

# 2020/21 第十一屆香港中學數學創意解難比賽

22/5/2021 (星期六) 10:25-11:30

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## 比賽時間：65 分鐘

參賽者須知：

1. 比賽時間：65 分鐘。建議在甲部用 50 分鐘作答，在乙部用 15 分鐘作答。
2. 本問題卷共 11 頁，甲部有 13 題數學題，乙部有 1 題創意解難題。
3. 每位參賽學生獲派一份問題卷及一份答題紙。
4. 比賽其間隊員可以討論題目，並於答題紙寫上議定的答案。  
\*\* 只有寫於隊長的答題紙上的答案方可得到評分。
5. 參賽隊伍需自備文具及計算機。為公平起見，比賽中只可使用非圖像計算機。本比賽中嚴禁使用電子字典、電腦、電話或其他有上網或通訊功能的工具。
6. 本試卷每頁的空白位置可作為草稿之用。每位參賽學生會獲派一張草稿紙，如有需要，可要求額外草稿紙。
7. 在筆試完結後，必須交回隊長的答題紙。

# 2020/21 The 11<sup>th</sup> Hong Kong Mathematics Creative Problem Solving Competition for Secondary Schools

22/5/2021(Saturday) 10:25-11:30

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**Time allowed : 65 minutes**

Instructions for participants :

1. **Time allowed: 65 minutes.** It is advised to spend 50 minutes in Section A and 15 minutes in Section B.
2. The question paper consists of 11 pages. There are 13 questions in Section A and 1 creative problem in Section B.
3. Each participant will get a set of question paper and a set of answer sheets.
4. Team members are allowed to discuss during the competition. The agreed answers should be written on the answer sheets.  
**\*\* Only the answers in the captain's answer sheet will be marked.**
5. Participating teams should bring their own stationery and calculators. For the purpose of fairness, please use only non-graphic calculators. Electronic dictionaries, computers, mobile phones and other online or communication devices are prohibited.
6. The blank space on each page of this question paper can be used for rough work. Each participant will get one rough work sheet. Extra rough work sheets will be provided upon request.
7. The captain's answer sheets will be collected after the competition.

甲部 (建議此部用 50 分鐘作答)

Section A (Suggested to use 50 minutes in this Section)

1.

將以下各數由小至大排列。

Rearrange the following numbers in ascending order.

$$2^{1190}, 3^{850}, 4^{680}, 5^{510}$$

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2.

若  $x_1, x_2, x_3, x_4, x_5$  是等差數列，即  $x_2 - x_1 = x_3 - x_2 = x_4 - x_3 = x_5 - x_4$ 。

圖一顯示了一塊  $5 \times 5$  正方形板。每行和列的數字都組成一條等差數列。已知板上的一些數字，求  $x$ 。

If  $x_1, x_2, x_3, x_4$  and  $x_5$  is an arithmetic sequence, then

$$x_2 - x_1 = x_3 - x_2 = x_4 - x_3 = x_5 - x_4.$$

Figure 1 shows a  $5 \times 5$  square board. The numbers in each row and column form an arithmetic sequence. Some of the numbers were given. Find the value of  $x$ .

				0
	4			
		10		
$x$			14	

圖一 / Figure 1

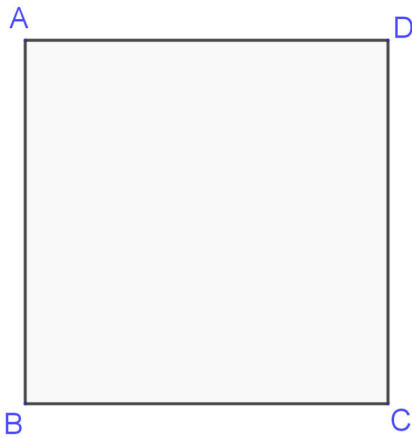
3.

