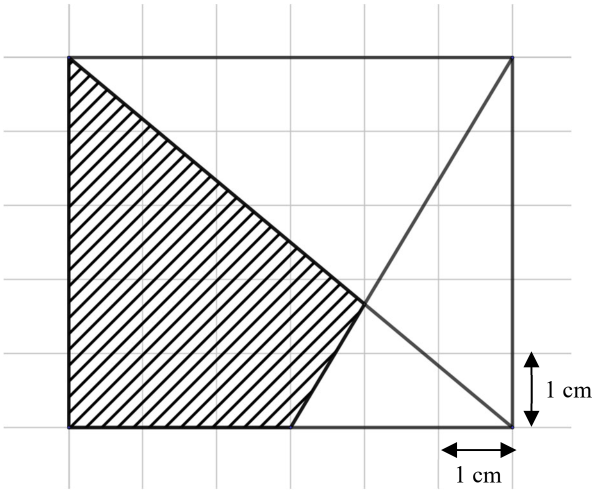
1.

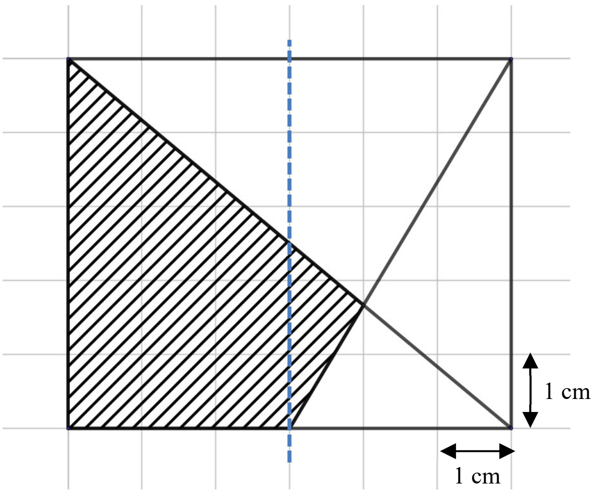


圖一 / Figure 1

圖一中每個小格子都是1 cm ×1 cm。求斜線部分面積。

In figure 1, each grid is 1 cm × 1 cm. Find the area of the shaded part.

建議題解：

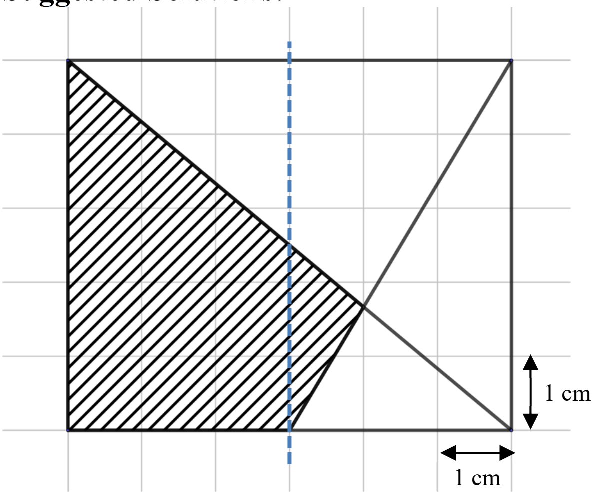


斜線部分面積: / Area of the shaded part:

$$= \frac{(5 + 2.5) \times 3}{2} + \frac{2.5 \times 1}{2}$$

$\underline{= 12.5\ \text{cm}^2}$

Suggested Solutions:



Area of the shaded part:

$$= \frac{(5 + 2.5) \times 3}{2} + \frac{2.5 \times 1}{2}$$

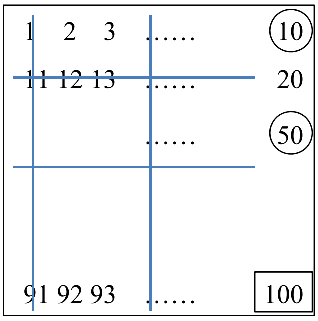
$\underline{= 12.5\ \text{cm}^2}$

2.

1 至 2019 間（包括 1 和 2019），共有多少個整數包含數字「1」或數字「5」？

For all integers between 1 and 2019 (inclusively), how many integers contain digit “1” or digit “5”?

建議題解：



1 至 100 間（包括 1 和 100）

符合題目條件的數字共有:

$= 40 - 4 + 1$ (四條橫直線 − 重複的數目 + 數字 100)

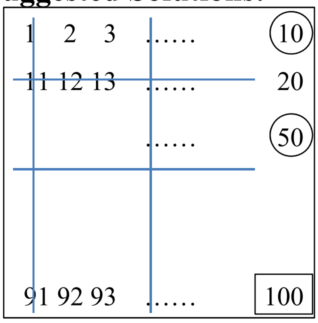
$= 37$

如此類推:

數字範圍	符合條件的數目
1 ~ 100	37
101 ~ 200	99
201 ~ 300	36
301 ~ 400	36
401 ~ 500	37
501 ~ 600	99
601 ~ 700	36
701 ~ 800	36
801 ~ 900	36
901 ~ 1000	37
1001 ~ 2000	999
2001 ~ 2019	12

總共有 $37 \times 3 + 36 \times 5 + 99 \times 2 + 99 + 999 + 12 = 1500$

Suggested Solutions:



From 1 to 100 inclusively

No. of integers satisfy the given conditions:

= 40 – 4 + 1 (4 hori./vert. lines – repeated + no. 100)

= 37

From the above, we have

Intervals	No. of integer
1 ~ 100	37
101 ~ 200	99
201 ~ 300	36
301 ~ 400	36
401 ~ 500	37
501 ~ 600	99
601 ~ 700	36
701 ~ 800	36
801 ~ 900	36
901 ~ 1000	37
1001 ~ 2000	999
2001 ~ 2019	12

Total no. of integers = $37 \times 3 + 36 \times 5 + 99 \times 2 + 99 + 999 + 12 = 1500$

3.

列出所有總和是 2019 的連續正整數組。

List all possible group(s) of positive consecutive integers having sum equal to 2019.

建議題解：

情況 I： 有奇數個連續正整數。

設連續正整數之和

$= (n-k) + (n-(k-1)) + \ldots + (n-1) + n + (n+1) + \ldots + (n+k)$, k 為正整數

$= (2k+1)\,n$

設

$(2k+1)\,n = 2019$

$(2k+1)\,n = 3 \times 673$ or $(2k+1)\,n = 1 \times 2019$

$n = 3, k = 336$ (rej.) ; $\underline{n = 673, k = 1}$; $n = 1, k = 1009$ (rej.) or $n = 2019, k = 0$ (rej.)

$\therefore$ $\underline{n = 673, k = 1}$

即連續正整數為 672，673，674。

情況 II： 有偶數個連續正整數。

設 N 為連續正整數的數目

若 N 能被 4 整除，其和必為雙數。

我們設 $N = 2(2K+1)$ K 是任意非負整數

設和為

$= (n-2K) + (n-(2K-1)) + \ldots + (n-1) + n + (n+1) + \ldots + (n+2K) + (n+2K+1)$

$= 2(2K+1)n + (2K+1)$

$= (2K+1)(2n+1)$

$(2K+1)(2n+1) = 2019$

$(2K+1)(2n+1) = 3 \times 673$ or $(2K+1)(2n+1) = 1 \times 2019$

$n = 1, K = 336$ (rej.) ; $\underline{n = 336, K = 1}$; $n = 0, K = 1009$ (rej.) or $\underline{n = 1009, K = 0}$

$\underline{\therefore\ \ n = 336, K = 1 \text{ or } n = 1009, K = 0}$

當 $n = 336, K = 1$

連續正整數的數目

$N = 2(2K + 1) = 2(2(1) + 1) = 6$

即連續正整數為 334，335，336，337，338，339。

當 $n = 1009, K = 0$

連續正整數的數目

$N = 2(2K + 1) = 2(2(0) + 1) = 2$

即連續正整數為 1009，1010。

∴ 共有 3 組。

Suggested Solutions:

Case I: The groups have odd number of consecutive integers.

