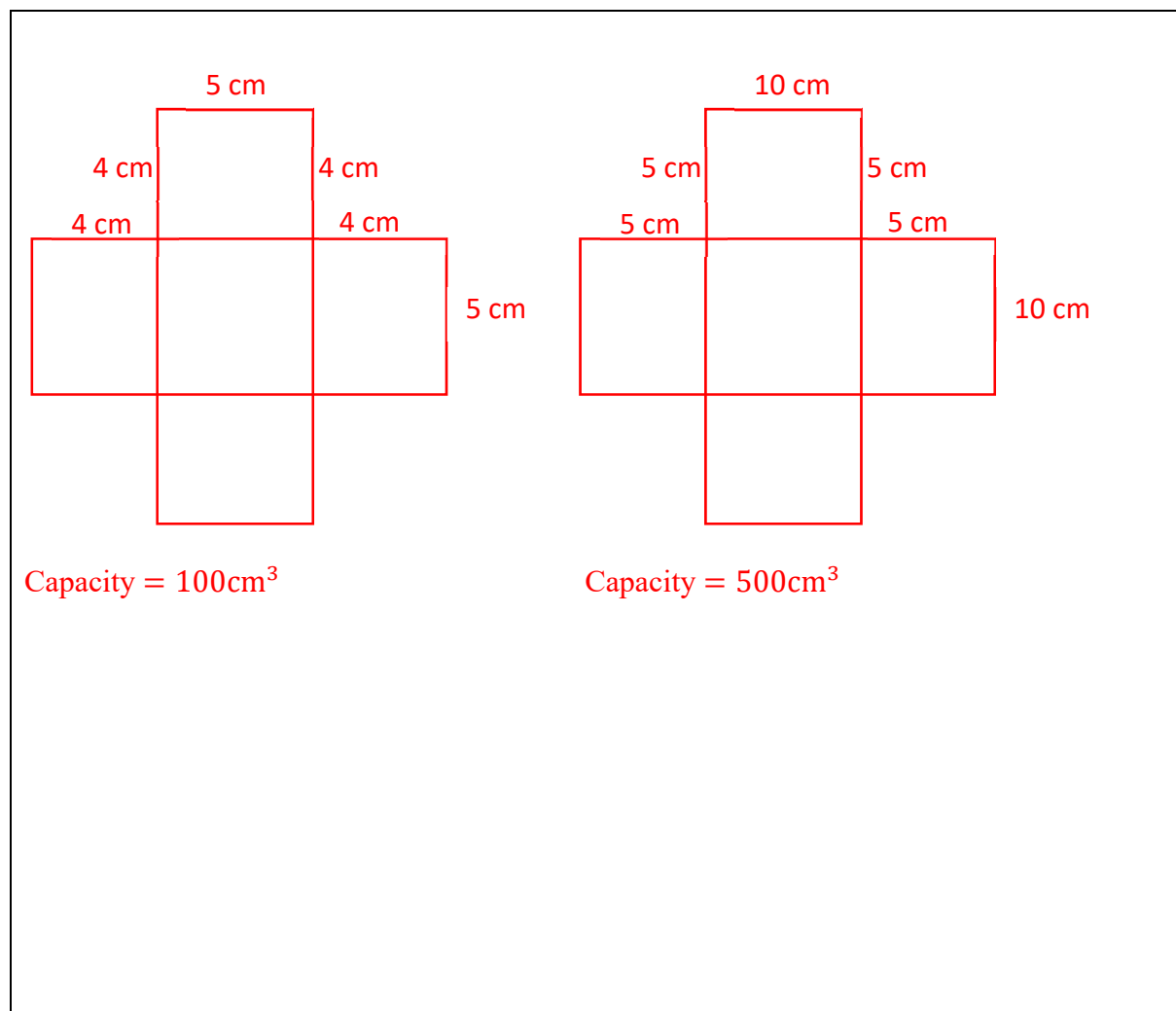
以兩張 A4 紙 (21 cm × 29.7 cm) 製作兩個開頂的長方體形狀容器，體積分別是 100 cm^3 和 500 cm^3 。你們可以用白米去測試。在貼紙上寫下容器的長、闊、高和體積並貼在容器上。

簡單解釋如何得出量度工具的體積。

Use two A4-size papers (21 cm × 29.7 cm) to make two open top containers in the shape of cuboids and having volumes of 100 cm^3 and 500 cm^3 . You may test your measuring tools with the rice.

You can make any measuring tools. Write down the dimension and the volume on the labels and stick it on the corresponding containers.

Explain briefly how you obtain the volume of the measuring tools.



Suggested Solution

Part B – Making open-containers – Method 1

乙部：製作開頂容器-方法一

方法一：開口容器

Method 1 - Open Top Container

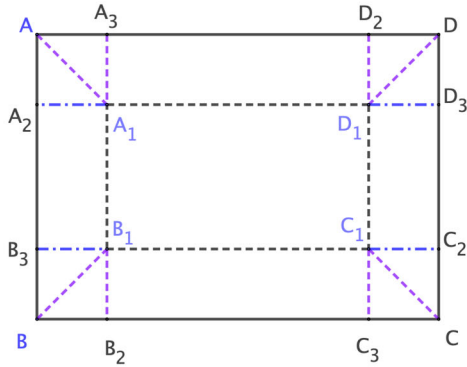


Figure 1a
圖 1a

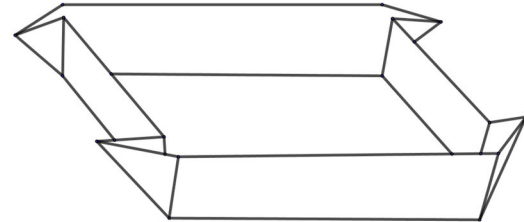


Figure 1b
圖 1b

圖 1a 顯示如何把一張 A4 紙摺成一個開頂的盒子(圖 1b)。接著，把四個角貼上，便製作出一個開頂的長方體。

Figure 1a shows how a piece of A4 paper could be folded to become an open top box (figure 1b). Then, stick the 4 corners to form an open cuboid.

跟據方法一的設計，改變 A_1A_2 的長度，試做一個容量最大的盒子，並計算或估算它的容量，以立方厘米表示。

(盒子會和其他隊伍造出來的比較。)

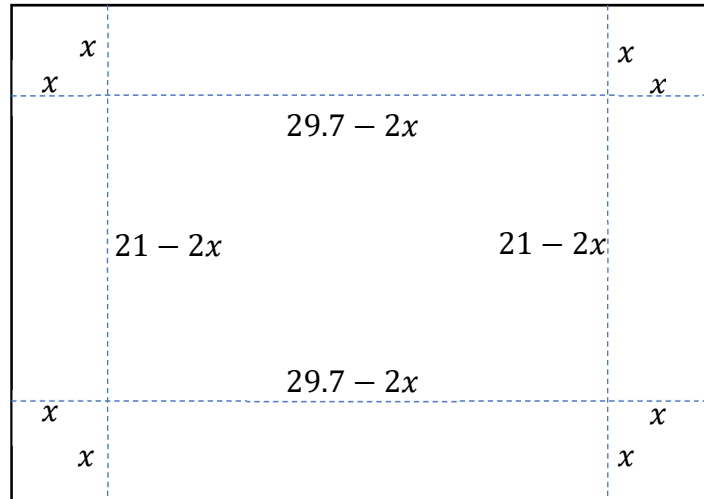
According to method 1, make an open container. Try to maximize the capacity of such a container by changing the length of A_1A_2 . Also, calculate / estimate its capacity in cm^3 .

(Your results will be compared with the results from the other teams.)

Suggested Solution

請以繪圖和計算去解釋一下你的設計。你可以用第九、十頁提供的長方形（合乎 A4 比例）來草擬你們的設計。你們可以量度、估算或計算容器的容量。

Give reasons for your design with drawings and calculations. You may use the rectangles (with ratio equal to that of A4 paper) pages 9 and 10 to draft your design. You may measure, estimate, or calculate the capacities of the containers.



The capacity of the container = $x(29.7 - 2x)(21 - 2x) \text{ cm}^3$

Suggested Solution

Write down the dimensions and the capacities of 3 different open boxes by method 1.
寫出由方法一製作得出的三個不同的開頂盒子的尺寸和容量。

A 盒子 Container A	B 盒子 Container B	C 盒子 Container C
尺寸 Dimension :	尺寸 Dimension :	尺寸 Dimension :
容量 Capacities :	容量 Capacities :	容量 Capacities :

Dimensions: As long as they are x , $29.7 - 2x$ and $21 - 2x$ and all dimensions are positive numbers

Capacities: The capacity should be calculated by $x(29.7 - 2x)(21 - 2x)$

方法一可以做到的最大容量：(清楚解釋如何得出最大容量。)

Maximum capacity by method 1: (Explain clearly how you obtain the maximum capacity.)

The maximum capacity $\approx 1128.5 \text{ cm}^3$

Students are expected to obtain the maximum capacity either through

(i) Trial and Error Method

Students should indicate an increase of capacity from Container A to Container B and a decrease of capacity from Container B to Container C for an increasing value of x

(ii) Calculus

Students should locate the local maximum of the capacity function by the first derivative / second derivative test

(iii) Other reasonable methods

Suggested Solution

題二: (30 分)

Question 2: (30 marks)

問題 1 的盒子是長立方體。如果容許其他形狀及設計，你們將會自由製作一個容量最大的開頂容器，要求如下：

In question 1, the basic shape of the box is a cuboid. If other shapes or designs are allowed, you are going to make any open top container with maximum capacity. The requirements are as follows:

- a) 製作一個容器—自由創作
可以使用膠紙；
其容量超過 1000 cm^3 ；
你只可以用一張 A4 紙作材料；
容器的形狀和尺寸不限。

Making Containers – Free Design

You can use adhesive tape;
its capacity $> 1000 \text{ cm}^3$;
you are only allowed to use a piece of A4 paper;
there is no restriction on the shape and dimension of the container.