寫出一個利用四個「3」組成的數值最大的數。

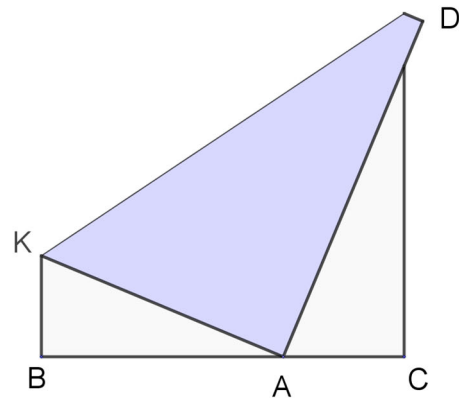
Write down the largest number formed by using four “3”s.

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4.



圖二 / Figure 2



圖三 / Figure 3

圖二的正方形紙ABCD按以下方式摺一次。(見圖三)

(I) A 點在 BC 上；

(II)  $BA : AC = 2 : 1$ 。

在圖三中，求  $KB : BA : AK$ 。

A piece of square paper ABCD in Figure 2 is folded once (see Figure 3) such that

(I) Point A lies on BC;

(II)  $BA : AC = 2 : 1$ 。

In Figure 3, find the ratio of  $KB : BA : AK$ .

5.

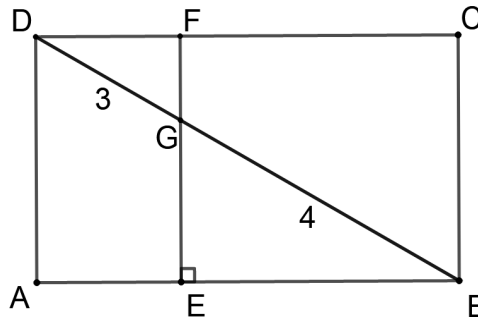
已知  $x$  ,  $y$  及  $z$  為非零實數，求  $x$  。

Given that  $x$ ,  $y$  and  $z$  are non-zero real numbers. Find the value of  $x$ .

$$\begin{cases} xy = 3(x + y) \\ yz = 6(y + z) \\ xz = 9(x + z) \end{cases}$$

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6.



圖四 / Figure 4

圖四顯示一個長方形  $ABCD$ 。  $E$  和  $F$  分別是  $AB$  和  $DC$  上的點，使得  $FE$  垂直  $AB$ 。 對角線  $DB$  與  $FE$  相交於  $G$ 。

已知  $DG = 3$  及  $GB = 4$ ，求  $FG \times GE + AE \times EB$ 。

Figure 4 shows a rectangle  $ABCD$ .  $E$  and  $F$  are points on  $AB$  and  $DC$  respectively such that  $FE$  is perpendicular to  $AB$ . The diagonal  $DB$  cuts  $FE$  at  $G$ .

Given that  $DG = 3$  and  $GB = 4$ , find  $FG \times GE + AE \times EB$ .

7.

四邊形 ABCD 的四條邊為  $AB=1$ ,  $BC=8$ ,  $CD=4$  及  $DA=7$ 。求該四邊形的最大面積。

The 4 sides of quadrilateral ABCD are  $AB=1$ ,  $BC=8$ ,  $CD=4$  and  $DA=7$ . Find the maximum area of the quadrilateral.

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8.

有 36 名學生參加田徑選拔賽，選拔賽中首 4 名學生將代表學校出賽。現時運動場有 6 條跑道，即每次只能讓 6 名學生同時較量，每場的先後排名會被紀錄。在沒有計時器的幫助下，最少要進行多少場跑步比賽才能選出 4 名代表？

There are 36 students in a preliminary selection of running race. The top 4 winners in the preliminary selection will be the school representatives. The running track has 6 lanes. It only allows 6 students run at the same time. What is the minimum number of races required to find the 4 representatives without using a timer?

9.

$$16^{2019 \cdot 2020} = a^b$$

a 和 b 是正整數，求 a 的可能值的數目。

a and b are positive integers. Find the number of possible values of a.

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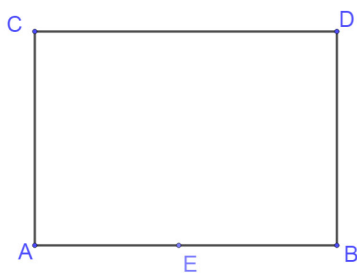
10.

圖五顯示了一張長方形紙 ABDC，有着邊比  $AB:AC = \sqrt{2}:1$ 。E 是 AB 的中點。沿着 CE 摺，再沿着  $\angle AEB$  的角平分線對摺（圖六）。角平分線與 DB 上的 F 點相交。將紙攤開，連接 CF（圖七）。

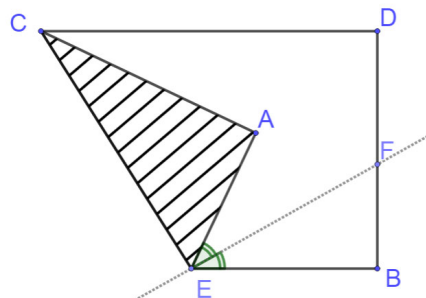
若長方形的面積是  $200 \text{ cm}^2$ ，求  $\triangle CFE$  的面積。

Figure 5 shows a piece of rectangle paper ABDC with ratio  $AB:AC = \sqrt{2}:1$ . E is the mid-point of AB. Fold along the line CE and then fold along the angle bisector of  $\angle AEB$  (Figure 6). The angle bisector intersects DB at F. Unfold the paper and join CF (Figure 7).

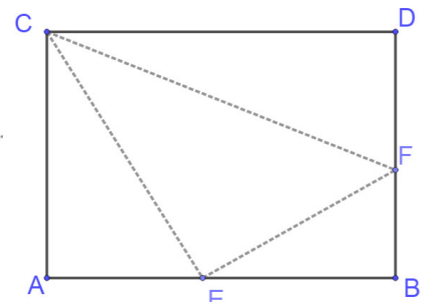
If the area of the rectangle is  $200 \text{ cm}^2$ , find the area of  $\triangle CFE$ .



圖五 / Figure 5

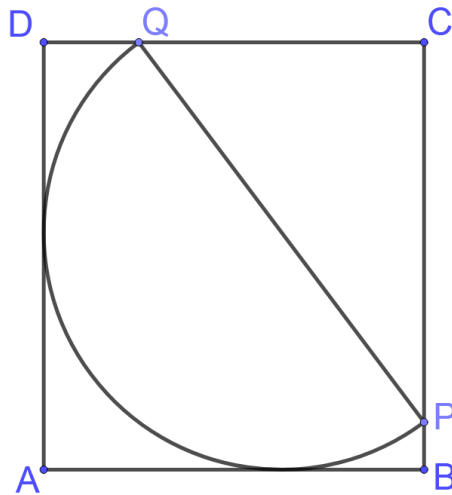


圖六 / Figure 6



圖七 / Figure 7

11.



圖八 / Figure 8

ABCD 是一個長方形。P 和 Q 分別是 BC 和 CD 上的一點。有一直徑為 PQ 的半圓與 AB 和 AD 相切(圖八)。已知  $PB = 1$ ， $QD = 2$  及  $PQ = 10$ ，求長方形 ABCD 的面積。

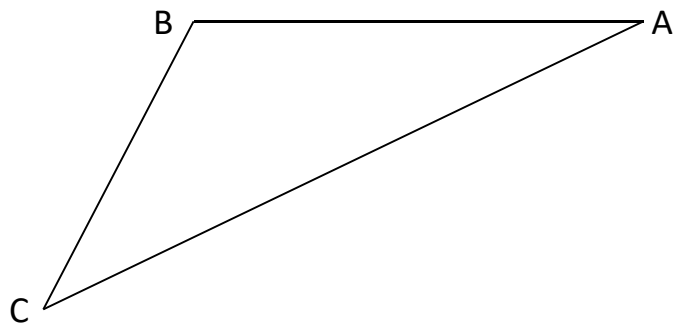
ABCD is a rectangle. P and Q are points on BC and CD respectively. A semi-circle with diameter PQ touches the side AB and AD (Figure 8). Given that  $PB = 1$ ,  $QD = 2$  and  $PQ = 10$ , find the area of rectangle ABCD.



