2021/22 第十二屆香港中學數學創意解難比賽

25/6/2022（星期六） 10:25-11:30

比賽時間：65 分鐘

參賽者須知：

1. **比賽時間：65 分鐘。**建議在甲部用 50 分鐘作答，在乙部用 15 分鐘作答。

2. 本問題卷共 12 頁，甲部有 13 題數學題，乙部有 1 題創意解難題。另外答題紙共 4 頁。

3. 每位參賽學生獲派一份問題卷及一份答題紙。

4. 比賽其間隊員可以討論題目，並於答題紙寫上議定的答案。

   ** 只有寫於隊長的答題紙上的答案方可得到評分。

5. 參賽隊伍需自備文具及計算機。為公平起見，比賽中只可使用非圖像計算機。本比賽中嚴禁使用電子字典、電腦、電話或其他有上網或通訊功能的工具。

6. 本試卷每頁的空白位置可作為草稿之用。每位參賽學生會獲派三張草稿紙，如有需要，可要求額外草稿紙。

7. 在筆試完結後，必須交回隊長的答題紙。

2021/22 The 12ᵗʰ Hong Kong Mathematics Creative Problem Solving Competition for Secondary Schools
25/6/2022 (Saturday)　10:25-11:30

Time allowed：65 minutes

Instructions for participants：

1. **Time allowed: 65 minutes.**　It is advised to spend 50 minutes in Section A and 15 minutes in Section B.

2. The question paper consists of 12 pages. There are 13 questions in Section A and 1 creative problem in Section B. It follows by 4 pages of answer sheets.

3．Each participant will get a set of question paper and a set of answer sheets.

4. Team members are allowed to discuss during the competition.　The agreed answers should be written on the answer sheets.
   ** Only the answers in the <u>captain's</u> answer sheet will be marked.**

5. Participating teams should bring their own stationery and calculators.　For the purpose of fairness, please use only non-graphic calculators.　Electronic dictionaries, computers, mobile phones and other online or communication devices are prohibited.

6. The blank space on each page of this question paper can be used for rough work. Each participant will get three rough work sheets.　Extra rough work sheets will be provided upon request.

7. The <u>captain's</u> answer sheets will be collected after the competition.

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甲部（建議此部用 **50** 分鐘作答）

Section A (Suggested to use 50 minutes in this Section)

1. 等差數列 11,14,17,…,341,344,347 的總項數是 _____

   The total number of terms in the arithmetic sequence 11, 14, 17, .., 341, 344, 347 is _____

2. 解方程組：$\begin{cases} 6751x + 3249y = 26751 \\ 3249x + 6751y = 23249 \end{cases}$。

   Solve the system of linear equations: $\begin{cases} 6751x + 3249y = 26751 \\ 3249x + 6751y = 23249 \end{cases}$.

3. 本年度香港中學數學創意解難比賽的舉行日期為 6 月 25 日。一般來說，把月份和日期均以兩位整數表達，再把它們拼在一起，即是以 MMDD 這種型式寫出來時，如果得出數字是一個平方數的話，那天便可稱為「好日子」。例如，6 月 25 日寫成 0625，而 625 是 25 的平方，所以 6 月 25 日是「好日子」。在 2022 年有多少天是好日子呢？

   The competition day of Hong Kong Mathematics Creative Problem Solving Competition for Secondary Schools this year is 25/6. In general, if the month number and the day number are considered as two-digit integers and when the four-digit number obtained by combining the two-digit month number with the two-digit day number becomes a square number, the day is called a "Good Day". For example, consider the date 25/6, the month number is 06 and the day number is 25, the four-digit number 0625 is obtained when they are combined. As 625 is the square of 25, hence 25/6 is a "Good Day".

   How many "Good Day" are there in the year 2022?

4. 圖中，*ABCD*，*EBFP* 及 *HPGD* 為長方形。*P* 是對角線 *AC* 上的一點。若 $AE = 2$ cm 及 *EBFP* 的面積 $= 18$ cm^2，求 *HPGD* 的面積。

In the figure, *ABCD*, *EBFP*, and *HPGD* are rectangles. *P* is a point on diagonal *AC*. If $AE = 2$ cm and area of *EBFP* $= 18$ cm^2, find the area of *HPGD*.
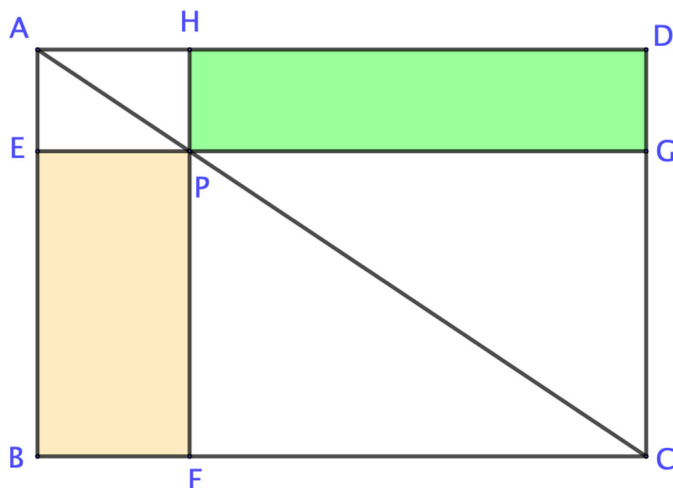
5. $2022 \times M \times N$ 是一個大於 20220625 的平方數，當中 *M* 和 *N* 為正整數。求 $M + N$ 的最小值。

$2022 \times M \times N$ is a square number larger than 20220625, where *M* and *N* are positive integers. Find the smallest value of $M + N$.

6. 有一間售賣地毯的店鋪，店主以地毯的面積和周界來定價，以下為各款地毯的售價：

In a carpet shop, the owner sets the price according to the area and perimeter of the carpet. The following are the prices of different types of carpets:

A: $720
B: $402
C: $530
D: $468
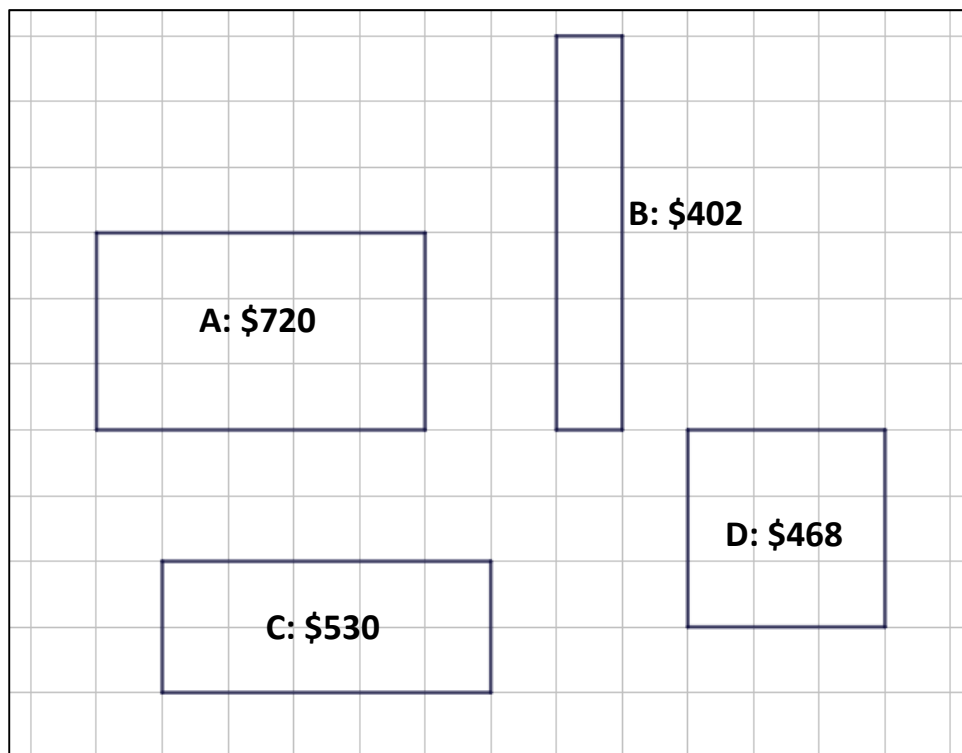
根據以上定價的規律，有客人想訂製以下款式的地毯，這款地毯的售價是多少？

A customer wants to order a carpet in the following shape. According to the information above, what is the price of this carpet?
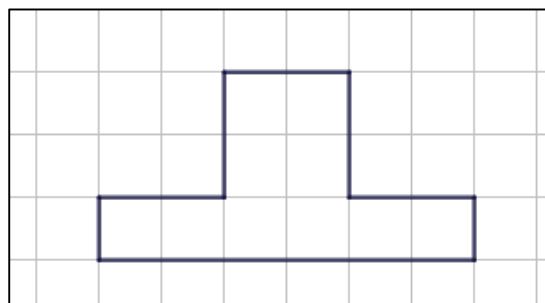
7. In the figure, the circle circumscribes the square *ABCD*. Another square *PQRS* is inscribed in the circle. Find $\dfrac{\text{area of shaded region 1}}{\text{area of shaded region 2}}$.

圖中的圓外接正方形 *ABCD*。另一正方形 *PQRS* 內接該圓。

求 $\dfrac{\text{陰影部分1的面積}}{\text{陰影部分2的面積}}$。
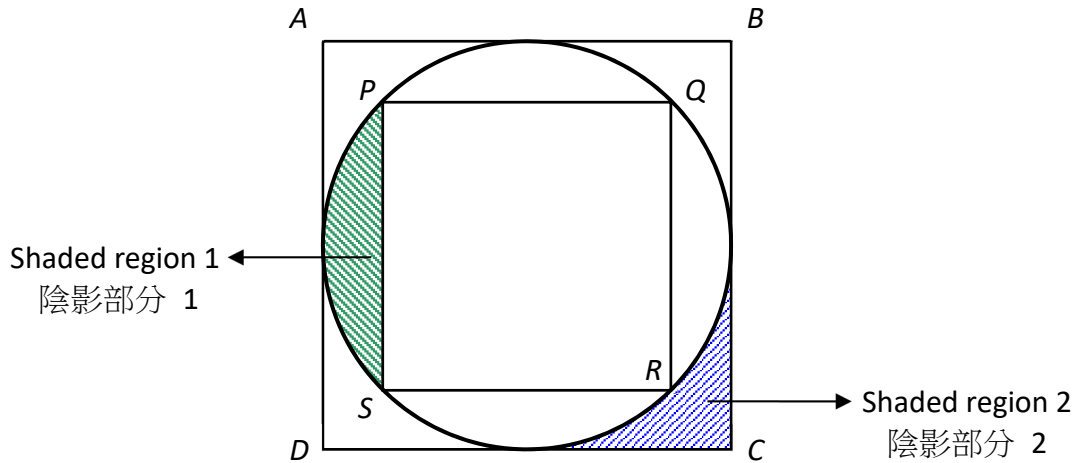
8. 設 x、y、z 為滿足 $x+y+z+xy+yz+zx+xyz=2021$ 的非零整數。求 xyz 的最大可能值。

Let x, y, z be non-zero integers satisfying
$x+y+z+xy+yz+zx+xyz=2021$. Find the greatest possible value of xyz.

9. 圖中，*ABCD* 為一正方形。以 *AD* 為直徑的半圓與弧*CFA*相交於*F*。已知 *E* 是 *AD* 的中點。若 Δ*BEF* 的面積是 25 cm^2，求正方形的面積。

In the figure, *ABCD* is a square. Semi-circle with diameter *AD* and circular arc *CFA* meet at *F*. Given that *E* is the mid-point of *AD*. If the area of Δ*BEF* is 25 cm², find the area of the square.
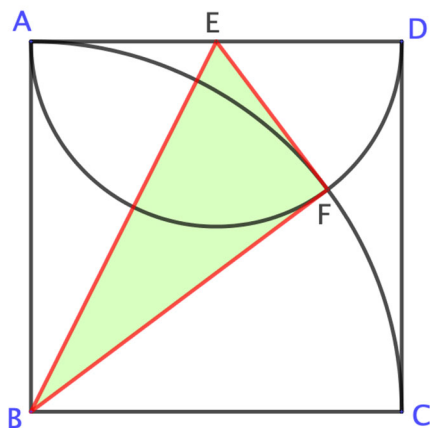
10. 以三種不同方式把 2、3、4、5、6、7、8、9 填到以下的空格內，使得不等式成立。

Fill in the following boxes by 2, 3, 4, 5, 6, 7, 8, 9 in three different ways to make the inequality valid.

(1) $\square^{\square} < 2022 < \square^{\square} < \square^{\square} < \square^{\square}$

(2) $\square^{\square} < \square^{\square} < 2022 < \square^{\square} < \square^{\square}$

(3) $\square^{\square} < \square^{\square} < \square^{\square} < 2022 < \square^{\square}$

11. 四位單車手在各自圓周為 3 公里的圓形軌道上踏單車。他們同時在圖中的黑點上出發，車速分別是每小時 6, 9, 12 及 15 公里。在 20 分鐘的旅程上，他們會同時到達各自的黑點多少次？

Four cyclists do their act on circular paths, each 3 km long. They start simultaneously at the black spots, with speeds of 6, 9, 12 and 15 km per hour. By the end of the act (20 minutes), how many times will they have simultaneously returned to the spots where they started?
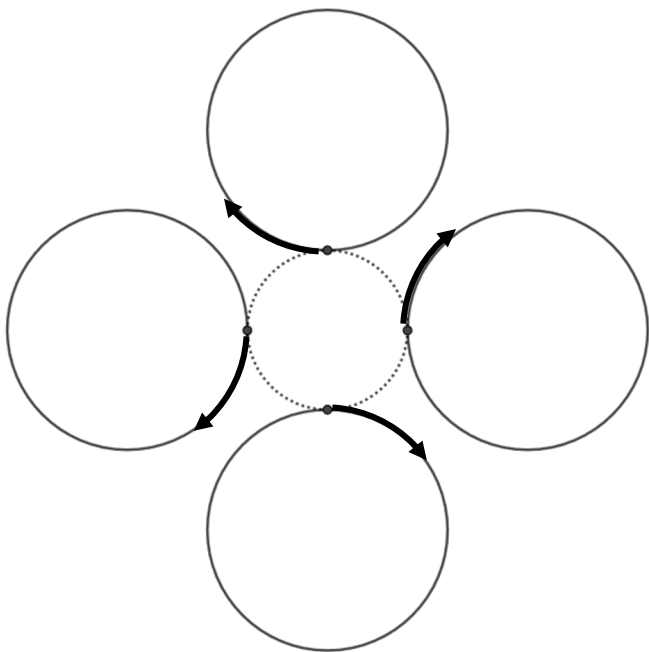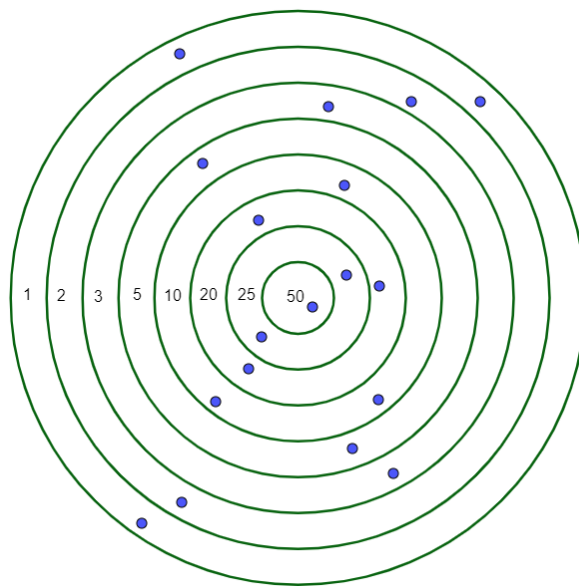
12. 在一個射擊比賽中，小創、小意及小蘭分別開了 6 槍並都得了 71 分。

