Sample (2)

Viviani's Triangle

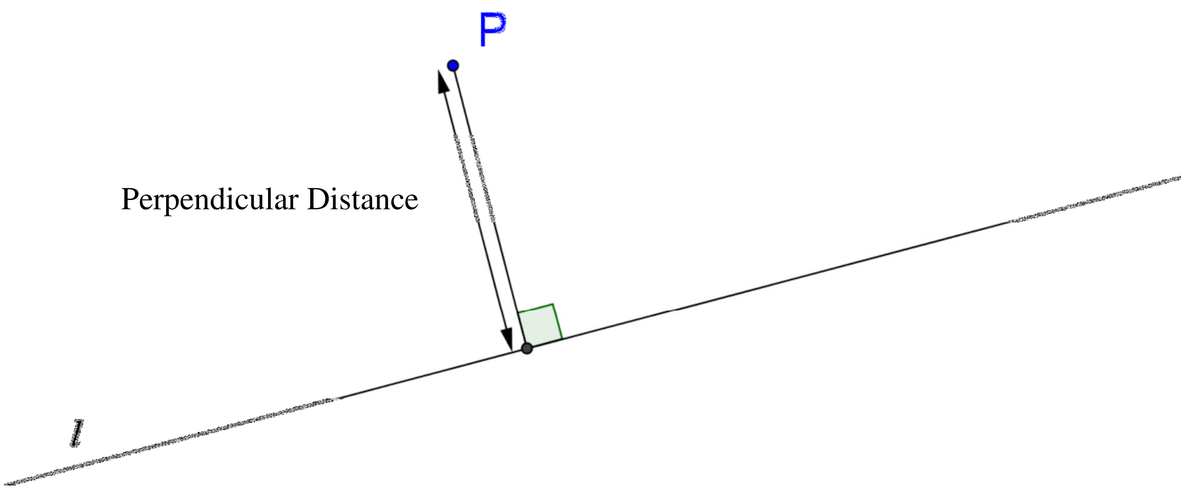
Vincenzo Viviani (1622 – 1703) was a famous Italian mathematician. With his exceptional intelligence in mathematics, Viviani became a pupil and associate of Galileo (1564-1642). Viviani's Theorem is one of his widely known achievements. This theorem is about equilateral triangles. We are going to explore this theorem by solving the following problem.

> An island in the shape of an equilateral triangle stands in the Pacific Ocean. Workers are repairing the seawalls on the three sides of the island in order to protect the coast from the effects of strong waves. They are also building a warehouse to store all the materials needed. For efficient management, the location of this warehouse is in a position such that the sum of its three perpendicular distances from the three sides of the island must be the smallest.
>
> Where should the warehouse be built on the island?

Glossary

Perpendicular Distance:

The perpendicular distance between a point P and a line l refers to the distance of the perpendicular line drawn between the point P and the line l, as shown in the following diagram.



Sample（2）

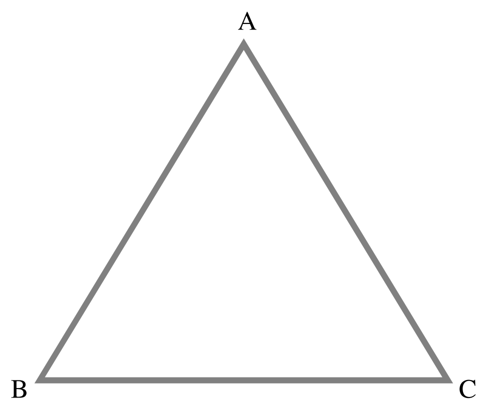
1. **To Calulate the Sum of Perpendicular Distances**

Calculate the sum of perpendicular distances by the following steps.

Steps:

（1）	Pick a point inside the equilateral triangle in Figure(a). This point is called P.
（2）	Draw a perpendicular line from the point P to the side AB, then draw a perpendicular line from the point P to the side BC, and finally draw a perpendicular line from the point P to the side AC.
（3）	Measure the sum of the three perpendicular distances between the point P and the three sides of the triangle.
（4）	Pick a point P inside the equilateral triangle in Figure(b). The location of this point P must be different from that of the point P in Figure (a). Then repeat steps (2) and (3).