12.

圖九顯示了三角形 ABC。AB = 112、BC = 88 及 AC = 160。D 為 AC 上的一點使得 BC = BD。求  $\frac{AD}{CD}$ 。

Figure 9 shows a triangle ABC. AB = 112, BC = 88 and AC = 160. D is a point on AC such that BC = BD. Find  $\frac{AD}{CD}$ .

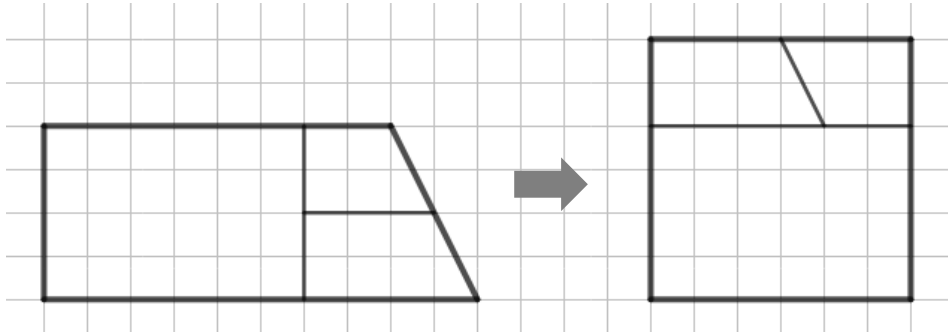


圖九 / Figure 9

13.

我們可以把梯形加上直線，把它分割成數份，並重新組合成一個正方形。  
圖十為一個分割的例子。

We can draw straight lines on the trapezium to cut it into several pieces, and rearrange the pieces to form a square. Figure 10 is an example.



圖十 / Figure 10

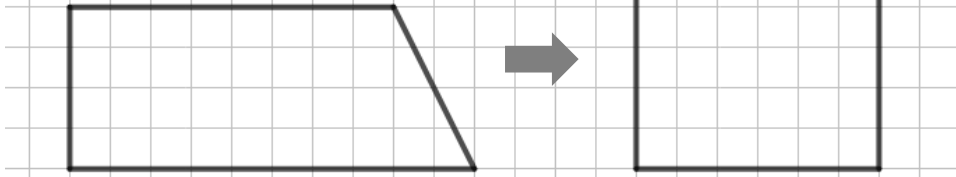
若兩種切割方法得出來的圖形組互為全等，則此兩種方法視之為相同的切割方法。

If the set of pieces obtained by two cutting methods are identical, we say these two cutting methods are the same.

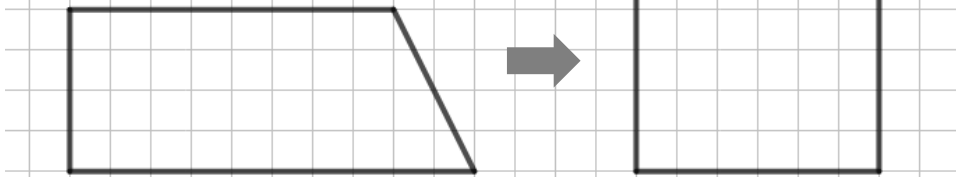
試設計兩種與例子不同的分割方法，把下圖的梯形加畫直線，分割成少於 5 塊，重新組合成一個正方形，並在正方形上畫直線顯示如何組合。

Design two cutting methods different from the example. Draw straight lines on each of the trapezium below to cut it into less than 5 pieces. Rearrange the pieces to form a square and draw straight lines on the squares to show the combinations.

方法一 Method 1:



方法二 Method 2:



乙部 (建議此部用 15 分鐘作答)

**Section B (Suggested to use 15 minutes in this Section)**

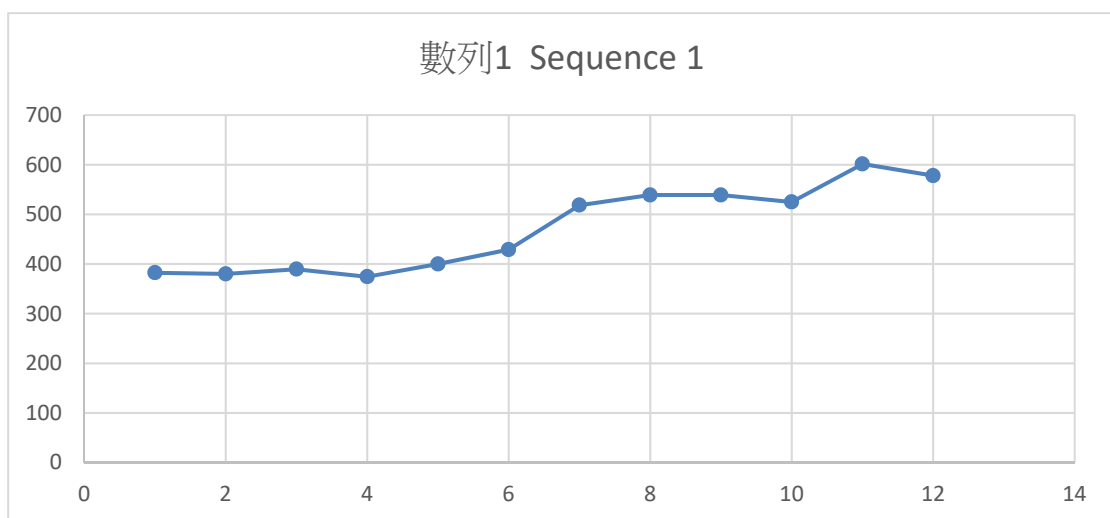
日常生活中有很多數值都會隨著時間而變化，而這些數值的變化往往都不大有規律，且有一些隨機性。量度這些數值的變化幅度是很重要的。