- b) 製作一個容器—自由創作
不可以使用膠紙；
其容量超過 800 cm^3 ；
你只可以用一張 A4 紙作材料；
容器的形狀和尺寸不限。

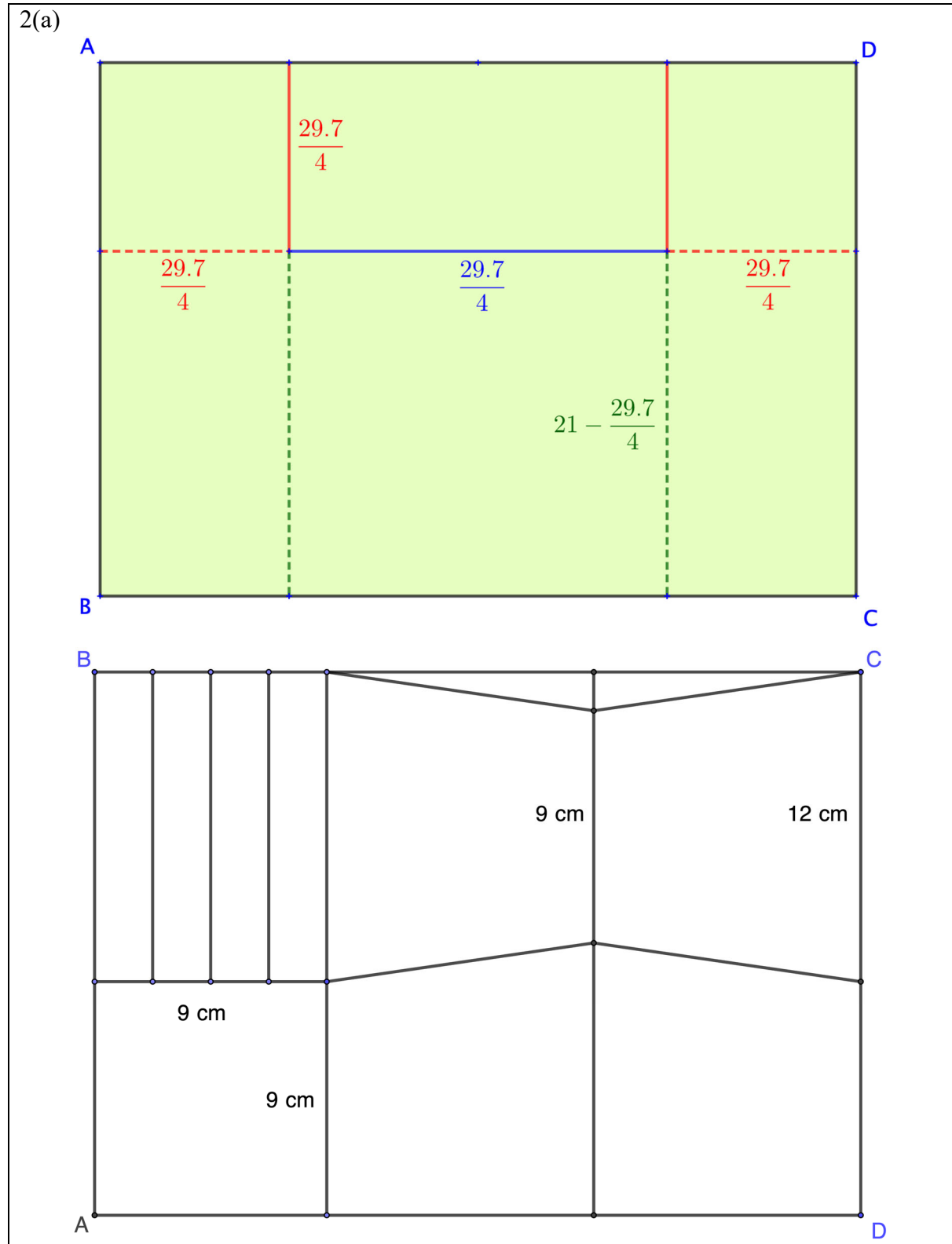
Making Containers – Free Design

You cannot use adhesive tape;
its capacity $> 800 \text{ cm}^3$;
you are only allowed to use a piece of A4 paper;
there is no restriction on the shape and dimension of the container.