Figure (a)

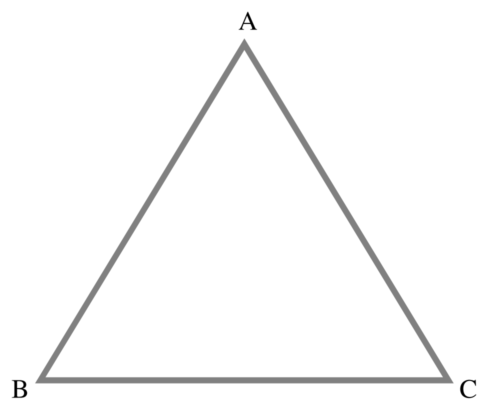


The sum of the three perpendicular distances between the point P and the three sides

= ______________________________

= ______________________________

Figure (b)



The sum of the three perpendicular distances between the point P and the three sides

= ______________________________

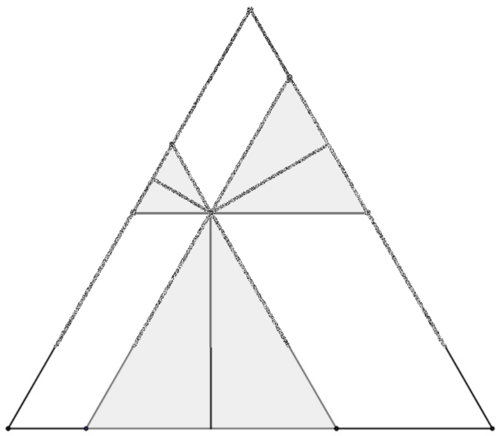
= ______________________________

Sample (2)

2. <u>To Explore Viviani's Theorem</u>

Viviani proposed Viviani's Theorem. It states that:

> For any given point inside an equilateral triangle, the sum of the perpendicular distances between the point and the three sides is always a fixed value.



A set of jigsaw puzzle, like the one shown on the left, is given. Try to join the pieces of the puzzle in a **<u>different</u>** way that can help you explain verbally to the judges why Viviani's Theorem is true.

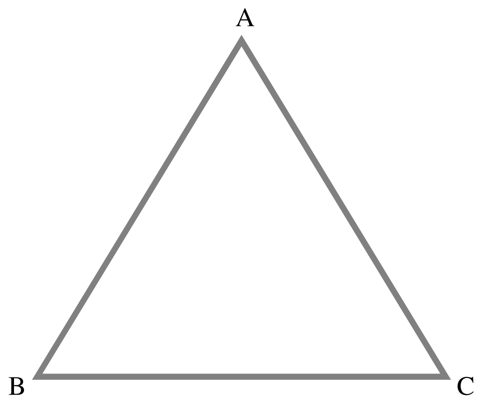
Sample (2)

3. **To Prove Viviani's Theorem**

Some people have suggested using the following strategy to prove Viviani's Theorem.

Consider any point P inside an equilateral triangle and the areas of the three triangles P forms with the three vertices of the equilateral triangle.

Prove that Viviani's therorem is true using the above strategy. Write the details of your proof on the following lines.

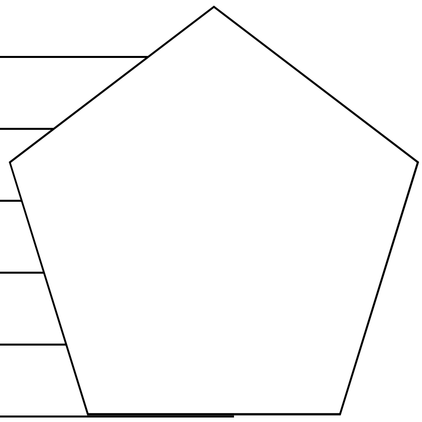


Sample (2)

4. **To Extend Viviani's Theorem**

Some people have suggested that the theorem should be extended to equilateral polygons such as squares, regular pentagons, regular hexagons, etc.

