1.

已知 $23! > 2019^n$，求 n 的最大可能整數值。

當中 23! 為 23 之階乘，是 1 至 23(包括 1 和 23) 內所有自然數的乘積。

例: $5! = 5 \times 4 \times 3 \times 2 \times 1$

Given that $23! > 2019^n$, find the largest possible integer n.

23! denotes 23 factorial, the product of all the natural numbers from 1 to 23 (inclusive).

For example, $5! = 5 \times 4 \times 3 \times 2 \times 1$

建議題解

$23! \approx 2.58 \times 10^{22}$

$2019^n \approx 2.019^n \times (10^3)^n$

當 $n = 7$,

$2019^7 = 2.019^7 \times (10^3)^7 = 2.019^7 \times 10^{21} \approx 2^7 \times 10^{21} \approx 128 \times 10^{21} \approx 1.28 \times 10^{23}$

當 $n = 6$,

$2019^6 \approx 2.019^6 \times (10^3)^6 \approx 2^6 \times 10^{18} \approx 64 \times 10^{18} \approx 6.4 \times 10^{19}$

$\therefore$ n 的最大值是 6。

Suggested Solutions:

$23! \approx 2.58 \times 10^{22}$

$2019^n \approx 2.019^n \times (10^3)^n$

When $n = 7$,

$2019^7 = 2.019^7 \times (10^3)^7 = 2.019^7 \times 10^{21} \approx 2^7 \times 10^{21} \approx 128 \times 10^{21} \approx 1.28 \times 10^{23}$

When $n = 6$,

$2019^6 \approx 2.019^6 \times (10^3)^6 \approx 2^6 \times 10^{18} \approx 64 \times 10^{18} \approx 6.4 \times 10^{19}$

$\therefore$ The largest value of n is 6.

2.

A、B、C、D 和 E 點分別是 5 個圓的圓心。各圓的半徑是 3 厘米。如圖 1 所示，所有圓相交。求整個圖形的周界**(實線部分)**。答案以π表示。

Points A, B, C, D and E are the centers of five circles respectively. The radius of each circle is 3 cm. The circles intersect as shown in figure 1. Find the perimeter of the entire figure **(marked in solid line)**. Give the answer in terms of π.

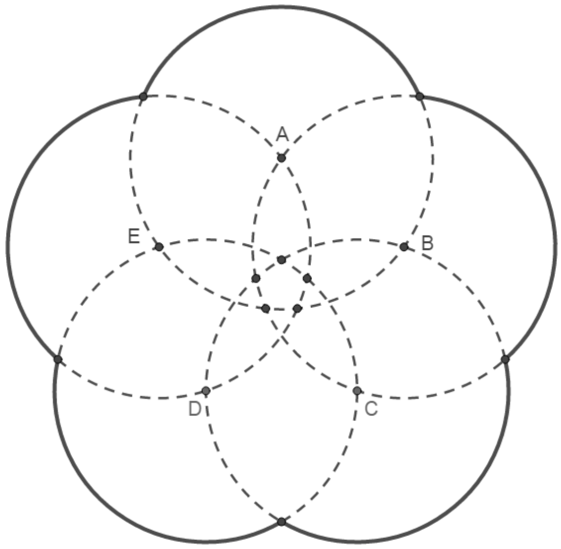
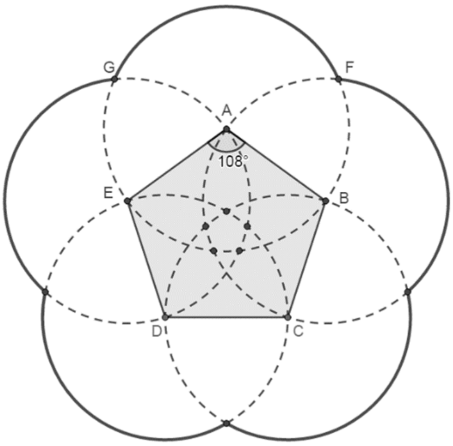
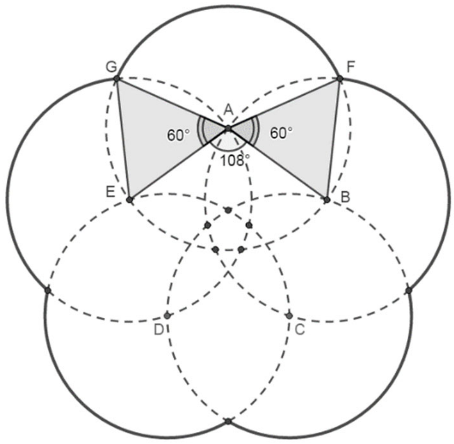


圖 1 / Figure 1





建議題解：

把點 A、B、C、D 和 E 連起來形成一個正五邊形。

正五邊形的內角: $[(5-2) \times 180°] \div 5 = 108°$ (多邊形內角和)

把點 A、F、B 和 A、G、E 連起來形成兩個等邊三角形。

等邊三角形的內角: $180° \div 3 = 60°$ (三角形內角和)

$\angle FAG = 360° - 60° - 60° - 108° = 132°$ (同頂角)

$\therefore$ 整個圖形的周界: $2\pi \times 3 \times \frac{132°}{360°} \times 5 = 11\pi$ 厘米

Suggested Solutions:

Connect points A, B, C, D and E to form a regular pentagon.

The interior angle of a regular pentagon: $[(5-2) \times 180°] \div 5 = 108°$

($\angle$ sum of polygon)

Connect points A, F, B and A, G, E to form two equilateral triangles.

The interior angle of an equilateral triangle: $180° \div 3 = 60°$ ($\angle$ sum of Δ)

$\angle FAG = 360° - 60° - 60° - 108° = 132°$ ($\angle$s at a point)

$\therefore$ The perimeter of the entire figure: $2\pi \times 3 \times \frac{132°}{360°} \times 5 = 11\pi$ (cm)

3.

在 $n \times n$ 的方格內，包含了數字格、地雷格（×）和安全格（○）。數字格中的值顯示周邊有多少個地雷格 (包括垂直、水平或對角)。圖 2a 是一個例子。

In $n \times n$ grid, there are Number grids, Mine grids (×) and Safe grids (○). The number inside the Number grid indicates the number of Mine grids surrounding it (vertically, horizontally, or diagonally). Figure 2a shows an example.

○	○	**1**	○
○	**3**	×	**2**
×	○	×	**2**
1	**2**	**1**	○

圖 2a / Figure 2a

圖 2b 空白的位置隱藏了地雷格和安全格，請把它們找出來，並分別以（×）和（○）表示。

In figure 2b, the Mine grids and Safe grids are hidden in the blank space. Find them out and label them by (×) and (○) respectively.

2			**2**	**3**	
	2				
			3		**1**
	4				
			1		
2					**1**

圖 2b / Figure 2b

建議題解/ Suggested Solutions:

2	×	○	**2**	**3**	×
×	**2**	○	×	×	○
○	○	○	**3**	○	**1**
×	**4**	×	○	○	○
×	×	○	**1**	○	×
2	○	○	○	○	**1**

4.

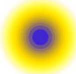
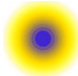
圖 3a 是一個由正四面體表面組成的立體迷宮，現把迷宮攤開成一摺紙圖樣（圖 3b）。請在圖 3b 上由起點畫出路徑至終點。(請清晰顯示路徑，並清除非路徑的線段。)

Figure 3a shows a 3D maze made by the surface of a regular tetrahedron. It is unfolded as a net (figure 3b). Draw the path from the starting point  to the end point  in figure 3b. (Show your path clearly and all the unwanted paths should be erased.)

