

2018/19 The 14th Hong Kong Mathematics
Creative Problem Solving Competition
for Primary School
(Heat – Written)

| | | | | | |
|------------------------|--|-----------------|--|------------------|--|
| School-ID: | | Session: | | Seat No.: | |
| Name of School: | | | | | |

Time allowed : 50 minutes

Instructions :

1. Time allowed: 50 minutes.
2. The question paper consists of 10 pages. There are 13 questions in this paper.
3. Each participant will get one set of question paper.
4. Each team will get one set of answer sheets (in Green) and four pieces of answer sheets (in Orange) (Appendix A).
5. Team members are allowed to discuss during the competition. The agreed answers should be written on the answer sheets (in Green). ** Only the answers in the answer sheets will be marked.
6. Participating teams should bring their own stationery and calculators. For the purpose of fairness, please use only non-graphic calculators. Electronic dictionaries, computers, mobile phones and other communication devices are prohibited.
7. The blank space on each page of this question paper can be used for rough work. Each participant will get one rough work sheet. Extra rough work sheets will be provided upon request.
8. The answer sheets, question papers and rough work sheets will be collected after the competition. Participants are not allowed to take away any of these papers or the team might risk disqualification.

2018/19 第十四屆香港小學數學創意解難比賽
(初賽-筆試)

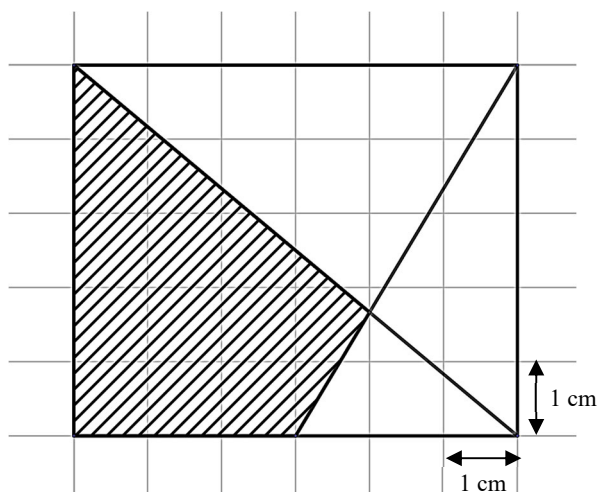
| | | | | | |
|--------------|--|--------------|--|--------------|--|
| 學校編號: | | 比賽場次: | | 座位編號: | |
| 學校名稱: | | | | | |

比賽時間：50 分鐘

參加者須知：

1. 比賽時間：50 分鐘。
2. 本問題卷共 10 頁，全卷共有 13 題。
3. 每位參賽學生獲派一份問題卷。
4. 每隊參賽隊伍獲派一份(黃色)答題紙及四張(橙色)答題紙(附件 A)。
5. 比賽其間隊員可以討論題目，並於(黃色)答題紙寫上議定的答案。
 ** 只有寫於答題紙上的答案方可得到評分。
6. 參賽隊伍需自備文具及計算機。為公平起見，比賽中只可使用非圖像計算機。
 本比賽中嚴禁使用電子字典、電腦、電話或其他有上網或通訊功能的工具。
7. 本試卷每頁的空白位置可作為草稿之用。每位參賽學生會獲派一張草稿紙，如有需要，可要求額外草稿紙。
8. 在筆試完結後，各學生必須交回所有問題卷、答題紙及草稿紙。參賽學生不得取走任何於比賽中所派發之紙張文具，違規者全隊可被取消資格。

1.



圖一 / Figure 1

圖一中每個小格子都是 $1\text{ cm} \times 1\text{ cm}$ 。求斜線部分面積。

In figure 1, each grid is $1\text{ cm} \times 1\text{ cm}$. Find the area of the shaded part.

2.

