學生課業：

第六課 「岩石調包」

案件發生地點：科學館

人物：維吉(科學館管理員)、警員和你

經過：

1. 有人舉報維吉將會在晚上值班時，偷走在展覽館的月球岩石。

2. 月球岩石的保安系統相當簡單，它只放在一個裝有重量傳感器上面，傳感器很靈敏，不可以有1g的誤差。

於是，警方展開調查，所錄口供對話如下：

警員(警)：維吉有人舉報你，說你打算晚上偷走價值差不多1 000萬元的月球岩石。據說在你的背包裡找到金屬砝碼，懷疑你打算用這些砝碼防止傳感器發出警報。(警員真的在維吉的背包找到多片砝碼。)

維吉(維)：這些砝碼不是我的！我也是剛見到這些砝碼。

你：真有趣。警察先生，你在維吉先生背包中找到的砝碼有幾重？

警：238g，我們剛用電子磅量度過。

你：月球岩石有多重？

警：95g

你：這些砝碼片是否一樣大？每片有多重？

警：砝碼片有2種，小的12片每片重9g，大的10片每片重13g，維吉有這麼多砝碼一定足以湊出95g。

你：(沉思一會，看著維吉)真奇妙，維吉先生。

維：(帶著微笑)簡直太玄妙了，這麼巧合。

這案件有趣的部分在於你和維吉最後的對話，有什麼意思呢？

I. 這個案件帶出一個有趣的問題，有2個砝碼﹝正整數9g (a)和13g (b)﹞，它們的線性組合，即c=ax+by　(x,y≥0)，有什麼特點？

這問題有點複雜，我們以簡單一點先了解，之後再破解案中問題。

從哪裡入手研究問題？如沒有明顯的線索，最好通過一些合適的例子做試驗。首先我們將2個砝碼設為a 和b (而a<b)。

假若 a=4 和 b=5，我們看看它們的線性組合能得出哪些數值(稱之為**好數**)，不能得出哪些數值(稱之為**壞數**)，並把它們填在下表。

a=4, b=5

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 數 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 好數 | 🗸 |  |  |  | 🗸 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 壞數 |  | 🗴 | 🗴 | 🗴 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

0=0×4+0×5

4=1×4+0×5

1. 我們可以猜 將是最後一個壞數，它之後的所有數都是好數。

2. 因此，我們稱 為第一個恆好數。

3. 為什麼我們知道 是最後一個壞數？

因a=4，我們只要把最先出現的a個（即4個）連續好數加上一個a，就可以生成一組新的a個（4個）連續好數，即 16=12+a；17=13+a；18=14+a；19=15+a。

由此得出以下2點假設：

1. 對任意a和b，總有一個最後的壞數；

2. 只要找到一串連續的a (a<b) 個好數，就能找到這個壞數。而這串數的第一個數叫第一個恆好數。

但當b是a的倍數，就不會得出恆好數，因此我們要把第一點修改：

　 對任意互質的整數a和b，總存在一個最後的壞數。(若兩個整數互質時，則它們之間的最大公因數為1。)

我們嘗試更多例子，看看能否找到一個規律。找出它們第一個恆好數。

a=2,b=5

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 數 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 好數 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 壞數 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

a=3,b=5

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 數 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 好數 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 壞數 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

a=5,b=6

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 數 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| 好數 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 壞數 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

a=5,b=7

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 數 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 |
| 好數 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 壞數 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

a=3,b=4

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 數 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 好數 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 壞數 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

a=4,b=7

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 數 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| 好數 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 壞數 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

a=4,b=9

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 數 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |
| 好數 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 壞數 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

把結果填在下表：

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | 2 | 3 | 4 | 5 | 5 | 3 | 4 | 4 |
| b | 5 | 5 | 5 | 6 | 7 | 4 | 7 | 9 |
| 第1個恆好數 |  |  |  |  |  |  |  |  |

2b. 從上表能找到規律嗎？

3. 現在回到案件中，對話中“似乎暗示着95有什麼特別之處，但到底特別在那兒呢？

延伸問題：

現在有3種不同重量的砝碼，你要作什麼假設？

挑戰題：現在有3種不同重量的砝碼，a=6, b=10, c=11 你能找到它們的第一個恆好數？

a=6, b=10, c=11

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 數 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |
| 好數 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 壞數 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 數 | 28 | 29 | 30 | 31 | 32 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 好數 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 壞數 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |