Education Bureau
Curriculum Support Division
School-based Curriculum Development (Secondary) Section
School Sharing (2022/23)

Mathematics Education

Promoting e-learning in Mathematics to develop students' ability to inquire and cater for learner diversity

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School Background

- Co-educational DSS School
- EMI
- Major Concerns:

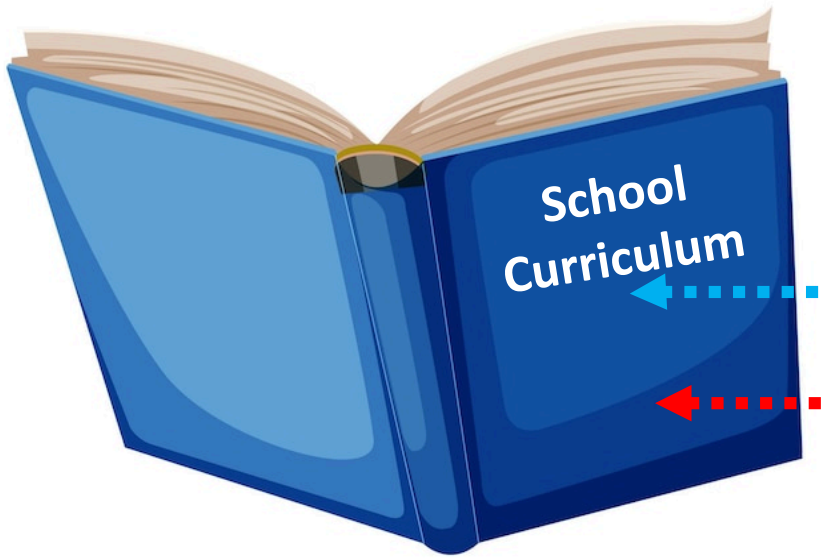
- Through-train School

Primary Section — Secondary Section

Curriculum and Enhance the Curriculum Acquaintance of

Differentiated Instruction

Positive Education



D.I. elements
- e-Learning Lesson Plan
- Game Activities
- Origami
- Inquiry and Investigation
- …

Positive Education Elements
- Positive Values
- Linkage between knowledge and daily life
- Appreciation of knowledge and individuals
- …

Triple E Framework

Engaged Learning
(Get students involved)

- Collect students' responses easily
- Provide more possibility for follow-up discussion
- Facilitate teacher-student and student-student interactions

Enhanced Learning
(Enhance the learning and teaching effectiveness)

- Understand the concepts through visualization
- Investigate the relationships through dynamic tools
- Make the concepts easier by scaffolding with the help of e-tools

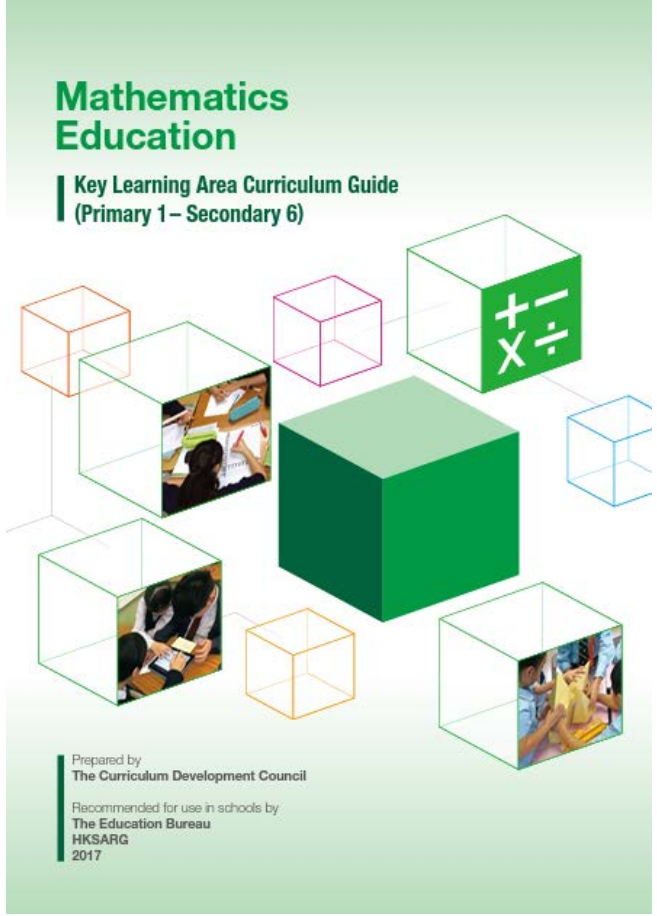
Extended Learning
(Create room for further investigation)

- Learn outside the classroom
- Create a bridge between school learning and daily life problems
- Build skills that cannot be learnt in traditional lessons

Source : https://www.tripleeframework.com/

Use of e-Learning Tools for Inquiry and Investigation

- "The dynamic geometry software facilitates the inquiry and investigation in many geometry topics in the Mathematics curriculum."
- "With the help of IT tools, students can easily plot graphs for visualising mathematical relations, propose conjectures and make connections between different areas of mathematics."



Source: Mathematics Education Key Learning Area Curriculum Guide (Primary 1 – Secondary 6) (2017), p.65

S1 Introduction to Geometry

Learning Objectives

- Students are able to perform some basic geometric constructions by using e-tools.
- Students are able to discover geometric properties by using e-tools.