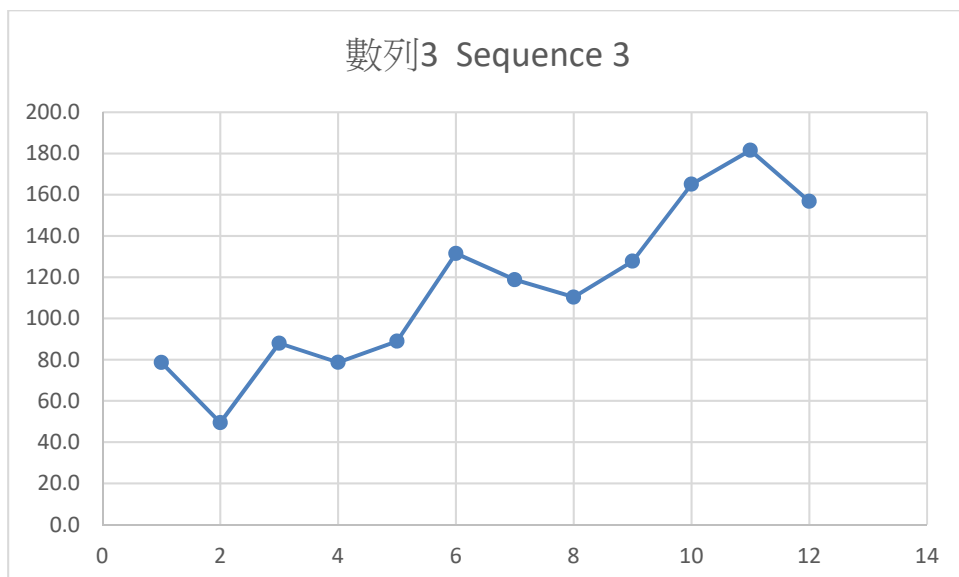
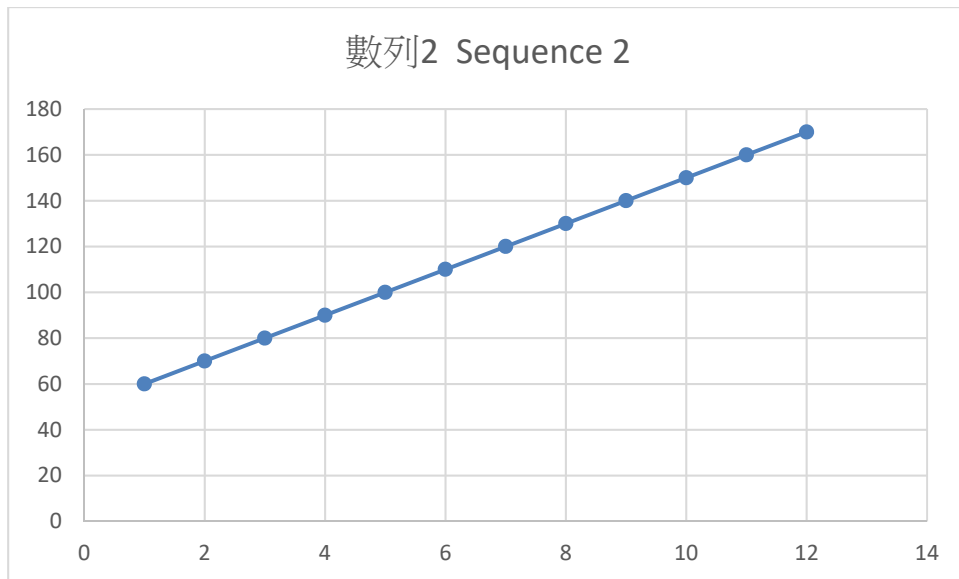
Many values in daily life will change over time, and the changes in these values are often not very regular and have some randomness. It is important to measure the magnitude of the variation of value.

數列 1 Sequence 1	
1	382.4
2	380
3	389.6
4	374.4
5	400
6	429
7	518.5
8	539
9	539
10	525
11	601.5
12	578

數列 2 Sequence 2	
1	60
2	70
3	80
4	90
5	100
6	110
7	120
8	130
9	140
10	150
11	160
12	170

數列 3 Sequence 3	
1	78.6
2	49.5
3	87.9
4	78.7
5	88.9
6	131.5
7	118.8
8	110.3
9	127.8
10	165.1
11	181.6
12	156.8





- (a) 參考以上三個數列，試引入一個用作量度這些數列的變化幅度的變數  $S$ ，並說明它的計算方法。例如  $S$  的數值越大，則數列的變化幅度越大。（注意：數列 2 的變化幅度是最少的。）

Referring to the above three sequences, introduce a new variable  $S$  to measure the magnitude of variation in a sequence, and explain its calculation method. For example, the greater the value of  $S$ , the greater the variation in the sequence. (Note: The variation in sequence 2 is the least.)

- (b) 就(a)部份引入的變數  $S$ ，計算數列 1、2、和 3 的  $S$  數值。  
Calculate the  $S$  value introduced in (a) for sequence 1, 2, and 3.

全卷完  
End of Paper



2020/21 第十一屆香港中學數學創意解難比賽

2020/21 The 11<sup>th</sup> Hong Kong Mathematics Creative Problem Solving Competition for  
Secondary Schools

答題紙 Answer sheets

學校名稱 School Name :

得分 Score :

/26

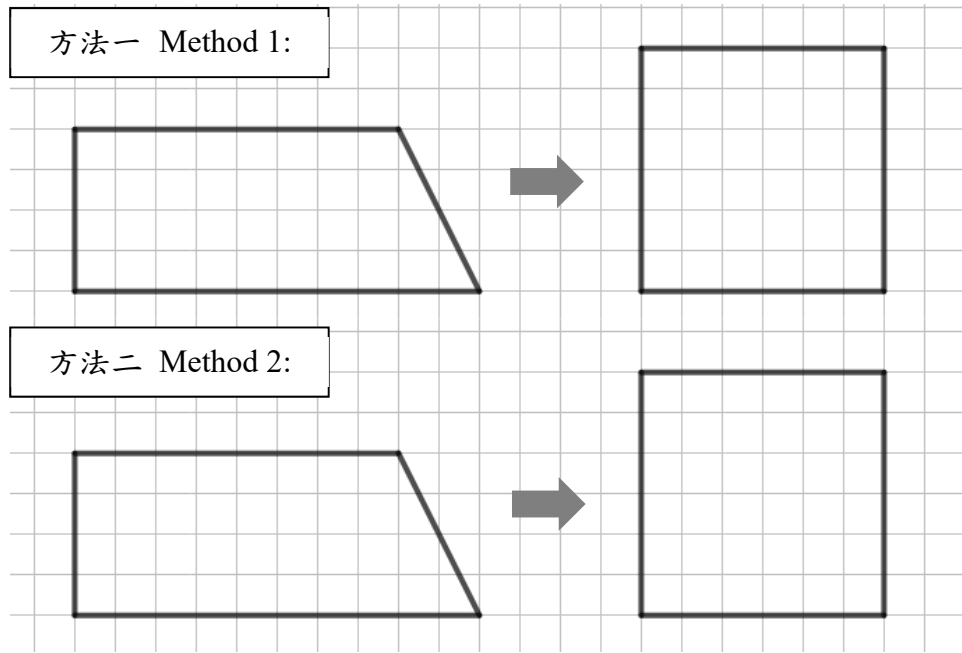
甲部 Section A

答案 Answers	評分 Marks
1. 將數字由小至大排列。 Rearrange the numbers in ascending order.  答/Ans: _____ < _____ < _____ < _____	/2
2. 求 x。 Find the value of x.  答/Ans: _____	/2
3. 寫出一個數值最大的數。 Write down the largest number.  答/Ans: _____	/2
4. 求 KB : BA : AK。 Find the ratio of KB: BA: AK.  答/Ans: _____	/2
5. 求 x。 Find the value of x.  答/Ans: _____	/2

<p>6. 求 <math>FG \times GE + AE \times EB</math>。 Find <math>FG \times GE + AE \times EB</math>.</p>	<p>答/Ans: _____ /2</p>
<p>7. 求該四邊形的最大面積。 Find the maximum area of the quadrilateral.</p>	<p>答/Ans: _____ /2</p>
<p>8. 最少要進行多少場跑步比賽? What is the minimum number of races required?</p>	<p>答/Ans: _____ /2</p>
<p>9. 求 a 的可能值的數目。 Find the number of possible value of a.</p>	<p>答/Ans: _____ /2</p>
<p>10. 求 <math>\triangle CFE</math> 的面積。 Find the area of <math>\triangle CFE</math>.</p>	<p>答/Ans: _____ <math>\text{cm}^2</math> /2</p>
<p>11. 求長方形 ABCD 的面積。 Find the area of rectangle ABCD.</p>	<p>答/Ans: _____ /2</p>
<p>12. 求 <math>\frac{AD}{CD}</math>。 Find <math>\frac{AD}{CD}</math>.</p>	<p>答/Ans: _____ /2</p>



13.



/2

**乙部 Section B**

(a)

(b)

全卷完  
**End of Paper**