

2021/22 第十六屆香港小學數學創意解難比賽

2021/22 The 16th Hong Kong Mathematics Creative Problem Solving
Competition for Primary Schools

題解 Solution

Section A 甲部

1. 六年級最少要有多少個學生，才能確保有 20 個學生在相同的月份生日？

At least how many primary six students are there in order to have 20 of them born in the same month?

建議答案 Suggested solutions:

$$19 \times 12 + 1 = 229$$

最少要有 229 個學生。

There should be at least 229 students.

2. 從 3、4、5、14、28、29 和 43 中選出 3 個不同的數，它們的和是奇數的方法共有多少個？

If three different numbers are selected among the integers 3, 4, 5, 14, 28, 29 and 43, how many of the combinations have the sum being odd numbers?

建議答案 Suggested solutions:

- 1) 3 個奇數：

3 odd numbers:

3, 5, 29

3, 5, 43

3, 29, 43

5, 29, 43

- 2) 1 個奇數及 2 個偶數：

1 odd number and 2 even numbers:

3, 4, 14 5, 4, 14 29, 4, 14 43, 4, 14

3, 4, 28 5, 4, 28 29, 4, 28 43, 4, 28

3, 14, 28 5, 14, 28 29, 14, 28 43, 14, 28

共有 16 個方法

There are 16 combinations

3. 用 5 元和 10 元硬幣換 50 元紙幣的方法能有多少種？

How many ways can a \$50 note be exchanged using \$5 and \$10 coins only?

建議答案 Suggested solutions:

情況 Case	5 元硬幣 \$5 Coin	10 元硬幣 \$10 Coin	共有 Total
1	10	0	50
2	8	1	50
3	6	2	50
4	4	3	50
5	2	4	50
6	0	5	50

共有 6 種方法

There are 6 ways

4. 小明順序寫出 1 至 100，他共寫了數字「9」多少次？

Billy writes down 1 to 100 in ascending order, how many digit '9' does he write?

建議答案 Suggested solutions:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

一共寫了 20 次。

He wrote 20 times.

5. 有 40 位學生，當中有 30 人是田徑隊，20 人是游泳隊。

There are 40 students. Among which 30 students join the athletics team and 20 students join the swimming team.

- a) 「最少」有多少位學生可能同時是田徑隊和游泳隊？

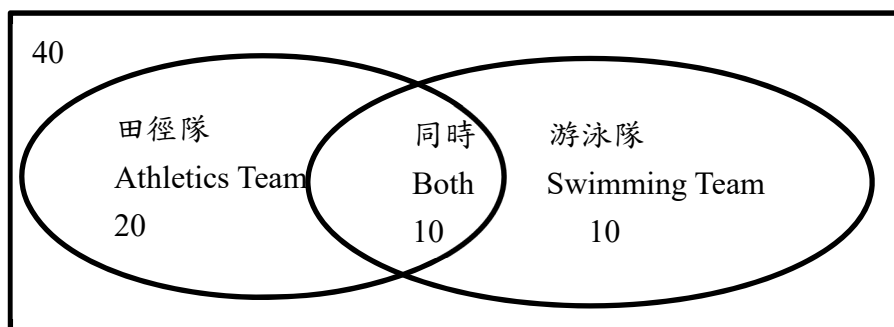
“At least” how many students are members of both athletics team and swimming team?

- b) 「最多」有多少位學生可能同時是田徑隊和游泳隊？

“At most” how many students are members of both athletics team and swimming team?

建議答案 Suggested solutions:

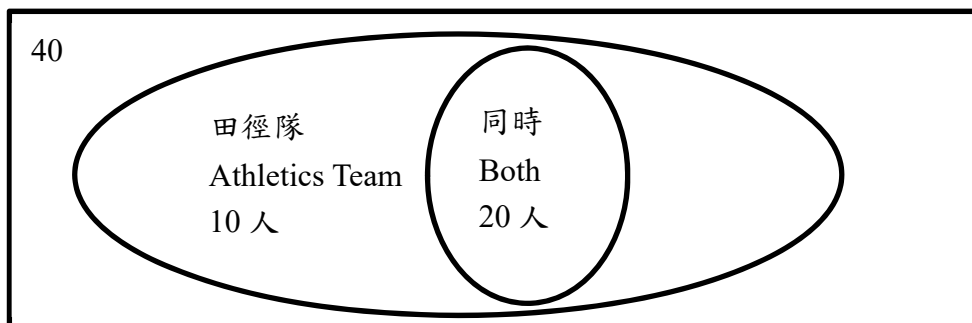
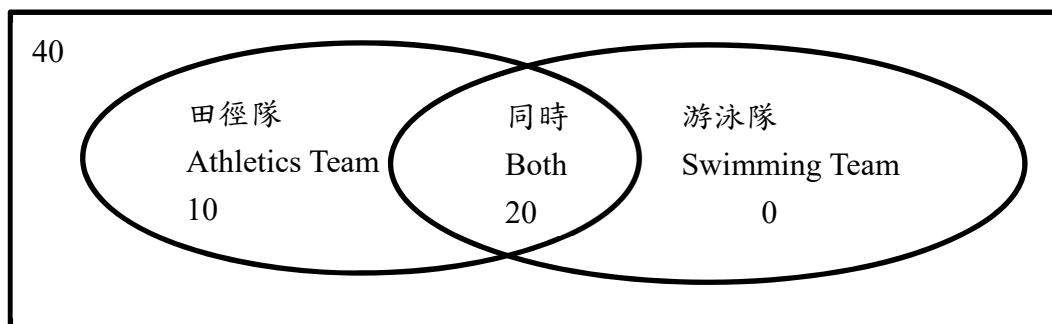
(a)



最少有 10 位學生。

At least 10 students.

(b)



最多有 20 位學生。

At most 20 students.

6. 在下式的方格內填上一個正分數，使得算式的結果化至最簡後是一個分母為 5 的真分數。
Fill in a positive fraction in the box of the expression below to make the result of the expression, in simplest form, a fraction with denominator equal to 5.

$$\frac{\frac{24}{1001}}{\frac{13}{77} - \boxed{}}$$

建議答案 Suggested solutions:

設方格內的正分數為 $\frac{p}{q}$ ，算式結果的真分數為 $\frac{n}{5}$ ，其中 n 為 1, 2, 3 或 4。

Let $\frac{p}{q}$ be the positive fraction in the box and $\frac{n}{5}$ be the resulting fraction in simplest form, where n is 1, 2, 3 or 4.

$$\frac{\frac{24}{1001}}{\frac{13}{77} - \frac{p}{q}} = \frac{n}{5}$$

$$\frac{120}{1001} = \left(\frac{13}{77} - \frac{p}{q} \right) n$$

$$120 = 1001 \left(\frac{13}{77} - \frac{p}{q} \right) n$$

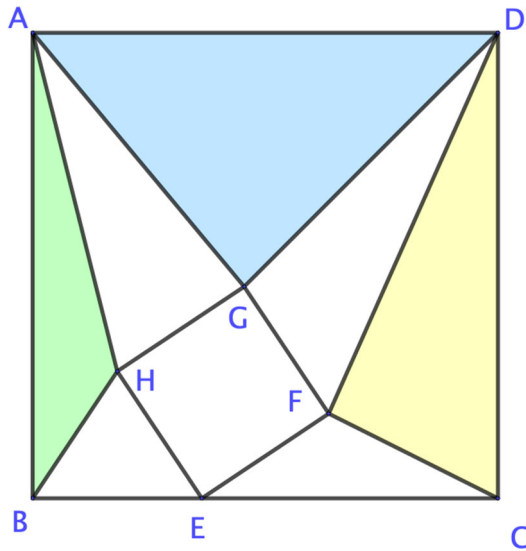
$$120 = \left(169 - \frac{1001p}{q} \right) n$$

$$\frac{120}{n} = 169 - \frac{1001p}{q}$$

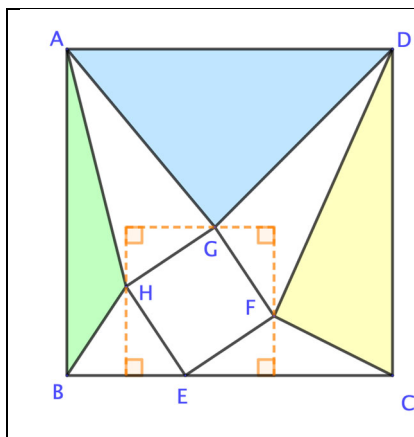
$n = 1$	$\frac{p}{q} = \frac{169 - 120}{1001} = \frac{7}{143}$
$n = 2$	$\frac{p}{q} = \frac{169 - 60}{1001} = \frac{109}{1001}$
$n = 3$	$\frac{p}{q} = \frac{169 - 40}{1001} = \frac{129}{1001}$
$n = 4$	$\frac{p}{q} = \frac{169 - 30}{1001} = \frac{139}{1001}$

7. 圖中 $ABCD$ 和 $EFGH$ 都是正方形。已知 E 點在線段 BC 上。如果 $\triangle ABH$ 和 $\triangle DAG$ 的面積分別是 11 cm^2 和 33 cm^2 ，求 $\triangle CDF$ 的面積。

In the figure, $ABCD$ and $EFGH$ are squares. Given that E is a point on BC . If the area of $\triangle ABH$ and $\triangle DAG$ are 11 cm^2 and 33 cm^2 respectively, find the area of $\triangle CDF$.



建議答案 Suggested solutions:



Add a square around $EFGH$. It is obvious that the area of the cyan \triangle = area of green \triangle + area of yellow \triangle

$$\text{Area of } \triangle CDF = 33 - 11 = 22 \text{ cm}^2$$

8. 小強與小聰在一條 100 公里的賽道上同時開始跑步，小強以每小時 3 公里的速度由起點跑到終點，小聰則以每小時 5 公里的速度由終點跑至起點。小強每跑 6 公里便休息 30 分鐘，而小聰每跑 5 公里便休息 15 分鐘。小強與小聰會在起跑後多久相遇？

Stone and Chung are running on a 100km track. They start at the same time. Stone runs from the start point to the end point at a speed of 3km/h. Chung runs from the end point to the start point at a speed of 5km/h. Stone has to take a 30 minute rest for every 6km he has run. Chung has to take a 15 minute rest for every 5km he has run. How long will it take for Stone and Chung to meet each other?

建議答案 Suggested solutions:

小強每跑 6 公里需 2 小時及休息 30 分鐘，即每 2.5 小時跑畢 6 公里。

小聰每跑 5 公里需 1 小時及休息 15 分鐘，即每 2.5 小時跑畢 10 公里。

Stone needs 2 hours of running time and 30 minutes of resting time for every 6km, i.e. he spends 2.5 hours for 6km.

Chung needs 1 hour of running time and 15 minutes of resting time for every 5km, i.e. he spends 2.5 hours for 10km.

小強及小聰之間的距離每 2.5 小時減少 16 公里。

The distance between Stone and Chung reduces by 16km every 2.5 hours.

15 小時後，小強距離起點 36 公里；小聰距離終點 60 公里。他們相差 4 公里。

After 15 hours, Stone is at a distance of 36km from the start point while Chung is at a distance of 60km from the end point. The distance between them is 4km.

0.5 小時後，小強多走 1.5 公里，小聰多走 2.5 公里。他們會在距離起點 37.5 公里的地點相遇。需時 **15.5** 小時。

After 0.5 hours, Stone runs 1.5km more while Chung runs 2.5km more. They will meet at a distance of 37.5km from the start point. The time they need is **15.5** hours.

9. 本年度香港小學數學創意解難比賽的舉行日期為 6 月 18 日。一般來說，如果日期數字是月份數字的倍數，那天便可稱為「吉日」。例如，6 月 18 日中日期數字為 18，月份數字為 6，而 18 是 6 的倍數，所以 6 月 18 日是「吉日」。

在 2022 年有多少天是吉日呢？

The competition day of Hong Kong Mathematics Creative Problem Solving Competition this year is 18/6. In general, if the day number is a multiple of the month number, the day is called a “Lucky Day”. For example, consider the date 18/6, the day number is 18 and the month number is 6, where 18 is a multiple of 6, hence 18/6 is a “Lucky Day”.

How many “Lucky Day” are there in the year 2022?

建議答案 Suggested solutions:

月份 Month	「吉日」數目 Number of “Lucky Day”
1	31
2	14
3	10
4	7
5	6
6	5
7	4
8	3
9	3
10	3
11	2
12	2
總數 Total	90

10. 某遊樂場的遊戲活動如下：每參加一次活動，參加者的積分會增加一倍，但活動後需收取 40 積分。

小明第一次參加後，他的積分多了一倍，需付 40 積分。如是者他參加三次活動後，發現自己只剩下 40 積分，剛好足夠付最後活動的積分費用。

小明原有積分多少？

The rule of an amusement part is as follow: For each game, the credit of a participant will first be doubled, but it will be reduced by 40 credits afterwards.

Billy's credit was doubled in his first participation of the game, but was reduced by 40 credits after the first participation. He has participated three times in the game and found that he has 40 credits left for the reduction after the last participation.

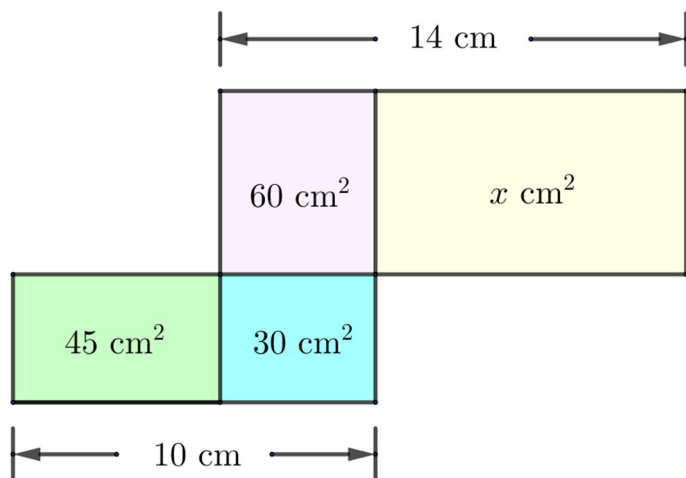
How many credits did Billy originally have?

建議答案 Suggested solutions:

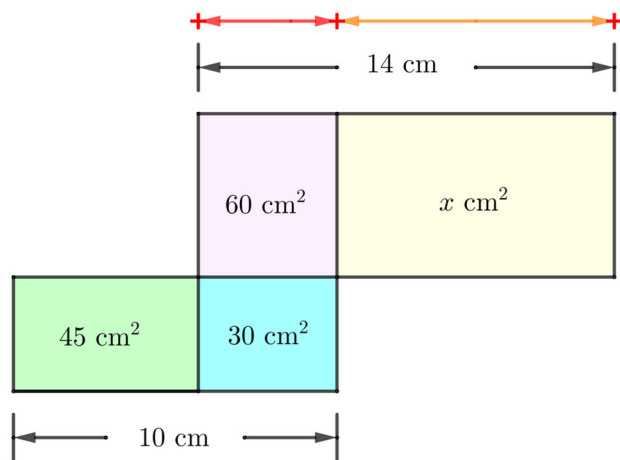
	活動前	活動後積分變雙倍	付 40 積分後餘下
第 3 次活動	20	40	0
第 2 次活動	30	60	20
第 1 次活動	35	70	30

11. 圖中的四個長方形以邊貼邊的方式連接。求 x 的值。

In the figure, there are four rectangles joining side by side as shown. Find x .



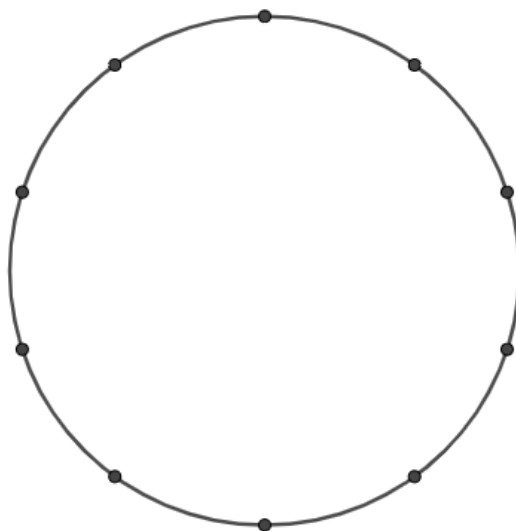
建議答案 Suggested solutions:



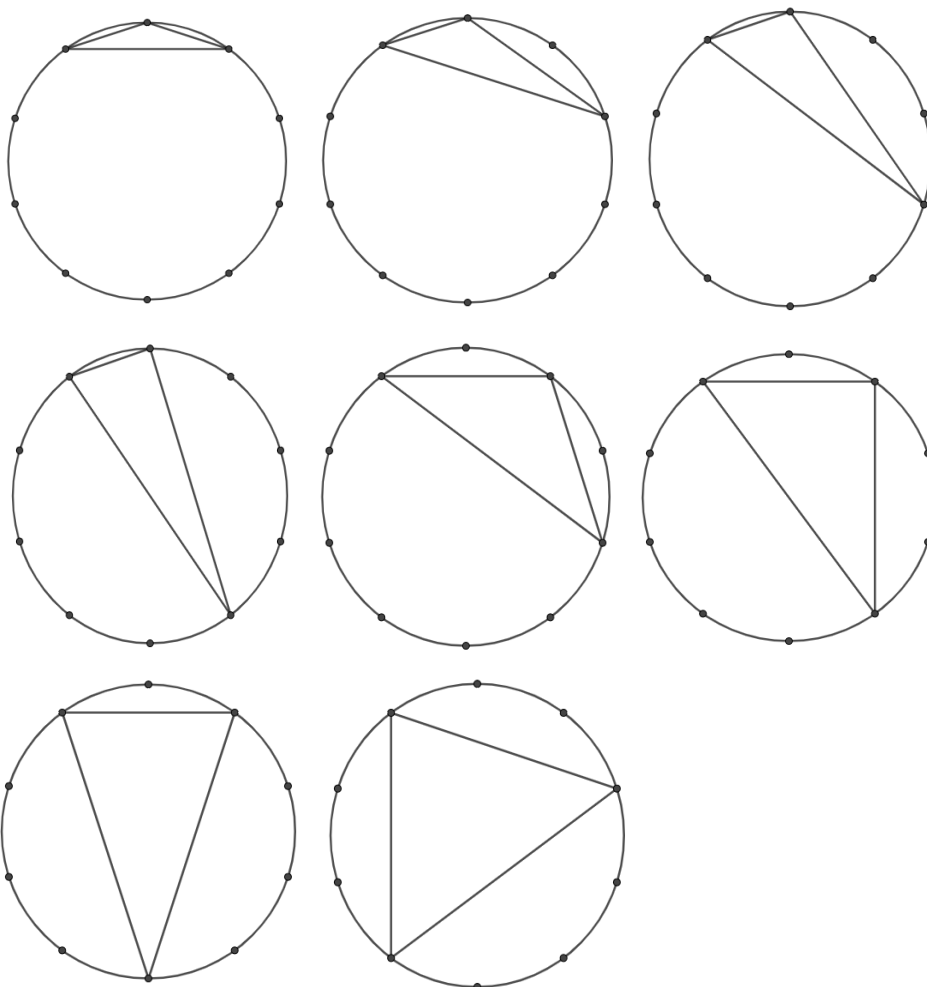
$$(10) \left(\frac{30}{45 + 30} \right) = 4 \text{ cm}$$

$$x = (60) \left(\frac{14 - 4}{4} \right) = 150$$

12. 在圓形釘板上有 10 個平均分佈的釘，在此圓形釘板上能夠製作出多少款不同的三角形？
There are 10 equally distributed pegs on a circular pegboard. How many different triangles can be made on this circular pegboard?

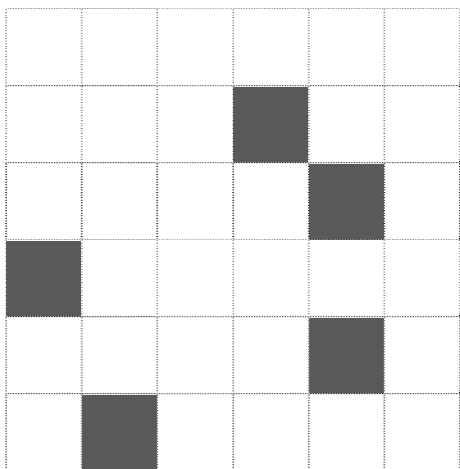


建議答案 Suggested solutions:



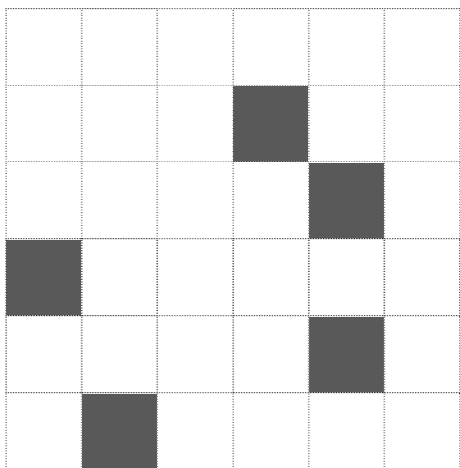
13. (a) 把下圖的 3 個方格塗黑，使得出的圖案成為一個軸對稱圖案，並畫出對稱圖案的對稱軸。

Blacken 3 boxes in the diagram below to obtain a diagram with reflectional symmetry, and draw the axis of reflection.