Developing students' ability to inquire

Professional development of Teachers

1. Acquire skills for utilising e-learning tools to:
    - enhance student engagement
    - help students explore geometric properties
    - make good use of electronic assessment data and provide immediate feedback

Developing students' ability to inquire

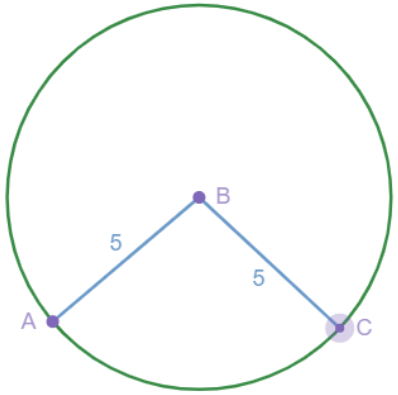
2. Design lessons to cater for students' diversified learning needs
    - Tiered tasks
    - Multiple solutions to a single problem

Catering for learner diversity

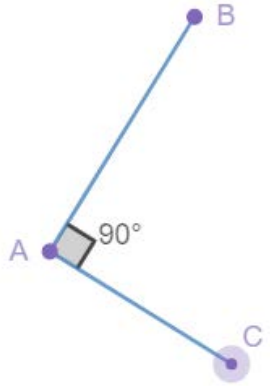
Pre-lesson e-Learning Tasks

- Revision on different types of angles
- Introduction to the use of e-learning tool to inquire and investigate
  - Measuring the size of an angle using the protractor
  - Copying line segments
  - Drawing perpendicular lines
  - Hiding unnecessary lines

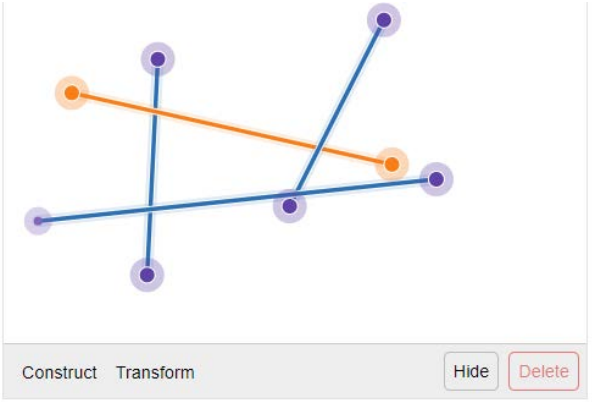
Developing students' ability to inquire before the lesson



Copying line segments

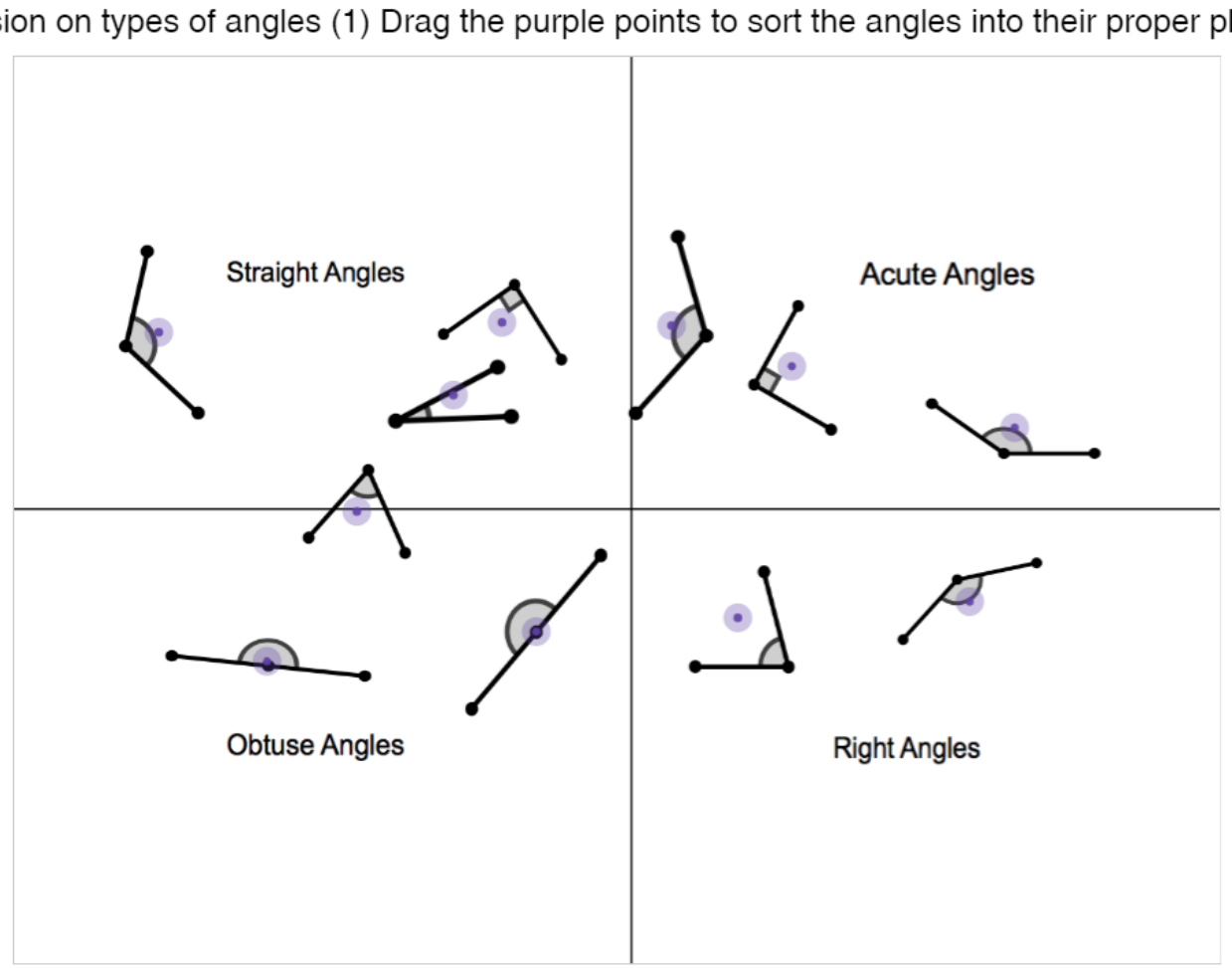


Drawing perpendicular lines



Hiding unnecessary lines

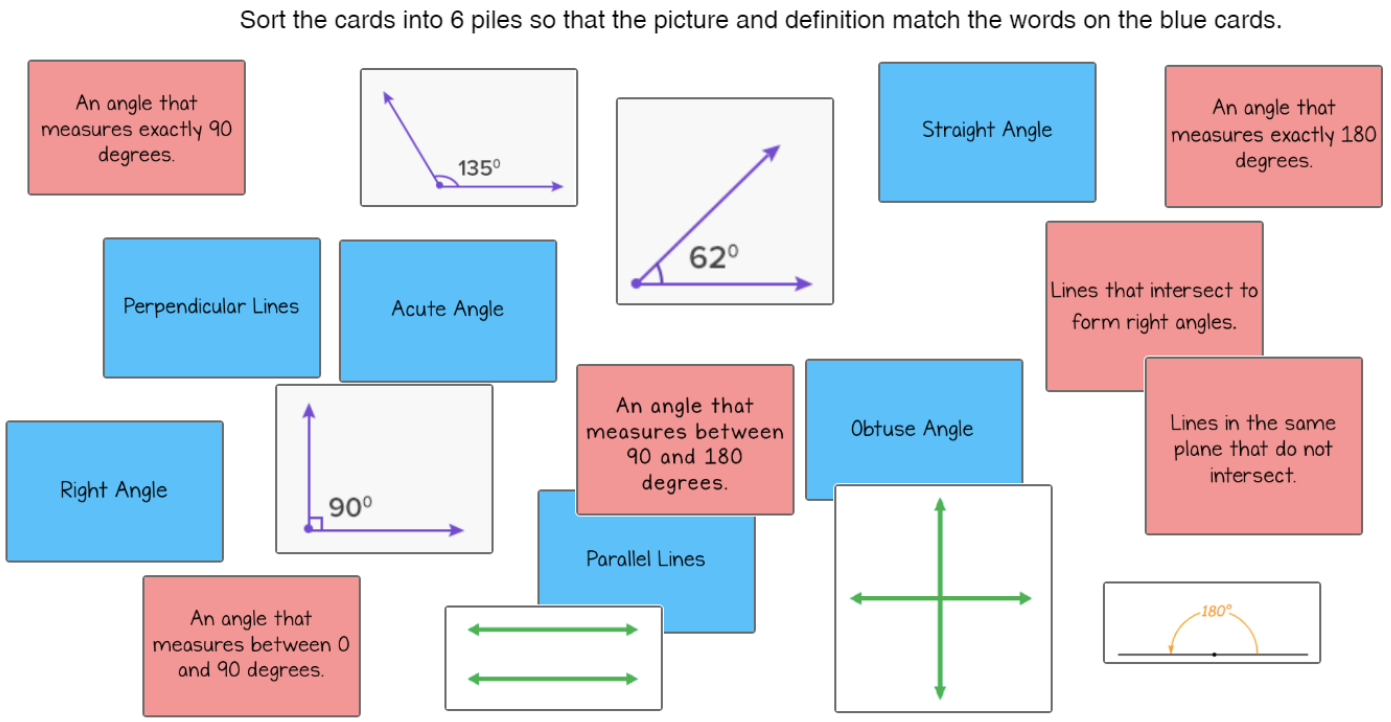
e-Learning Tasks During Lesson



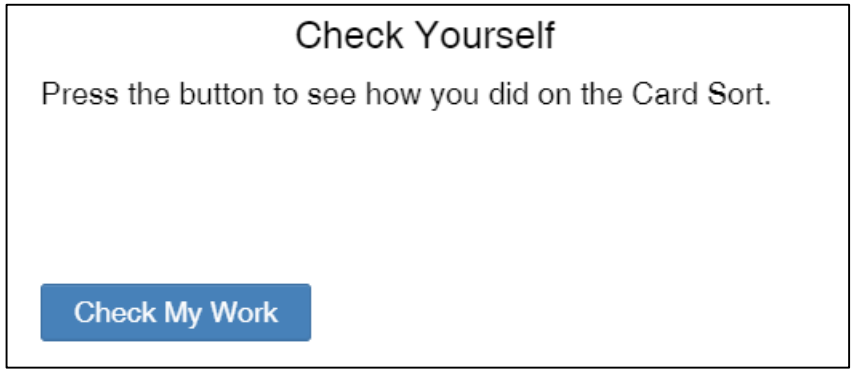
Grouping

Assessing students' prior knowledge by engaging them in diversified e-learning tasks

e-Learning Tasks During Lesson



Card Sort



Self-checking

e-Learning Tasks During Lesson

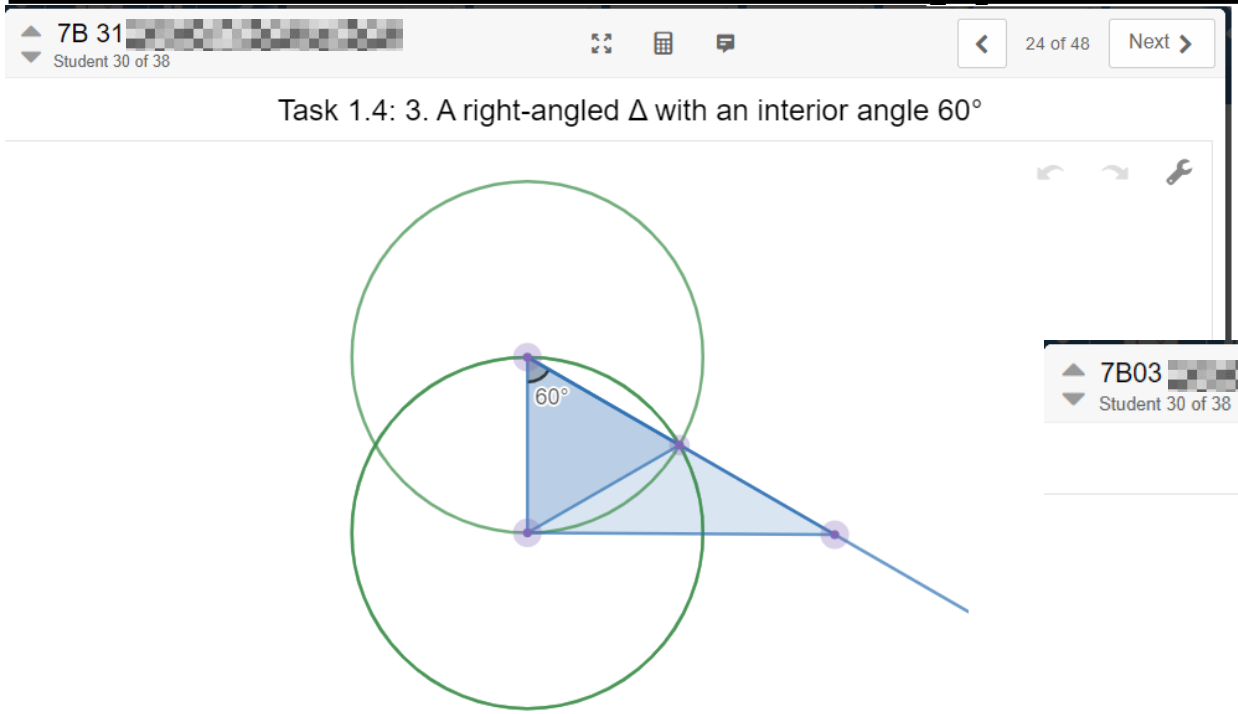
Problem-solving tasks
- Draw an angle with specific size (e.g. 30°, 60°, 90°).
- Draw triangles satisfying specific conditions.

😊😊Congratulations! Now you can draw a 30 degree, 60 degree or 90 degree angle using the Desmos tools!

1. An obtuse-angled Δ with an interior angle 120°
2. An obtuse-angled Δ with an interior angle 60°
3. A right-angled Δ with an interior angle 60°
4. An isosceles Δ with an interior angle 120°

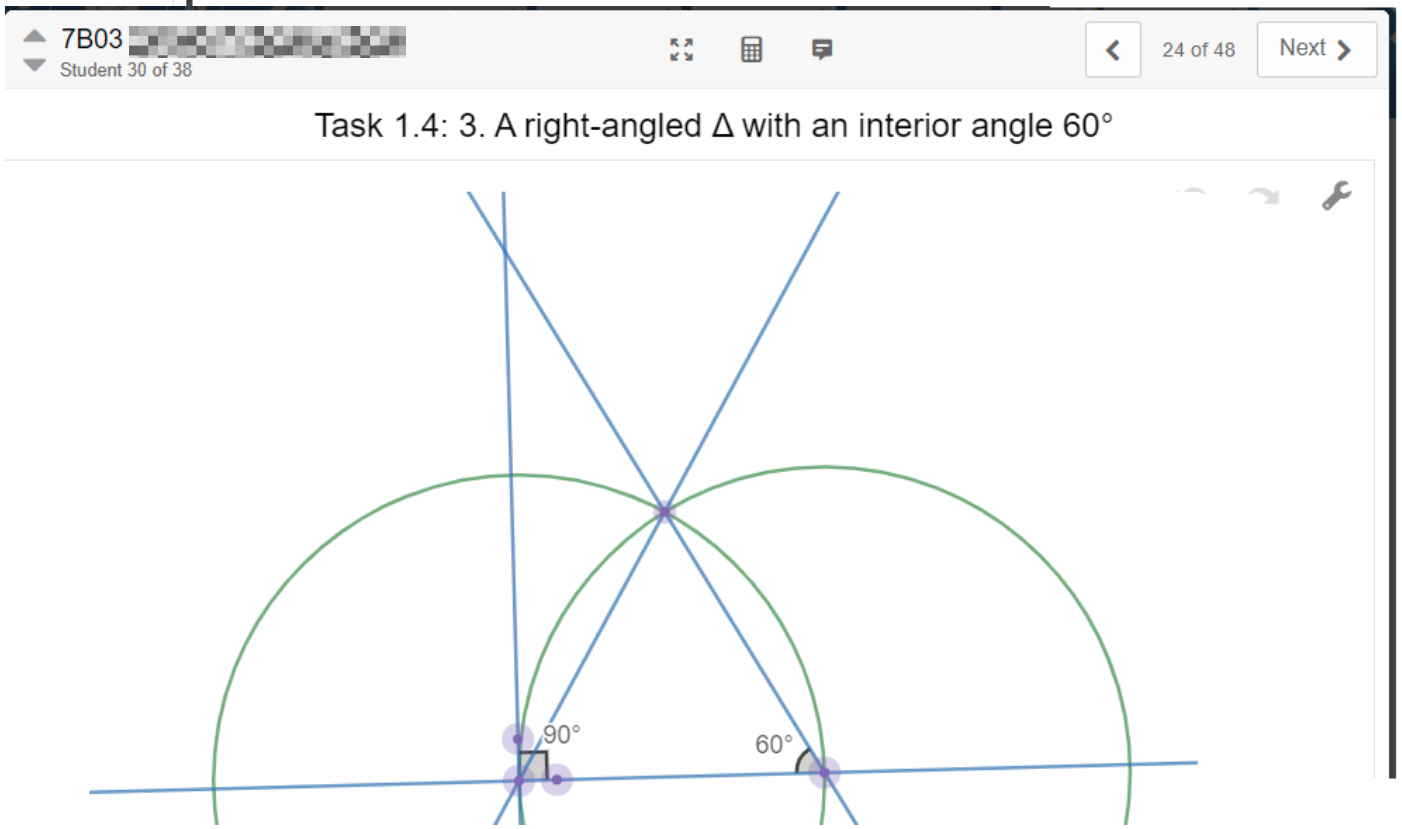
Catering for learner diversity
- **Tasks with different levels of difficulty**
- **Two tasks chosen by students**

Students' Learning Outcome

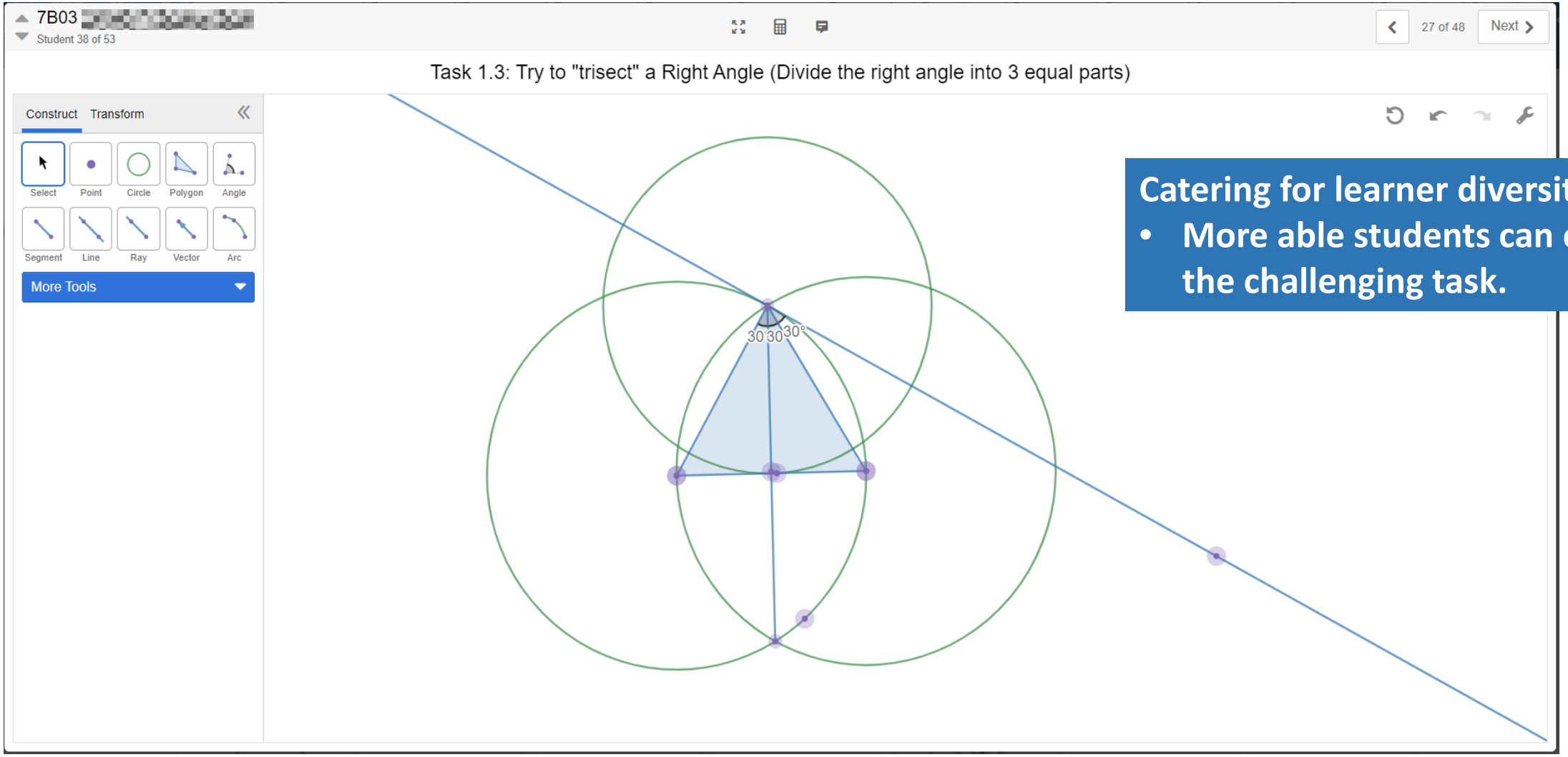


Problem-solving skills

- Students can construct triangles using different strategies.



Students' Learning Outcome



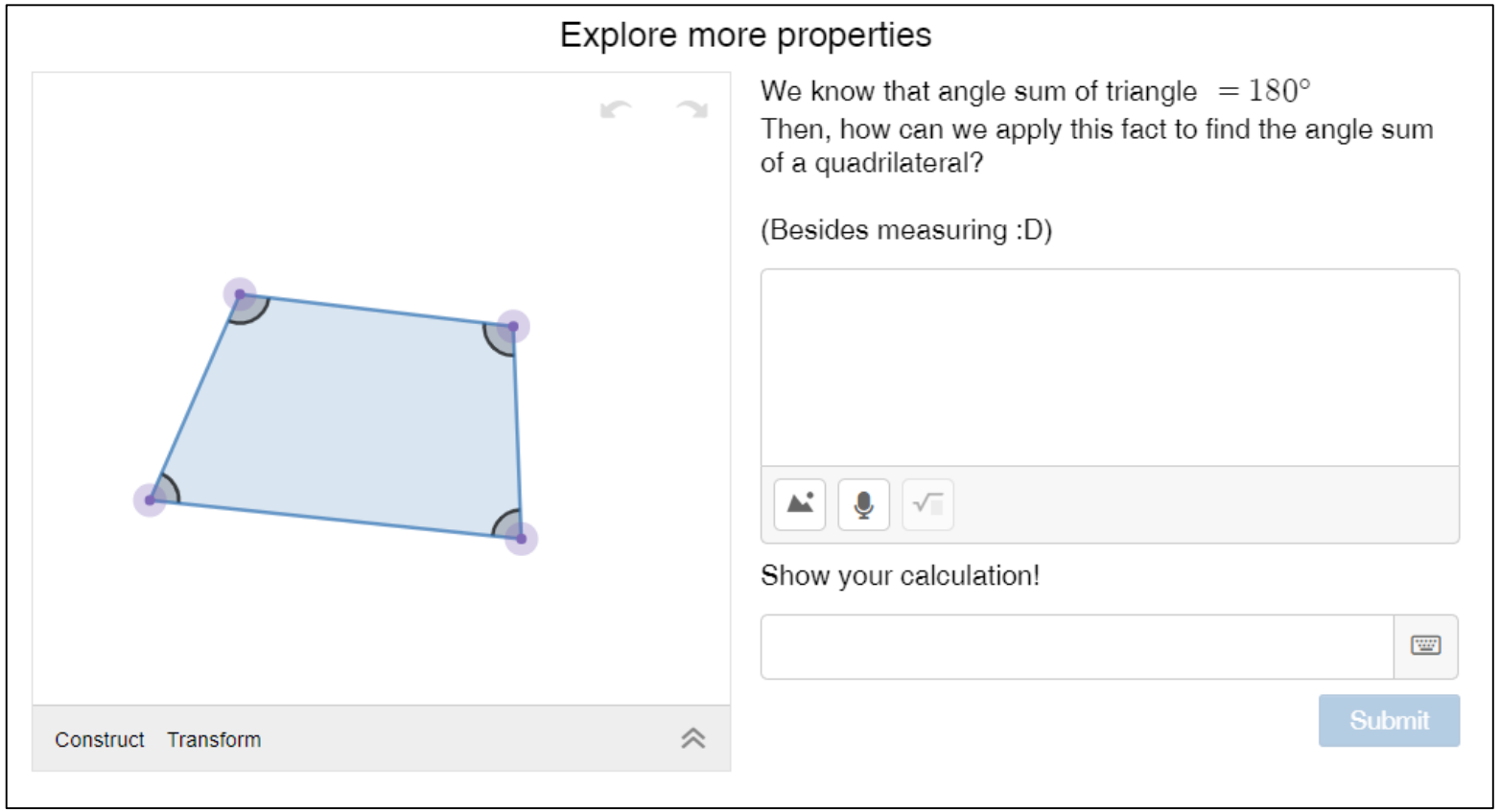
Catering for learner diversity

- More able students can complete the challenging task.

e-Learning Tasks During Lesson

Inquiry and Investigation
- Angle sum of “reflex angles” of a triangle
- Angle sum of a quadrilateral
- Angle sum of “reflex angles” of a quadrilateral

Catering for learner diversity
- **Tasks with different levels of difficulty**



Developing students’ ability to inquire
- **Students may**
  - **measure the size of each angle to find the angle sum of a specific quadrilateral;**
  - **move the vertices to observe the angle sum of a quadrilateral;**
  - **draw auxiliary line segments and use the fact about angle sum of triangle to find the angle sum of a quadrilateral.**

Evaluation using Triple E Framework

	Evaluation
Engagement	• This can be achieved through the use of interactive e-learning tasks, as well as through collaborative activities that encourage communication and teamwork among students.
Enhancement	• Through the problem solving tasks, students develop a deeper understanding of the properties of equilateral triangles. • Students are allowed to demonstrate their understanding of geometry using the platform of the e-learning tool.
Extension	• Using the platform, students can further investigate the properties about the angle sum of a polygon.

Teacher's Reflection (Catering for learner diversity)

Student engagement:

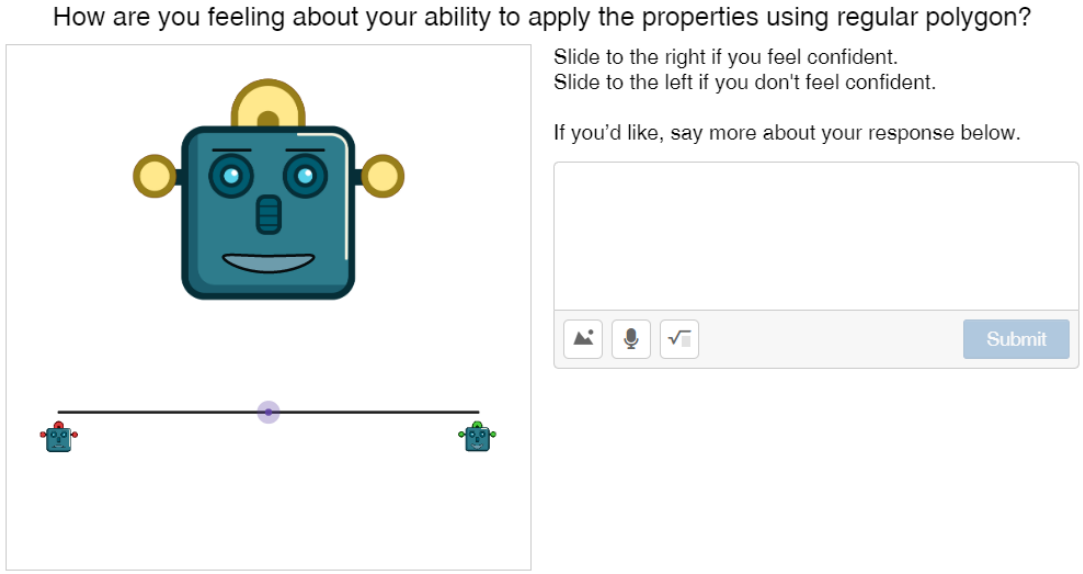
- Students use e-learning platforms to complete diversified e-learning tasks.
- Students are allowed to express their reflections on the learning process in different forms.

Tiered tasks:

- Learning tasks are designed with different levels of difficulty (including challenging questions).
- Students are allowed to choose the problems they want to solve in the e-platform according to their interests/abilities.

Multiple solutions:

- Students are allowed to use different methods to draw angles.
- Students have chances to present their way of thinking through the e-platform. Teachers can show students' works easily and facilitate further discussion among students.

