

Mathematics Creative Problem Solving Competition 2012-2013

Final Event (Secondary)

Marking Scheme

Part A

1. a. $x + 13$ [1A]

b. $-x^2 + 7x - 7$ [1A]

2. $k(x+1)(x-1)(x-8)$ or $k(x^3 - 8x^2 - x + 8)$ [1A]

3. a. $(x+1)x(x-1)(x-3)(x-8)/(-5)$ [2A]

b. $\frac{(x+1)x(x-1)(x-3)(x-8)}{-5} + \frac{(x+2)x(x-1)(x-3)(x-8)}{4} + \frac{(x+2)(x+1)(x-1)(x-3)(x-8)}{-8}$

+ $\frac{(x+2)x(x+1)x(x-3)(x-8)}{-4} + \frac{(x+2)(x+1)x(x-1)(x-3)}{56}$ [3A]

Note: Lagrange Interpolation

$$f(x) \approx f_1 L_1(x) + \dots + f_n L_n(x),$$

where

$$L_i(x) = \frac{(x - x_1) \dots (x - x_{i-1})(x - x_{i+1}) \dots (x - x_n)}{(x_i - x_0) \dots (x_i - x_{i-1})(x_i - x_{i+1}) \dots (x_i - x_n)}$$

4.

$i =$	1	2	3	4	5	6
$x_i =$	-2	-2	0	1	3	8
$y_i =$	60	12	-6	-21	0	450

[2A]

5.

- Calculate the average humidity of each month. (i.e. We end up with 12 data points)
- Use the above method and one can end up with a degree 11 polynomial. [1M+1M]

Part B

1.
 - The moving averaged curve is smoother.
 - There is a time lag in the moving averaged curve. If the number of averaged dates is increased, the time lag increased.
 - The number of peaks and troughs decrease as the number of averaged day increases. (The local peaks are diminishing.)

[2A]+ [2A]+ [2A]

2.
$$Y = X + \frac{p-q}{n}$$

[2A]

3. a. Regular average methods always give average values which are smaller than the maximum value and higher than the minimum value. In the cases of stock market crashes, the values virtually go down monotonically in short period of time. Therefore regular average methods do not work.

[2A]

b. Method 1:

- Take the difference of the consecutive daily prices.
- Study the moving average curves of the difference of prices.
- Compare the moving average curves of the difference of prices with the curves of previously known stock market crashes.

Method 2: Assume there is a crash, the divergence, at a particular day. Study the properties of the divergence and see if there is any sign of divergence within the error.

Or other plausible methods.

[Reasonable approach 2A]

[Logical presentation 2A]