Let the sum

$= (n - k) + (n - (k - 1)) + \ldots + (n - 1) + n + (n + 1) + \ldots + (n + k)$ where k is any positive integer

$= (2k + 1)\, n$

Let

$(2k + 1)\, n = 2019$

$(2k + 1)\, n = 3 \times 673$ or $(2k + 1)\, n = 1 \times 2019$

$n = 3, k = 336$ (rej.) ; $\underline{n = 673, k = 1}$; $n = 1, k = 1009$ (rej.) or $n = 2019, k = 0$ (rej.)

$\therefore$ $\underline{n = 673, k = 1}$

<u>i.e. The consecutive integers are 672, 673, 674.</u>

Case II: The groups have even number of consecutive integers.

Let N be the number of consecutive integers.

If N is divisible by 4, the sum must be an even number.

We let $N = 2(2K + 1)$ where K is any non-negative integer.

Let the sum

$= (n - 2K) + (n - (2K - 1)) + \ldots + (n - 1) + n + (n + 1) + \ldots + (n + 2K) + (n + 2K + 1)$

$= 2(2K + 1)n + (2K + 1)$

$= (2K + 1)(2n + 1)$

$(2K + 1)(2n + 1) = 2019$

$(2K + 1)(2n + 1) = 3 \times 673$ or $(2K + 1)(2n + 1) = 1 \times 2019$

$n = 1, K = 336$ (rej.) ; $\underline{n = 336, K = 1}$; $n = 0, K = 1009$ (rej.) or $\underline{n = 1009, K = 0}$

$\underline{\therefore \quad n = 336, K = 1 \text{ or } n = 1009, K = 0}$

When $n = 336, K = 1$

No. of consecutive integers

$N = 2(2K + 1) = 2(2(1) + 1) = 6$

<u>i.e. The consecutive integers are 334, 335, 336, 337, 338, 339.</u>

When $n = 1009, K = 0$

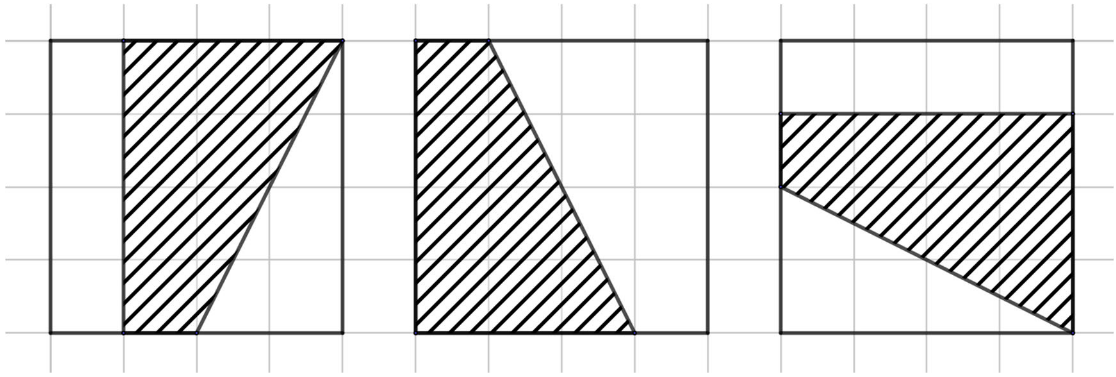
No. of consecutive integers

$N = 2(2K + 1) = 2(2(0) + 1) = 2$

<u>i.e. The consecutive integers are 1009, 1010.</u>

$\therefore$ There are 3 groups

4.



圖二 / Figure 2

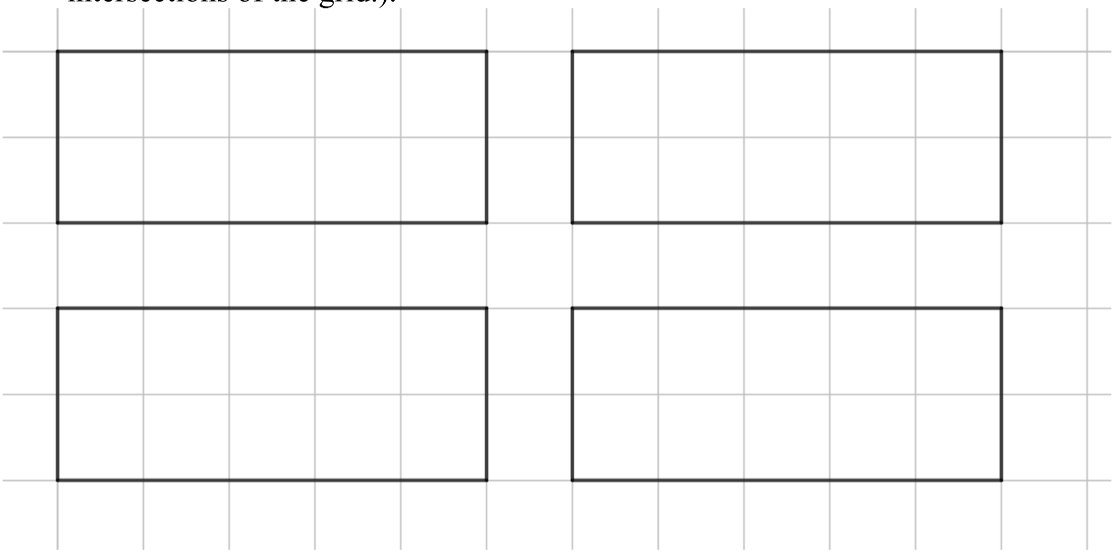
圖二中有 3 個形狀相同的四邊形，它們的所有頂點位於正方形周界上的格點。

Figure 2 shows three quadrilaterals with the same shape. All the vertices locate on the perimeter of the squares and lie on the intersections of the grid.

以下是外框為 2×5 的長方形:

The followings are the rectangles with dimensions 2×5:

(a) 請在答題紙上畫出四個**形狀不同**的四邊形，使得它們各自的面積等於長方形外框面積的一半。(四邊形的頂點必須位於外框的格點上)
Draw four quadrilaterals with **different shapes** such that each of their area is half of the rectangle. (All the vertices must locate on the perimeter of the rectangle and lie on the intersections of the grid.).

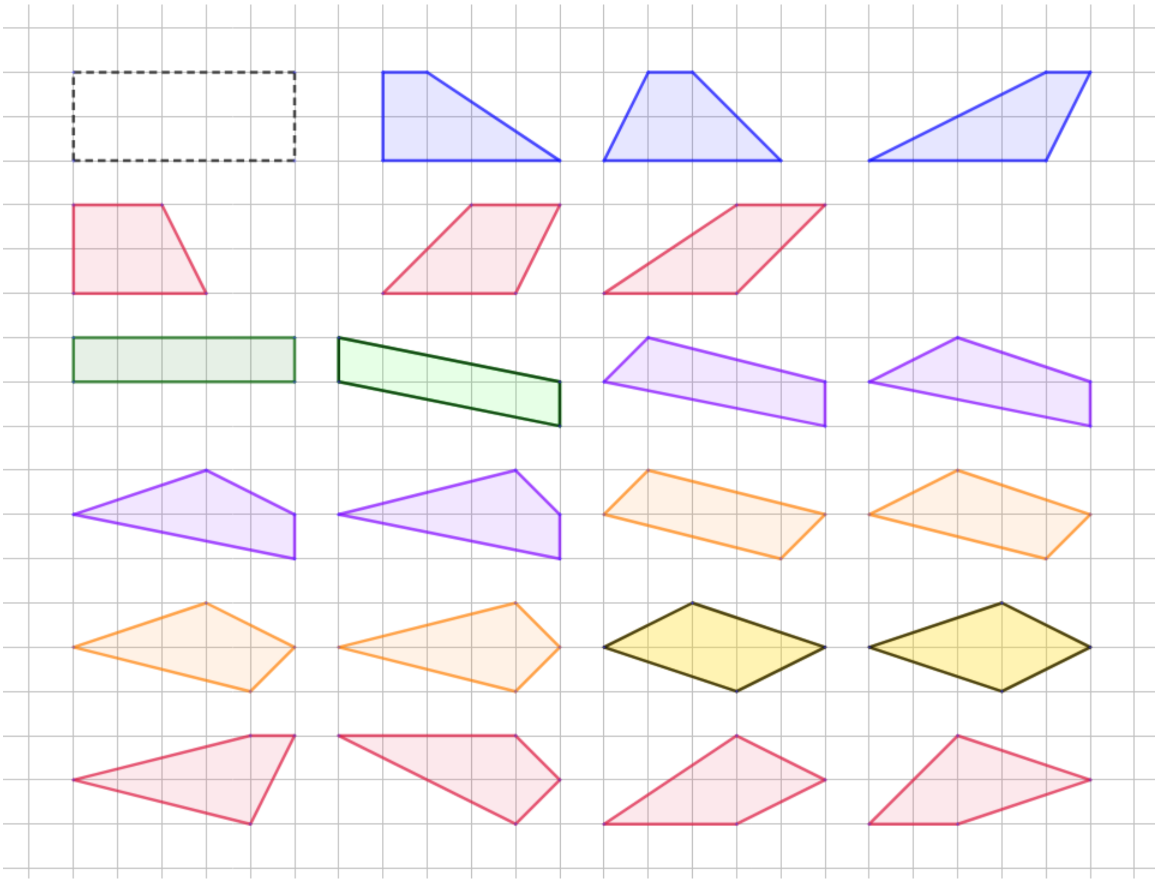


(b) 共有多少個不同形狀的四邊形能符合(a)部提及的條件？
How many quadrilaterals with different shapes which fulfill the conditions stated in (a)?

建議題解/ Suggested Solutions:

共有 22 個不同形狀的四邊形能符合(a)部提及的條件。

There are 22 quadrilaterals with different shapes which fulfill the conditions stated in (a).

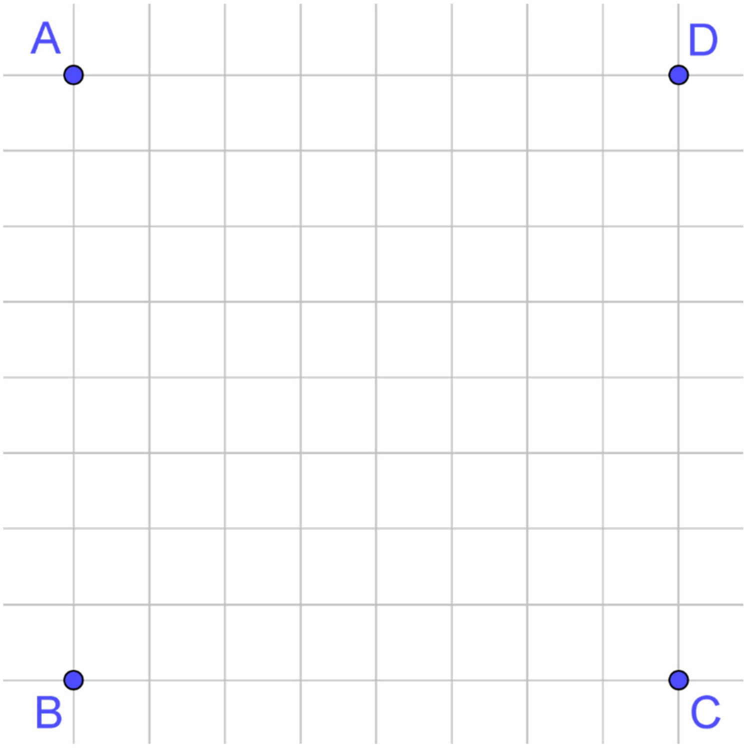


5.

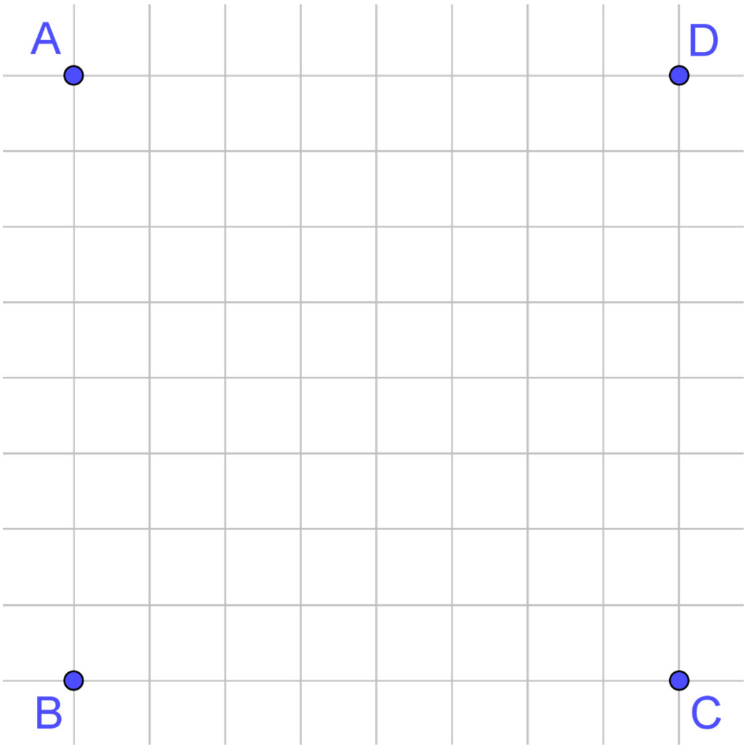
八邊形的每一條邊長皆為整數，而所有頂點皆在圖中格點上。A、B、C 和 D 為其中四個頂點。請根據以下的指示畫出八邊形：

Each of the sides of octagon is integer. All the vertices are located on the intersections of the grid. A, B, C and D are four of the vertices. Draw the octagons with the following instructions:

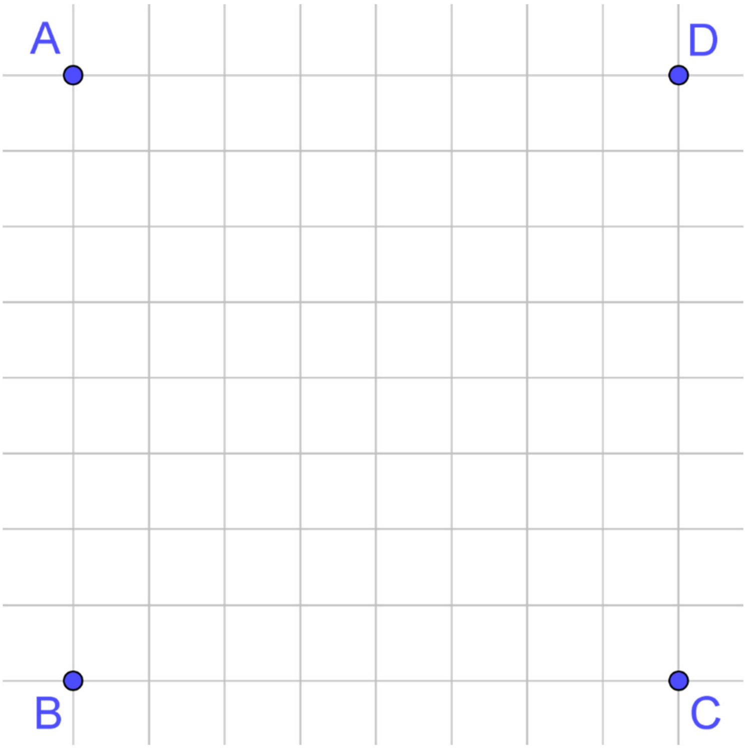
（a）周界最長　The longest perimeter



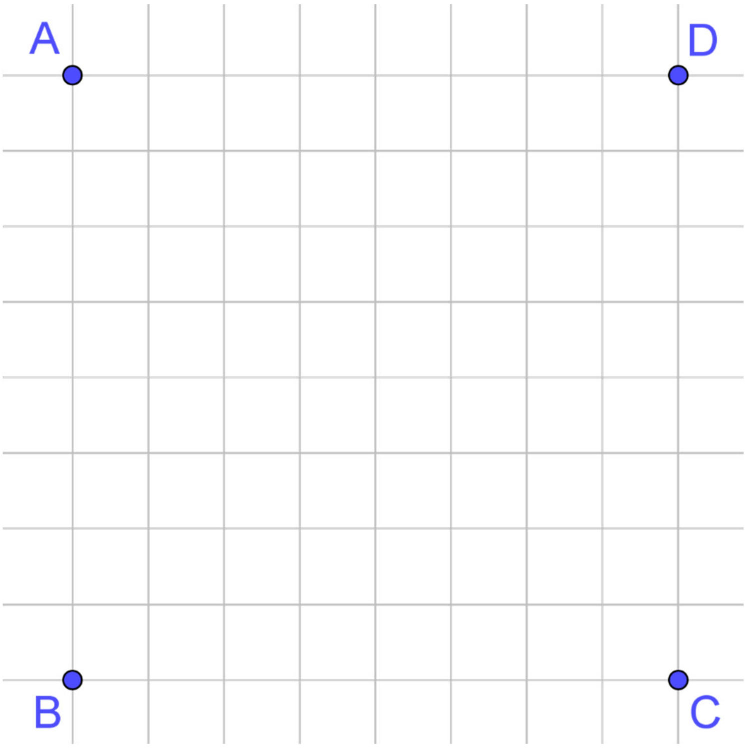
（b）周界最短　The shortest perimeter



（c）面積最大 The largest area

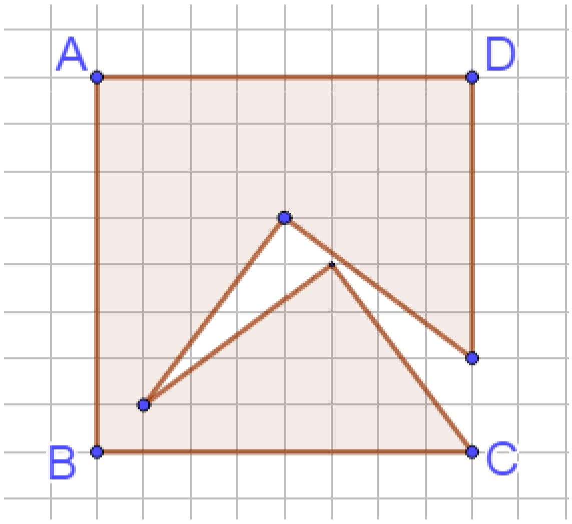


（d）面積最小 The smallest area

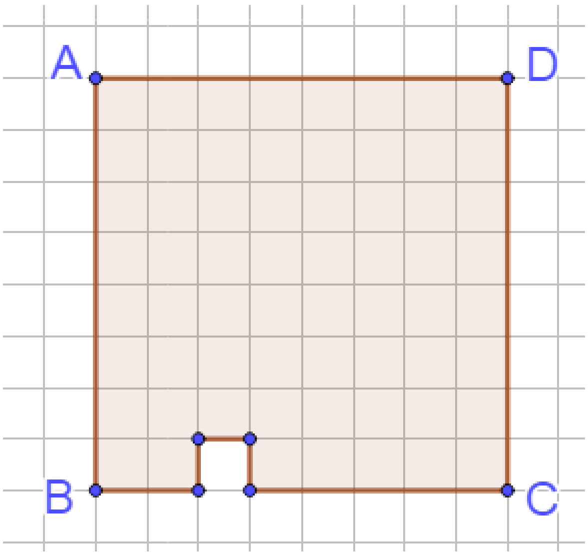


建議題解：/ Suggested Solutions:

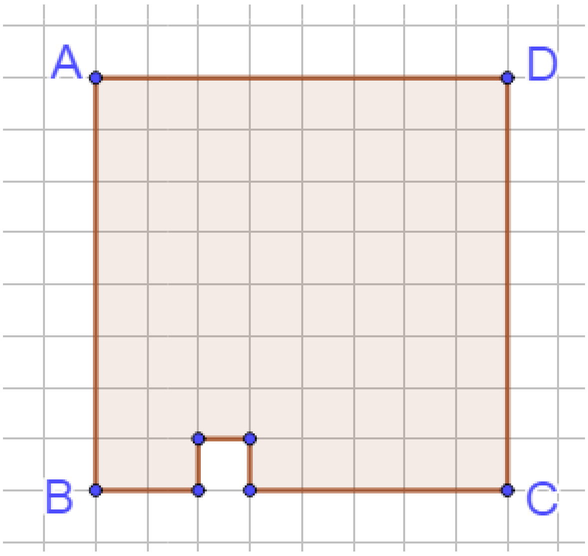
（a）周界最長　The longest perimeter



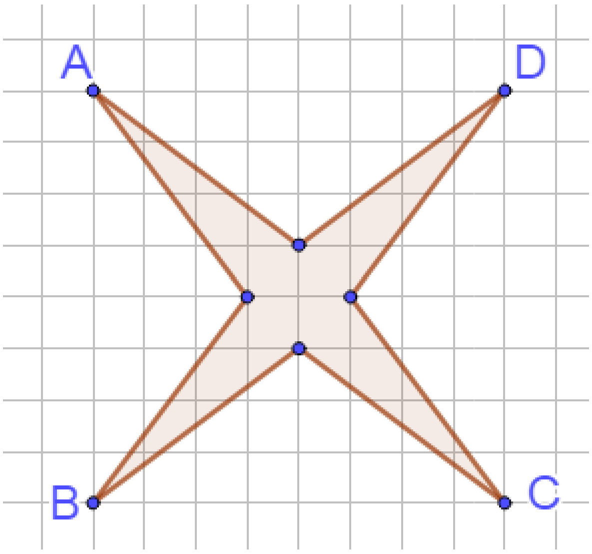
（b）周界最短 The shortest perimeter



（c）面積最大 The largest area



（d）面積最小 The smallest area



6.

