Social Distancing and the Spread of COVID-19

Junior Secondary Mathematics

Curriculum Development Institute, Education Bureau

Background

To minimise the risk of COVID-19 spreading in the community, we all need to work together on social distancing. While the Government will adopt the "Suppress and Lift" approach (張弛有度、能收能放), members of the public are advised strongly to observe the announced measures that could help enhance **social distancing**.

Why is social distancing so important?

In this activity, we are going to use **a simple mathematical model** to study the spread of virus with/without social distancing.

Government website on social distancing:
https://www.coronavirus.gov.hk/eng/social_distancing.html

Data and Assumptions

Before interpreting the effect of social distancing, we need to collect related data and make appropriate assumptions. Data below are needed for modelling the spread of COVID-19:

1. The average number of infections from an infected person
2. The average effective period of infection by an infected person

It is important to collect information and data only from **reliable sources**, such as websites of government departments concerned or well-established academic institutions.

Data and Assumptions

1. The average number of infections from an infected person

We may consider the "**real time effective** reproduction number **for local cases**" announced by the LKS Faculty of Medicine, HKU (i.e. the current average number of persons who will be infected by an infected person).

To simulate the scenario at the beginning of the local outbreak of the disease, we adopt the first peak value on 27 Jan 2020 in this activity, which is roughly 1.8.

Real-time Dashboard by School of Public Health, The University of Hong Kong:
https://covid19.sph.hku.hk

Data and Assumptions

2. The average effective period of infection by an infected person

Assumption 1: We may assume that the incubation period equals to the **period of infection**.

[Incubation period means the time between catching the virus and beginning to have symptoms of the disease]

Q1) Please give a reason to support the above assumption.

Q2) How long is the incubation period for COVID-19?

Data and Assumptions

2. The average effective period of infection by an infected person

Assumption 1: We may assume that the incubation period equals to the period of infection.

Q1) Please give a reason to support the above assumption.

During the incubation period, the infected person may not notice his/her infection and would go on contacting other persons. After the symptoms developed, the person would be consequently quarantined, and become no longer infective to the society.

Data and Assumptions

2. The average effective period of infection by an infected person

Assumption 1: We may assume that the incubation period equals to the **period of infection**.

Q2) How long is the incubation period for COVID-19?

From the website of Centre for Health Protection, most estimates of the incubation period for COVID-19 range from 1 to 14 days, most commonly around **5 days**.

FAQ 6, Frequently Asked Questions on COVID-19, Centre for Health Protection:
https://www.chp.gov.hk/en/features/102624.html#FAQ6

Data and Assumptions

Lastly, we make an assumption on effect of social contact to the reproduction number:

Assumption 2: It is assumed that the reproduction number will be *directly proportional* to the number of persons closely contacted during the period of infection.

For example, if the number of persons closely contacted by an infected person is reduced by half, the reproduction number will also be reduced by half.

Building the Model

To start with, assume that there is one infected person at day 0.

Let y be the total number of infected person.

In 5 days (i.e. after 1 period of infection),

$$y = \underbrace{1}_{\text{Already infected}} + \underbrace{1 \times 1.8}_{\text{Newly infected}} = 2.8$$

The number is derived from statistical data. It does not imply fractional infection.

Question: *How about 5 more days later?*

Building the Model

By assumptions, while the original infected person would be quarantined, each of the 1.8 newly infected persons will infect another 1.8 persons within a period of infection. Hence, in 10 days (2 periods of infection), the total number of infected persons y is

$$y = \underbrace{1+1.8}_{\text{Already infected}} + \underbrace{1.8\times 1.8}_{\text{Newly infected}} = 1+1.8+1.8^2 = 6.04$$

Building the Model

Q3) By observing the above calculations, try to calculate the total number of infected persons, y , after 15, 20, 25 and 30 days using a spreadsheet.

no. of periods	no. of days	Total no. of infected persons (without social distancing, cor. to 1 d.p.)
0	0	1
1	5	2.8
2	10	6.0
3	15	
4	20	
5	25	
6	30	

Remarks: Non-integral numbers of persons are derived from statistical data.

Building the Model

Q3) By observing the above calculations, try to calculate the total number of infected persons, y , after 15, 20, 25 and 30 days using a spreadsheet.

no. of periods	no. of days	Total no. of infected persons (without social distancing, cor. to 1 d.p.)
0	0	1
1	5	2.8
2	10	6.0
3	15	11.9
4	20	22.4
5	25	41.3
6	30	75.3

Remarks: Non-integral numbers of persons are derived from statistical data.

Building the Model (Extension)

Q3*) From the formula used in the spreadsheet for the calculation of total number of infected persons, can you write down a formula in mathematics?

Answer:

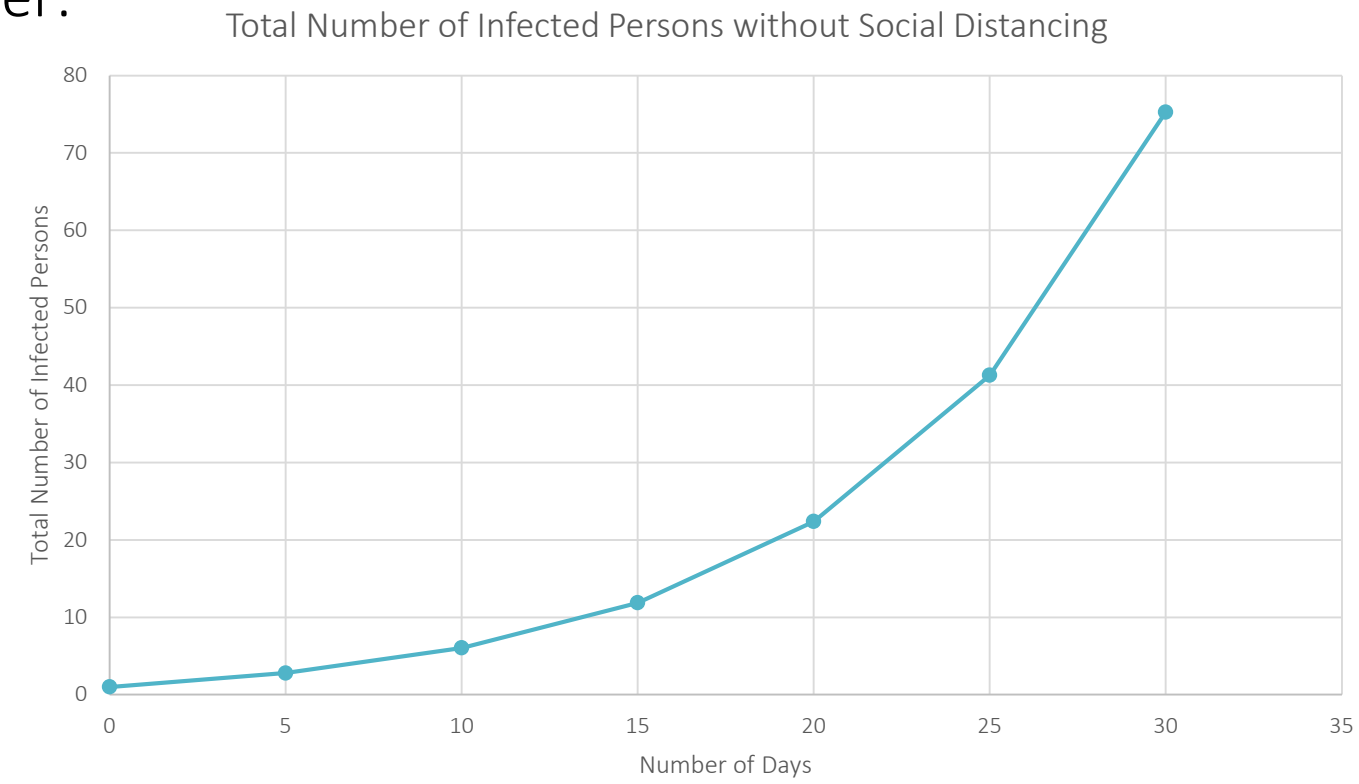
After n periods of infection, the total number of infected persons is

$$y = 1 + 1.8 + 1.8^2 + 1.8^3 + \cdots + 1.8^n$$

Building the Model

Q4) Using the spreadsheet in Q3, present the total number of infected persons by a suitable statistical graph.

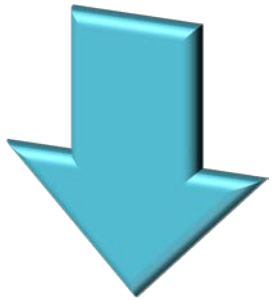
Answer:



Effect of social distancing

Suppose the measures that could enhance social distancing reduce our amount of social gatherings by half, let's see what will happen.

By Assumption 2, the reproduction number is also reduced by half, that is, 1.8 is now reduced to 0.9



Effect of social distancing

Q5) By modifying the formula used in the spreadsheet for Q3, find the total number of infected persons y in 5 , 10 , 15, 20 , 25 and 30 days time with measures that could enhance social distancing.

no. of periods	no. of days	Total no. of infected persons (cor. to 1 d.p.)	
		without social distancing	with social distancing
0	0	1	1
1	5	2.8	
2	10	6.0	
3	15	11.9	
4	20	22.4	
5	25	41.3	
6	30	75.3	

Remarks: Non-integral numbers of persons are derived from statistical data.

Effect of social distancing

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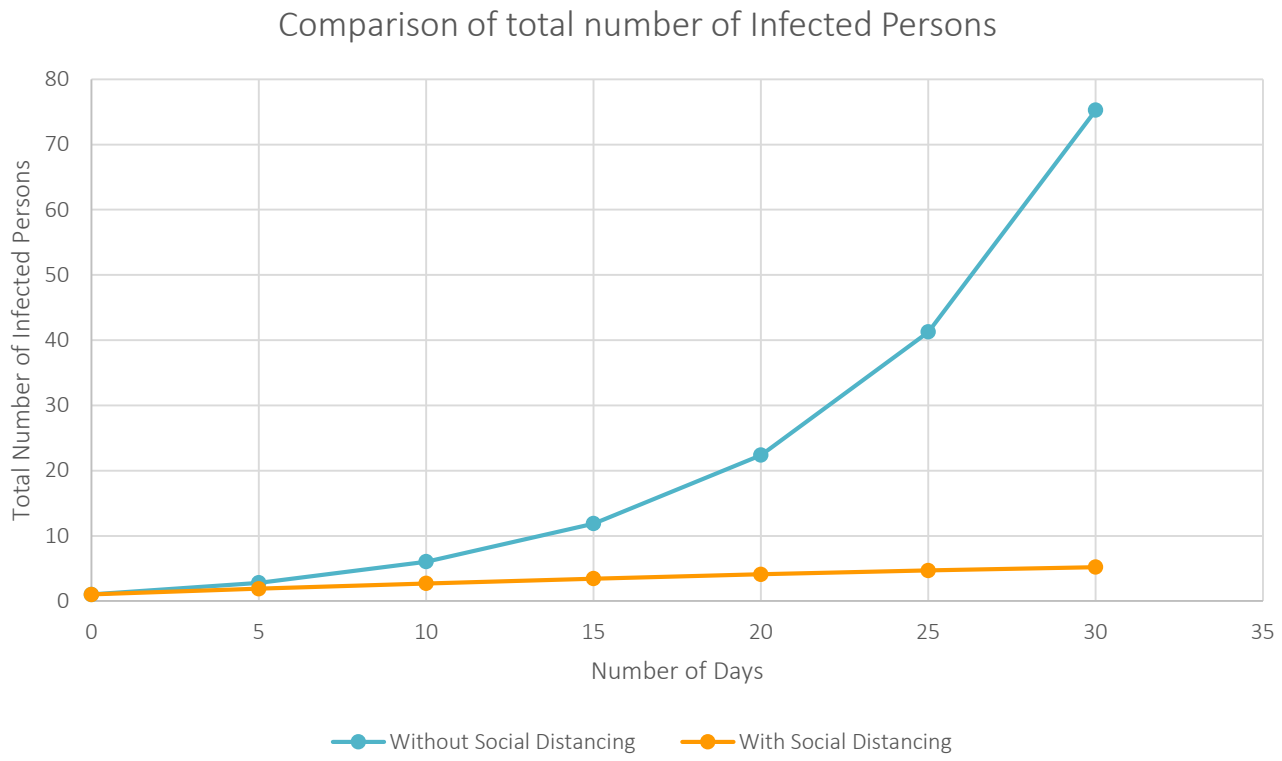
no. of periods	no. of days	Total no. of infected persons (cor. to 1 d.p.)	
		without social distancing	with social distancing
0	0	1	1
1	5	2.8	1.9
2	10	6.0	2.7
3	15	11.9	3.4
4	20	22.4	4.1
5	25	41.3	4.7
6	30	75.3	5.2

Remarks: Non-integral numbers of persons are derived from statistical data.

Effect of social distancing

Q6) Construct a graph presenting the numbers of infections for the two cases in Q5, and compare the two cases.

Answer:



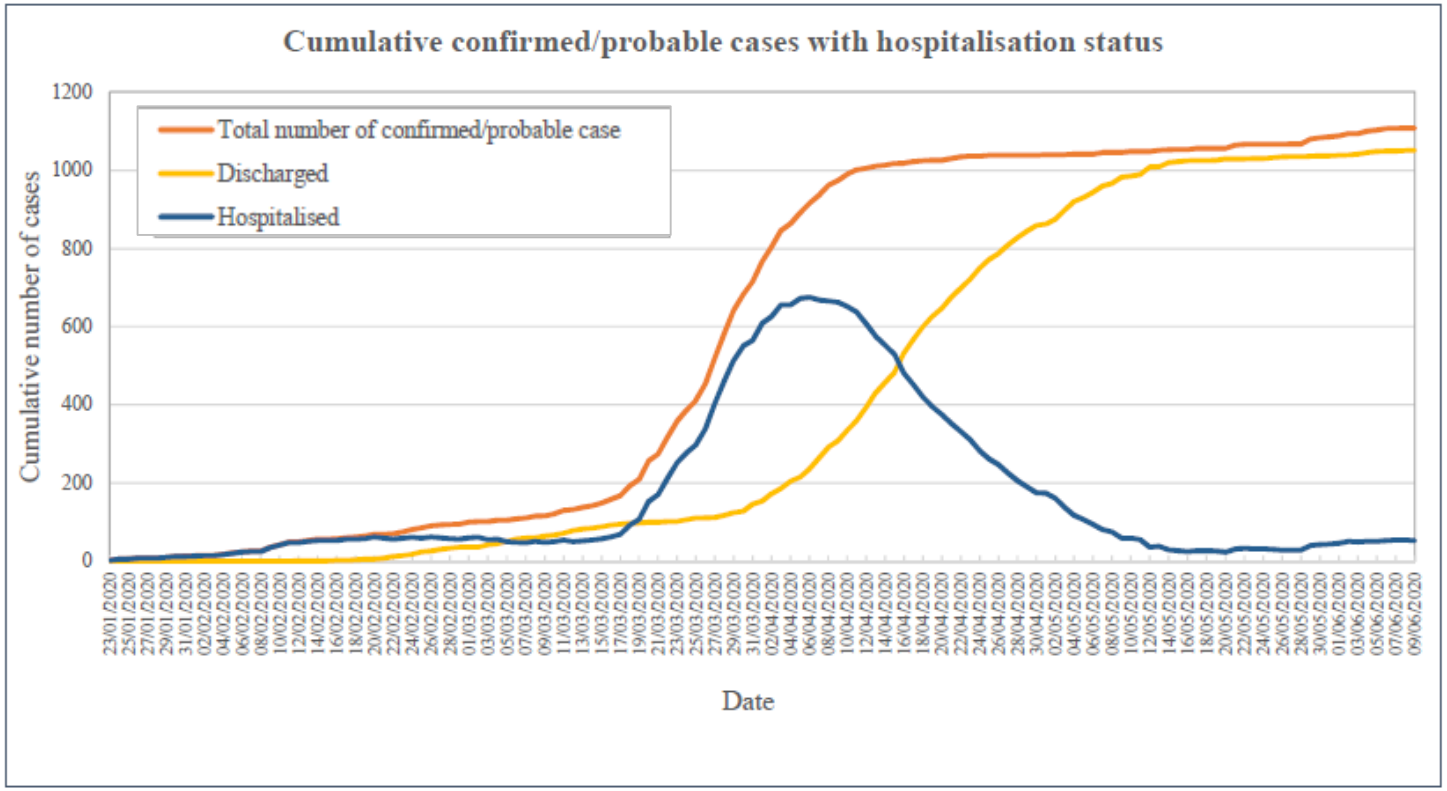
Effect of social distancing

Q7) Briefly describe the effect of social distancing (assuming reducing social contact by half) as observed from the graph for Q6.

As observed from the graph, without social distancing, the total number of infected persons increases at a greater rate. An infected person will result in a total of around 75 infected persons in 30 days.