小創頭兩槍合共得了 22 分而小蘭第一槍得了 3 分。

他們當中誰人射中了紅心呢？

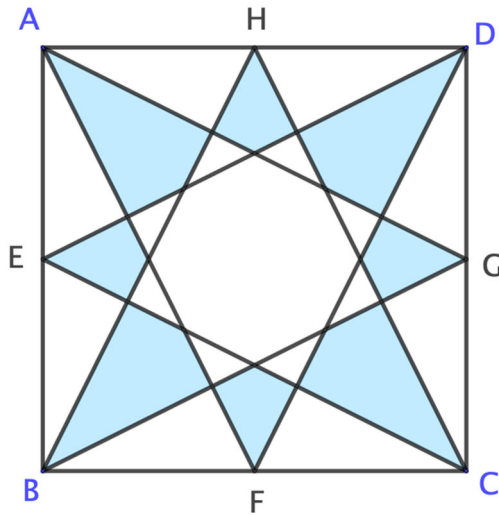In a shooting match, Ada, Ben and Christine each fired 6 shots and each got 71 points.

Ada's first two shots got 22 points and Christine's first shot got only 3 points.

Who hit the bull's-eye?

13. 圖中，$ABCD$ 是一個正方形，E、F、G、H 分別是 AB、BC、CD、DA 的中點。若 $AB = 30$ cm，求藍色部份的面積。

In the figure, $ABCD$ is a square and E, F, G, H are mid-points of AB, BC, CD, DA respectively. If $AB = 30$ cm, find the area of the blue parts.

乙部（建議此部用 15 分鐘作答）

Section B (Suggested to use 15 minutes in this Section)

1. 必勝和當勞玩一個遊戲。兩人輪流放下 1 至 10 元的整數硬幣，第一個放到 100 元正的便獲勝。

   必勝爭取第一個並放下 1 元後，他便滿有信心說：「我必能勝出這場比賽。」

   Wynner and Donald play a game together. They put down coins of value ranges from \$1 to \$10 alternately. The one who puts the coins which makes the total value \$100 will be the winner of the game.

   Wynner is the first one to put down coins. After he puts down a \$1 coin, he claims that he must be able to win this competition.

   (a) 試以數學的方法解釋必勝必能獲得勝利的原因。

      Explain, with mathematical arguments, why Wynner is confident with winning the game.

   (b) 若遊戲規則不改變，但改為第一個放到 200 元正的便獲勝。試解釋必勝是否應該繼續爭取第一個放下硬幣，使得他必能勝出這場比賽。

      If the other regulations are unchanged, but the one who puts the coins which makes the total value \$200 will be the winner of the game. Explain whether Wynner should put down coins first to ensure that he could win the game.

(c) 若遊戲規則不改變，但改為第一個放到 T 元正$(T>200)$的便獲勝。試寫下 3 個不同的 n 並解釋原因，使得第一個放下硬幣的人**必不能**勝出這場比賽。

If the other regulations are unchanged, but the one who puts the coins which makes the total value $\$T$ will be the winner of the game, where $T>200$. Write down 3 different values of T, with explanation, so that the first one who put down coins **must not** be the winner of the game.

(d) 必勝改為與小創及小意玩遊戲，三人輪流放下硬幣，若小創及小意可放下 1 至 8 元的整數硬幣，而必勝可放下 1 至 n 元的整數硬幣，第一個放到 T 元正$(T>200)$的便獲勝。

必勝爭取第一個並放下 a 元後，他便滿有信心說：「我必能勝出這場比賽。」

試寫出一組 n、T 及 a 的數值，或以數學的方法表達 n、T 及 a 之間的關係，其中 n 為能確保必勝能勝出比賽的最小值，並解釋必勝如何能勝出這場比賽。

Wynner plays the game with Ada and Ben. They put down coins alternately. Ada and Ben can put down coins of values range from $\$1$ to $\$8$, while Wynner can put down coins of values range from $\$1$ to $\$n$. The one who puts the coins which makes the total value $\$T$, where $T>200$, will be the winner of the game.

Wynner is the first one to put down coins. After he puts down coins of value $\$a$, he claims that he must be able to win this competition.

Write down a set of values of n, T and a, or express the mathematical relation between n, T and a, where n is the smallest value to ensure Wynner to win the competition. Explain, with mathematical arguments, why Wynner is confident with winning the game.

全卷完
End of Paper

答題紙 Answer sheets

學校名稱 School Name：	得分 Score：
	/27

甲部 Section A

答案 Answers		評分 Marks
1.	等差數列 11,14,17,…,341,344,347 的總項數是 The total number of terms in the arithmetic sequence 11, 14, 17, .., 341, 344, 347 is _____ 答/Ans:_____	/2
2.	解方程組：$\begin{cases} 6751x + 3249y = 26751 \\ 3249x + 6751y = 23249 \end{cases}$。 Solve the system of linear equations: $\begin{cases} 6751x + 3249y = 26751 \\ 3249x + 6751y = 23249 \end{cases}$. 答/Ans: $x =$ _____; $y =$ _____	/2
3.	在 2022 年有多少天是好日子呢? How many "Good Day" are there in the year 2022? 答/Ans:_____	/2
4.	求 $HPGD$ 的面積。 Find the area of $HPGD$. 答/Ans:_____ cm²	/2
5.	求 $M + N$ 的最小值。 Find the smallest value of $M + N$. 答/Ans:_____	/2

6. 這款地毯的售價是多少? What is the price of this carpet?	答/Ans: $_____	/2
7. 求 $\dfrac{陰影部分1的面積}{陰影部分2的面積}$ 。 Find $\dfrac{\text{area of shaded region 1}}{\text{area of shaded region 2}}$.	答/Ans:_____	/2
8. 求 xyz 的最大可能值。 Find the greatest possible value of xyz .	答/Ans:_____	/2
9. 求正方形的面積。 Find the area of the square.	答/Ans:_____ cm^2	/2
10. (1) $\square^{\square} < 2022 < \square^{\square} < \square^{\square} < \square^{\square}$ (2) $\square^{\square} < \square^{\square} < 2022 < \square^{\square} < \square^{\square}$ (3) $\square^{\square} < \square^{\square} < \square^{\square} < 2022 < \square^{\square}$		/3
11. 他們會同時到達各自的黑點多少次? How many times will they have simultaneously returned to the spots where they started?	答/Ans:_____	/2
12. 他們當中誰人射中了紅心呢? Who hit the bull's-eye?	答/Ans:_____	/2
13. 求藍色部份的面積。 Find the area of the blue parts.	答/Ans:_____ cm^2	/2

1. (a) 試以數學的方法解釋<u>必勝</u>必能獲得勝利的原因。

   Explain, with mathematical arguments, why Wynner is confident with winning the game.

   (b) 試解釋<u>必勝</u>是否應該繼續爭取第一個放下硬幣，使得他必能勝出這場比賽。

   Explain whether Wynner should put down coins first to ensure that he could win the game.

(c) 試寫下 3 個不同的 n 並解釋原因，使得第一個放下硬幣的人<u>必不能</u>勝出這場比賽。

Write down 3 different values of T, with explanation, so that the first one who put down coins must not be the winner of the game.

(d) 試寫出一組 n、T 及 a 的數值，或以數學的方法表達 n、T 及 a 之間的關係，並解釋必勝如何能勝出這場比賽。

Write down a set of values of n, T and a, or express the mathematical relation between n, T and a. Explain, with mathematical arguments, why Wynner is confident with winning the game.

答題紙完

End of Answer Sheets