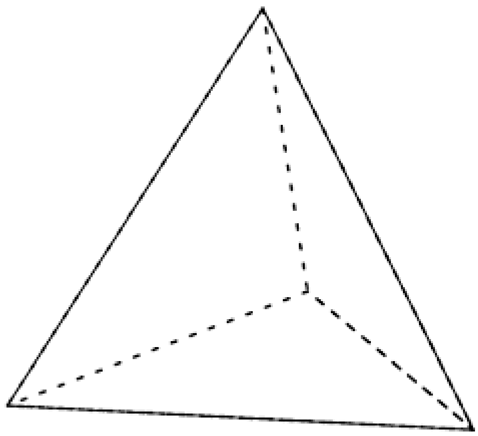


圖 3a / Figure 3a

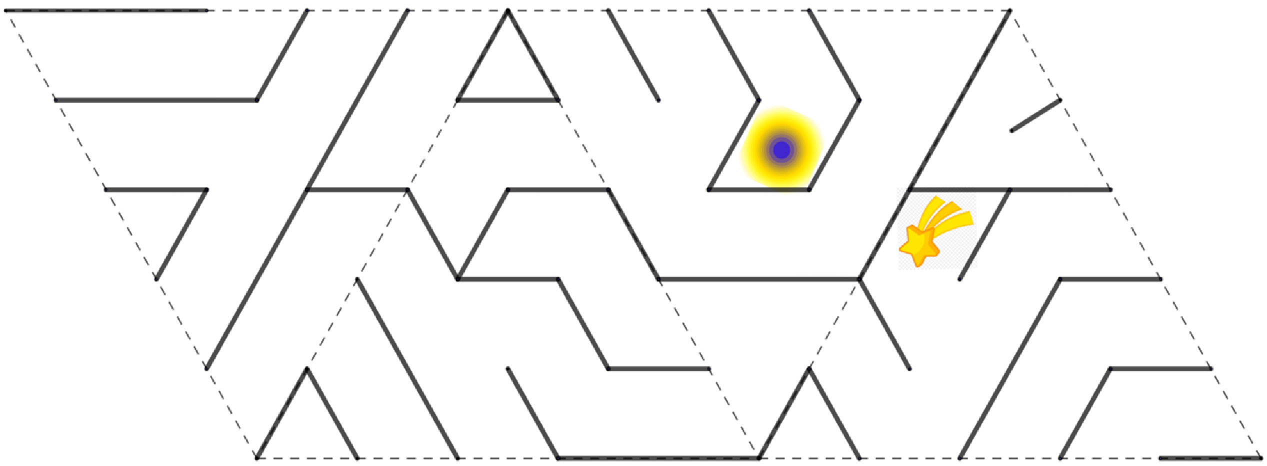
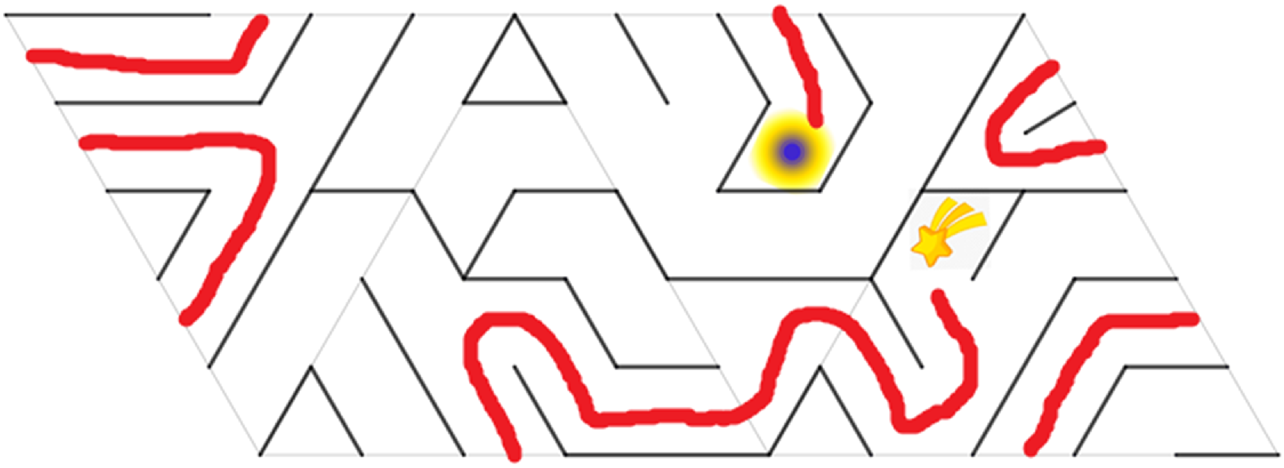


圖 3b / Figure 3b

建議題解/ Suggested Solutions:



5.

巴士路線 23 號由觀塘碼頭開往順天，服務時間為早上 06:10 至午夜 00:25，每 12 至 25 分鐘開出一班。其中一巴士準確地於某時某分正離開車站後，以每小時 44 公里的平均車速行駛了 12 公里。那時司機瞥見手錶上的分針和時針重疊。

Bus route number 23, from KWUN TONG FERRY to SHUN TIN, serves from morning 06:10 to midnight 00:25 in every 12 to 25 minutes. A bus left the terminal precisely on the minute. The average speed over the first 12 km was 44 km/h. Meanwhile, the driver consulted his watch and saw the hour-hand was overlapping with the minute-hand.

(a) 求巴士行駛該 12 公里路段所需的時間，以分鐘作單位，並以真分數表達。
Find the time needed for the bus travelled 12 km in minutes and express your answer in proper fraction.

(b) 請問巴士是何時離開車站? (以 24 小時報時制式表達)
At what time did the bus leave the terminal? (express your answer in 24-hour notation)

建議題解：

(a) 行車時間 $= \frac{12\text{ 公里}}{\text{每小時 }44\text{ 公里}} = \frac{12}{44} \times 60$ 分鐘 $=16\frac{4}{11}$ 分鐘

(b)

時針　12 小時行走 360°	分針　1 小時行走 360°
1 小時行走 $\frac{360^o}{12} = 30°$	1 分鐘行走 $\frac{360^o}{60} = 6°$
1 分鐘行走 $\frac{360^o}{12\times60} = \frac{1^o}{2}$	

假設司機於 a 時 b 分(當中 a 為 0 至 11 間之整數，b 則為 0 至 60 間之實數)瞥見手錶上的分針和時針重疊，此刻時針所指方向為 $30°(a) + \frac{1^o}{2}(b)$，亦為分針所指之方向 $6°\times(b)$。

得
$$30°(a) + \frac{1^o}{2}(b) = 6°\times(b)$$
$$60a = 11b$$
$$b = \frac{60a}{11}$$

因行車班次準確地於某時某分正離開車站，故 b 之小數部份與行車時間之小數部份相同，

即$\frac{60a}{11} - 16\frac{4}{11}$ 為整數，得 $60a - 180$ 為 11 之倍數。求得 $a = 3$、$b =16\frac{4}{11}$。

司機於 3 時$16\frac{4}{11}$分瞥見手錶上的分針和時針重疊，行車時間為$16\frac{4}{11}$ 分鐘。

得出巴士於 15:00 離開車站。

Suggested Solutions:

(a) Time needed = $\frac{12\ \text{km}}{44\text{km/h}} = \frac{12}{44} \times 60\ \text{min} = 16\frac{4}{11}\ \text{min}$

(b)

hour-hand 12 hours for 360°

1 hour for $\frac{360^{\circ}}{12} = 30^{\circ}$

1 min $\frac{360^{\circ}}{12\times 60} = \frac{1^{\circ}}{2}$

minute-hand 1 hour for 360°

1 min for $\frac{360^{\circ}}{60} = 6^{\circ}$

Assume the driver saw the hour-hand was overlapping with the minute-hand at b minutes pass a , where $0 \le a \le 11$, $0 \le b \le 60$, a is an integer, b is a real number.

Hour-hand is pointing to $30^{\circ}(a) + \frac{1^{\circ}}{2}(b)$ which is the same as the minute-hand $6^{\circ}\times(b)$.

Therefore, we have

$$30^{\circ}(a) + \frac{1^{\circ}}{2}(b) = 6^{\circ}\times(b)$$

$$60a = 11b$$

$$b = \frac{60a}{11}$$

As the bus left terminal precisely on the minute, the decimals part of b equals to the decimals part of the time needed in part (a).

i.e.

$$\frac{60a}{11} - 16\frac{4}{11} \text{ is an integer}$$

$$60a - 180 \text{ is a multiple of } 11$$

$\therefore\ a = 3$ 、 $b = 16\frac{4}{11}$.

The driver saw the hour-hand was overlapping with the minute-hand at $16\frac{4}{11}$ minutes pass 3, the time for travelling 12 km is $16\frac{4}{11}$ min.

The bus left at 15:00 .

6.

已知 $x<y<z$ ，求下列方程組之解。

Given that $x<y<z$, solve the following system of equations.

$$\begin{cases} x^2+y^2+z^2=1341 \ \ldots\ldots\ldots\ldots (1) \\ x+y+z=63 \quad \ldots\ldots\ldots\ldots (2) \\ x-y=y-z \ldots\ldots\ldots\ldots (3) \end{cases}$$

建議題解：

由 (2) 及 (3)，得 $3y=63$

$y=21$

設 $d=x-21=21-z$,

$x=21+d$, $z=21-d$ (4)

由 (4) 及 (1)，得

$(21+d)^2+21^2+(21-d)^2=1341$

$2d^2=18$

$d=3$ (捨去) or -3

$\therefore$ $x=18$, $y=21$, $z=24$

Suggested Solutions:

By (2) and (3), we have $3y=63$

$y=21$

Let $d=x-21=21-z$,

$x=21+d$, $z=21-d$ (4)

By (4) and (1), we have

$(21+d)^2+21^2+(21-d)^2=1341$

$2d^2=18$

$d=3$ (rej.) or -3

$\therefore$ $x=18$, $y=21$, $z=24$

7.