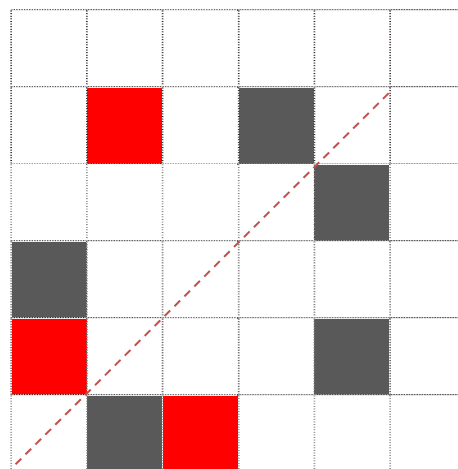
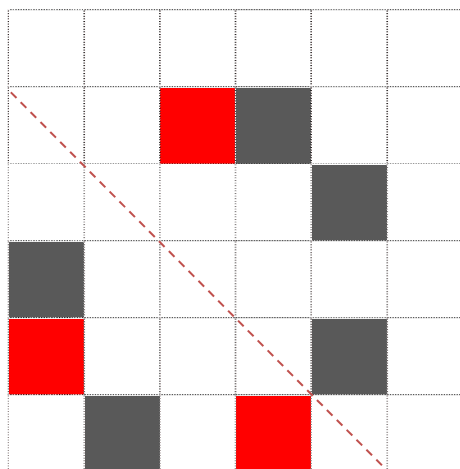
- (b) 把下圖的 4 個方格塗黑，使得出的圖案成為一個軸對稱圖案，並畫出對稱圖案的對稱軸。

Blacken 4 boxes in the diagram below to obtain a diagram with reflectional symmetry, and draw the axis of reflection.

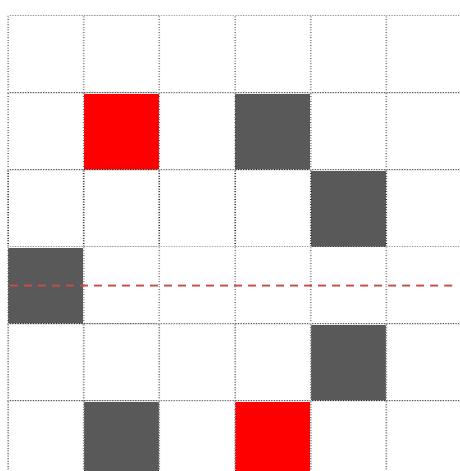
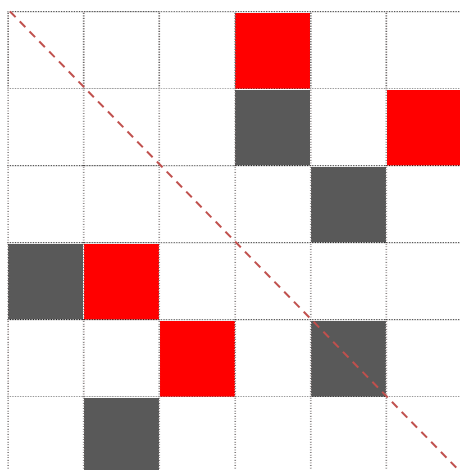


建議答案 Suggested solutions:

(a)



(b)



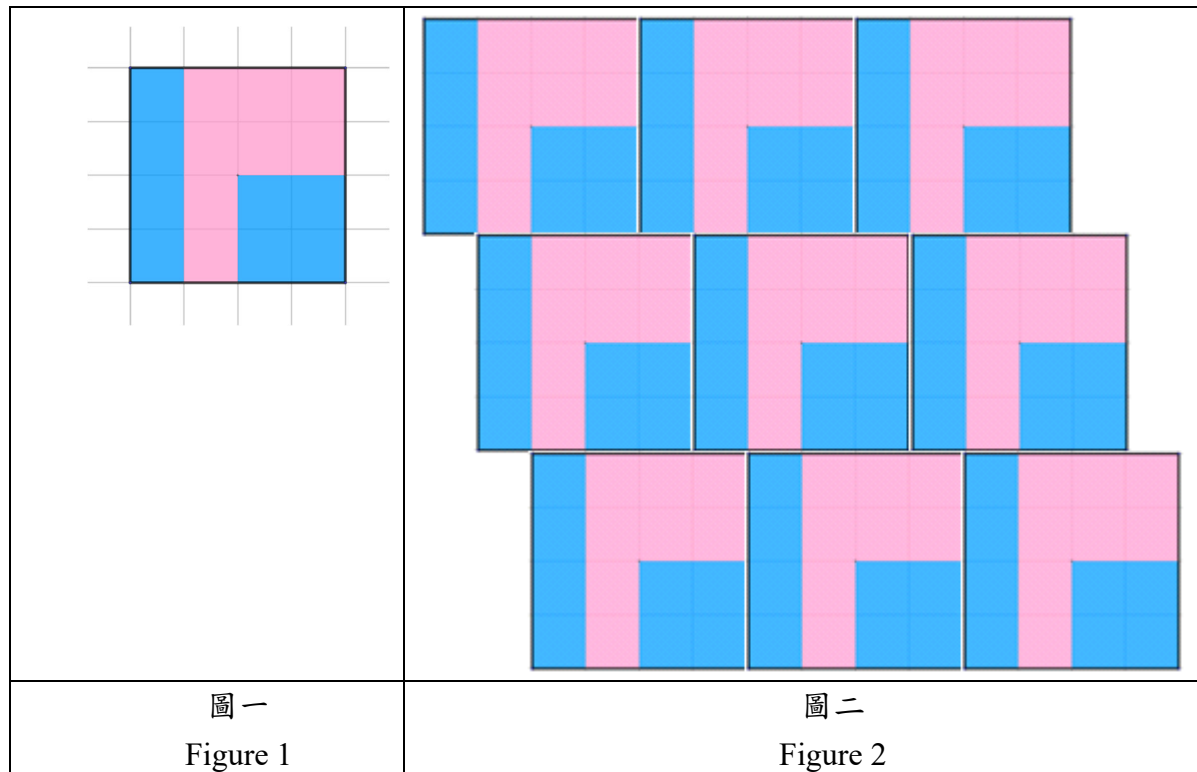
(其餘兩格只需符合以紅色虛線為對稱軸便可自由選填。)

The other two boxes can be freely chosen if they are reflectional symmetry about the red dotted line.)

Section B 乙部

1. 圖一顯示一塊只有兩種顏色的正方形瓷磚，兩種顏色均佔正方形的 50%。若把這塊瓷磚以圖二的方式排列，則會形成一款由兩種不同顏色但相同形狀及大小的圖案密鋪而成的設計。

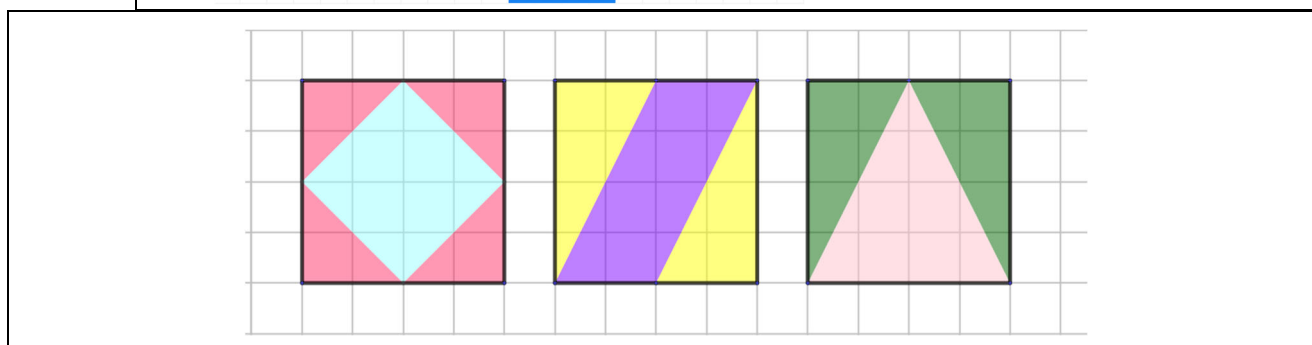
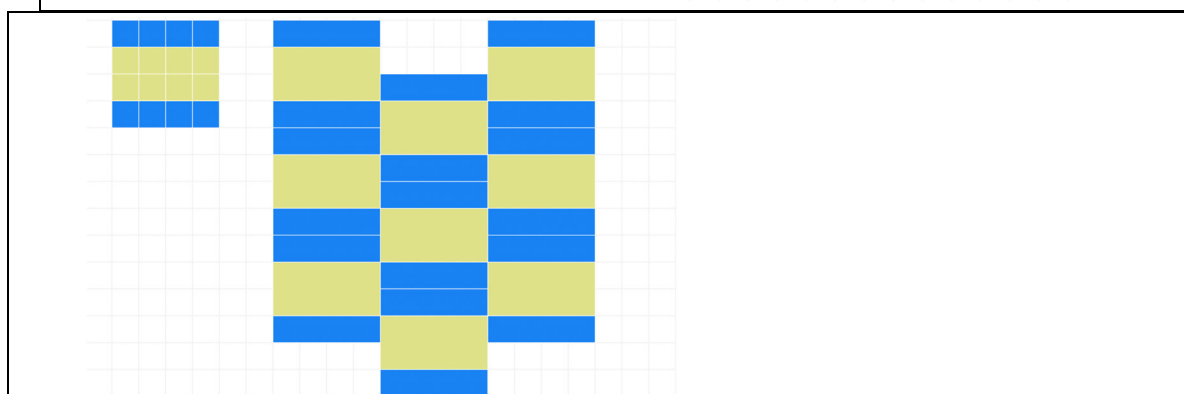
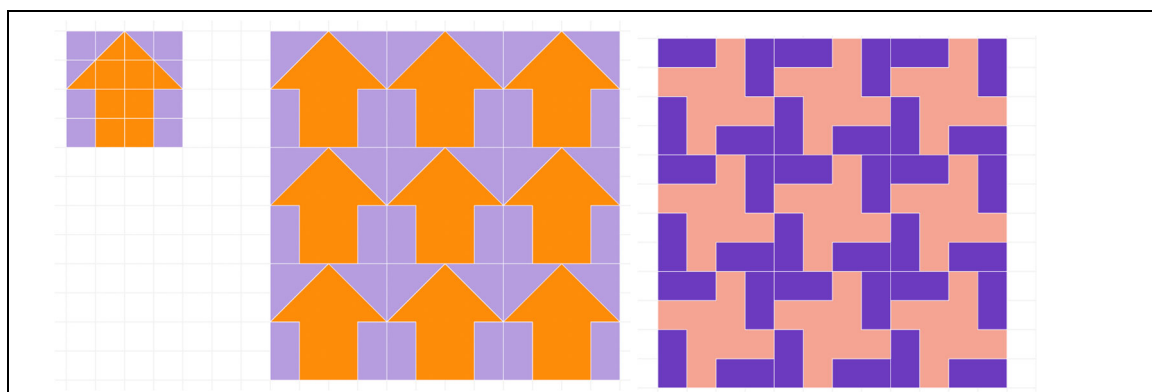
Figure 1 shows the design of a square tile with two different colours. Each colour occupies 50% of the area of the square. If such square tiles are arranged as shown in figure 2, a two-colour tessellation tile pattern having the same shape and the same pattern can be designed.



- (a) 設計三款只有兩種顏色的正方形瓷磚，兩種顏色均佔正方形的 50%，並可砌成如圖二般的密鋪圖案。請畫出正方形瓷磚的設計及密鋪平面。

Design 3 square tiles containing two different colours with each colour occupying 50% of the area of the square tile. Then, tessellate with the square tiles like the one as shown in figure 2. Draw your square tile designs as well as the tessellated plane.

建議答案 Suggested solutions:



- (b) 設計一款可填上多於是兩種顏色的正方形瓷磚圖案（註：同一圖案可填上不同的顏色。），利用這瓷磚密鋪平面後，會得出只有一種圖案的磚密鋪平面，顏色不同而形狀與大小相同，在密鋪平面中每種顏色的圖案所佔的面積是正方形面積的 50%。請畫出正方形瓷磚的設計及密鋪平面。

Design a square tile with patterns filled with more than 2 colours. (Note that different colour can be filled in the same pattern). After tessellating a plane with these tiles, you obtain only one pattern of different colours in the tessellated plane in which the area of the pattern is 50% of the area of the original square tile.

Draw your square tile designs indicating the colour of each pattern as well as the tessellated plane.

建議答案 Suggested solutions:

