Education Bureau
Curriculum Support Division
School-based Curriculum Development (Secondary) Section

School Sharing in 2022/23

Strengthening Students' Problem Solving Skills Through Mathematical Modelling

Mathematics Education

St. Margaret's Co-educational English Secondary & Primary School
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Contents

School Background

- Direct Subsidy Scheme (DSS) through-train school
- English as Medium of Instruction
- Major concern in school development cycle (2018/19 - 2022/23): Nurturing Global Citizenship
    - → Implement inquiry learning to promote the three focuses of global citizenship in solving global issues
    - → Promote the habit on reading global citizenship related texts

Student Characteristics

- Background: Students of different ethnic groups
- Thought: No strong connection between Mathematics and daily life situation
- Curiosity: Wonder the applications of Mathematics in various areas
- Lesson behaviour: Afraid of giving “wrong” answers / waiting for correct answers

School-based support service - Timeline

Sept 2022 **1st meeting**

- **Introduce the support service**
- **Discuss the support level and focus**
  - **Support level:** **S.3**
  - **Support focus:** Using an **inquiry approach** and **explanatory activities** to enhance students' learning experience and motivation

Co-planning (1)

- Identify key concepts in mathematical modelling
- Plan learning activities and discuss expected students' learning outcomes

Sept 2022

Oct 2022

Co-planning (2)

- Refine learning and teaching materials
- Predict learning difficulties and modify respective classroom instructions

Oct 2022 Lesson observation

Curriculum planning

1. Connect student pre-requisite knowledge and daily life experience to lesson activities

- Topic
  - Using the spread of COVID-19 as an example to strengthen students' problem solving skills through mathematical modelling
- Context
  - Fifth wave pandemic of COVID-19 in Hong Kong
- Mathematical knowledge
  - S.3 Percentage (II) – Growth and Decay

Curriculum planning

2. Identify key concepts

- Mathematical modelling
  - Its definition - Galbraith , P., & Holton, D. (2018). Mathematical modelling: a guidebook for teachers and teams .

    Mathematical modelling refers to using mathematical concepts and language to describe a real world situation, to test ideas and make estimations about the situation by mathematical computation and analysis. Teachers may explain to students that at a very basic and simplistic level, "mathematical model" has been used in the sense of a formula (Galbraith & Holton, 2018).
  - Source: EDB Resources – STEAM Examples – Mathematical modelling on the accommodation demand of visitors to Hong Kong

    To formulate a mathematical model to represent the number of visitor arrivals at the *n*th years after a particular year by assuming constant growth rate

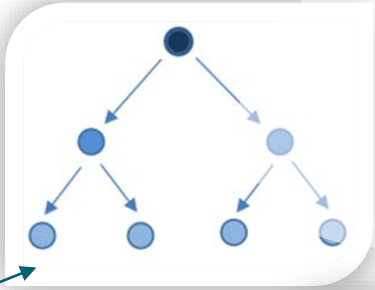
    The model for Activity 1 is $N_1 = V(1 + 5\%)^n$.

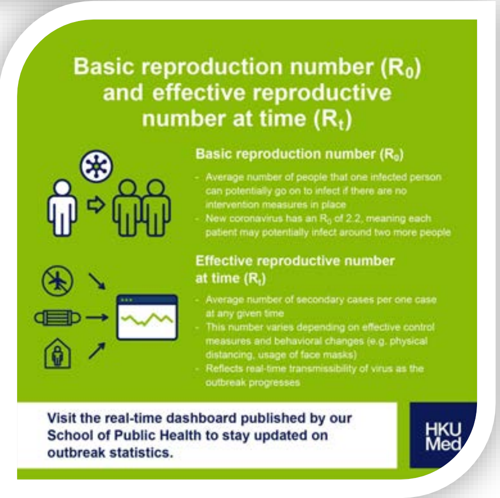
Curriculum planning

2. Identify key concepts

- Spread of disease
  - Simple epidemic model
    - Source: EDB Resources – STEAM Examples – [Modelling the spread of a disease]
  - Basic reproduction number (R_0) and

    Effective reproductive number at time (R_t)
    - Source: HKU Medicine

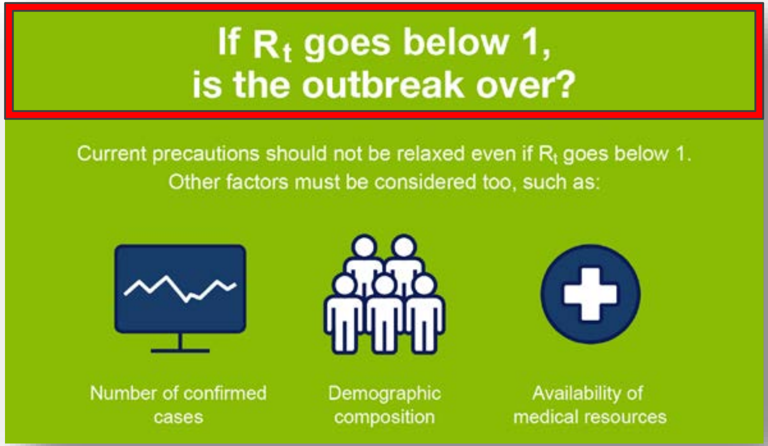




Curriculum planning

3. Incorporate the knowledge of mathematical modelling to reading tasks

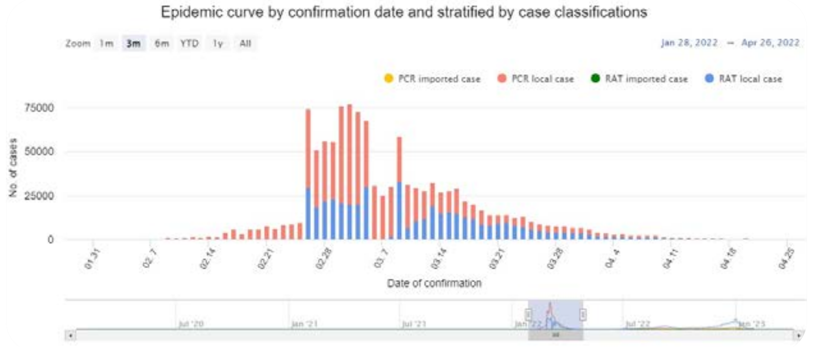
- Article 1: Recognise the stages of mathematical modelling
- Article 2: Understand the meaning of basic reproduction numbers (R_0) and effective reproductive numbers (R_t)
- Questions for reading tasks



Curriculum planning

4. Conduct a case study with authentic data

- Authentic data
    - Daily confirmed cases of COVID -19 from 4/2/2022 to 10/2/2022
    - Source: HKU Medicine – Real-time Dashboard



Learning and Teaching strategies

Pre-lesson	• Reading tasks
Lesson (Part 1)	• Inquiry activities + discussion (in-class)
Lesson (Part 2)	• Inquiry activities + reflection (after class)

Learning and Teaching strategies

Pre-lesson: Reading tasks

Learning objectives :

- Recognise the stages of mathematical modelling through reading article 1.
- Understand the meaning of basic reproductive numbers and effective reproductive numbers through reading article 2.

Learning and Teaching strategies

Pre-lesson: Reading tasks

Question design:

- Assess their understanding
  - Name three applications of mathematical modelling
- Explanatory questions
  - Summarise main stages involved in mathematical modelling
  - Meanings of effective reproductive numbers ($R_t > 1$, $= 1$ & < 1)
- Give reasons to support their answer
  - Assuming $R_t < 1$, is the outbreak over?

Student Performance – Pre-lesson task

Pre-lesson: Reading tasks

Some students paraphrased and summarised the stages involved in mathematical modelling.

Q2. Summarize those five main stages involved in mathematical modelling.

The first stage: Identify the problem and make variables and assumptions to solve the problem

The second stage : Develop the model using algorithms and numerical methods.
The third stage : Analyse the results from the model and express and evaluate.

The forth stage: Apply real data to the model to test the validation of the model

The fith stage: To further modify the model to improve the complexity and/or the efficiency

Student Performance – Pre-lesson task

Pre-lesson: Reading tasks

Many students showed their understanding about R_t of different magnitude.

Q3. Obviously, R_t reflective of the current pandemic progression. Then what are the meanings of $R_t > 1$, $R_t = 1$ and $R_t < 1$?

If $R_t > 1$, then one case has more than 1 secondary case on average.

If $R_t = 1$, then one case has 1 secondary case on average

If $R_t < 1$, then one case has less than 1 secondary case on average.

Student Performance – Pre-lesson task

Pre-lesson: Reading tasks

Students demonstrated thinking from different views.

Q4. Assuming R_t goes below 1, is the outbreak over?

No, it is not. Only at a given time is $R_t < 1$. After some time, the outbreak ean spread and can increase, due to other possible factors. By then, R_t can be 1 or larger than one and the outbreak will continue.

Q4. Assuming R_t goes below 1, is the outbreak over?

The outbreak is not over. Although the number of secondary cases given is less than the average number of cases, there are still people infected every day, but the number of cases is not much as before.

Learning and Teaching strategies

Lesson Part 1 – learning objectives

Inquiry Approach

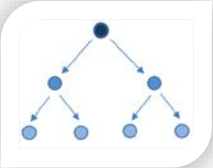
1. Investigate the effect of R_0 on the spread of disease.
2. Apply the simple epidemic model to construct the mathematical model.

Learning and Teaching strategies

Lesson Part 1 – learning objective 1 (Investigate the effect of R_0)

Inquiry Approach

- 3 scenarios were provided for investigating the relationship between R_0 and the spread of disease.
  - 1^{st} scenario: $R_0 = 2$ → introducing the simple epidemic model
  - 2^{nd} scenario: $R_0 = 8$, i.e. R_0 of Omicron [Source: CityU Research News]
  - 3^{rd} scenario: $R_0 = 12$



Relate mathematical modelling to daily life

CityU About CityU Admissions Academics Re

The aim was to simulate the number of infections that may be caused by the fifth wave under different intervention policies: assuming (i) 65% of the population has received at least one dose of the vaccine, (ii) vaccine effectiveness against infection is 50% (Sinovac) and 70% (BioNTech), (iii) the basic reproduction number (R0) for Omicron is 8 (when the entire population is susceptible, not vaccinated or there is no exercising of social distancing measures), and (iv) antibodies wane across time.

Learning and Teaching strategies

Lesson Part 1 - learning objective 1 (Investigate the effect of R_0)

Inquiry Approach

- Similar questions were set in 3 scenarios to let students observe the effect of R_0 on spread of disease.
  - Scenario 1 ($R_0 = 2$): After n generations of spread, the newly infected cases first exceeds 10000. Find n.
  - Scenario 2 ($R_0 = 8$): How many generations are needed when the newly infected cases first exceeds 10000?
  - Scenario 3 ($R_0 = 12$): Find the number of generations needed if the newly infected cases first exceeds 10000.

Compare the three scenarios, what can you conclude about the disease with different R_0?

Learning and Teaching strategies

Lesson Part 1 - learning objective 1 (Investigate the effect of R_0)

Scenario 1

Suppose $R_0 = 2$ and there is only 1 infected person in Hong Kong.

2. What is the number of newly infected cases in the 1st, 2nd and 3rd generation of spread respectively?

3. After n generations of spread, the newly infected cases first exceed 10 000. Find n. Briefly explain your answer.

- **Q2:** Observe the **growth pattern** in the first three generations
- **Q3: Set up an equation**, it will be used **for comparison** with different R_0 later

Learning and Teaching strategies

Lesson Part 1 - learning objective 1 (Investigate the effect of R_0)

Use authentic data

Scenario 2

A highly transmissible variant of COVID-2019, Omicron, appeared. Suppose its R_0 is equal to 8 [Source: https://www.cityu.edu.hk/media/news/2022/02/09/accurate-estimate-5th-wave-covid-19-infections-using-cityu-mathematical-model-predicted-peak-yet-come]. Assume there is only 1 infected person in Hong Kong.

2. How many generations of spread are needed when the newly infected cases first exceed 10 000?

3. What is the total number of people infected after 5 generations of spread?

- **Q2:** Similar to Scenario 1 (Set up equation for comparison later)
- **Q3:** Test their understanding of the relationship between newly infected cases and total infected cases

Student Performance - Lesson Part 1

Lesson Part 1 - learning objective 1 (Investigate the effect of R_0)

- Students applied the concept of growth to find the number of new cases.
- More able students rewrote the expression as A x $(1+r\%)^n$.

Scenario 1

Suppose $R_0 = 2$ and there is only 1 infected person in Hong Kong.

2. What is the number of newly infected cases in the 1st, 2nd and 3rd generation of spread respectively?

In the first generation = $1 \times (1+100\%) = 2$

second generation $= \boxed{1 \times (1+100\%)^2} = 4$ / $2 \times (1+100\%)$

third generation $= \boxed{1 \times (1+100\%)^3} = 8$ / $4 \times (1+100\%)$

Student Performance - Lesson Part 1

Lesson Part 1 - learning objective 1 (Investigate the effect of R_0)

Without the knowledge of logarithm, students set up an equation and found n by testing.

3. After n generations of spread, the newly infected cases first exceed 10 000. Find n. Briefly explain your answer.

$1 \times (1 + 100\%)^n = 10000$

$2^n = 10000$

$2^{13} = 8192 < 10000$

$2^{14} = 16384 > 10000$

Therefore the least value of n is 14

Student Performance - Lesson Part 1

Lesson Part 1 - learning objective 1 (Investigate the effect of R_0)

Scenario 2

A highly transmissible variant of COVID-2019, Omicron, appeared. Suppose its R_0 is equal to 8 [Source: https://www.cityu.edu.hk/media/news/2022/02/09/accurate-estimate-5th-wave-covid-19-infections-using-cityu-mathematical-model-predicted-peak-yet-come]. Assume there is only 1 infected person in Hong Kong.

1. Express R_0 in the form of $1 + r\%$. $1+700\%$

2. How many generations of spread are needed when the newly infected cases first exceed 10 000?

$1 \times (1+700\%)^n = 10000$

$8^n = 10000$

$\boxed{\begin{array}{l} 8^4 = 4096 < 10000 \\ 8^5 = 32768 > 10000 \end{array}}$

$\therefore$ The least value of n is 5

- Similar to Scenario 1, students set up an equation and found the answer by **testing**.

Scenario 3

For different infectious disease, their R_0 are different. Suppose there is only 1 infected person in Hong Kong. An unknown virus appears in Hong Kong. Suppose $R_0 = 12$. $1 + 1100\% = 1 + r$

1. Find the number of generations needed if the newly infected cases first exceed 10 000.

$12^n = 10000$

$\boxed{\begin{array}{l} n = 3 = 1728 \text{ newly infected cases} \\ n = 4 = 20736 \text{ newly infected cases} \end{array}}$ $\therefore$ 4 generation needed

Student Performance - Lesson Part 1

Lesson Part 1 - learning objective 1 (Investigate the effect of R_0)

- Students analysed the effects of R_0 and wrote a conclusion from different views.

2. Compared with Scenario 1, 2 and 3, what can you conclude about the disease with different R_0?

The higher the R_0, the faster it spreads.

Rate of spread

The larger R_0 is, the less generations it takes for the respective disease to exceed 10 000 newly infected cases, the larger the difference between each generation

the number of generations needed to attain a certain number of new cases

Student Performance - Lesson Part 1

Lesson Part 1 - learning objective 1 (Investigate the effect of R_0)

- Students analysed the effects of R_0 and wrote a conclusion from different views.

2. Compared with Scenario 1, 2 and 3, what can you conclude about the disease with different R_0?

When the value of Ro is higher, more people is infected in each generation.

number of infected people in each generation

Larger Ro, spread faster. Larger Ro difference between each generation is larger.

difference between each generation

Design school - based L&T strategies

Lesson Part 1 - learning objective 2 (Construct a mathematical model)

Inquiry Approach

- Using scenario 1 - 3, generalise and construct a model for the disease.
- For more able students: test their understanding of the difference between newly infected cases and total infected cases

Our mathematical model (formula) for COVID-2019:

Suppose there are A infected people in Hong Kong originally and $R_0 = 1 + r\%$. After n generations of spread,

number of newly infected cases = ______________________

Total number of infected cases = ______________________

Challenging question for more able students

Student Performance - Lesson Part 1

Lesson Part 1 - learning objective 2 (Construct a mathematical model)

Our mathematical model (formula) for COVID-2019:

Suppose there are A infected people in Hong Kong originally and $R_0 = 1 + r\%$. After n generations of spread,

number of newly infected cases = $\boxed{A(1+r\%)^n \ / \ AR_0^n}$

Total number of infected cases = $\underset{\text{0gen}}{A} + \underset{\text{1st gen}}{A \times (1+r\%)} + \underset{\text{2nd gen}}{1A(1+r\%)^2} + \underset{3,4,5\ldots}{\ldots} + A(1+r\%)^n$

--- End of Lesson Worksheet (Part 1) ---

Our mathematical model (formula) for COVID-2019:

Suppose there are A infected people in Hong Kong originally and $R_0 = 1 + r\%$. After n generations of spread,

number of newly infected cases = $A(1+r\%)^n$

Total number of infected cases = $\boxed{\sum_{k=0}^{n} A(1+r\%)^k} \ / \ A + A(1+r\%)^1 + \ldots + A(1+r\%)^n$

- Less able students:
  - needed more time
  - needed more assistance to get the n th term
- More able students:
  - constructed a model (even in different presentations)

Learning and Teaching strategies

Lesson Part 2 - learning objectives

1. Observe the difference between the constructed model and the authentic data through 2 case studies.
2. Recognise the assumptions and limitations of mathematical modelling.
3. Appreciate the importance of mathematical modelling.

Inquiry Approach
+
Explanatory Activities

Learning and Teaching strategies

Lesson Part 2 – learning objective 1

Two case studies were introduced to let students observe the difference between the constructed model and the authentic data.

Case 1: Without any preventive measures (assumption)

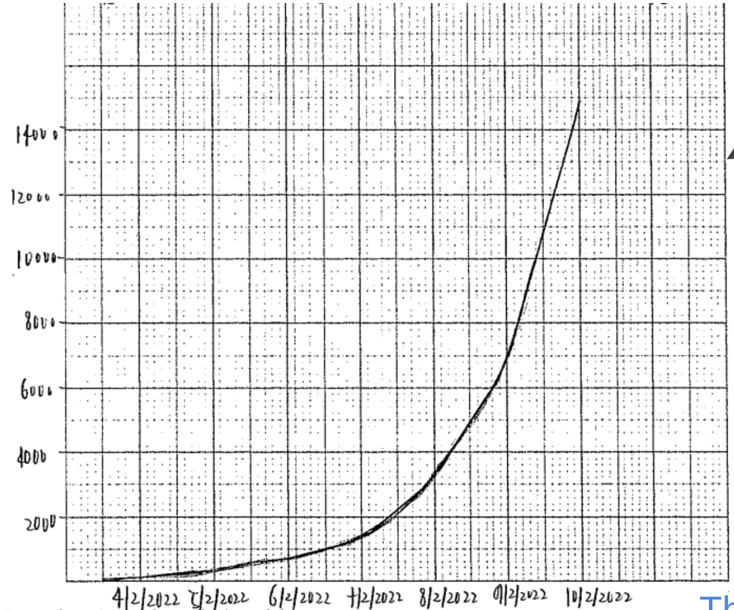
Assumed factors:

1. Assumed R_0 as 2.2 (one person will continue infecting 2.2 people).
2. There are no any preventive measures.
3. One day as one generation of spread.

Case 2: With preventive measures (authentic data from 4/2/2022 to 10/2/2022)

Learning and Teaching strategies

Lesson Part 2 – learning objective 1



Formulate the mathematical model:

Number of new cases = $131(1+20\%)^n$

With the assumed factors, students were able to plot the graph that is exponential, while the number of cases will never drop.

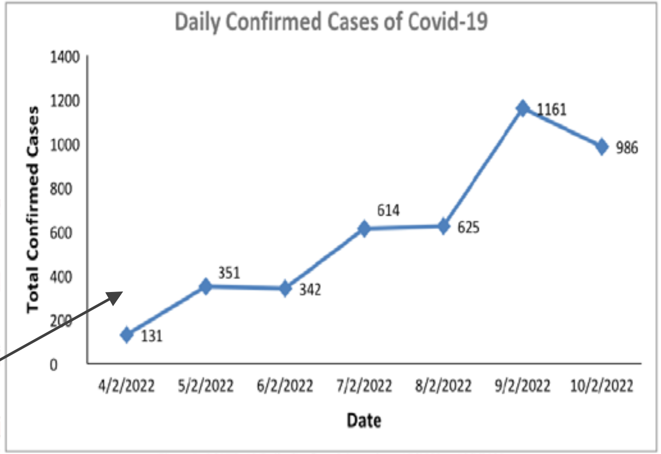


Figure 1: COVID-19 Daily Confirmed Cases from 4/2/2022 to 10/2/2022
Data Source: HKU LKS Faculty of Medicine School of Public Health

Then the graph was used to compare with the graph of authentic data

Learning and Teaching strategies

Lesson Part 2 - learning objective 2

Reflection & Evaluation by students

1. What reasons behind will lead to the difference between the two graphs?
2. What are the limitations of the model?
3. Why do we need mathematical modelling?

Student Performance - Lesson Part 2

Lesson Part 2 - learning objective 2

Assumptions witnessed by students:

2. What reasons will lead to the difference observed in the two graphs?

External factors (e.g. change in government policies, decrease of reported cases). If the government tighten the social distancing policy, the number of new cases may decrease, therefore causing the graph to slightly go down in Part 2. However, the no. of new cases predicted in Part 1 tends to increase, widening the gap of the no. of cases in the 2 parts.

2. What reasons will lead to the difference observed in the two graphs?

Actual interventions $\therefore R_t < R_0$

assumption 1 day is each generation

unexpected cases

Student Performance - Lesson Part 2

Lesson Part 2 - learning objective 2

Limitations witnessed by students:

3. What are the limitations of the model in Part 1?

There can be lack of information and data monitoring as our understanding of phenomenon may not be complete.

3. What are the limitations of the model in Part 1?

This model may be far off from the actual number of cases. Since it is graph with the consideration of no epidemic intervention measures, it is hard to guarantee certainty on how the pandemic will envelope in the current period. There are many uncontrollabe variables such as human behaviour, mutations of the virus, the virus's biology.

Student Performance - Lesson Part 2

Lesson Part 2 - learning objective 3

Students appreciated the applications of mathematical modelling

5. Why do we need to use modelling?

Because it gives us precision and strategy for problem sollution, so it can help us develop accurate ideas and assumptions sometimes. Understanding a situation better can let us come up with better solutions.

5. Why do we need to use modelling?

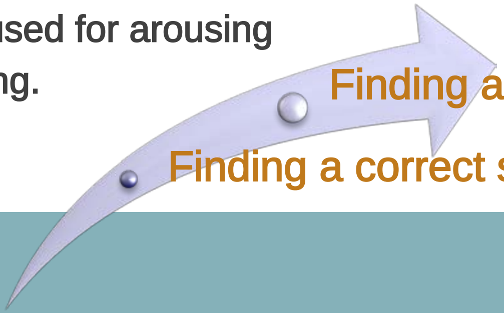
Because we need to predict what is going to happen next (e.g. no. of new cases) to alert people what is going to happen in the worst case.

Effectiveness and Reflection

- Students enhanced their generic skills such as
    - mathematical skills
        - Apply various mathematical concepts in authentic situation (e.g. set formula, solve equations, test hypothesis)
        - Handle statistical data and make reasonable interpretation of results (e.g. case studies)
    - communication skills
        - Use appropriate mathematical language in verbal and written communication to present information and different points of view (e.g. draw a conclusion, reflection and evaluation)
- Students were more active in lesson activities (e.g. when introducing the simple epidemic model).
- Students showed their appreciation for the applications of mathematical modelling.

Effectiveness and Reflection

- Limited preparation time
  - Questions can be more adaptive to diverse student abilities.
  - Reading passages can be used for arousing students’ interest in learning.
- Change in the teaching belief



“All models are wrong, but some are useful.”
George Box

The way forward

- Improvement in task design (Simpler language? Change in guiding questions? More interactive?)
- Enriching STEAM elements (Handling a large data set with the use of technology?)
- Cross-subject collaboration? (PSHE Project learning/ other STEAM related subjects?)

End