400 至 800 間的整數（包括 400 和 800），共有多少個能符合以下**所有**條件：

(I) **不能被** 5 整除

(II) **不能被** 6 整除

(III) **不包含**數字「5」

(IV) **不包含**數字「6」

For all integers between 400 and 800 (inclusively), how many of them fulfill **ALL** the following conditions:

(I) **NOT** divisible by 5

(II) **NOT** divisible by 6

(III) **DO NOT** contain digit “5”

(IV) **DO NOT** contain digit “6”

建議題解：

5 和 6 的 L.C.M. 是 30。

由 400 開始，30 最小的倍數是 420。

我們先數由 400 至 420 的數字。

由 400 至 420

~~400~~ 401, ~~402~~, 403, 404, ~~405~~, ~~406~~, 407, ~~408~~, 409, ~~410~~ 411, 412, 413, ~~414~~, ~~415~~, ~~416~~, 417, 418, 419, ~~420~~	餘下數字: 11 個

由 421 至 450

421, 422, 423, 424, ~~425~~, ~~426~~, 427, 428, 429, ~~430~~ 431, ~~432~~, 433, 434, ~~435~~, ~~436~~, 437, ~~438~~, 439, ~~440~~ 441, 442, 443, ~~444~~, ~~445~~, ~~446~~, 447, 448, 449, ~~450~~	餘下數字: 18 個

由 451 至 480 (留意要刪去十位是「5」或「6」的數字)

(~~451~~), (~~452~~), (~~453~~), (~~454~~), ~~455~~, ~~456~~, (~~457~~), (~~458~~), (~~459~~), ~~460~~ (~~461~~), ~~462~~, (~~463~~), (~~464~~), ~~465~~, ~~466~~, (~~467~~), ~~468~~, (~~469~~), ~~470~~ 471, 472, 473, ~~474~~, ~~475~~, ~~476~~, 477, 478, 479, ~~480~~	餘下數字: 18 –7 – 5 = 6 個

範圍	餘下數字
由 481 至 510	18 – 6 = 12
由 511 至 690	0
由 691 至 720	18 – 7 = 11
由 721 至 750	18
由 751 至 780	18 –7 – 5 = 6
由 781 至 800	12

<u>總數: 11 + 18 + 6 + 12 + 11 + 18 + 6 + 12 = 94 個</u>

Suggested Solutions:

The least common multiple of 5 and 6 is 30.

Start from 400, 420 is the least multiple of 30.

We count the numbers between 400 and 420 first.

From 400 to 420

~~400~~ 401, ~~402~~, 403, 404, ~~405~~, ~~406~~, 407, ~~408~~, 409, ~~410~~ 411, 412, 413, ~~414~~, ~~415~~, ~~416~~, 417, 418, 419, ~~420~~	Remain: 11 numbers

From 421 to 450

421, 422, 423, 424, ~~425~~, ~~426~~, 427, 428, 429, ~~430~~ 431, ~~432~~, 433, 434, ~~435~~, ~~436~~, 437, ~~438~~, 439, ~~440~~ 441, 442, 443, ~~444~~, ~~445~~, ~~446~~, 447, 448, 449, ~~450~~	Remain: 18 numbers

From 451 to 480 (Notice that the digit in tens place: “5” or “6”)

(~~451~~), (~~452~~), (~~453~~), (~~454~~), ~~455~~, ~~456~~, (~~457~~), (~~458~~), (~~459~~), ~~460~~ (~~461~~), ~~462~~, (~~463~~), (~~464~~), ~~465~~, ~~466~~, (~~467~~), ~~468~~, (~~469~~), ~~470~~ 471, 472, 473, ~~474~~, ~~475~~, ~~476~~, 477, 478, 479, ~~480~~	Remain: 18 –7 – 5 = 6 numbers

Region	Remain
From 481 to 510	18 – 6 = 12
From 511 to 690	0
From 691 to 720	18 – 7 = 11
From 721 to 750	18
From 751 to 780	18 –7 – 5 = 6
From 781 to 800	12

<u>The total numbers: 11 + 18 + 6 + 12 + 11 + 18 + 6 + 12 = 94 .</u>

8.

(a) 圖 4a 為一正方形，頂點分別為 A，B，C 和 D。開始時，4 隻靜止的螞蟻分別位於各頂點上。一會兒後，每隻螞蟻都會隨機選擇一條邊，沿着該邊走到相鄰的頂點。問螞蟻之間有多少種不會相遇的走法？

Figure 4a shows a square with vertices A, B, C and D. In the beginning, 4 ants sit at different vertices. After a while, each ant moves to the adjacent vertex by randomly choosing and following a side. How many ways can the ants move such that they will not meet each other?

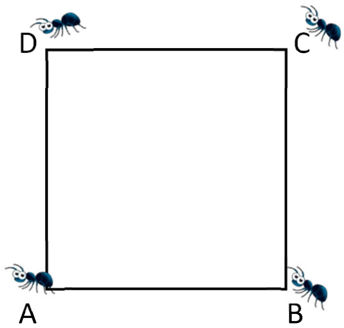


圖 4a / Figure 4a

(b) 若(a)部的正方形改為正立方體 *ABCDEFGH*（圖 4b）及螞蟻數量由 4 隻改為 8 隻。螞蟻行走的方法與(a)部相同。問螞蟻之間有多少種不會相遇的走法？

Suppose the square in part (a) is changed into a cube *ABCDEFGH* (figure 4b) and the number of ants is increased from 4 to 8. The movement of the ants follows the rules as part (a). How many ways can the ants move such that they will not meet each other?

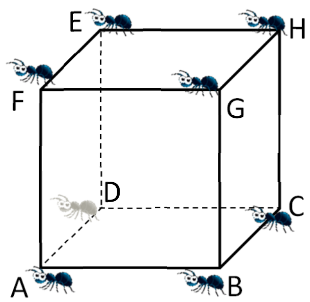


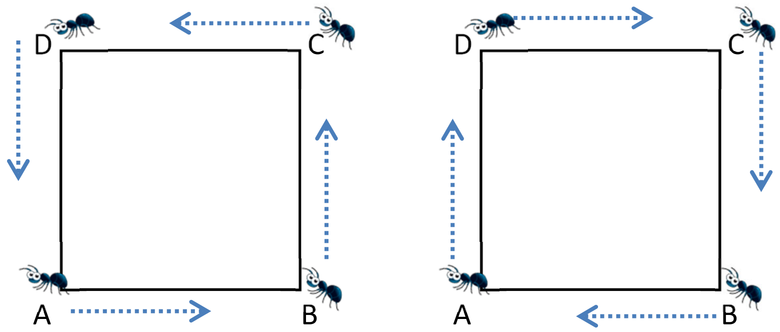
圖 4b / Figure 4b

Suggested solution:

(a) 2

正方形組成一個環，我們有順時針及逆時針兩個方向。

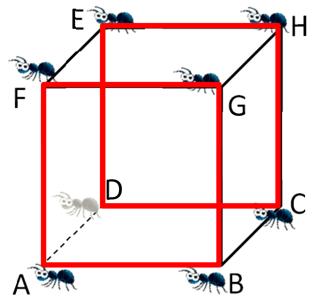
The given square form a ring and we have clockwise and anticlockwise directions.



(b) 24

情況一 (2 個平行的環)

Case I (2 parallel rings)



不會相遇走法的數量

= 環擺置方法的數量 × 螞蟻在第一環上走向的數量× 螞蟻在第二環上走向的數量

$= 3 \times 2 \times 2$

$= 12$

Number of ways

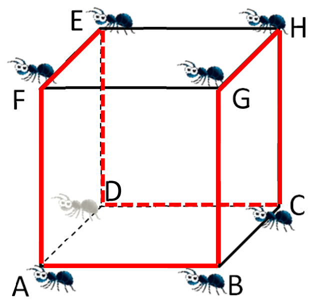
= orientation of the rings × the direction of the ants 1st ring × the direction of the ants 2nd ring

$= 3 \times 2 \times 2$

$= 12$

情況二 (1 個大環)

Case II (1 big ring)



不會相遇走法的數量

= 環擺置方法的數量×螞蟻走向的數量

$= 6 \times 2$

$= 12$

Number of ways

= orientation of the rings × the direction of the ants

$= 6 \times 2$

$= 12$

不會相遇走法的總數量 / Total number of ways = 12 + 12 = 24

9.

