1. Consider $15600 = 2\times2\times2\times2\times3\times5\times5\times13$, it can be written as $15600 = (2\times2\times2\times3)\times(5\times5)\times(2\times13) = 24\times25\times26$

The total surface area is $3748 \text{ cm}^2$

OR

consider $\sqrt[3]{15600} \approx 24.99$, the three sides are of values near to 24 and 25.

2. Let the distance travelled in the remaining half journey be $x$ km.

$$\frac{x}{250} - \frac{x}{350} = 1 + 0.5$$

$$\frac{x}{5} - \frac{x}{7} = 75$$

$$2x = 35(75) = 2625$$

The distance between $A$ and $B$ is $2625 \text{ km}$.

3. In figure 1, the equilateral triangle $ABC$ and the regular hexagon $PQCRSB$ have the same perimeter. Dissecting both figures into smaller equilateral triangles as shown in figure 2, it can be seen that area $\Delta ABC : \text{area PQCRSB} = 4:6$.

$$\text{Area of the hexagon} = \text{area of triangle} \times \frac{6}{4}$$

$$= 3 \times \frac{6}{4} = 4.5$$

Therefore, area of the hexagon $DEFGHI$ is $4.5$. 
4. Consider the expression as \[ (A + (B) - (C))^2 \times (D) \div (E). \]

In order that the value is the greatest, the value should be positive. Since \[ (A + (B) - (C))^2 \] must be positive, (D) and (E) should be of the same sign.

Case 1: \[ D = +2 \text{ and } E = +1. \]
For the value to be the greatest, make \[ (A + (B) - (C)) \] as negative as possible, i.e. \[ \{(-3) + (-2) - (-1)\} \]
\[ R = [(-3) + (-2) - (-1)]^2 \times 2 + 1 = 32 \]

Case 2: \[ D = -3 \text{ and } E = -1. \]
Take \[ (A + (B) - (C)) \] as \[ \{(+2) + (+1) - (-2)\} \].
\[ R = [(+2) + (+1) - (-2)]^2 \times (-3) \div (-1) = 75 \]

Case 3: \[ D = -2 \text{ and } E = -1. \]
Take \[ (A + (B) - (C)) \] as \[ \{(+2) + (+1) - (-3)\} \].
\[ R = [(+2) + (+1) - (-3)]^2 \times (-2) \div (-1) = 72 \]

Case 4: \[ D = -3 \text{ and } E = -2. \]
Take \[ (A + (B) - (C)) \] as \[ \{(+2) + (+1) - (-1)\} \].
\[ R = [(+2) + (+1) - (-1)]^2 \times (-3) \div (-2) = 24 \]

Case 2 gives the greatest value of \( R \). Greatest value of \( R \) is 75.

5. \( \Phi \frac{2013}{2013} \) is a value between 0.595 and 0.605.
Consider \( 2013 \times 0.595 = 1197.735 \) and that \( 2013 \times 0.605 = 1217.865 \)
\[ M = 1217, N = 1198 \]
\[ M + N = 2415 \]

6. In packet A, the amount of sesame = \( 1000 \text{ g} \times 3\% = 30 \text{ g} \).
There must also be 30 g of green bean in packet B.

\[ \therefore \text{Percentage of green bean in packet B} = \frac{30}{600} \times 100\% = 5\% \]
7. a. For \((1 \times 5) + (2 \times 10) + (3 \times 15) + (4 \times 20) + \ldots + (99 \times 495)\):

Among the 99 terms to be added, the values of \((1 \times 5), (3 \times 15), \ldots, (99 \times 495)\) are 5, i.e. 50 terms are 5, the others are 0.