在 1 至 2019 間（包括 1 和 2019），共有多少個整數包含數字「1」或數字「5」？

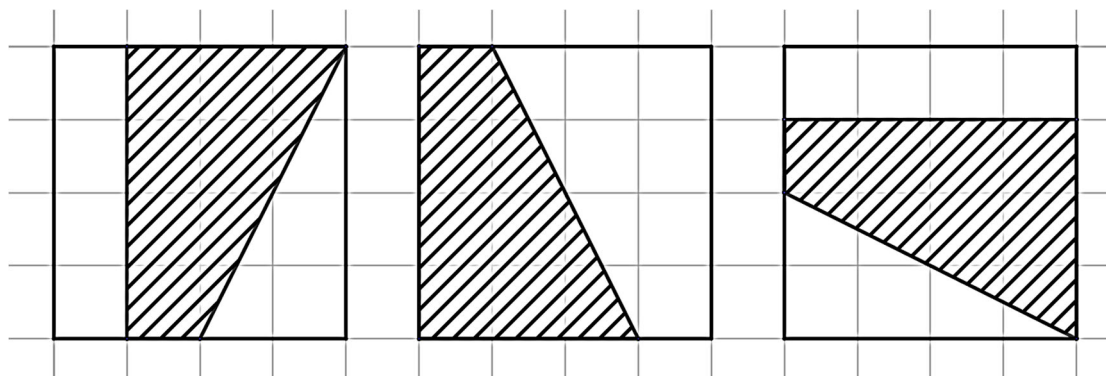
For all integers between 1 and 2019 (inclusively), how many integers contain digit “1” or digit “5”?

3.

列出所有總和是 2019 的連續正整數組。

List all possible group(s) of positive consecutive integers having sum equal to 2019.

4.



圖二 / Figure 2

圖二中有 3 個形狀相同的四邊形，它們的頂點都位於正方形周界並且是在格線的交點上。

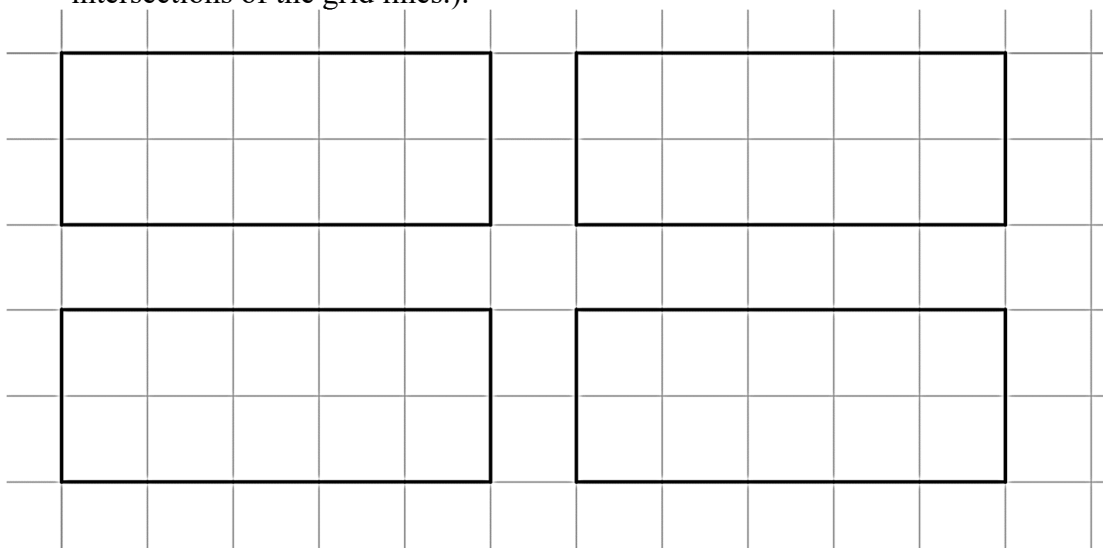
Figure 2 shows three quadrilaterals with the same shape. All their vertices locate on the perimeter of the squares and lie on the intersections of the grid lines.