100 以內的正整數中，哪個/哪些數字有最多不同的因數？

For all positive integers less than 100, which integer(s) has/have the greatest number of different factors?

建議題解：

$60 = 2^2 \times 3 \times 5$,因數的數量 $= 3 \times 2 \times 2 = 12$

$72 = 2^3 \times 3^2$, 因數的數量 $= 4 \times 3 = 12$

$84 = 2^2 \times 3 \times 7$, 因數的數量 $= 3 \times 2 \times 2 = 12$

$90 = 2 \times 3^2 \times 5$, 因數的數量 $= 2 \times 3 \times 2 = 12$

$96 = 2^5 \times 3$, 因數的數量 $= 6 \times 2 = 12$

Suggested Solutions:

$60 = 2^2 \times 3 \times 5$,number of factors $= 3 \times 2 \times 2 = 12$

$72 = 2^3 \times 3^2$, number of factors $= 4 \times 3 = 12$

$84 = 2^2 \times 3 \times 7$, number of factors $= 3 \times 2 \times 2 = 12$

$90 = 2 \times 3^2 \times 5$, number of factors$= 2 \times 3 \times 2 = 12$

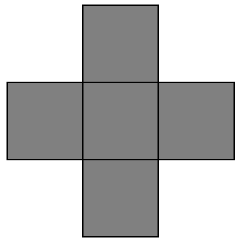
$96 = 2^5 \times 3$, number of factors $= 6 \times 2 = 12$

7.
圖三顯示出一數字板及圖四顯示一由 5 個小正方形組成的十字架方塊。

Figure 3 shows a numbered square board and figure 4 shows a cross tile formed by 5 small squares.

1	**2**	**3**	**…**	**…**	**9**	**10**
11	**12**	**13**	**…**	**…**	**19**	**20**
:	**…**	**…**	**…**	**…**	**…**	**:**
:	**…**	**…**	**…**	**…**	**…**	**:**
91	**92**	**93**	**…**	**…**	**…**	**100**

圖三 / Figure 3



圖四 / Figure 4

我們把十字架方塊放到數字板上剛好覆蓋其中五格。若被遮蓋的數字之和是 310，求十字架方塊中心方格覆蓋的數字。

We place the cross tile on the board and it covers 5 tiles completely. If the sum of the covered numbers is 310, find the number covered by the center of the tile.

建議題解：

設十字架方塊中心方格覆蓋的數字為 x。

$$5x = 310$$

$$x = 62$$

<u>十字架方塊中心方格覆蓋的數字是 62。</u>

Suggested Solutions:

Let x be the number covered by the center of the tile.

$$5x = 310$$

$$x = 62$$

<u>The number covered by the center of the tile is 62.</u>

8.

在下列數字之間加入「＋」或「－」，或把相鄰數字合併，使算式得出指定答案。

By adding “+” or “−”, or combining the adjacent numbers, construct equations with the following answers.

例子/ Example:

98 – 76 – 5 – 4 – 3 – 2 – 1 – 0 = 7

答案/Answers:

(a) 9 8 7 6 5 4 3 2 1 0 = 2

(b) 9 8 7 6 5 4 3 2 1 0 = 0

(c) 9 8 7 6 5 4 3 2 1 0 = 1

(d) 9 8 7 6 5 4 3 2 1 0 = 9

建議題解：/ Suggested Solutions:

9 − 8 + 7 + 6 – 5 + 4 – 3 + 2 – 10 = 2

9 − 87 + 6 + 54 – 3 + 21 + 0 = 0

9 − 8 + 7 – 6 – 5 + 4 – 3 + 2 + 1 + 0 = 1

9 − 8 + 7 + 6 – 5 – 4 + 3 + 2 – 1 + 0 = 9

(接受其他合理答案/Accept any possible answers)

9.

在不用直尺、不撕開紙張、不繪畫線條的情況下，利用提供的長方形紙，分別摺出以下圖形：

Without using ruler, tearing of paper and drawing any line, use the pieces of rectangular paper provided to fold the following figures:

(a) 面積為原來 50% 的不等邊非直角三角形；
A scalene triangle (without right angle) having 50% of the area of the paper.

(b) 面積為原來 37.5% 的菱形；
A rhombus having 37.5% of the area of the paper.

(c) 面積為原來 62.5% 的（沒有直角的）平行四邊形；
A parallelogram (without right angle) having 62.5% of the area of the paper.

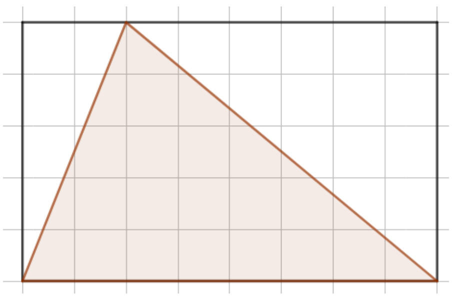
(d) 面積為原來 $\frac{9}{16}$ 的非對稱五邊形。
An asymmetric pentagon having $\frac{9}{16}$ of the area of the paper.

(把摺好的圖形放入文件夾內。)

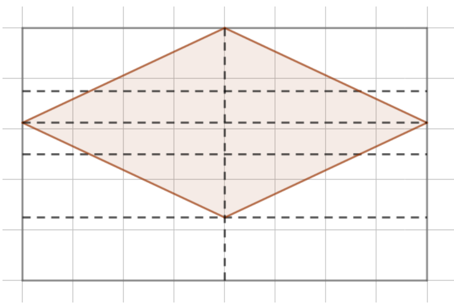
(Put all the folded figures into the folder.)

建議題解：/ Suggested Solutions:

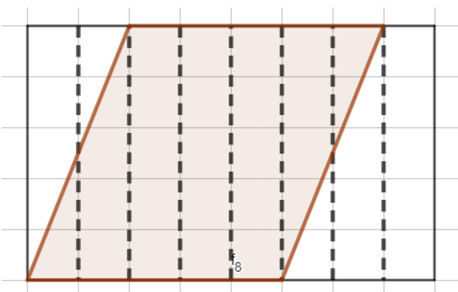
(a)



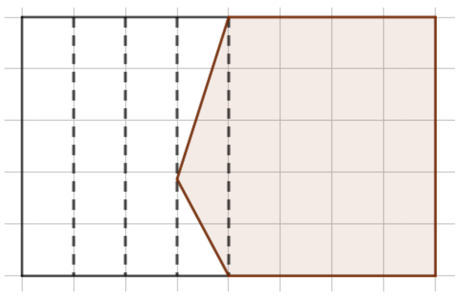
(b)



(c)



(d)



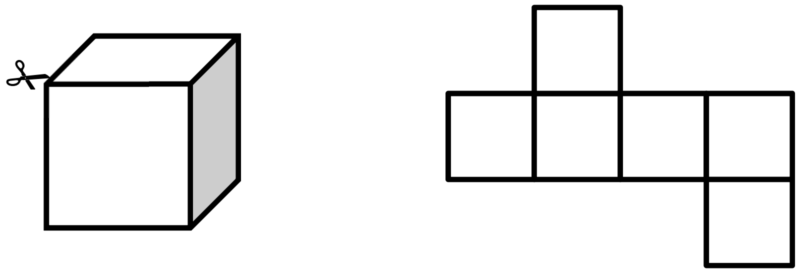
(接受其他合理答案/Accept any possible answers)

10.

沿着多面體上特定的棱剪開，可以把它翻開變成展開圖。

Cut along some edges of a polyhedron, we can unfold it and form a net.

例子/Example:



正立方體和它的其中一個展開圖

A Cube and its net

圖五顯示一個六條棱的長度皆不同的三角錐體。

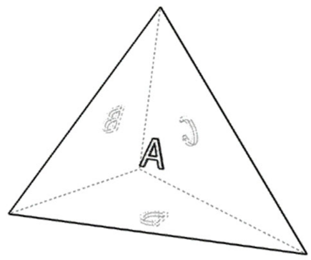
Figure 5 shows a triangular pyramid with the 6 edges of different lengths.

(a) 試畫出其中兩個可能的展開圖。(展開圖內側朝上方)

Draw two possible nets. (The inner side faces up)

(b) 求此三角錐體共有多少個不同的展開圖(展開時內側朝上)？

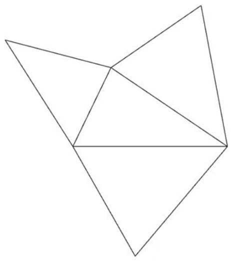
How many nets can be formed by unfolding the triangular pyramid with the inner side facing up?



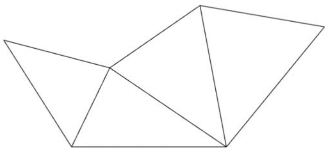
圖五 / Figure 5

建議題解：

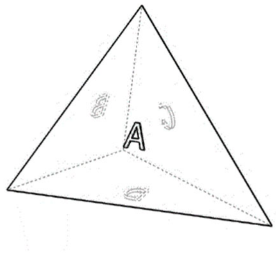
(a)



展開圖 *A*



展開圖 *B*



(b)

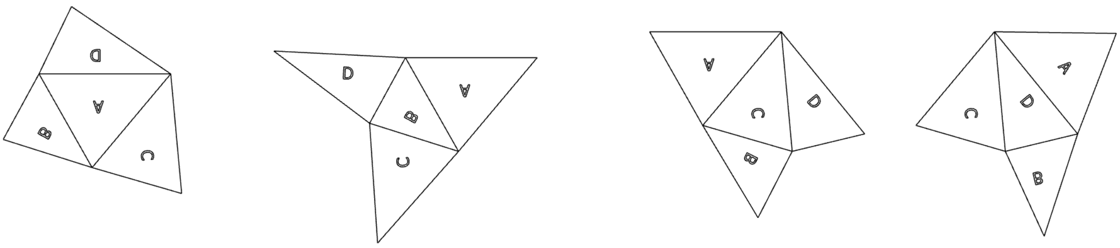
展開圖圖案有兩類:

展開圖 *A*: 2 層。

展開圖 *B*: 1 層，所有面都連成一行。

<u>**展開圖 *A*:**</u>

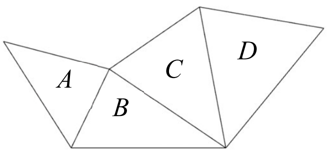
先固定最底的三角形，然後翻開，我們得出以下展開圖:



(以上的<u>展開圖</u>均是內側朝上。)

共有 4 個展開圖。

<u>**展開圖 *B*:**</u>



展開圖 *B*

展開圖 B 的數量

= 把 A、B、C 和 D 填進展開圖的排列數目

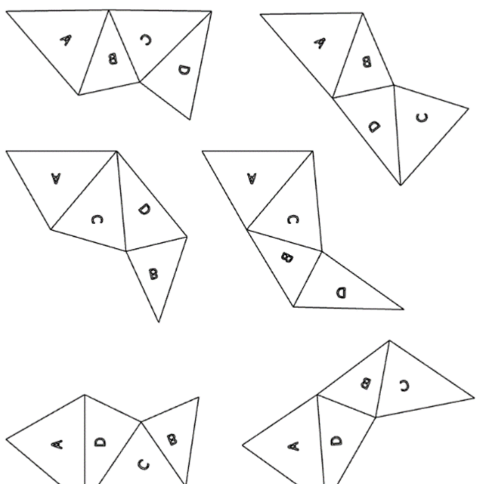
= 4!

答案: 4! + 4 = 28

以下列出部份展開圖 B 的排列:

ABCD, *ABDC* ; *ACDB*, *ACBD* ; *ADBC* , *ADCB*

(以上的展開圖均是內側朝上。)



11.

A、B、C和D四人一起到小食店。

A、B和C共吃了20粒魚蛋;

A、B和D共吃了16粒魚蛋;

A、C和D共吃了18粒魚蛋;

B、C和D共吃了21粒魚蛋。

吃得最多魚蛋的人要替自己和吃得最少魚蛋的人結帳,每粒魚蛋售1元5角,他共需付多少元?

A, B, C and D went to a snack shop.

A, B and C ate 20 fish balls in total.

A, B and D ate 16 fish balls in total.

A, C and D ate 18 fish balls in total.

B, C and D ate 21 fish balls in total.

The one who ate the most fish balls has to pay for him and the one who ate the least. The price of each fish ball is $1.5, how much should he pay?

建議題解：

$$\begin{cases} A+B+C=20 & \dots(1) \\ A+B+D=16 & \dots(2) \\ A+C+D=18 & \dots(3) \\ B+C+D=21 & \dots(4) \end{cases}$$

(1) + (2) + (3) + (4)

$3(A + B + C + D) = 75$

$A + B + C + D = 25$

$\therefore$ A = 4, B = 7 , C = 9 and D = 5

$\therefore$ $(C + A) \times 1.5 = (9 + 4) \times 1.5 = 13 \times 1.5 = 19.5$

<u>他共需付 19 元 5 角 / 19.5 元。</u>

Suggested Solutions:

$$\begin{cases} A + B + C = 20 & \dots (1) \\ A + B + D = 16 & \dots (2) \\ A + C + D = 18 & \dots (3) \\ B + C + D = 21 & \dots (4) \end{cases}$$

(1) + (2) + (3) + (4)

$3(A + B + C + D) = 75$

$A + B + C + D = 25$

$\therefore$ A = 4, B = 7 , C = 9 and D = 5

$\therefore$ (C + A) × 1.5 = (9 + 4) × 1.5 = 13 × 1.5 = 19.5

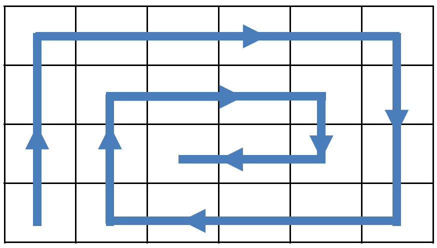
<u>He should pay $19.5.</u>

12.

$M \times N$ 的方格內，在其中一格為起點，以橫直方向移動到另一方格內，直至經過所有方格，當中每次移動不會重覆踏上之前走過的方格，我們稱該有方向性的移動路徑為哈密頓路徑 (Hamiltonian path)。

In $M \times N$ grid, we pick up a grid to be the starting grid, then move vertically or horizontally to another until we walk through all the grids. For each move, we are not allowed to walk on the grids we passed. The directed moving path is called Hamiltonian path.

