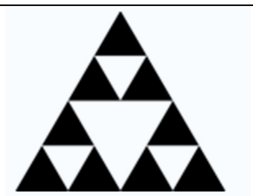
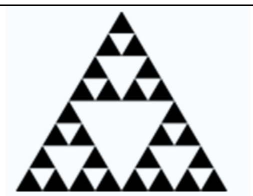
Sample (3)

The Magical Sierpinski Triangle and Carpet

Glossary:

*** Before attempting the questions, you are advised to read Definitions 1-3 below for better understanding of the questions.**

1. The definition of Sierpinski Triangle:

 T_0	T_0 is a solid equilateral triangle.
 T_1	On the solid equilateral triangle T_0, join the mid points on each of its sides so that the solid equilateral triangle is divided into 4 smaller triangles. Remove the small triangle at the centre, which is the white part in the figure on the left. The new shape is called <u>**Triangle T_1**</u>。
 T_2	On each solid equilateral triangle in T_1, join the mid points on each of its sides so that the solid equilateral triangle is divided into 4 smaller triangles. Remove the small triangle at the centre, which is the white part in the figure on the left. The new shape is called <u>**Triangle T_2**</u>。
 T_3	On each solid equilateral triangle in T_2, join the mid points on each of its sides so that the solid equilateral triangle is divided into 4 smaller triangles. Remove the small triangle at the centre, which is the white part in the figure on the left. The new shape is called <u>**Triangle T_3**</u>。

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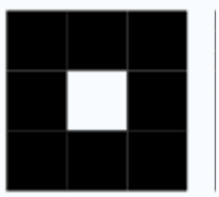
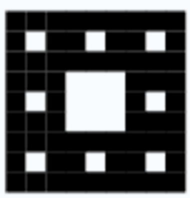
 T_4	On each solid equilateral triangle in T_3, join the mid points on each of its sides so that the solid equilateral triangle is divided into 4 smaller triangles. Remove the small triangle at the centre, which is the white part in the figure on the left. The new shape is called **<u>Triangle T_4</u>**。

*In summary, by applying the "dividing and removing" process once, we get T_1. By applying the process twice, we get T_2. By applying the process three times, we get T_3. By applying the process n times, we get T_n. By applying the process infinitely, we get a Sierpinski Triangle, which is T_∞. ("∞" means infinity)

Glossary (cont.)

2. The definition of Sierpinski Carpet:

 C_0	C_0 is a solid square.
 C_1	Divide the solid square C_0 into 3×3 = 9 smaller equal squares. Remove the small square at the centre, which is the white part in the figure on the left. The new shape is called **<u>Sqaure C_1</u>**。
 C_2	Divide each solid square in C_1 into 3×3 = 9 smaller equal squares. Remove the small square at the centre, which is the white part in the figure on the left. The new shape is called **<u>Sqaure C_2</u>**。

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 C_3	Divide each solid square in C_2 into 3×3 = 9 smaller equal squares. Remove the small square at the centre, which is the white part in the figure on the left. The new shape is called **<u>Sqaure C_3</u>** 。
 C_4	Divide each solid square in C_3 into 3×3 = 9 smaller equal squares. Remove the small square at the centre, which is the white part in the figure on the left. The new shape is called **<u>Sqaure C_4</u>** 。

*In summary, by applying the "dividing and removing" process once, we get C_1. By applying the process twice, we get C_2. By applying the process three times, we get C_3. By applying the process n times, we get C_n. By applying the process infinitely, we get a Sierpinski Carpet, which is C_∞. ("∞" means infinity)

Glossary (cont.)

3. The definition of dimension (D):

In Mathematical Physics, the mass of a one dimensional object has a linear relationship with its length. This means that if the length becomes 2 times of the original length, the mass will become $2^D=2^1=2$ times of the original mass. That is D=1.

An iron wire is a one dimensional object. It is because when the length of the iron wire becomes two times of the original length, the mass of the wire will become 2 times of the original mass. That is $2^1=2$.

e.g.

length = 1 cm，mass = 1 g　　　　length = 2 cm，mass = 2 g

The mass of a two dimensional object has a square relationship with its length. This means that if the length becomes 2 times of the original length, the mass will become $2^D=2^2=4$ times of the original mass. That is D=2.

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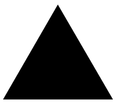
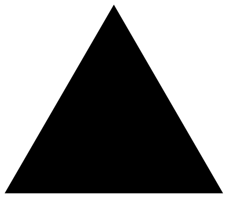
A wooden equilaterial triangular board is a two dimensional object. It is because when the length of the wooden board becomes two times of the original length, the mass of the wooden board will become 4 times of the original mass. That is 2^2=4.

e.g.



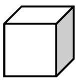


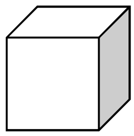
length of the side = 1 cm，mass = 1 g　　length of the side = 2 cm，mass = 4 g

The mass of a three dimensional object has a cubic relationship with its length. This means that if the length becomes 2 times of the original length, the mass will become 2^D=2^3=8 times of the original mass. That is D=3.

A plastic cube is a three dimensional object. It is because when the length of the cube becomes two times of the original length, the mass of the cube will become 8 times of the original mass. That is 2^3=8.

e.g.





length of the side = 1 cm，mass = 1 g　　length of the side = 2 cm，mass = 8 g

*Therefore the dimensions of usual objects are 1, 2 or 3.　However, there are some objects whose dimensions are not equal to these three numbers.

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The Magical Sierpinski Triangle and Carpet

Question:

Experimental Apparatus:

A plastic plate with a hollow equilateral triangle (side length of triangle = 5cm), some rice, a ruler, some paper, a calculator

Answer the following questions:

(a) By using the apparatus provided, estimate how many pieces of rice can fill up, as much as possible, an equilateral triangle with side length of 5 cm.

Answer:

(b) Estimate how many pieces of rice can fill up, as much as possible, an equilateral triangle with side length of 10 cm.

Answer:

(c) Use an equilateral triangle with side length of 640 cm to make a Triangle T_k (Refer to Definition 1, Glossary). If the side length of the smallest black triangle in T_k cannot be shorter than 4 cm, calculate the value of K and estimate how many pieces of rice can fill up, as much as possible, the black part of Triangle T_k with side length of 640 cm.

Answer:

(d) Use an equilateral triangle with side length of 640 cm to make a Triangle T_m (Refer to Definition 1, Glossary). If the smallest back triangle in T_m can still be filled up, as much as possible, by rice (but T_{m+1} cannot be filled up by rice), calculate the value of m and estimate how many pieces of rice can fill up, as much as possible, the black part of Triangle T_m with side length of 640 cm.

Answer:

(e) Use an equilateral triangle with side length of 512 cm to make a Triangle T_n (Refer to Definition 1, Glossary). If the side length of the smallest black triangle in T_n cannot be shorter than 8 cm, estimate how many pieces of rice can fill up, as much as possible, the black part of Triangle T_n with side length of 512 cm.

Answer:

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(f) Estimate or calculate the dimension of a Sierpinski Triangle (i.e. T_∞; refer to Definition 1, Glossary), correct to 2 decimal places.

Answer:

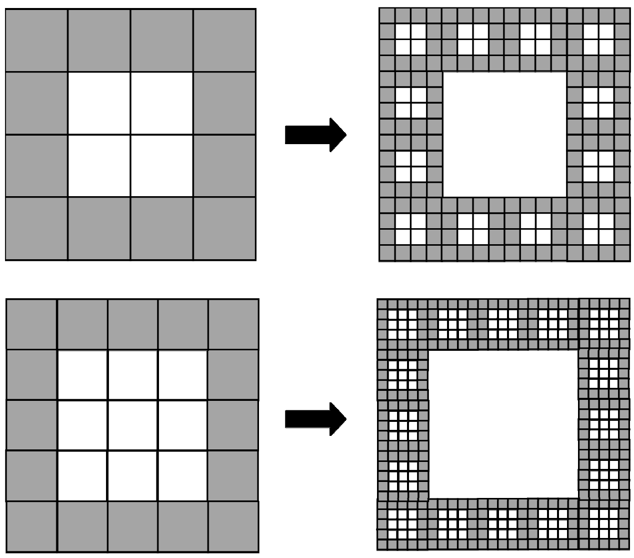
(g) Estimate the total area of the black part of a Sierpinski Triangle (i.e. T_∞).

Answer:

(h) Which has a higher dimension, Sierpinski Triangle or Sierpinski Carpet (Refer to Definition 2, Glossary)? Why?

Answer:

(i) There are other forms of Sierpinski Carpet, e.g.,



Construct a Sierpinski Carpet which has the same dimension as a Sierpinski Triangle.

Answer:

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(j) Construct another Sierpinski Triangle which has a dimension different from that of the original Sierpinski Triangle (i.e. the Sierpinski Triangle mentioned in Definition 1, Glossary).

Answer:

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