For any point P inside any regular polygon, what is the relationship between the number of sides of the polygon and the sum of the perpendicular distances of P from the sides? Try to prove this relationship in the case of a regular pentagon, and write the details of your proof on the following lines. [Hint: Consider the "central point" of the polygon and the sum of its perpendicular distances from the sides.]



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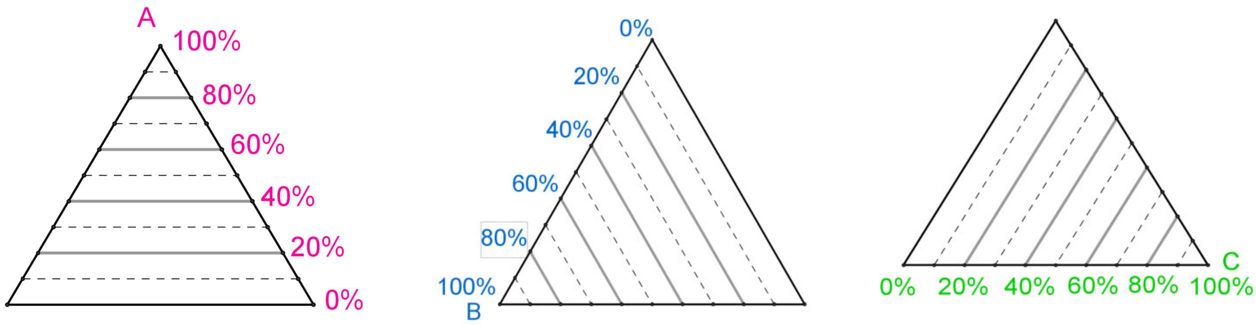
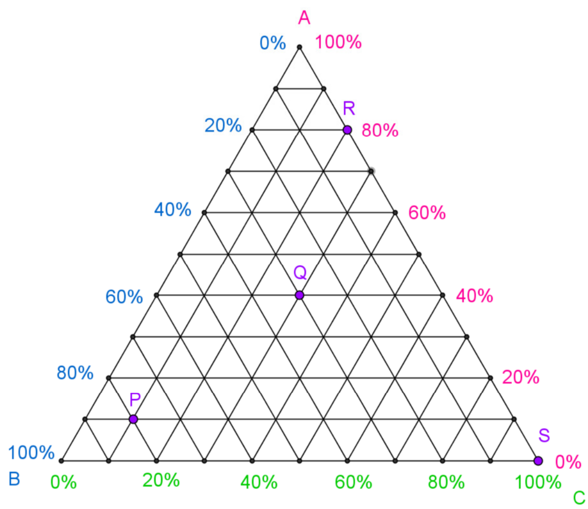
5. **Viviani's Theorem and Ternary Plot**

As Viviani's Theorem is true, we can conclude that for any point P inside an equilateral triangle, the sum of the perpendicular distances of P from the sides of the triangle is always a fixed value. Some people have suggested using this property to represent problems about distribution of a fixed sum.



By superimposing the above three equilateral triangles, we get the following Ternary Plot.

(5a) In the ternary plot below, four mixtures P, Q, R and S are formed by mixing with different percentages of the three elements A, B and C. Write down the corresponding percentages of A, B and C in Table (1).



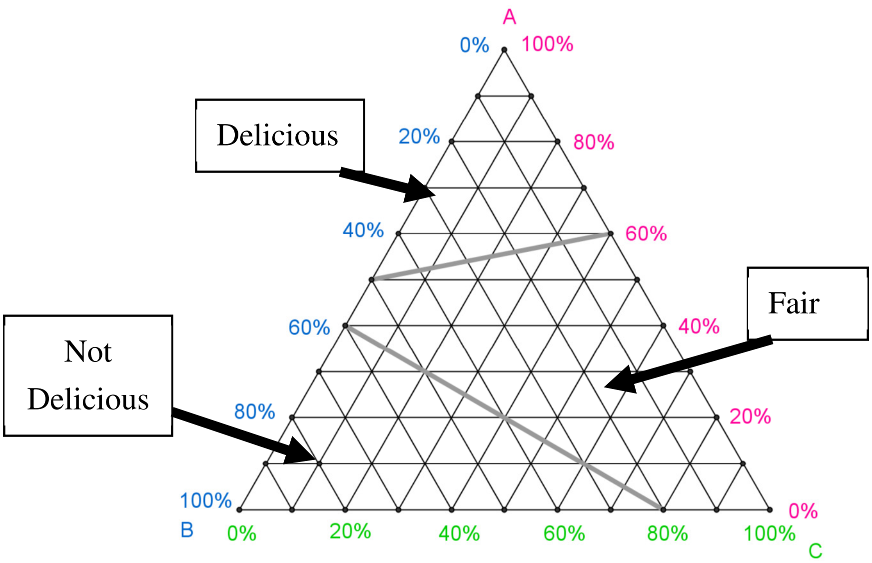
Elements / Mixtures	A	B	C
P			
Q			
R			
S			

Table (1)

/ 4

Sample (2)

(5b) A cookie company produces cookies of different flavors based on different combinations (in percentage) of ingredients A, B & C. According to a customer survey, the company found that customers commented these cookie as 'delicious', 'fair' and 'not delicious' for the different combinations of the ingredients. The company has drawn the following ternary plot for reference. If the company produces cookies with reference to the combinations listed in Table (2), what comments will customers give on the cookies respectively? Write the answers in Table (2).



Customers' comments on cookies

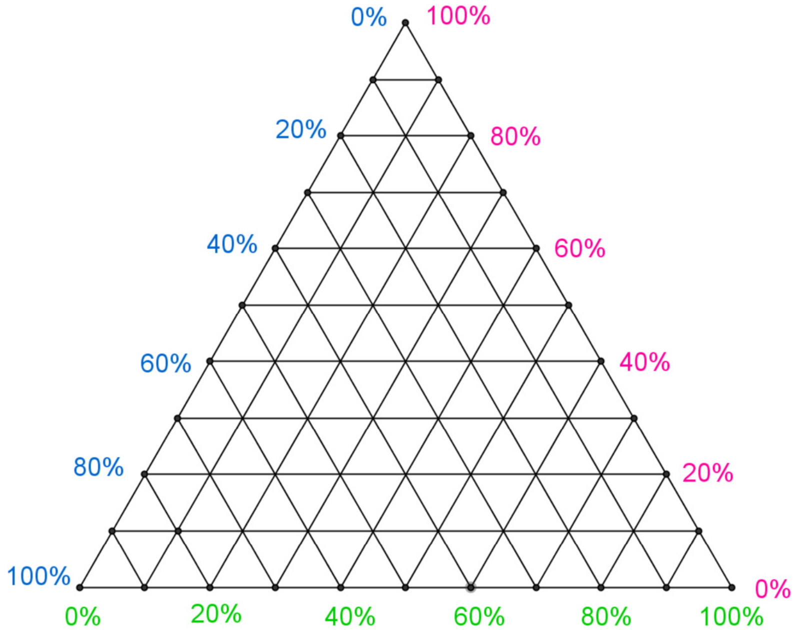
Ingredients / Comments	A	B	C
	20%	30%	50%
	40%	20%	40%
	60%	10%	30%
	80%	0%	20%

Table (2)

4

Sample (2)

(5c) Suppose ***'Leaning Mathematics Successfully'*** is composed of three elements. What will the three elements be ? What are your secrets of success? Use the following ternary plot to express your answer.



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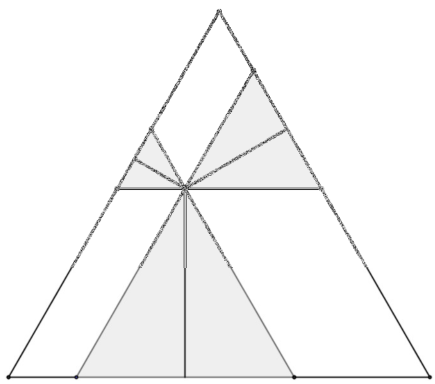
6. Bonus Question: Other Proofs of Viviani's Theorem

Do you have other ways to prove Viviani's Theorem? Please write the details of your proofs in the following space.



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- End of Paper -