With measures that could reduce social contact by half, the total number of infected persons will be around 5 in 30 days. There is a great difference in the rate of increase.

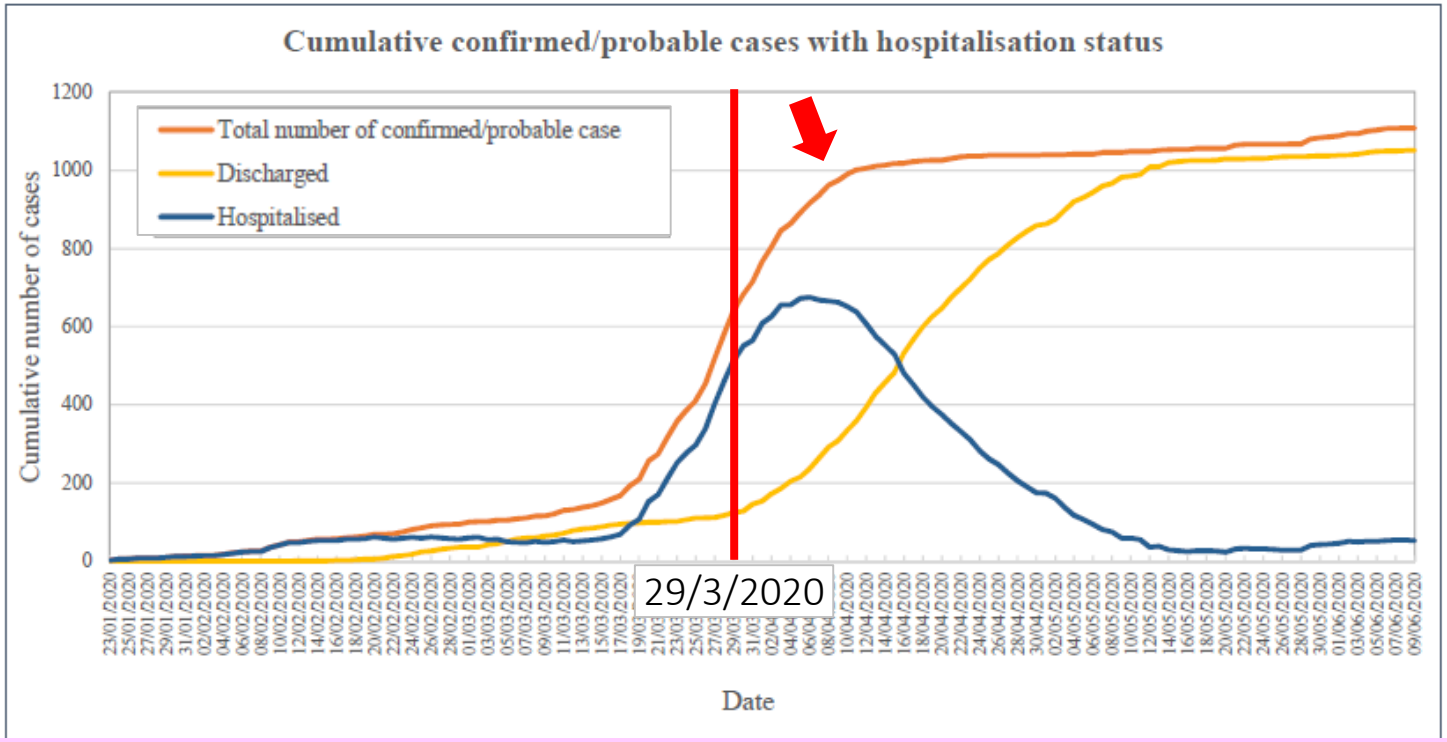
Prohibition on group gathering



Retrieved from https://www.chp.gov.hk/files/pdf/local_situation_covid19_en.pdf

Figure above shows the cumulative number of confirmed/probable cases in Hong Kong (up and until 11/6/2020).

Prohibition on group gathering



With all our concerted effort to follow the regulations put in place to reduce group gatherings from 29/3/2020 onwards, the total number of confirmed/probable cases began to flatten out in around 10 days!

Together, we fight the virus!

To conclude, let's be responsible citizens helping oneself and the others, keeping all of us away from the spread of COVID-19 by continuously following the regulations on reducing social gatherings.