(a) 求有多少種把九個相同的球分作三份的方法，使每一份至少有一個球，而每份中球的數量皆不同。

Find the number of ways to divide 9 identical balls into 3 groups, such that each group has at least 1 ball and the numbers of balls in each group are different.

(b) 試利用下圖的等距方格來畫一個邊長分別為 1、2、3、4、5 和 6 的六邊形。

Using the isometric grid paper below, draw a hexagon with sides 1, 2, 3, 4, 5 and 6 respectively.

(c) 求符合(b)部而面積最大的六邊形的面積。

Find the area of hexagon which has the largest area and satisfies the conditions in part (b).

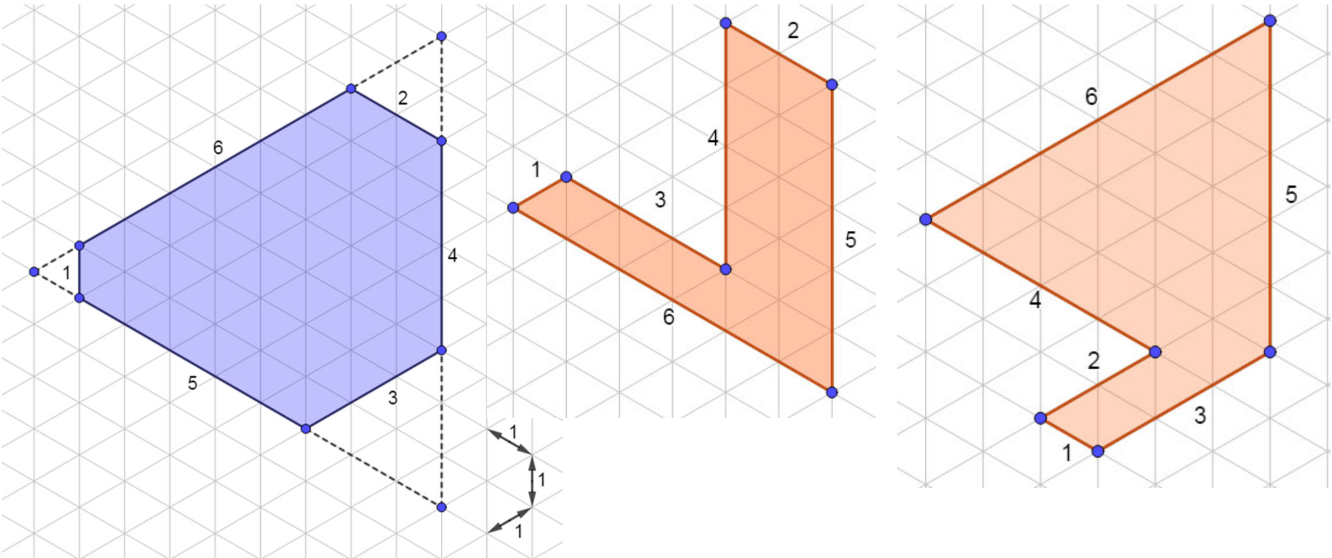
建議題解/ Suggested Solutions:

(a) 3 個分法/ ways

1, 2, 6; 1, 3, 5; 2, 3, 4

(b) (Accept any other reasonable answers)

(接受其他合理答案)



(c) 面積/Area = $\frac{9^2\sqrt{3}}{4} - \frac{\sqrt{3}}{4}(1 + 2^2 + 3^2) = \frac{67\sqrt{3}}{4}$ sq. unit.

10.

數名同學到食肆一起吃小籠包，當中一些「肚餓」的同學，他們每人會吃六隻或七隻小籠包，其他同學則每人吃一隻或兩隻。每一籠小籠包有六隻。若他們點三籠，則不能夠滿足所有同學；若他們點四籠，就會吃不完所有小籠包。

問共有多少同學到食肆吃小籠包?當中有多少是「肚餓」的同學?

Several students go to a restaurant to eat steamed pork buns in bamboo steamer. Each “hungry” student eats either 6 or 7 pork buns. Everyone else eats only 1 or 2 pork buns. Each bamboo steamer has 6 steamed pork buns. Three bamboo steamers are not sufficient to serve all students while students cannot finish all the pork buns if four bamboo steamers are ordered.

How many students went to the restaurant? How many of them were "hungry"?

建議題解：

設「肚餓」同學的人數為 x 和「不肚餓」同學的人數為 y 。

如果可以的話，「肚餓」同學會吃七只小籠包，否則會吃六只。

$18 < 6x + y$ 及 $7x + 2y < 24$

即 $19 \le 6x + y$ ~~~~~❶ 及 $7x + 2y \le 23$ ~~~~~❷

❷−❶，得出 $x + y \le 4$

若 $x = 4$，$7x = 28 \nless 24$，捨去

若 $x = 3$，$7x = 21, y = 0,$ 或 1，得出 $7x + 2y < 24$

$6x = 18,$ 只有當 $y = 1$ 能夠符合 $19 \le 6x + y$

若 $x = 2$， y 的最大值 $= 2$， $6x + y$ 的最大值$= 14 \ngtr 18$.

若 $x = 1$， y 的最大值$= 3$， $6x + y$ 的最大值$= 9 \ngtr 18$.

若 $x = 0$， y 的最大值$= 4$， $6x + y$ 的最大值$= 4 \ngtr 18$.

$\therefore$ $x = 3$，$y = 1$

到食肆吃小籠包的同學人數 $= 4$

「肚餓」同學的人數 $= 3$

Suggested solution:

Let number of “hungry” pupils be x and number of “non-hungry” pupils be y.

The “hungry” pupils would eat 6 pork buns but would eat 7 if they were available.

$18 < 6x + y$ and $7x + 2y < 24$

i.e. $19 \le 6x + y$ and $7x + 2y \le 23$

Taking the difference, we have $x + y \le 4$

If $x = 4$, $7x = 28 \nless 24$, rejected.

If $x = 3$, $7x = 21$, $y = 0$, or 1, we have $7x + 2y < 24$

$6x = 18$, only $y = 1$ can make $19 \le 6x + y$

If $x = 2$, maximum value of $y = 2$, maximum value of $6x + y = 14 \ngtr 18$.

If $x = 1$, maximum value of $y = 3$, maximum value of $6x + y = 9 \ngtr 18$.

If $x = 0$, maximum value of $y = 4$, maximum value of $6x + y = 4 \ngtr 18$.

$\therefore$ $x = 3, y = 1$.

Number of pupils went to the restaurant = 4

Number of "hungry" pupils = 3

11.

圖 5 顯示一個希羅五邊形，其邊長、對角線及面積均為自然數。AB = AE = 65，AC = AD = 156 及 BCDE 是一個長方形。求 BD 的長度。

Figure 5 shows a Heron pentagon in which the sides, the diagonals and the area are natural numbers. AB = AE = 65, AC = AD = 156 and BCDE is a rectangle. Find the length of BD.

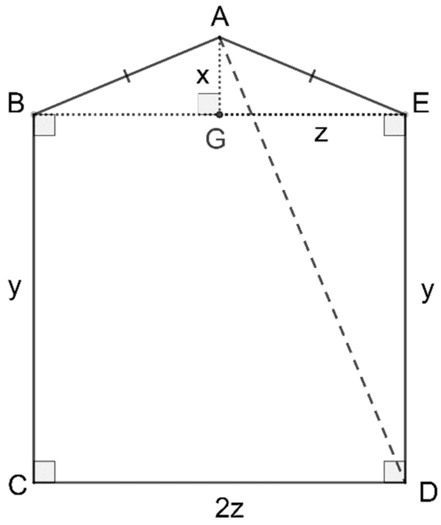


圖 5 / Figure 5

建議題解/ Suggested Solutions:

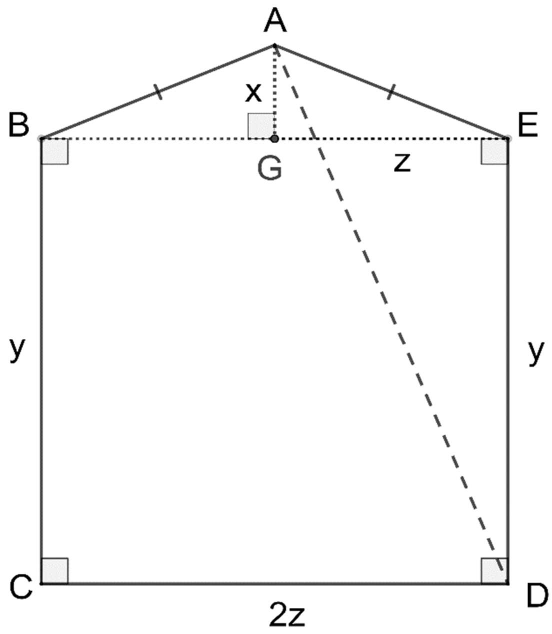
$$(x+y)^2+z^2 = 156^2 \sim\sim\sim\sim\sim ❶$$

$$x^2+z^2 = 65^2 \sim\sim\sim\sim\sim ❷$$

❶ − ❷ : $2xy+y^2 = (156+65)(156-65)$

$y(2x+y) = 221\times 91$

$=13\times 17\times 13\times 7$



Since all x, y and z are integers, we have

由於 x、 y 和 z 都是整數，因此得出:

y	$2x+y$	x	z	
17	13×13×7	$\frac{1}{2}(13\times13\times7-17)>65$		rejected 捨去
13	13×17×7	$\frac{1}{2}(13\times17\times7-13)>65$		rejected 捨去
7	13×13×17	$\frac{1}{2}(13\times13\times17-7)>65$		rejected 捨去
17×13	13×7	$0>\frac{1}{2}(13\times7-17\times13)$		rejected 捨去
13×7	13×17	$\frac{1}{2}(13\times17-13\times7)=65$	0	rejected 捨去
7×17 = 119	13×13 = 169	$\frac{1}{2}(13\times13-7\times17)=25$	60	
13×17×7	13	$\frac{1}{2}\times(13-13\times17\times7)<0$		rejected 捨去
13×13×7	17	$\frac{1}{2}\times(17-13\times13\times7)<0$		rejected 捨去
13×13×17	7	$\frac{1}{2}\times(7-13\times13\times17)<0$		rejected 捨去

∴ $x=25$, $y=119$, $z=60$

$BD^2 = 119^2+120^2$

$BD = 169$

12.

有一位工程師正步行穿越一條隧道作結構檢查，在隧道中每兩逃生出口之間都有 4 個距離相等的標距指示牌作位置標示。當他行經其中兩逃生出口 G 和 H 時，控制中心通知他有一輛工程車向他迎面而來。在此刻他正位於由出口 G 去 H 之間的第 3 個標距指示牌，他判斷若向前跑，剛好趕及走進逃生出口 H；若向後跑，也剛好趕及走進逃生出口 G。若他每分鐘能跑 300 米，問工程車之速度每小時有多少公里？

An engineer is passing a tunnel on foot for structure inspection. Inside the tunnel, 4 distance marks are evenly distributed between every two safety exits. When he is walking on the way between safety exits G and H, control center informs him that a truck is coming toward him head-on. At that moment, he is at the third mark on the way from exit G to H. He realizes that he has just enough time to run toward the truck and get into the safety exit H or to run away from the truck and get into another safety exit G. If he can run 300 meters per minute, how fast is the truck going in km per hour?

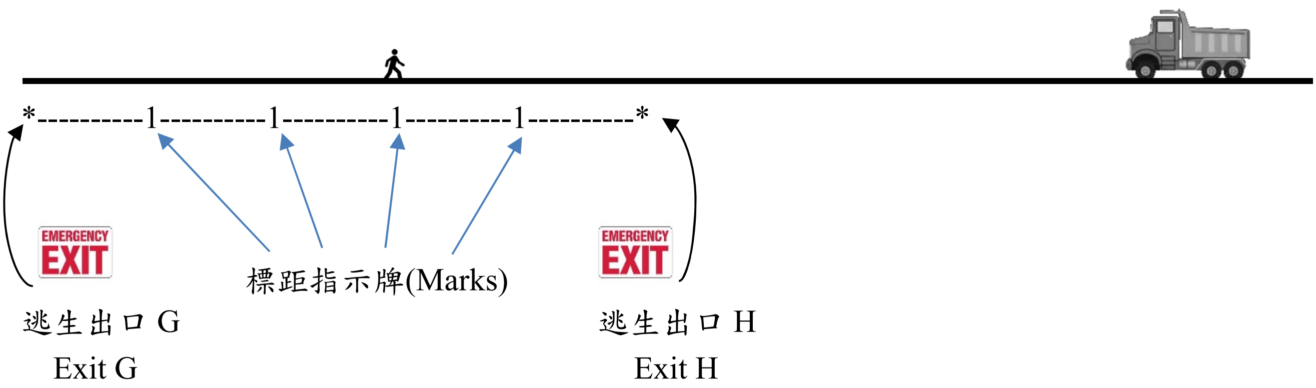
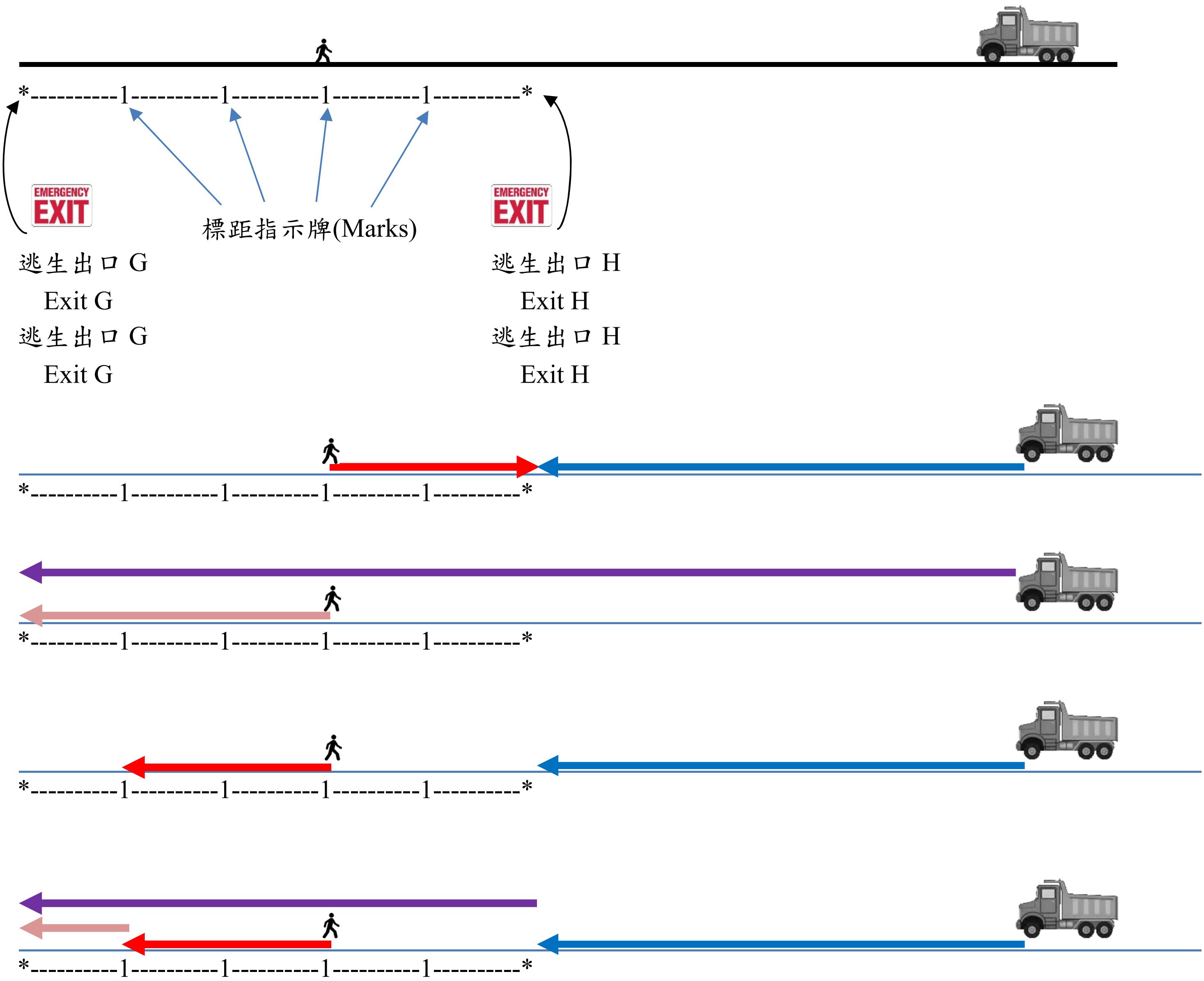


圖 6 / Figure 6

建議題解/ Suggested Solutions:



Speed of truck = 5×speed of engineer = 5× 300 m/min = 1500 m/min = 1.5 km/min = 90 km/hour

工程車的速度 ＝5×工程師的號步速度 ＝5× 300 米/分鐘 ＝1500 米/分鐘= 1.5 公里/分鐘 = 90 公里/小時

13.