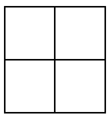
例子/Example：



圖六 / Figure 6

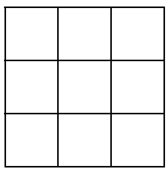
(a) 在 2 × 2 的方格內，共有多少條哈密頓路徑？

In 2×2 grid, how many Hamiltonian paths are there?



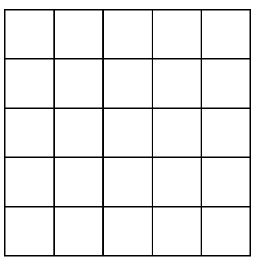
(b) 在 3 × 3 的方格內，共有多少條哈密頓路徑？

In 3×3 grid, how many Hamiltonian paths are there?



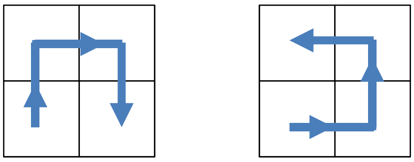
(c) 在 5 × 5 的方格中，美玲認為以任何一格作為起點，都能畫出哈密頓路徑。你同意嗎？若不同意，請在下圖以「×」表示出所有不可能作為起點的方格。

In 5×5 grid, Mary thinks that any grids can be the starting grid of the Hamiltonian path. Do you agree? If not, put a "X" to denote all the grids that cannot be the starting grids.



建議題解：

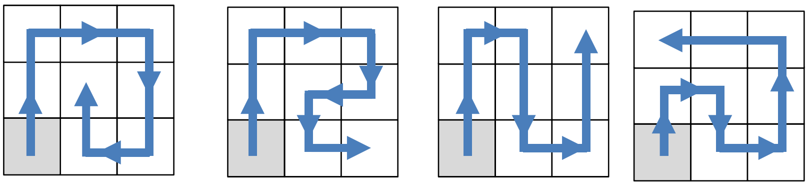
(a) 若我們由左下角的格內作為起點，得出以下兩條路徑。



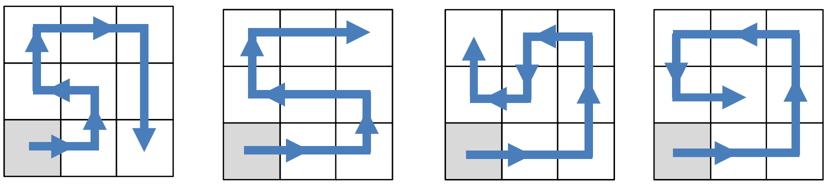
同樣地，我們在其他的角也會得到兩條路徑。

<u>∴ 共有 $2 \times 4 = 8$ 路徑。</u>

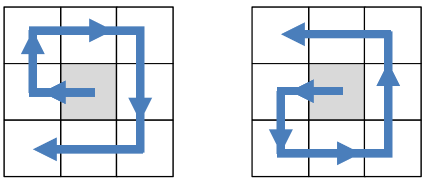
(b) *起點格為角*



上圖沿着 $y = x$ 反轉，我們得出另一組路徑。



起點格為中心

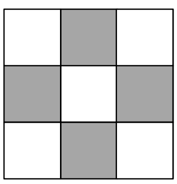


第一步去東，

然後左／右轉。

同樣地，向北，南或西走，會得出 2 條路徑。

其他起點格

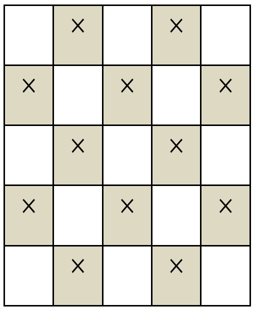


左圖中，陰影的格會得出 0 條路徑。

因為圖中有 5 個白格和 4 個黑格，同時我們以黑格為起點。

<u>∴ 共有 $8 \times 4 + 0 \times 4 + 8 = 40$ 路徑。</u>

(c)

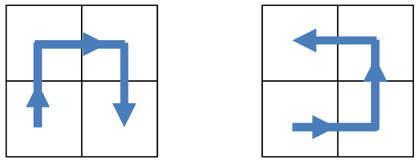


∵ 13 white squares and 12 black squares

∴ we can only start from a white square and end at a white square.

Suggested Solutions:

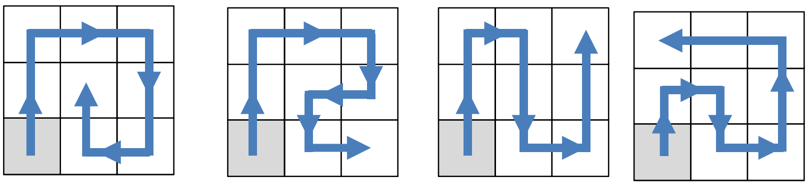
(a) If we start from the lower left hand corner, we have 2 paths shown below.



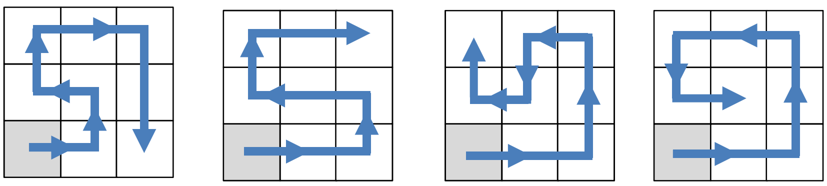
Similarly, we can have 2 paths starting from any corner.

$\therefore$ <u>There are total $2 \times 4 = 8$ paths.</u>

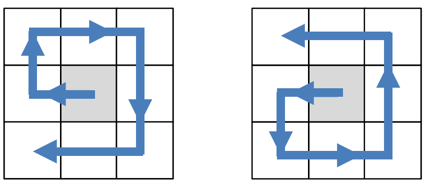
(b) *Corner Starting grids*



We can obtain another set by flipping the figures above by $y = x$.



Centre Starting grids

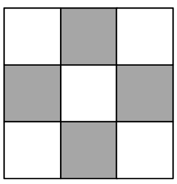


1st step moving east,

then turn left/right

Similarly, we have 2 ways for moving north, south and west.

Other Starting grids

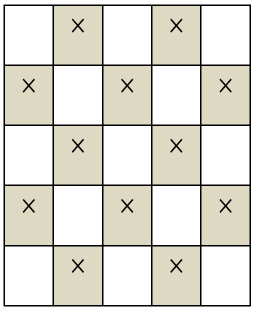


The shaded starting grids on the left have 0 paths.

Because there are 5 white squares and 4 black squares and we start from the black squares.

