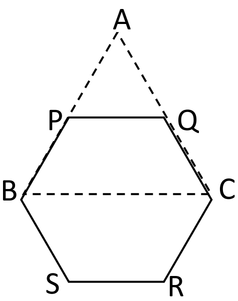
Secondary (Heat)   Solutions

<table>
<tr><td>1.</td><td>Consider 15600 = 2×2×2×2×3×5×5×13,
it can be written as
15600 = (2×2×2×3)×(5×5)×(2×13) = 24×25×26

Total surface area = 2 × (24×25 + 25×26 + 24×26)
= 3748

Total surface area is 3748 cm^2</td><td>OR
consider $\sqrt[3]{15600} \approx 24.99$,
the three sides are of values near to 24 and 25.</td></tr>
<tr><td>2.</td><td colspan="2">Let the distance travelled in the remaining half journey be x km.
$\frac{x}{250} - \frac{x}{350} = 1 + 0.5$
$\frac{x}{5} - \frac{x}{7} = 75$
$2x = 35(75) = 2625$

The distance between A and B is 2625 km.</td></tr>
<tr><td>3.</td><td colspan="2">In figure 1, the equilateral triangle ABC and the regular hexagon PQCRSB have the same perimeter. Dissecting both figures into smaller equilateral triangles as shown in figure 2, it can be seen that area ΔABC : area PQCRSB = 4:6.

figure 1


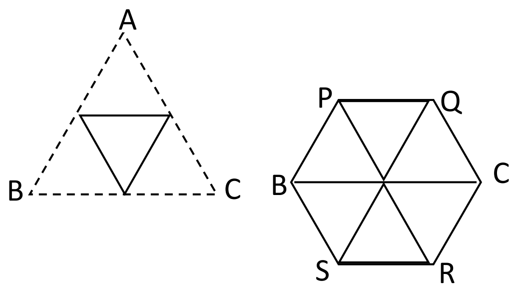


figure 2

Area of the hexagon = area of triangle × $\frac{6}{4}$
$= 3 \times \frac{6}{4} = 4.5$

Therefore, area of the hexagon DEFGHI is 4.5 .</td></tr>
</table>

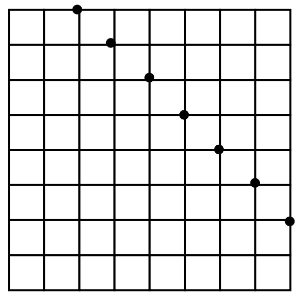
4.	Consider the expression as $[\,(A) + (B) - (C)\,]^2 \times (D) \div (E)$. In order that the value is the greatest, the value should be positive. Since $[\,(A) + (B) - (C)\,]^2$ must be positive, (D) and (E) should be of the same sign. Case 1: D = +2 and E = +1. For the value to be the greatest, make [(A) + (B) − (C)] as negative as possible, i.e. $[(-3) + (-2) - (-1)]$ $R = [(-3) + (-2) - (-1)]^2 \times 2 \div 1 = 32$ Case 2: D = −3 and E = −1. Take [(A) + (B) − (C)] as [(+2) + (+1) − (−2)]. $R = [\,(+2) + (+1) - (-2)\,]^2 \times (-3) \div (-1) = 75$ Case 3: D = −2 and E = −1. Take [(A) + (B) − (C)] as [(+2) + (+1) − (−3)]. $R = [\,(+2) + (+1) - (-3)\,]^2 \times (-2) \div (-1) = 72$ Case 4: D = −3 and E = −2. Take [(A) + (B) − (C)] as [(+2) + (+1) − (−1)]. $R = [\,(+2) + (+1) - (-1)\,]^2 \times (-3) \div (-2) = 24$ Case 2 gives the greatest value of *R*. Greatest value of *R* is $\underline{\underline{75}}$.
5.	$\frac{\Phi}{2013}$ is a value between 0.595 and 0.605. Consider $2013 \times 0.595 = 1197.735$ and that $2013 \times 0.605 = 1217.865$ $M = 1217$, $N = 1198$ $M + N = \underline{\underline{2415}}$
6.	In packet A, the amount of sesame = 1000 g × 3% = 30 g. There must also be 30 g of green bean in packet *B*. $\therefore$ Percentage of green bean in packet B = $\frac{30}{600} \times 100\% = \underline{\underline{5\%}}$

7.	a. For (1☆5) + (2☆10) + (3☆15) + (4☆20) + … + (99☆495): Among the 99 terms to be added, the values of (1☆5), (3☆15), …, (99☆495) are 5, i.e. 50 terms are 5, the others are 0. ∴ (1☆5) + (2☆10) + (3☆15) + (4☆20) + … + (99☆495) = 50 × 5 = 250 b. For (1☆3) + (3☆5) + (5☆7) + (7☆9) + … + (2011☆2013): the terms (1☆3) + (3☆5) + (5☆7) + (7☆9) + (9☆11) = 3 + 5 + 5 + 3 + 9 = 25 same for (11☆13) + (13☆15) + (15☆17) + (17☆19) + (19☆21) and (21☆23) + (23☆25) + (25☆27) + (27☆29) + (29☆31) ….. and (2001☆2003)+(2003☆2005)+(2005☆2007)+(2007☆2009)+(2009☆2011) ∴ The sum = (3 + 5 + 5 + 3 + 9) × 201 + 3 = 5028	
8.	a. By observation, for 1 more ‘ring’ of triangles added, there are 3 more layer. When *N* rings are added, the number of layers is (1 + 3*N*). = 100. In the end, 1 + 3*N* = 100. ∴ *N* = 33. There are 34 different colors. OR Let L_N be the number of layers in the n-th pattern. $L_1 = 1$, $L_2 = 4$, $L_3 = 7$, $L_4 = 7+3 = 10$, ⋯ The general term is $L_N = 1 + 3(N-1) = 3N-2$ For $L_N = 3N - 2 = 100$, N = 34., i.e. The final pattern is the 34th pattern.	
	b. When the outermost ring of triangles are added, there are 100 layers. Before adding this ring, there are 97 layers. Number of triangles added to the three sides = (2×97 + 3)×3 = 591	Or consider, [1+3+5+ … + (1+99×2)] – [1+3+5+…+ (1+96×2)] = 199 + 197 + 195 = 591

9.	a. At each vertex, there are 6 angles. On all the six vertices, there are 36 angles. b. The eight faces are equilateral triangles. Their interior angles are 60°. 24 of the angles are 60°. Considering the symmetry in the regular octahedron: *ABCD*, *PDQB*, *APCQ* are squares. Their interior angles are 90°. 12 of the angles are 90°. The biggest angles are of size 90°. There are 12 of them. The smallest angles are of size 60°. There are 24 of them.
10.	Since 17+18=35， the squares formed must be 1, 4, 9,16 or 25. For 18, 17 and 16, the only possible pairing arrangements are 18+7= 25, 17+8= 25 and 16 + 9 = 25. Consider the remaining numbers: 1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15 15 might be with 10 or 1, 14 might be with 11 or 2, 13 might be with 12 or 3, 12 might be with 13 or 4, 11 might be with 14 or 5, 10 might be with 15 or 6, 6 might be with 10 or 3, 5 might be with 11 or 4, 4 might be with 12 or 5, 3 might be with 13, 6 or 1, 2 might be with 14 or 7, 1 might be with 15, 8 or 3. But considering the case of '2', since '7' has to pair with '18', '2' has to pair with '14'. Since '14' has to pair with '2', '11' has to pair with '5'. Since '5' has to pair with '11', '4' has to pair with '12'….. With similar reasoning, we have the pairing arrangement as follows: (18, 7)，(17, 8)，(16, 9)，(15, 1)，(14, 2)，(13, 3)，(12, 4)，(11, 5)，(10, 6). 2 pairs with 14, 4 pairs with 12, 6 pairs with 10, 8 pairs with 17 and 10 pairs with 6.

11.

a. The figure shows the furthest point that the beetle can reach by moving at most 10 units. Points further than those marked cannot be reached.



Number of unreachable points

$= 1 + 2 + 3 + 4 + 5 + 6 = 21$

$\therefore$ Number of reachable lattice points

$= 9 \times 9 - 21 = \underline{\underline{60}}$

b. If a lattice point can be reached by the beetle moving for 2, 4, 6, 8 or 10 units, it can be reached in exactly 10 units. For example, in figure a, the beetle can move from O to point A in a minimum of 4 moves (3 ups and 1 right).

A path can also be formed by adding 3 left and 3 right which totally consists of exactly 10 moves.

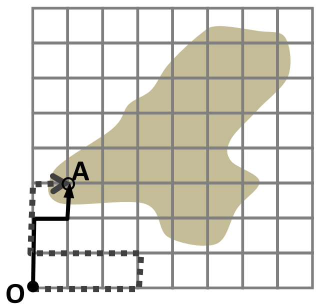


figure (a)

There are totally $\underline{\underline{9}}$ lattice points in the shaded region as shown in figure b.

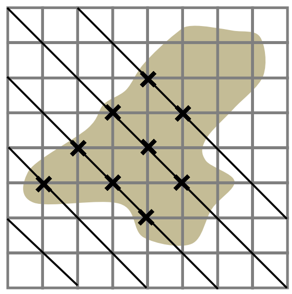
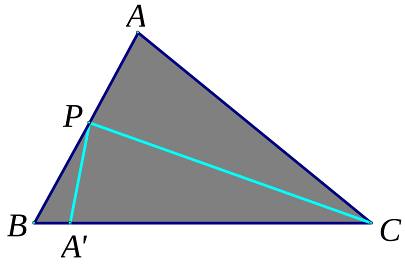


figure (b)

12

Consider the figure:

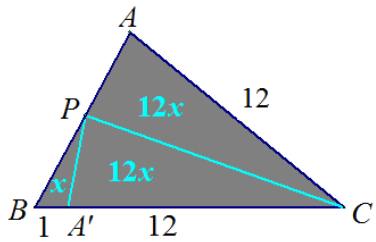


$\Delta PAC \cong \Delta PA'C$

$\therefore\ A'C = AC = 12, \qquad BA' = 13 - 12 = 1$

Comparing $\Delta BPA'$ and $\Delta CPA'$ as two triangles sharing the same height (from P), with respectively BA' and $A'C$ as bases,

area of $\Delta PBA'$: area of $\Delta PCA'$ = BA' : CA' = 1 : 12



Let area of $\Delta PBA'$ be x.

area of $\Delta PCA'$ = area of $\Delta PBA'$ = $12x$

$\therefore$ area of $\Delta ABC = x + 12x + 12x = 25x$.

Percentage of visible grey region = $\dfrac{x}{25x} \times 100\% = 4\%$.

b. Comparing ΔAPC and ΔBPC as two triangles sharing the same height (from C) with respectively AP and BP as bases,

area of ΔAPC : area of ΔBPC = 12 : 13

$\therefore\ AP : BP = 12 : 13$

$BP = 10 \times \dfrac{13}{12+13} = 5.2$ cm

13.

a. The number of matches needed

$= 20 \times (13+1) + 13 \times (20+1)$

$= \underline{\underline{553}}$

b. Suppose an $m \times n$ rectangle is formed with the 87 matches.

$m \times (n + 1) + n \times (m + 1) = 82$, where m and n are both integers

$2mn + m + n = 82$

$4mn + 2m + 2n = 164$

$4mn + 2m + 2n + 1 = 165$

$(2m + 1)(2n + 1) = 165 = 3 \times 5 \times 11$

For $2m + 1 = 3$, $2n + 1 = 55$: $m = 1, n = 27$, there are 27 unit squares

For $2m + 1 = 5$, $2n + 1 = 33$: $m = 2, n = 16$, there are 32 unit squares

For $2m + 1 = 11$, $2n + 1 = 15$: $m = 5, n = 7$, there are 35 unit squares

The rectangle consists of at most $\underline{\underline{35}}$ unit squares.

OR

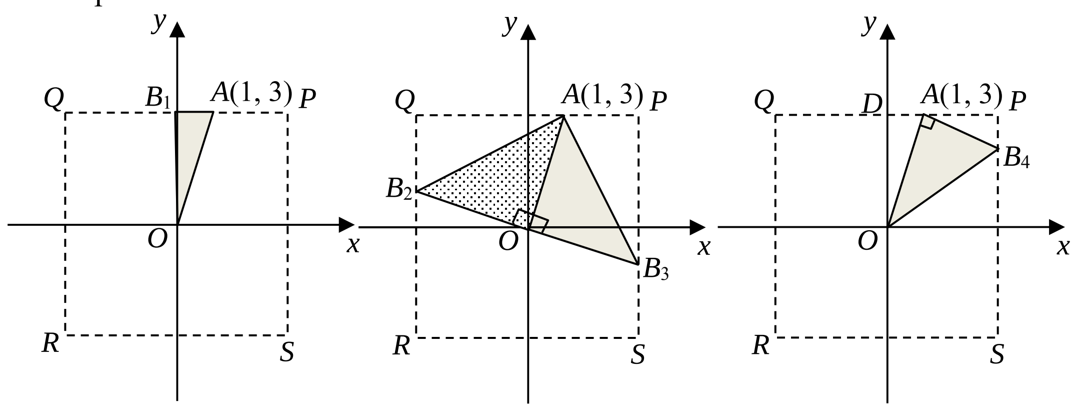
Consider $2mn + m + n = 82$ i.e. $m = \dfrac{82-n}{2n+1}$

With $n = 1$, $m = 27$.
With $n = 2$, $m = 16$.
With $n = 5$, $m = 7$.
As n gets larger, m gets smaller.
The other possibilities of m to be considered are $m =$ 6, 5, 4, 3, 2, 1

Since $n = \dfrac{82-m}{2m+1}$, n will be an integer when m = 5, 2, 1.

Possible pairs of m, n are (1, 27), (2, 16), (5, 7), (7, 5), (16, 2) and (27, 1).
Greatest value of $m \times n = 7 \times 5 = \underline{\underline{35}}$.

14. a. All possible locations of B are as shown:



The three figures shows the triangles ΔOAB formed with 90° at respectively vertices B, vertex O and vertex A.
B_1 is (0, 3).
Consider the rotation of $A(1, 3)$ by 90°, B_2 is (−3, 1) and B_3 is (3, −1).
Consider similar triangles: ΔODA and ΔAPB

$$\frac{PB}{PA} = \frac{1}{3}$$

$$PB = \frac{2}{3}$$

Coordinates of $B_4 = (3,\ 3-\frac{2}{3}) = (3,\ 7/3)$

b. Take the case above with M at (1, 3), B_1, B_2 and B_3 can be taken as N.
area of the right-angled triangles are

$\frac{1}{2}\times 1\times 3 = 1.5$ or $\frac{1}{2}(3+4)\times 3 - \frac{1}{2}\times 1\times 3 - \frac{1}{2}\times 2\times 4 = 5$

Consider the case with M at (2, 3),
the right-angled triangles formed with N at (0, 3), (3, −2) or (−2, 3)
Area of the right-angled triangles are

$\frac{1}{2}\times 2\times 3 = 3$ or $\frac{1}{2}(3+5)\times 3 - \frac{1}{2}\times 2\times 3 - \frac{1}{2}\times 1\times 5 = 6.5$

	Consider the case with M at (3, 3), N can be at (3, −3) or (0, 3) Area of the right-angled triangles are $\frac{1}{2}\times 3\times 3 = 4.5$ or $\frac{1}{2}\times 6\times 3 = 9$ The possible values of x are <u>1.5, 3, 4.5, 5, 6.5 or 9.</u>
15.	In order to cross out the least possible number of integers, 2013 should be the sum of the as many integers as possible, i.e. integers remaining should be as small as possible. $\therefore$ consider $1 + 2 + 3 + \ldots$. Consider the sum $1 + 2 + 3 + \ldots + N = \frac{(N+1)N}{2} \approx 2013$ [considering $\sqrt{2\times 2013} = 63.45$ for reference] When $N = 63$, $1 + 2 + 3 + \ldots + 63 = 64 \times 63 \div 2 = 2016$ $\therefore$ The number of remaining integers must be smaller than 63. When $N = 60$, $1 + 2 + 3 + \ldots + 60 = 61 \times 60 \div 2 = 1830$ $2013 - 1830 = 183$ which can be $83 + 100$ or $91 + 92$ etc. e.g. $1 + 2 + 3 + \ldots + 60 + (83 + 100) = 2013$ $\therefore$ 2013 can at most be the sum of 62 integers in the list. $\therefore$ At least <u>38</u> integers are to be crossed out. b. Consider the sum of a, $a+1$, $a+2$, … to N terms. Sum $= \frac{(a+a+N-1)N}{2} = 2013$ where $N < 62$, $a < 100$, $a + N - 1 \le 100$, $2a + N - 1 \le 200$ $4026 = 2\times 3\times 11\times 61$ $(= 2013 \times 2 = 1324 \times 3 = 671 \times 6 = 366 \times 11 = 183 \times 22 = 122 \times 33 = 66 \times 61 ..)$ To compare the above products to $(2a + N - 1) \times N$, only 183×22, 122×33, 66×61 are to be considered. For $N = 22$, $2a+N-1 = 183$, then $a = 81$ and $a + N - 1 = 102$. $\therefore$ Not a solution. For $N = 33$, $2a+N-1 = 122$, then $a = 45$ and $a + N - 1 = 77$. For $N = 61$, $2a + N-1 = 66$, then $a = 3$ and $a + N - 1 = 63$. Leaving the numbers from <u>45 to 77</u> or from <u>3 to 63</u> can complete the task.