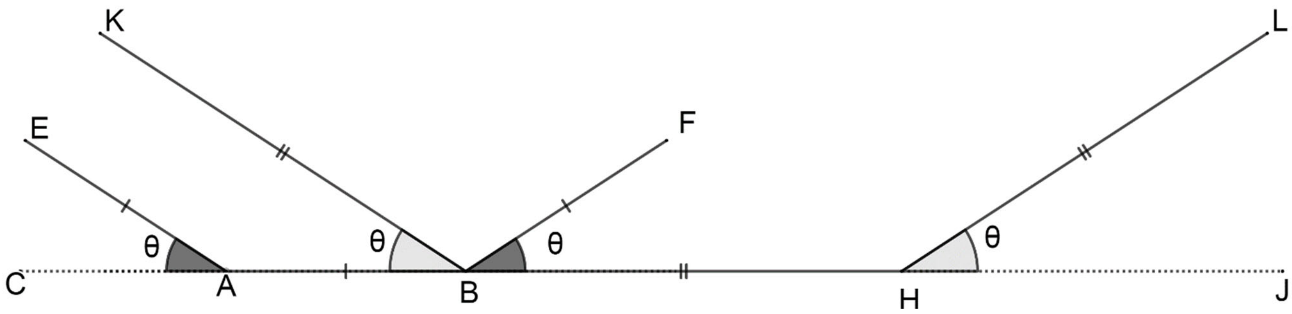


圖 7 / Figure 7

已知 EA = AB = BF 和 KB = BH = HL 使得 AB : BH = 2 : $(1+\sqrt{5})$。

$\angle$EAC = $\angle$KBA = $\angle$FBH = $\angle$LHJ = θ。

Given that EA = AB = BF and KB = BH = HL such that AB : BH = 2 : $(1+\sqrt{5})$.

$\angle$EAC = $\angle$KBA = $\angle$FBH = $\angle$LHJ = θ.

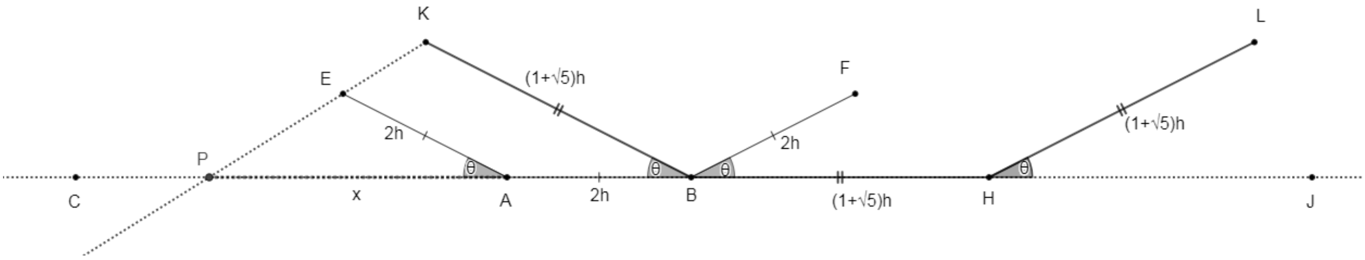
(a) KE 延線和 JC 延線相交於 P 點，LF 延線和 JC 延線相交於 Q 點，求 PQ 的長度。
Let P be the intersection of the extends of KE and JC, Q be the intersection of the extends of LF and JC. Find the length of PQ.

(b) 求$\frac{\text{KE}}{\text{FH}}$之值，準確至三位有效數字。

Find the value of $\frac{\text{KE}}{\text{FH}}$, correct to 3 significant figures.

建議題解：

(a)

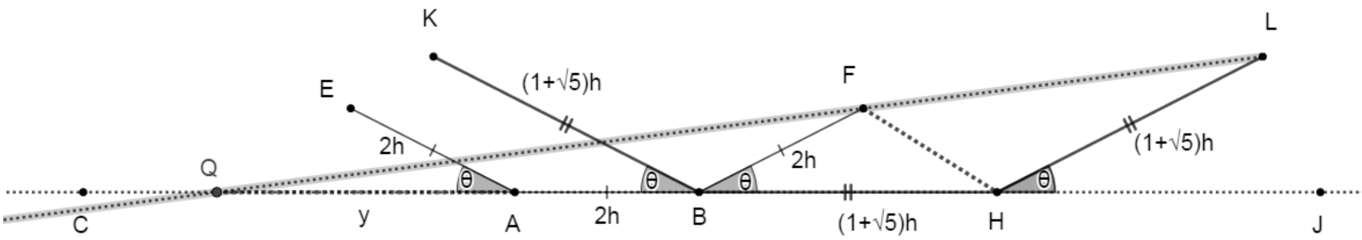


設 PA 為 x, AB 為 $2h$, KB 為$(1+\sqrt{5})h$. △PAE ~ △PBK.

$$\frac{x}{2h} = \frac{x+2h}{(1+\sqrt{5})h}$$

$$(\sqrt{5}\ -1)\,x = 4h$$

$$x = \frac{4}{\sqrt{5}-1}h$$



設 QA 為 y, BF 為 $2h$, HL 為 $(1+\sqrt{5})h$. △PAE ~ △PBK.

$$\frac{y+2h}{2h} = \frac{y+2h+(1+\sqrt{5})h}{(1+\sqrt{5})h}$$

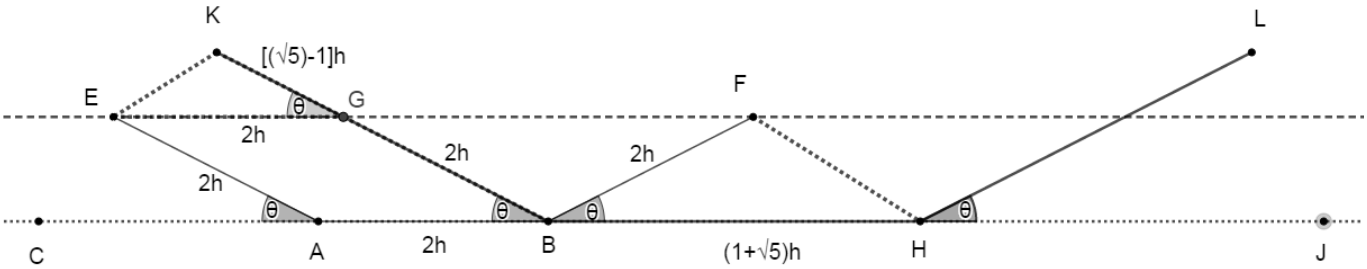
$$(\sqrt{5}\ -1)\,y = 4h$$

$$y = \frac{4}{\sqrt{5}-1}h$$

∵ $x = y$

∴ PQ = 0

(b)



設 EG // CJ

∵ $5 - 1 = 4$

$(\sqrt{5})^2 - 1^2 = 2^2$

$(\sqrt{5}\ -1)\ (\sqrt{5}\ +1) = 2\times 2$

$\frac{\sqrt{5}-1}{2} = \frac{2}{\sqrt{5}+1}$

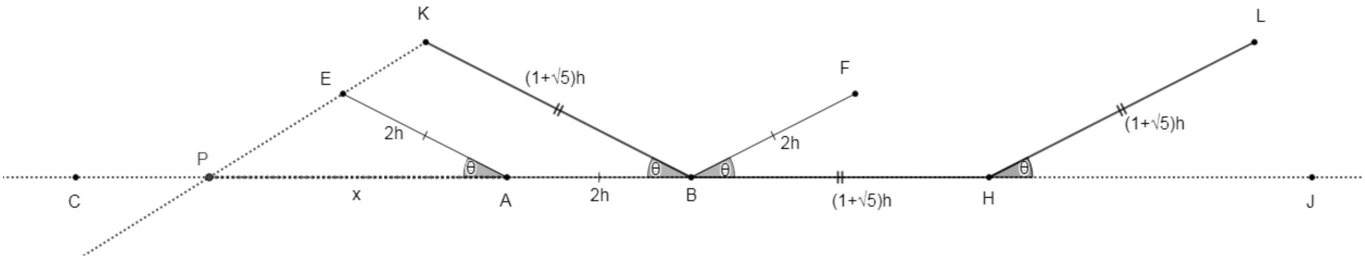
$\therefore\ \frac{\mathrm{KG}}{\mathrm{FB}} = \frac{EG}{HB}$ 及 $\angle \mathrm{KGE} = \angle \mathrm{FBH}$

$\triangle \mathrm{KGE} \sim \triangle\ \mathrm{FBH}$

得出 $\frac{\mathrm{KE}}{\mathrm{FH}}\ =\ \frac{\sqrt{5}-1}{2} = \frac{2}{\sqrt{5}+1}\ =0.618$

Suggested solution:

(a)

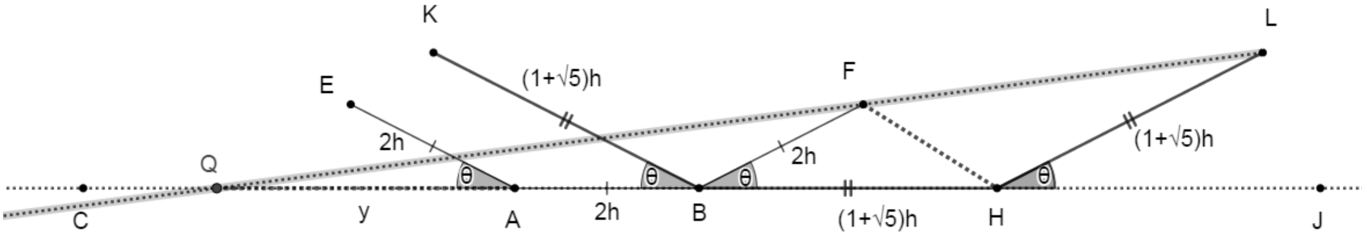


Let PA be x, AB be $2h$, KB be $(1+\sqrt{5})h$. △PAE ~ △PBK.

$$\frac{x}{2h} = \frac{x+2h}{(1+\sqrt{5})h}$$

$$(\sqrt{5}\ -1)\,x = 4h$$

$$x = \frac{4}{\sqrt{5}-1}h$$



Let QA be y, BF be $2h$, HL be $(1+\sqrt{5})h$. △PAE ~ △PBK.

$$\frac{y+2h}{2h} = \frac{y+2h+(1+\sqrt{5})h}{(1+\sqrt{5})h}$$

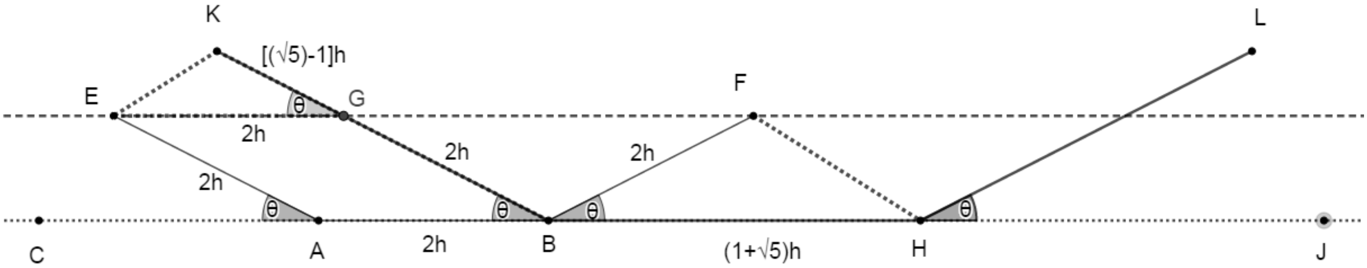
$$(\sqrt{5}\ -1)\,y = 4h$$

$$y = \frac{4}{\sqrt{5}-1}h$$

∵ $x = y$

∴ $\mathrm{PQ} = 0$

(b)



Let EG // CJ

∵ $5 - 1 = 4$

$(\sqrt{5})^2 - 1^2 = 2^2$

$(\sqrt{5}\ - 1)(\sqrt{5}\ + 1) = 2\times 2$

$$\frac{\sqrt{5}-1}{2} = \frac{2}{\sqrt{5}+1}$$

$\therefore\ \frac{\mathrm{KG}}{\mathrm{FB}} = \frac{EG}{HB}$ and also ∠KGE = ∠FBH

△KGE ~ △ FBH

We have $\frac{\mathrm{KE}}{\mathrm{FH}} = \frac{\sqrt{5}-1}{2} = \frac{2}{\sqrt{5}+1}$ =0.618

14.

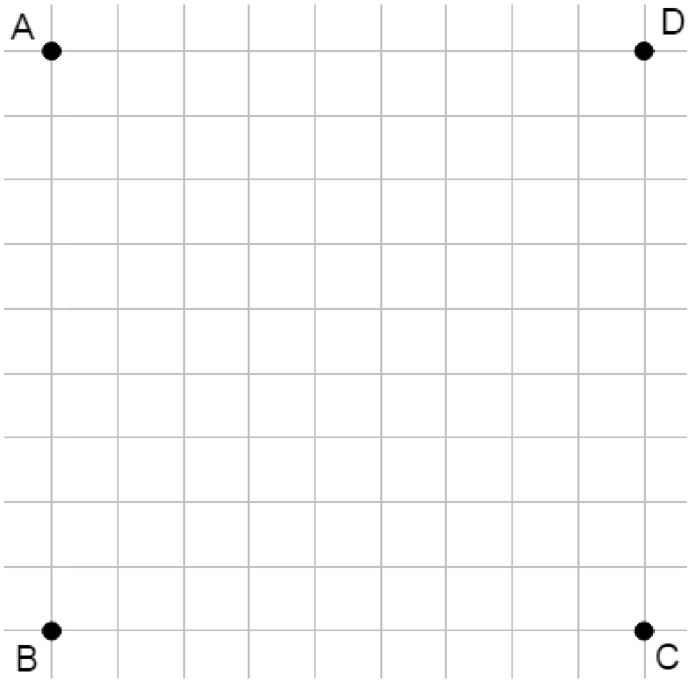
有一八邊形的每條邊長皆為整數，而所有頂點皆在圖中格點上。

A、B、C 和 D 為其中四個頂點。請根據以下的指示畫出八邊形:

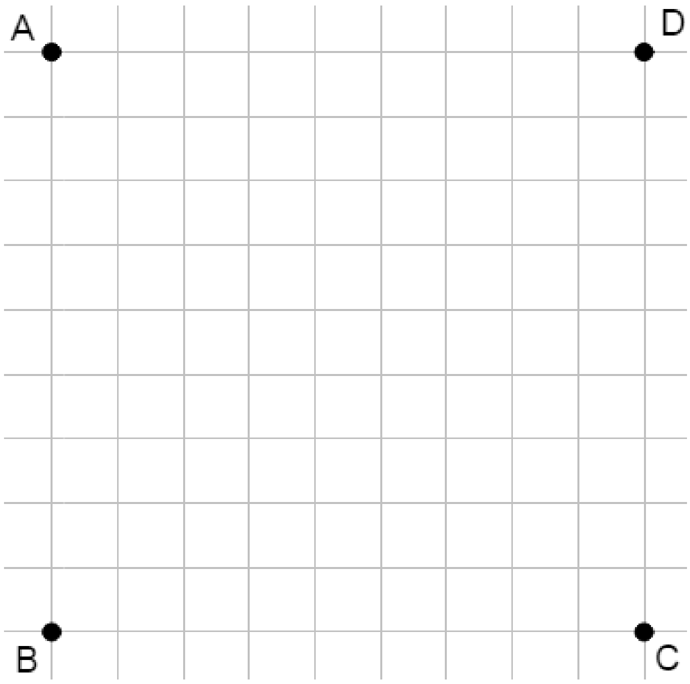
The length of each side of an octagon is integer. All the vertices are located on the intersections of the grid.

A, B, C and D are four of the vertices. Draw the octagons with the following instructions:

(a) 面積最小的八邊形 The octagon with smallest area



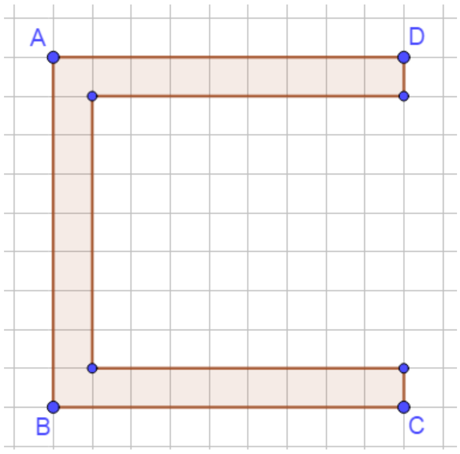
(b) 周界最長的八邊形 The octagon with longest perimeter



建議題解/ Suggested Solutions:

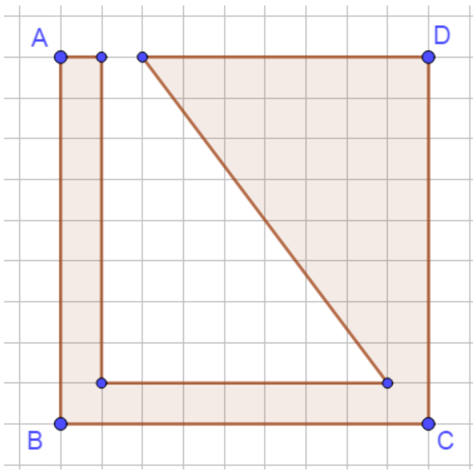
(a) 面積最小的八邊形 The octagon with smallest area

Area = 25 sq. units



(b) 周界最長的八邊形 The octagon with longest perimeter

Perimeter = 60



15.

(a) 圖 8a 有一單面顏色正方形紙。它可摺成另一正方形 ABCD，使得它有四個全等長方形和一正方形在中央（圖 8b）。

在不用直尺、不撕開/剪開紙張、不繪畫線條的情況下，利用大會提供的正方形紙（附件 A）摺出與圖 8b 條件相同的圖形，但其中央正方形的面積為紙張面積的$\frac{1}{16}$。

Figure 8a shows a square paper with one side colored. It is folded to form another square ABCD which contains four congruent rectangles and a small square at the center as shown in figure 8b.

Without using ruler, tearing/cutting of paper or drawing any line, use the square paper provided (Appendix A) to fold the figure with the same conditions as figure 8b, but the area of the square at the center is $\frac{1}{16}$ of the area of original paper.

圖 8a/ Figure 8a

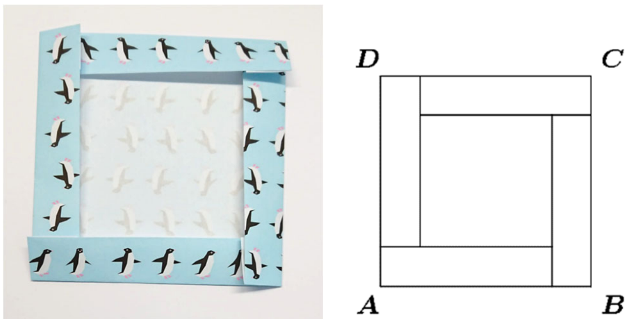


圖 8b/ Figure 8b

(b) 圖 8c 有一單面顏色正方形紙。它可摺成另一正方形 EFHG，使得它有四個全等直角三角形和一正方形在中央（圖 8d）。

在不用直尺、不撕開/剪開紙張、不繪畫線條的情況下，利用大會提供的正方形紙（附件 A）摺出與圖 8d 條件相同的圖形，但其中央正方形的面積為紙張面積的$\frac{1}{16}$。

Figure 8c shows another square paper with one side colored. It is folded to form another square EFHG which contains four congruent right-angled triangles and a small square at the center as shown in figure 8d.

Without using ruler, tearing/cutting of paper or drawing any line, use the square paper provided (Appendix A) to fold the figure with the same conditions as figure 8d, but the area of the square at the center is $\frac{1}{16}$ of the area of original paper.

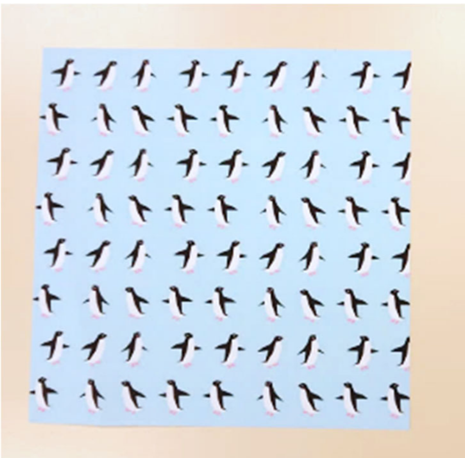


圖 8c/ Figure 8c

➔➔

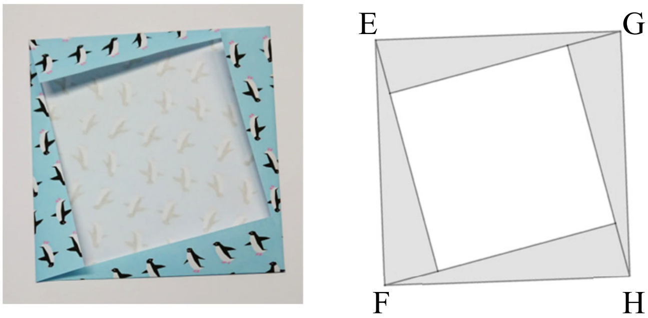
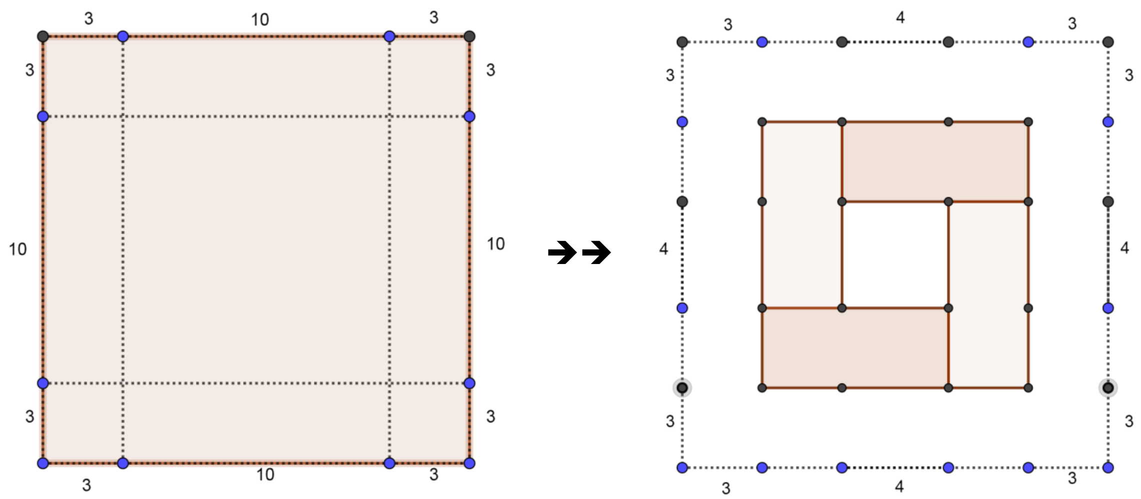


圖 8d/ Figure 8d

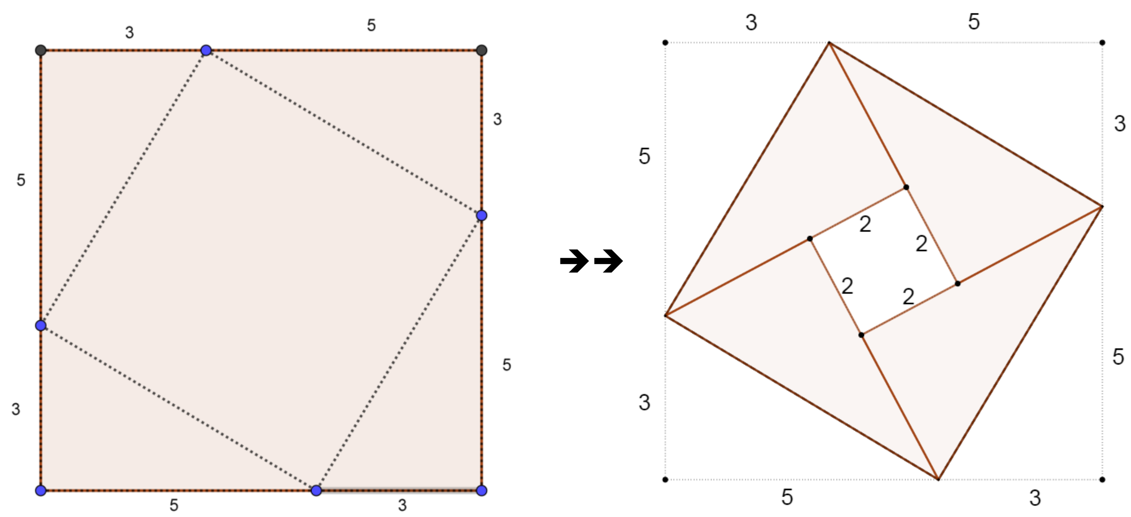
(＊*把摺好的圖形紙放入文件夾內。*)
(＊*Put all the folded papers into the folder.*)

建議題解/ Suggested Solutions:

(a)



(b)



全卷完 [End of Paper]