\[ \therefore (1 \times 5) + (2 \times 10) + (3 \times 15) + (4 \times 20) + \ldots + (99 \times 495) \]
\[ = 50 \times 5 \]
\[ = 250 \]

b. For \((1 \times 3) + (3 \times 5) + (5 \times 7) + (7 \times 9) + \ldots + (2011 \times 2013)\):

the terms \((1 \times 3) + (3 \times 5) + (5 \times 7) + (7 \times 9) + (9 \times 11) = 3 + 5 + 5 + 3 + 9 = 25\)
same for \((11 \times 13) + (13 \times 15) + (15 \times 17) + (17 \times 19) + (19 \times 21)\)
and \((21 \times 23) + (23 \times 25) + (25 \times 27) + (27 \times 29) + (29 \times 31)\)

\[ \ldots \]

\[ \therefore \text{The sum} = (3 + 5 + 5 + 3 + 9) \times 201 + 3 = 5028 \]

8. a. By observation, for 1 more ‘ring’ of triangles added, there are 3 more layer. When \(N\) rings are added, the number of layers is \((1 + 3N)\). = 100.

In the end, \(1 + 3N = 100\).

\[ \therefore N = 33. \]

There are 34 different colors.

OR Let \(L_N\) be the number of layers in the \(n\)-th pattern.

\[ L_1 = 1, L_2 = 4, L_3 = 7, L_4 = 7 + 3 = 10, \ldots \]

The general term is \(L_N = 1 + 3(N-1) = 3N-2\)

For \(L_N = 3N - 2 = 100\), \(N = 34\), i.e. The final pattern is the 34th pattern.

b. When the outermost ring of triangles are added, there are 100 layers.

Before adding this ring, there are 97 layers.

Number of triangles added to the three sides
\[ = (2 \times 97 + 3) \times 3 = 591 \]

Or consider,

\[ [1+3+5+\ldots+(1+99\times2)] - [1+3+5+\ldots+(1+96\times2)] \]
\[ = 199 + 197 + 195 \]
\[ = 591 \]
9. a. At each vertex, there are 6 angles. 
   On all the six vertices, there are \(36\) angles.

   b. The eight faces are equilateral triangles. Their interior angles are \(60^\circ\). 
   \(24\) of the angles are \(60^\circ\).

   Considering the symmetry in the regular octahedron: 
   \(ABCD, PDQB, APCQ\) are squares. Their interior angles are \(90^\circ\). 
   \(12\) of the angles are \(90^\circ\).

   The biggest angles are of size \(90^\circ\). There are \(12\) of them. 
   The smallest angles are of size \(60^\circ\). There are \(24\) of them.

10. Since \(17+18=35\), the squares formed must be \(1, 4, 9, 16\) or \(25\).

    For \(18, 17\) and \(16\), the only possible pairing arrangements are 
    \(18+7=25, 17+8=25\) and \(16+9=25\).

    Consider the remaining numbers: \(1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15\)
    
    15 might be with 10 or 1,
    14 might be with 11 or 2,
    13 might be with 12 or 3,
    12 might be with 13 or 4,
    11 might be with 14 or 5,
    10 might be with 15 or 6,
    6 might be with 10 or 3,
    5 might be with 11 or 4,
    4 might be with 12 or 5,
    3 might be with 13, 6 or 1,
    2 might be with 14 or 7,
    1 might be with 15, 8 or 3.

    But considering the case of ‘2’,
    since ‘7’ has to pair with ‘18’, ‘2’ has to pair with ‘14’.
    Since ‘14’ has to pair with ‘2’, ‘11’ has to pair with ‘5’.
    Since ‘5’ has to pair with ‘11’, ‘4’ has to pair with ‘12’…..

    With similar reasoning, we have the pairing arrangement as follows:
    \((18, 7), (17, 8), (16, 9), (15, 1), (14, 2), (13, 3), (12, 4), (11, 5), (10, 6)\).

    2 pairs with \(14\), 4 pairs with \(12\), 6 pairs with \(10\), 8 pairs with \(17\) and 10 pairs with \(6\).
11. a. The figure shows the furthest point that the beetle can reach by moving at most 10 units. Points further than those marked cannot be reached.

Number of unreachable points
= 1 + 2 + 3 + 4 + 5 + 6 = 21

∴ Number of reachable lattice points
= 9 × 9 – 21 = 60

b. If a lattice point can be reached by the beetle moving for 2, 4, 6, 8 or 10 units, it can be reached in exactly 10 units. For example, in figure a, the beetle can move from O to point A in a minimum of 4 moves (3 ups and 1 right).

A path can also be formed by adding 3 left and 3 right which totally consists of exactly 10 moves.

There are totally 9 lattice points in the shaded region as shown in figure b.
12. Consider the figure:

\[ \Delta PAC \cong \Delta PA'C \]
\[ \therefore A'C = AC = 12, \quad BA' = 13 - 12 = 1 \]

Comparing \( \Delta BPA' \) and \( \Delta CPA' \) as two triangles sharing the same height (from \( P \)), with respectively \( BA' \) and \( A'C \) as bases,

area of \( \Delta PBA' \): area of \( \Delta PCA' \) = \( BA' : CA' = 1 : 12 \)

Let area of \( \Delta PBA' \) be \( x \).
area of \( \Delta PCA' \) = area of \( \Delta PBA' = 12x \)
\[ \therefore \text{area of} \ \Delta ABC = x + 12x + 12x = 25x. \]

Percentage of visible grey region = \( \frac{x}{25x} \times 100\% = 4\% \).

b. Comparing \( \Delta APC \) and \( \Delta BPC \) as two triangles sharing the same height (from \( C \)) with respectively \( AP \) and \( BP \) as bases,

area of \( \Delta APC \) : area of \( \Delta BPC = 12 : 13 \)
\[ \therefore \frac{AP}{BP} = 12 : 13 \]
\[ BP = 10 \times \frac{13}{12+13} = 5.2 \text{ cm} \]

13. a. The number of matches needed
\[ = 20 \times (13+1) + 13 \times (20+1) \]
\[ = 553 \]

b. Suppose an \( m \times n \) rectangle is formed with the 87 matches.

\[ m \times (n + 1) + n \times (m + 1) = 82, \quad \text{where} \ m \text{ and } n \text{ are both integers} \]
\[ 2mn + m + n = 82 \]
\[ 4mn + 2m + 2n = 164 \]
\[ 4mn + 2m + 2n + 1 = 165 \]
\[ (2m + 1)(2n + 1) = 165 = 3 \times 5 \times 11 \]

For \( 2m + 1 = 3, \ 2n + 1 = 55: \quad m = 1, \ n = 27, \text{there are 27 unit squares} \]
For \( 2m + 1 = 5, \ 2n + 1 = 33: \quad m = 2, \ n = 16, \text{there are 32 unit squares} \]
For \( 2m + 1 = 11, \ 2n + 1 = 15: \quad m = 5, \ n = 7, \text{there are 35 unit squares} \]

The rectangle consists of at most 35 unit squares.
Consider \( 2mn + m + n = 82 \) i.e. \( m = \frac{82-n}{2n+1} \)

With \( n = 1, m = 27. \)
With \( n = 2, m = 16. \)
With \( n = 5, m = 7. \)
As \( n \) gets larger, \( m \) gets smaller.
The other possibilities of \( m \) to be considered are \( m = 6, 5, 4, 3, 2, 1 \)

Since \( n = \frac{82-m}{2m+1}, n \) will be an integer when \( m = 5, 2, 1. \)

Possible pairs of \( m, n \) are \((1, 27), (2, 16), (5, 7), (7, 5), (16, 2) \) and \((27, 1)\).
Greatest value of \( m\times n = 7\times 5 = 35. \)

14. a. All possible locations of \( B \) are as shown:

The three figures shows the triangles \( \Delta OAB \) formed with \( 90^\circ \) at respectively vertices \( B, \) vertex \( O \) and vertex \( A. \)
\( B_1 \) is \((0, 3).\)
Consider the rotation of \( A(1, 3) \) by \( 90^\circ, B_2 \) is \((-3, 1) \) and \( B_3 \) is \((3, -1).\)
Consider similar triangles: \( \Delta ODA \) and \( \Delta APB \)
\[
\frac{PB}{PA} = \frac{1}{3} \\
PB = \frac{2}{3}
\]
Coordinates of \( B_4 = (3, 3 - \frac{2}{3}) = (3, 7/3) \)

b. Take the case above with \( M \) at \((1, 3), B_1, B_2 \) and \( B_3 \) can be taken as \( N. \)
Area of the right-angled triangles are
\[
\frac{1}{2} \times 1 \times 3 = 1.5 \quad \text{or} \quad \frac{1}{2} (3 + 4) \times 3 - \frac{1}{2} \times 3 - \frac{1}{2} \times 2 \times 4 = 5
\]
Consider the case with \( M \) at \((2, 3), \)
the right-angled triangles formed with \( N \) at \((0, 3), (3, -2) \) or \((-2, 3)\)
Area of the right-angled triangles are
\[
\frac{1}{2} \times 2 \times 3 = 3 \quad \text{or} \quad \frac{1}{2} (3 + 5) \times 3 - \frac{1}{2} \times 2 \times 3 - \frac{1}{2} \times 1 \times 5 = 6.5
\]
Consider the case with $M$ at $(3, 3)$, $N$ can be at $(3, -3)$ or $(0, 3)$.

Area of the right-angled triangles are
\[
\frac{1}{2} \times 3 \times 3 = 4.5 \quad \text{or} \quad \frac{1}{2} \times 6 \times 3 = 9
\]

The possible values of $x$ are 1.5, 3, 4.5, 5, 6.5 or 9.

15. In order to cross out the least possible number of integers, 2013 should be the sum of the as many integers as possible, i.e. integers remaining should be as small as possible.

\[\therefore\text{ consider } 1 + 2 + 3 + \ldots\]

Consider the sum 1 + 2 + 3 + \ldots + $N = \frac{(N+1)N}{2} \approx 2013$

[ considering $\sqrt{2 \times 2013} = 63.45$ for reference]

When $N = 63$, 1 + 2 + 3 + \ldots + 63 = 64 \times 63 \div 2 = 2016
\[\therefore \text{ The number of remaining integers must be smaller than 63.}\]

When $N = 60$, 1 + 2 + 3 + \ldots + 60 = 61 \times 60 \div 2 = 1830
2013 – 1830 = 183 which can be 83 + 100 or 91 + 92 etc.
\[\text{e.g. } 1 + 2 + 3 + \ldots + 60 + (83 + 100) = 2013\]
\[\therefore 2013 \text{ can at most be the sum of 62 integers in the list.}\]

\[\therefore\text{ At least 38 integers are to be crossed out.}\]

b. Consider the sum of $a, a+1, a+2, \ldots$ to $N$ terms.

\[\text{Sum} = \frac{(a + a + N - 1)N}{2} = 2013\]

where $N < 62, a < 100, a + N - 1 \leq 100, 2a + N - 1 \leq 200$

4026 = 2 \times 3 \times 11 \times 61
\[
(= 2013 \times 2 = 1324 \times 3 = 671 \times 6 = 366 \times 11 = 183 \times 22 = 122 \times 33 = 66 \times 61 \ldots)
\]

To compare the above products to $(2a + N - 1) \times N$,
only 183 \times 22, 122 \times 33, 66 \times 61 are to be considered.

For $N = 22, 2a + N - 1 = 183$, then $a = 81$ and $a + N - 1 = 102$. \[\therefore\text{ Not a solution.}\]
For $N = 33, 2a + N - 1 = 122$, then $a = 45$ and $a + N - 1 = 77$.
For $N = 61, 2a + N - 1 = 66$, then $a = 3$ and $a + N - 1 = 63$.

Leaving the numbers from 45 to 77 or from 3 to 63 can complete the task.