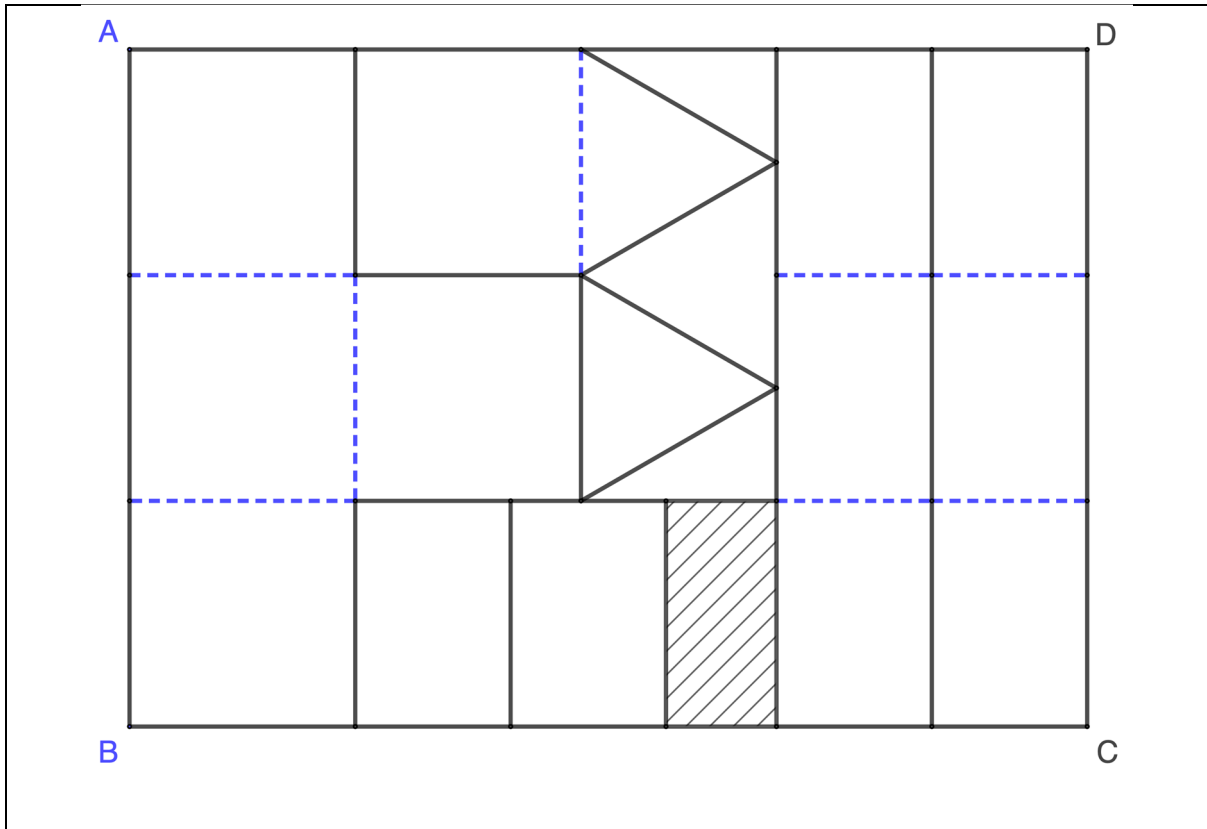
Suggested Solution

用繪圖解釋你的設計並用估算或計算問題 2(a) 和 2(b)容器的容量。

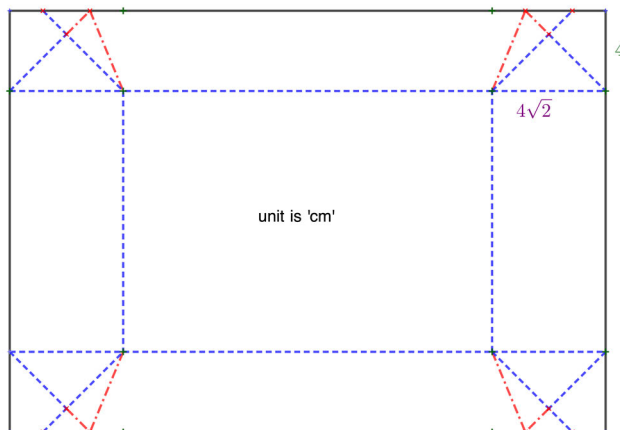
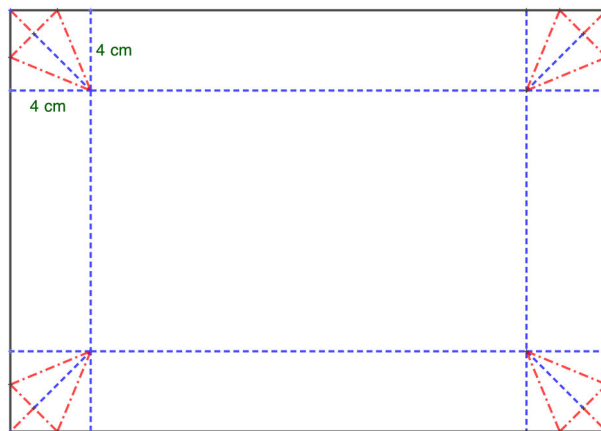
Explain your design with drawings and find the capacities of the containers by estimation/calculation for question 2(a) and 2(b).



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2(b)



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- c) 假如把 2(a)的容器來放花生並容許花生堆得高於容器的高度。請清楚解釋如何估算新的容量。

If the container in 2(a) is used to hold some peanuts and the level of peanuts is allowed to be higher than the height of the container. Explain clear how you estimate the new capacity.

Any reasonable answers are acceptable

Factors for consideration:

- (i) The rigidity of the container
- (ii) The volume of the peanuts higher than the height of the container
- (iii) How the peanuts higher than the height of the container should be organised to avoid falling out from the container

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