2017/18

The Thirteenth Hong Kong Mathematics Creative Problem Solving Competition for Primary School

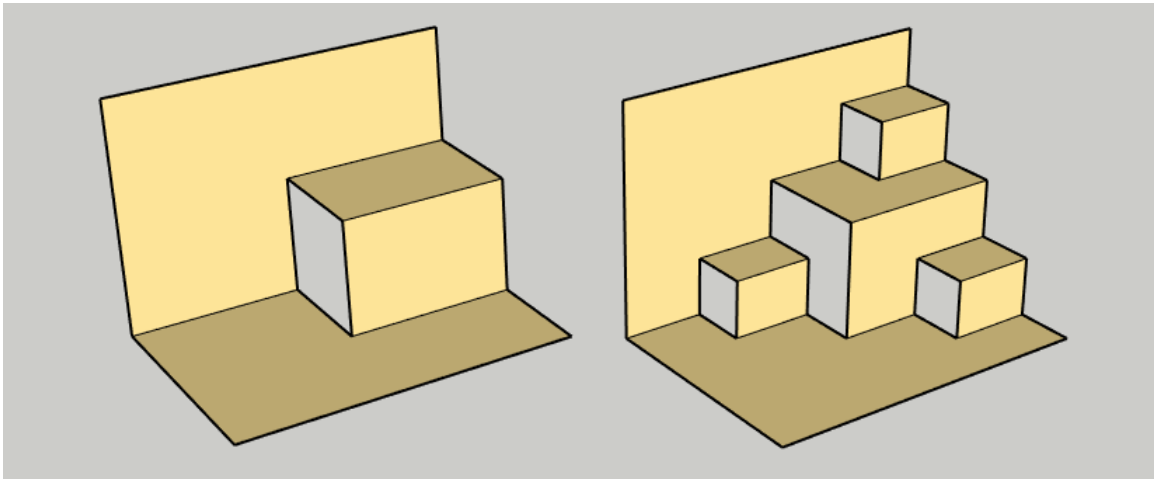
Final Cum Invitation Tournament – Mathematics Problem Solving Experiment and Debate

Between One Dimension and Three Dimensions

Suggested solutions

Section A - Mathematics in Paper Art

1. (10 marks) By folding and cutting a paper repeatedly, we can obtain a 3-dimensional paper art model. The volume of the paper remains unchanged but it will occupy extra space.



Model 1 Model 2

Figure 1

(a) Please calculate the space occupied by the paper art Model 1 and Model 2 provided. (*The dimensions of the paper are 16 cm × 20 cm.)

Space occupied by Model 1 $= \left(\frac{20}{4}\right)\left(\frac{20}{4}\right)\left(\frac{16}{2}\right) = 200\ \text{cm}^3$

Space occupied by Model 2

$= \left(\frac{20}{4}\right)\left(\frac{20}{4}\right)\left(\frac{16}{2}\right)\left(1 + 3 \times \frac{1}{8}\right) = 275\ \text{cm}^3$

(2+2)

(b) Please follow the pattern from Model 1 to Model 2, imagine the next model and calculate the space occupied by this new model.

Capacity of Model 3

$$= \left(\frac{20}{4}\right)\left(\frac{20}{4}\right)\left(\frac{16}{2}\right)\left(1+\frac{3}{8}+\frac{9}{64}\right) = 303.125\ cm^3$$

（2+1）

(c) Please make this paper art model using the paper provided.

(3)

2. (6 marks) If the model pattern goes on, what would the models look like? Please describe the trend of the models.

- The Capacities increases but bounded by an upper limit.

  The no. of cuboid increases is 3 times of the previous one.

  The capacities of the cuboid increases is 3/8 of the previous one

- The shape tends to be a tetrahedron
- Self-similar
- …

Section B – The mathematics in paper ring

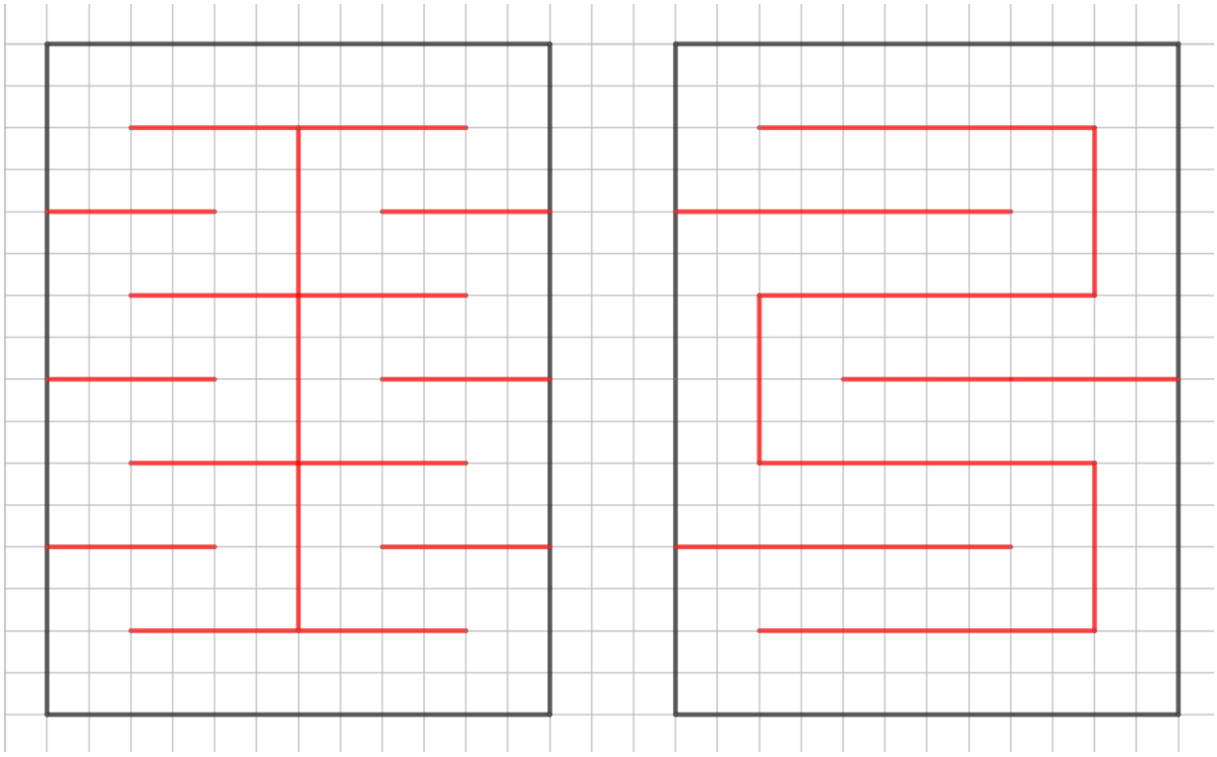


Figure 2a

Figure 2b

1. (6 marks) You are given 2 pieces of rectangular paper. Please cut the paper into paper rings according to figure 2a and 2b. Please estimate the largest possible length surrounded by the paper ring. Explain your answer, with diagrams if necessary.

   Figure 2a

   The length of the paper ring

   $= \left(8\ \text{cm} \times 2 + \sqrt{2^2 + 2^2}\ \text{cm} \times 6 \times 2\right) \approx 49.9\ \text{cm}.$

   or

   $= (8\ \text{cm} \times 2 + 2.8\ \text{cm} \times 6 \times 2) \approx 49.6\ \text{cm}$

Lower limit of the acceptable range $= (8\ \text{cm} \times 2 + 2\ \text{cm} \times 6 \times 2) \approx 40\ \text{cm}$

(1+1)

Figure 2b

The approximate length of the paper ring

$= 8\ \text{cm} \times 2 + 4\ \text{cm} \times 3 + \sqrt{6^2 + 2^2}\ \text{cm} \times 6 = 65.9\ \text{cm}$

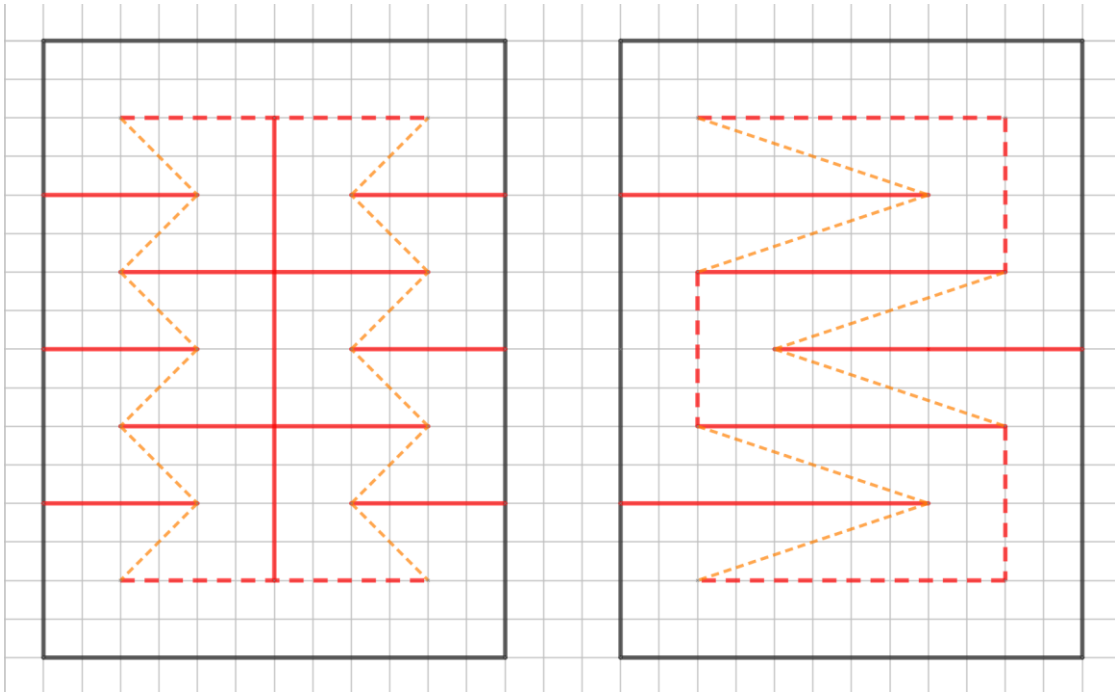
or

$= 8\ \text{cm} \times 2 + 4\ \text{cm} \times 3 + 6.3\ \text{cm} \times 6 = 65.8\ \text{cm}$

Lower limit of the acceptable range

$= 8\ \text{cm} \times 2 + 4\ \text{cm} \times 3 + 6\ \text{cm} \times 6 = 64\ \text{cm}$

(1+1)

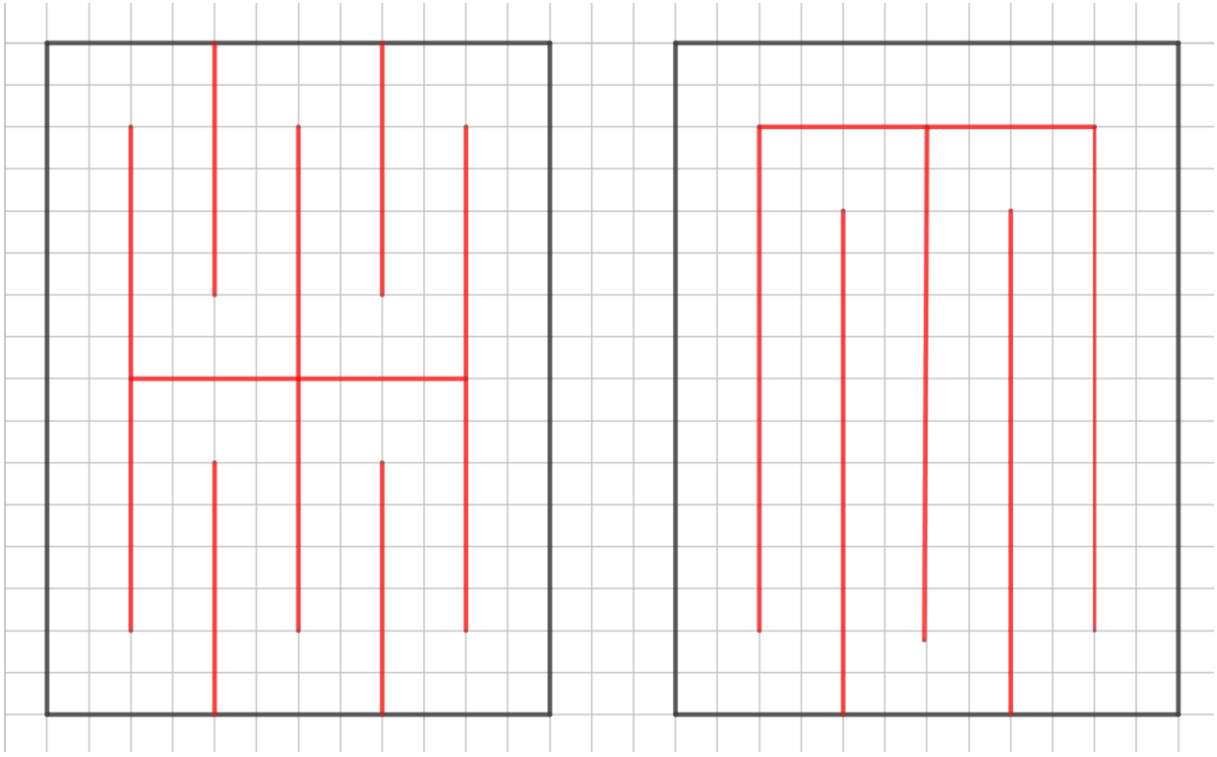


(1+1)

2. (6 marks) The lengths of the 2 pieces of paper ring are different. It requires many steps of cutting, too. In figure 2b, you will need 10 steps in maximum to cut the pattern, given that you need not to turn around in each step.

Please improve the design of the paper ring or the cutting process such that the width of the paper ring remains unchanged.

Please draw a diagram to illustrate your solution.



Both figures above reduce the steps of cutting and ,at the same time, increase the length of the paper ring.

The length of the figure on the left

$= \sqrt{4^2 + 2^2} \times 4 \times 2 + 12 \times 2 \approx 59.8$ (3)

The length of the figure on the right

$= \sqrt{10^2 + 2^2} \times 4 + 12 \times 2 + 8 \approx 72.8$ (4)

Fold before cutting can also reduce the steps of cutting. (2)

3. (8 marks) You are required to cut a paper ring from a piece of A4 paper and make it as large as possible. Your design of cutting can be based on the result of Q.2.

(a) Please draw a diagram to illustrate your solution.

(b) Please make a paper ring with the A4 paper provided. This paper ring will be used to surround an object provided by the judges. Marks will be given to the team who made a paper ring successfully. Bonus marks will be given to the team having the shortest paper ring to surround the object successfully or the longest paper ring that failed to surround the object.

* Each team can only observe the object from a given position.

3(a)	Design	(1)
	If the width is reduced into half, the length of the paper can be doubled	(2)
3(b)	Paper ring with shortest length that can surround an object	(5)
	Paper ring can surround an object	(4)
	Paper ring with the longest length that cannot surround an object	(3)
	Paper ring cannot surround an object	(2)
	Fail to make a paper ring	(0)