以下是外框為 2×5 的長方形：

The followings are the rectangles with dimensions 2×5 :

- (a) 請在答題紙上畫出四個形狀不同的四邊形，使得它們各自的面積等於長方形外框面積的一半。（四邊形的頂點必須位於外框的格線的交點上）

Draw four quadrilaterals with **different shapes** such that each of their area is half of the rectangle. (All the vertices must locate on the perimeter of the rectangle and lie on the intersections of the grid lines.).



- (b) 共有多少個不同形狀的四邊形能符合(a)部提及的條件？

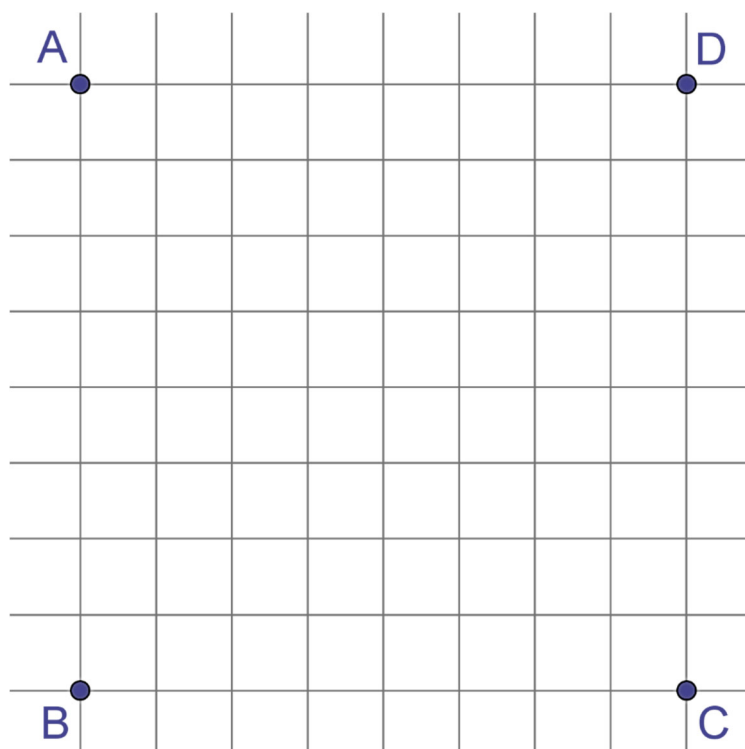
How many quadrilaterals with different shapes fulfill the conditions stated in (a)?

5.

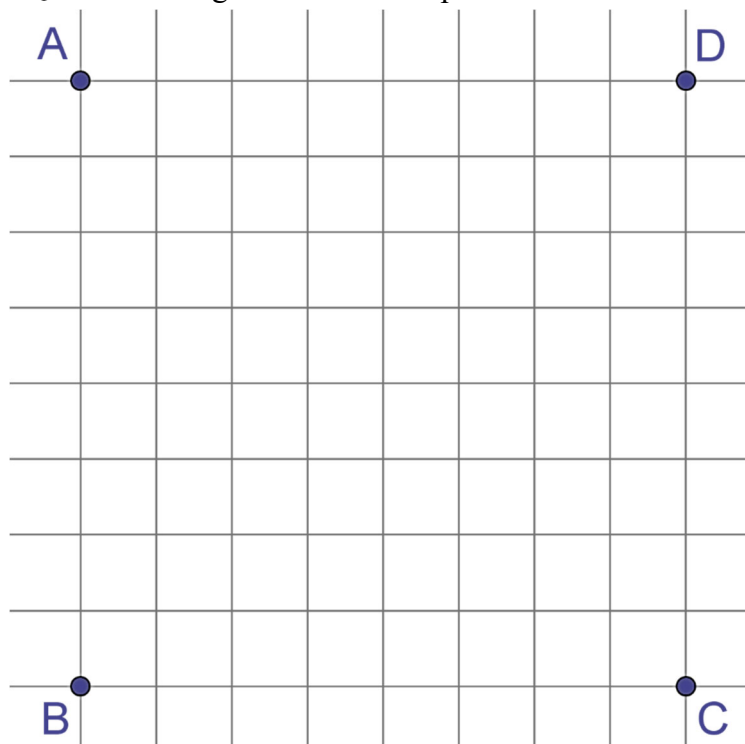
八邊形的每一條邊長皆為整數，而所有頂點皆在圖中格點上。A、B、C 和 D 為其中四個頂點。
請根據以下的指示畫出八邊形：

Each of the sides of octagon is integer. All the vertices are located on the intersections of the grid.
A, B, C and D are four of the vertices. Draw the octagons with the following instructions:

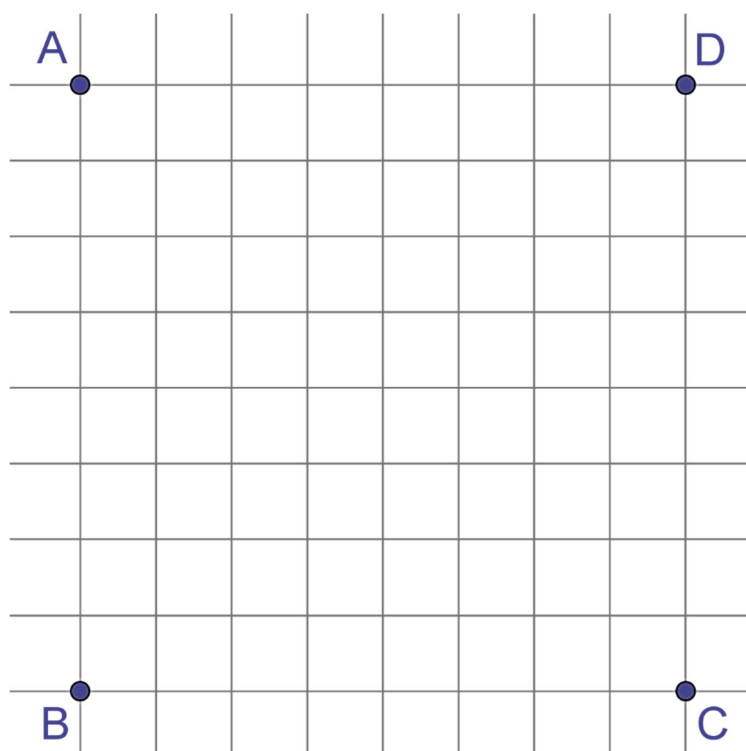
(a) 周界最長的八邊形 The octagon with longest perimeter



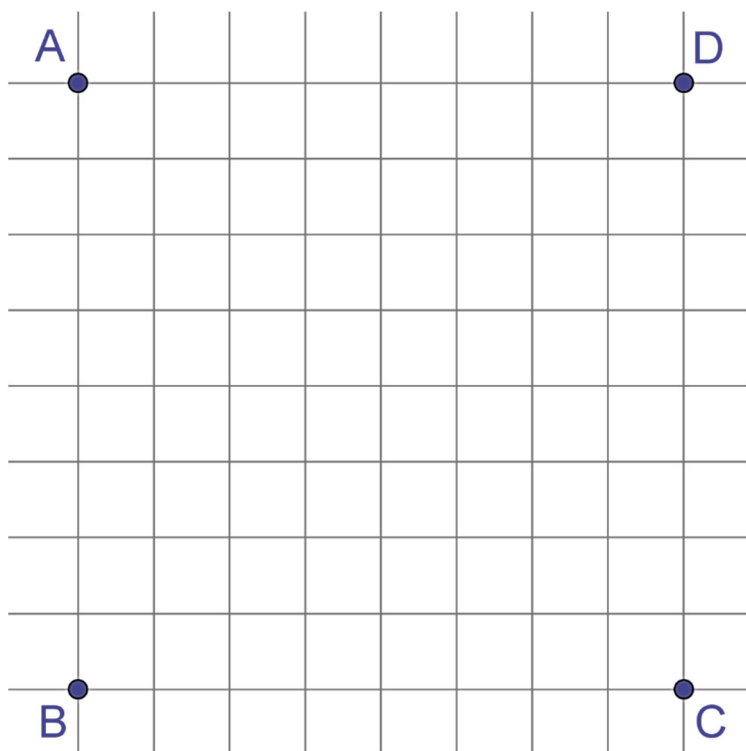
(b) 周界最短的八邊形 The octagon with shortest perimeter



(c) 面積最大的八邊形 The octagon with largest area



(d) 面積最小的八邊形 The octagon with smallest area



6.

100 以內的正整數中，哪個/哪些數字有最多不同的因數？

For all positive integers less than 100, which integer(s) has/have the greatest number of different factors?

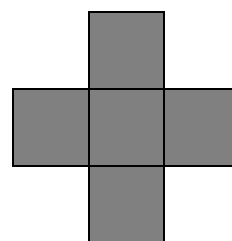
7.

圖三顯示出一數字板及圖四顯示一片由 5 個小正方形組成的十字架方塊。

Figure 3 shows a numbered square board and figure 4 shows a cross tile formed by 5 small squares.

| | | | | | | |
|----|-----|-----|-----|-----|-----|-----|
| 1 | 2 | 3 | ... | ... | 9 | 10 |
| 11 | 12 | 13 | ... | ... | 19 | 20 |
| : | ... | ... | ... | ... | ... | : |
| : | ... | ... | ... | ... | ... | : |
| 91 | 92 | 93 | ... | ... | ... | 100 |

圖三 / Figure 3



圖四 / Figure 4

我們把十字架方塊放到數字板上剛好覆蓋其中五格。若被遮蓋的數字之和是 310，求十字架方塊中心方格覆蓋的數字。

We place the cross tile on the board and it covers 5 tiles completely. If the sum of the covered numbers is 310, find the number covered by the center of the tile.

8.

在下列數字之間加入「+」或「-」，或把相鄰數字合併，使算式得出指定答案。

By adding “+” or “-”, or combining the adjacent numbers, construct equations with the following answers.

例子/ Example:

$$98 - 76 - 5 - 4 - 3 - 2 - 1 - 0 = 7$$

答案/Answers:

$$(a) \quad 9 \quad 8 \quad 7 \quad 6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad 0 \quad = \quad 2$$

$$(b) \quad 9 \quad 8 \quad 7 \quad 6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad 0 \quad = \quad 0$$

$$(c) \quad 9 \quad 8 \quad 7 \quad 6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad 0 \quad = \quad 1$$

$$(d) \quad 9 \quad 8 \quad 7 \quad 6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad 0 \quad = \quad 9$$

9.

在不用直尺、不撕開紙張、不繪畫線條的情況下，利用提供的長方形紙(附件 A)，分別摺出以下圖形：(把摺好的圖形放入文件夾內。)

Without using ruler, tearing of paper or drawing any line, use the pieces of rectangular paper provided (Appendix A) to fold the following figures: (*Put all the folded figures into the folder.*)

(a) 面積為原來 50% 的不等邊非直角三角形；

A scalene triangle (without right angle) having 50% of the area of the paper.

(b) 面積為原來 37.5% 的菱形；

A rhombus having 37.5% of the area of the paper.

(c) 面積為原來 62.5% 的（沒有直角的）平行四邊形；

A parallelogram (without right angle) having 62.5% of the area of the paper.

(d) 面積為原來 $\frac{9}{16}$ 的非對稱五邊形。

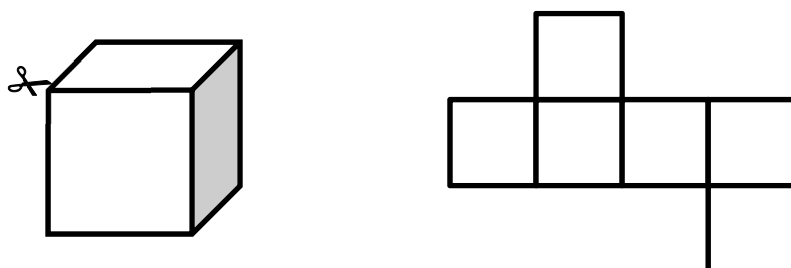
An asymmetric pentagon having $\frac{9}{16}$ of the area of the paper.

10.

沿着多面體上特定的稜剪開，可以把它翻開變成展開圖。

Cut along some edges of a polyhedron, we can unfold it and form a net.

例子/Example:



正立方體和它的其中一個展開圖

A Cube and its net

圖五顯示一個六條稜的長度皆不同的三角錐體。

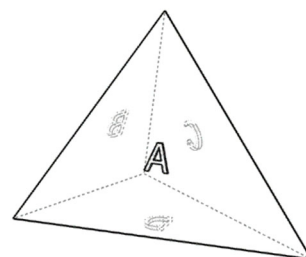
Figure 5 shows a triangular pyramid with the 6 edges of different lengths.

(a) 試畫出其中兩個可能的展開圖。(展開圖內側朝上方)

Draw two possible nets. (The inner side faces up)

(b) 求此三角錐體共有多少個不同的展開圖(展開時內側朝上)?

How many nets can be formed by unfolding the triangular pyramid with the inner sides facing up?



圖五 / Figure 5

11.

A、B、C 和 D 四人一起到小食店。

A、B 和 C 共吃了 20 粒魚蛋；

A、B 和 D 共吃了 16 粒魚蛋；

A、C 和 D 共吃了 18 粒魚蛋；

B、C 和 D 共吃了 21 粒魚蛋。

吃得最多魚蛋的人要替自己和吃得最少魚蛋的人結帳，每粒魚蛋售 1 元 5 角，他/共需付多少元？

A, B, C and D went to a snack shop.

A, B and C ate 20 fish balls in total.

A, B and D ate 16 fish balls in total.

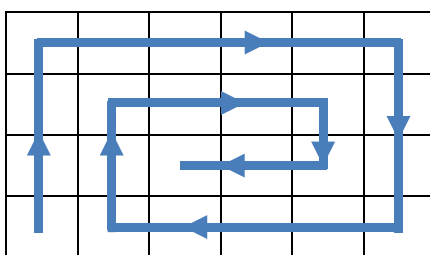
A, C and D ate 18 fish balls in total.

B, C and D ate 21 fish balls in total.

The one who ate the greatest amount fish balls has to pay for him and the one who ate the least. The price of each fish ball is \$1.5, how much should he/she pay?

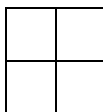
$M \times N$ 的方格內，在其中一格為起點，以橫直方向移動到另一方格內，直至經過所有方格，當中每次移動不會重覆踏上之前走過的方格，我們稱該有方向性的移動路徑為哈密頓路徑 (Hamiltonian path)。

例子/Example :

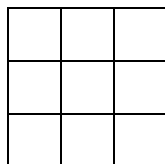


圖六 / Figure 6

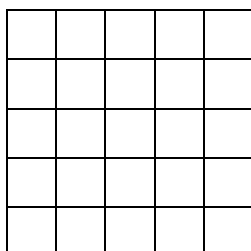
In 2×2 grid, how many Hamiltonian paths are there?



In 3×3 grid, how many Hamiltonian paths are there?



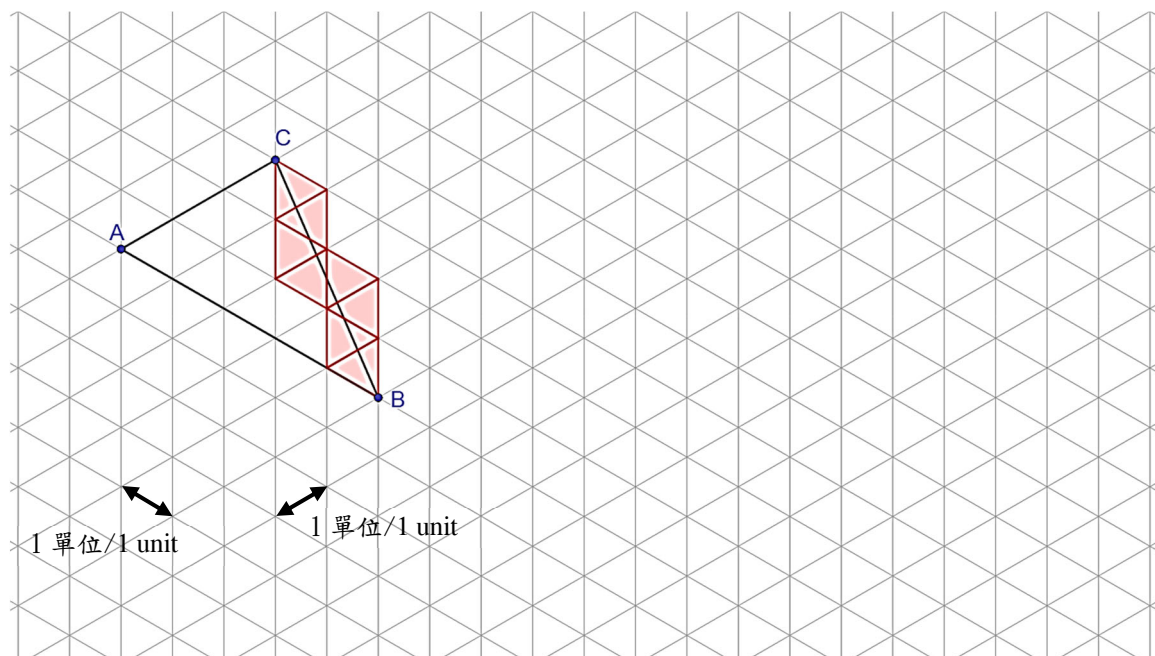
In 5×5 grid, Mary thinks that any grids can be the starting grid of a Hamiltonian path. Do you agree? If not, put a "X" to denote all the grids that cannot be the starting grids.



13.

在圖七中，A 為固定點，設 $AC = 3$ 單位， $AB = 5$ 單位，則 BC 經過 8 個小三角形。

In figure 7, A is a fixed point. Let $AC = 3$ units, $AB = 5$ units, then BC cuts across 8 triangles.



圖七/ Figure 7

- (a) 分別延伸 AB 及 AC 至 B' 及 C'，使得 $AB' = 8$ 單位， $AC' = 5$ 單位。問 B'C' 會經過多少個小三角形？

Suppose AB and AC are produced to B' and C' respectively such that $AB' = 8$ units and $AC' = 5$ units. How many triangles will be cut across by B'C'?

- (b) 如(a)部所示，若 $AB' = 10$ 單位及 $AC' = 6$ 單位。問 B'C' 會經過多少個小三角形？

According to part (a), if $AB' = 10$ units and $AC' = 6$ units. How many triangles will be cut across by B'C'?

- (c) 如(a)部所示，若 $AB' = 96$ 單位及 $AC' = 60$ 單位。問 B'C' 會經過多少個小三角形？

According to part (a), if $AB' = 96$ units and $AC' = 60$ units. How many triangles will be cut across by B'C'?

全卷完 [End of Paper]