$\therefore$ <u>There are total $8 \times 4 + 0 \times 4 + 8 = 40$ paths.</u>

(c)

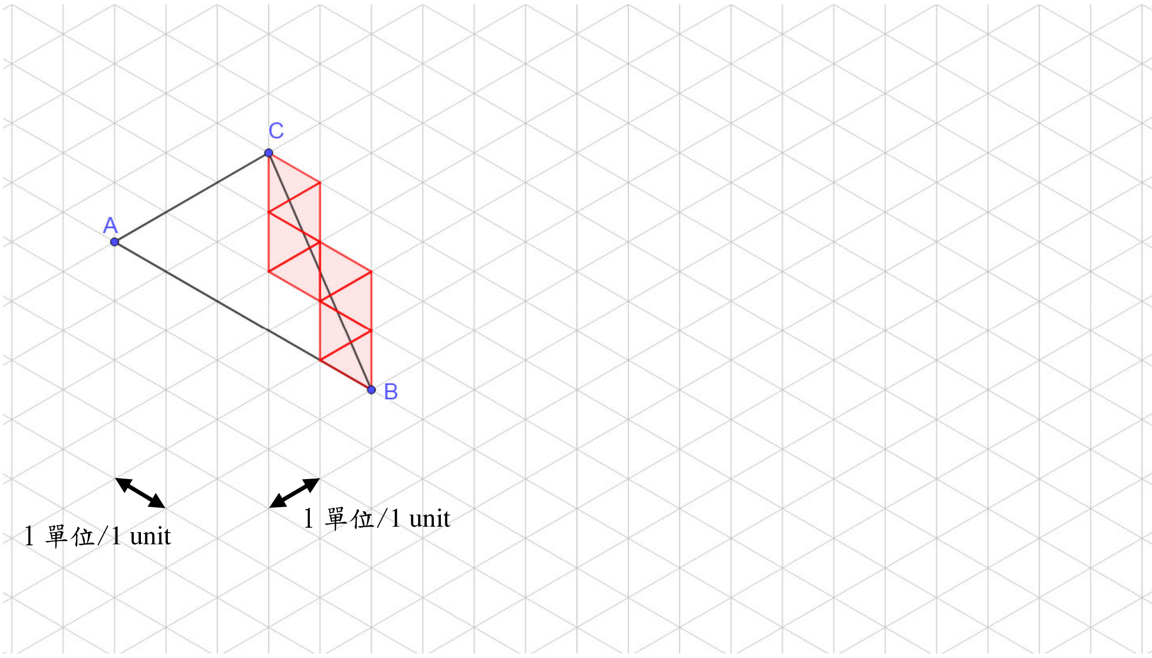


∵ 13 white squares and 12 black squares

∴ we can only start from a white square and end at a white square.

13.
在圖七中，A 為固定點，設 AC = 3 單位，AB = 5 單位，我們稱 BC 經過 8 個小三角形。

In figure 7, A is a fixed point. Let AC = 3 units, AB = 5 units, we say BC cuts across 8 triangles.



圖七/ Figure 7

(a) 分別延伸 AB 及 AC 至 B′及 C′，使得 AB′ = 8 單位，AC′ = 5 單位。問 B′C′會經過多少個小三角形？

AB and AC are produced to B′ and C′ respectively such that AB′ = 8 units and AC′= 5 units. How many triangles have been cut across by B′C′?

(b) 如(a)部所示，若 AB′ = 10 單位及 AC′ = 6 單位。問 B′C′會經過多少個小三角形？

According to part (a), if AB′ = 10 units and AC′= 6 units. How many triangles have been cut across by B′C′?

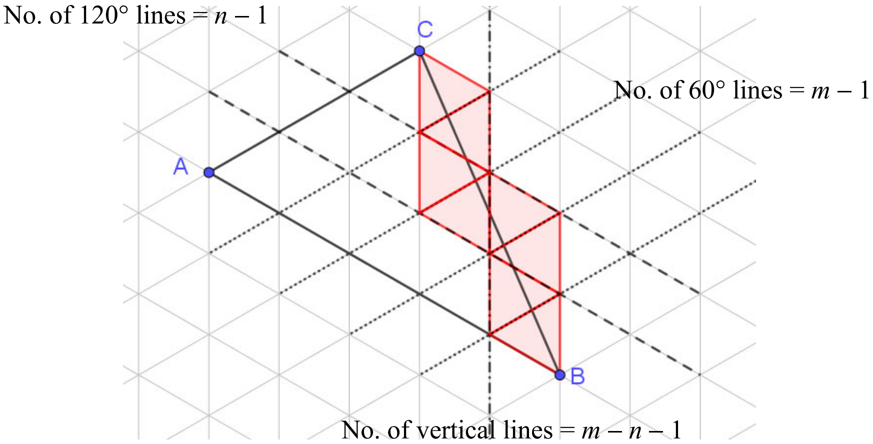
(c) 如(a)部所示，若 AB′ = 96 單位及 AC′ = 60 單位。問 B′C′會經過多少個小三角形？

According to part (a), if AB′ = 96 units and AC′= 60 units. How many triangles have been cut across by B′C′?

建議題解：

設 $AB = m$，$AC = n$

若 m 和 n 互質，線段 BC 不會穿過任何格點。



總共經過：

= $[1 + (\text{no. of } 120° \text{ lines crossed}) + (\text{no. of } 60° \text{ lines crossed}) + (\text{no. of vertical lines crossed})]$

= $[1 + (n - 1) + (m - 1) + (m - n - 1)]$ $m > n$ 及 m 和 n 互質

(a) $m = 8$, $n = 5$

總共經過三角形 $= [1 + 4 + 7 + 2] = 14$

(b) $m = 10$, $n = 6$, g.c.d.(10, 6) = 2

g.c.d. (10,6) × Number of triangles cut across when $m = 5$, $n = 3$

$= 2 \times [1 + 4 + 2 + 1]$

$= 16$

(c) $m = 96$, $n = 60$, g.c.d.(96, 60) = 12

g.c.d. (96,60) × Number of triangles cut across when $m = 8$, $n = 5$

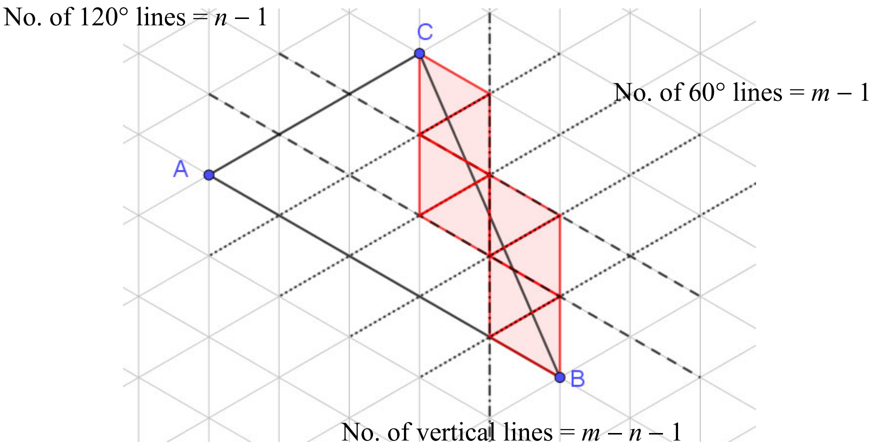
$= 12 \times$ part (a)

$= 168$

Suggested Solutions:

Let $AB = m$, $AC = n$,

When m and n are relative prime, the line segment BC will not pass through any grid points other that BC.



Number of triangles cut across

= [1 + (no. of 120° lines crossed) + (no. of 60° lines crossed) + (no. of vertical lines crossed)]

= $[1 + (n - 1) + (m - 1) + (m - n - 1)]$ where $m > n$, m and n are relative prime.

(a) $m = 8$, $n = 5$

Number of triangles cut across = [1 + 4 + 7 + 2] = 14

(b) $m = 10$, $n = 6$, g.c.d.(10, 6) = 2

g.c.d. (10,6) × Number of triangles cut across when $m = 5$, $n = 3$

= 2 × [1 + 4 + 2 + 1]

= 16

(c) $m = 96$, $n = 60$, g.c.d.(96, 60) = 12

g.c.d. (96,60) × Number of triangles cut across when $m = 8$, $n = 5$

= 12 × part (a)

= 168

全卷完 [End of Paper]