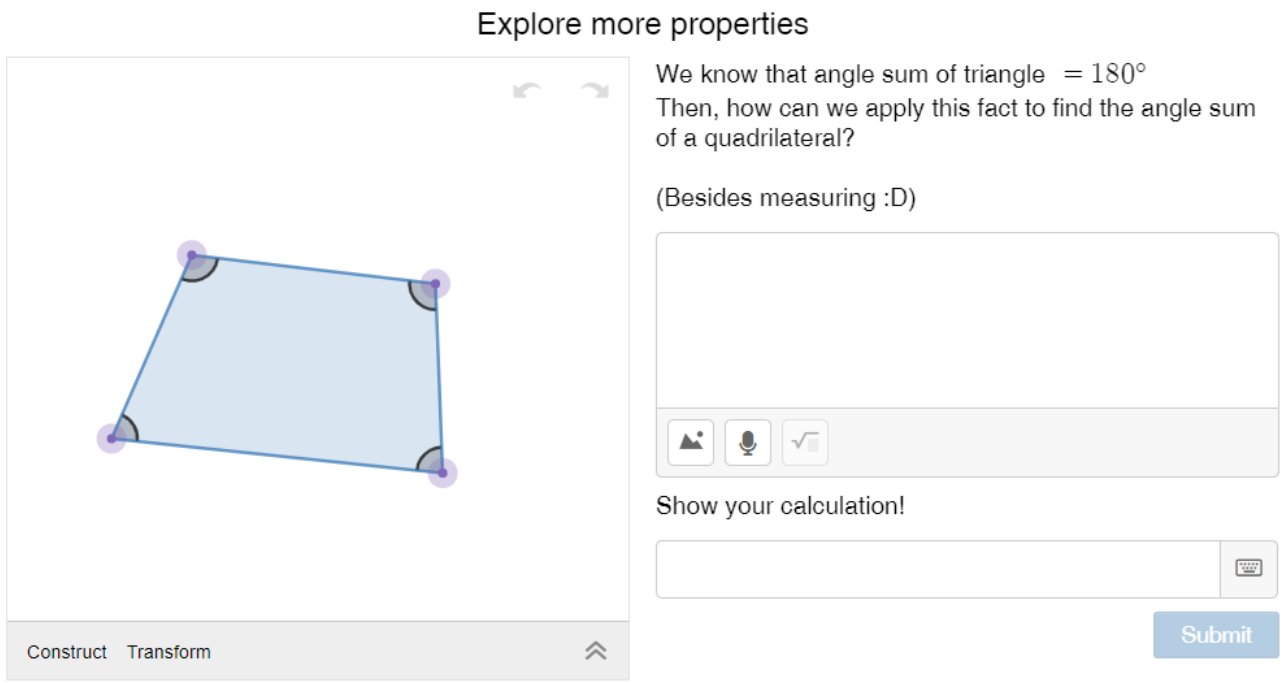


Proper use of e-learning tools and careful design of e-learning tasks can cater for learner diversity effectively.

Teacher's Reflection (Inquiry and investigation)

Helping students explore geometric properties using e-learning tools:

- Students use the e-learning tools to observe patterns and make conjectures about the geometric properties.
- Students use the e-platform to explain their conjectures.

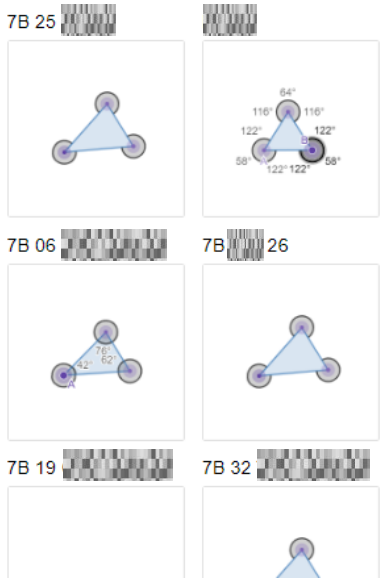


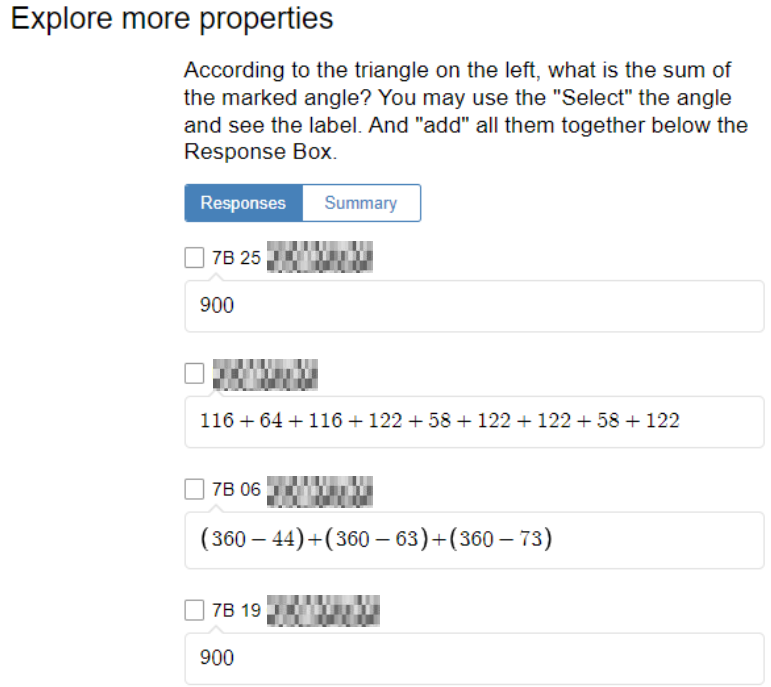
The dynamic geometry software facilitates the inquiry and investigation in many geometry topics in the Mathematics curriculum.

Teacher's Reflection (Effectiveness of e-learning strategies)

Utilising electronic assessment data to provide immediate feedback:

- Teacher uses the data in the pre-lesson e-learning task to understand whether students have mastered the drawing skills.
- Based on students' performance in the e-learning activities, teacher understands how students think mathematically and provides immediate feedback (asking and probing questions).





The use of e-assessment can enhance learning and teaching by providing instant feedback to both students and teachers.

S2 Trigonometric Ratios

Learning Objectives

- Understand the properties of trigonometric ratios
  - effect of angle size on the ratios ($x \uparrow, \sin x \uparrow$)
  - range of values of trigonometric ratios($0 < \sin x < 1$ for $0° < x < 90°$)
- Use trigonometric ratios to solve problems involving plane figures
  - Find the length of sides of a right-angled triangle using trigonometric ratios

- Suitable resources found

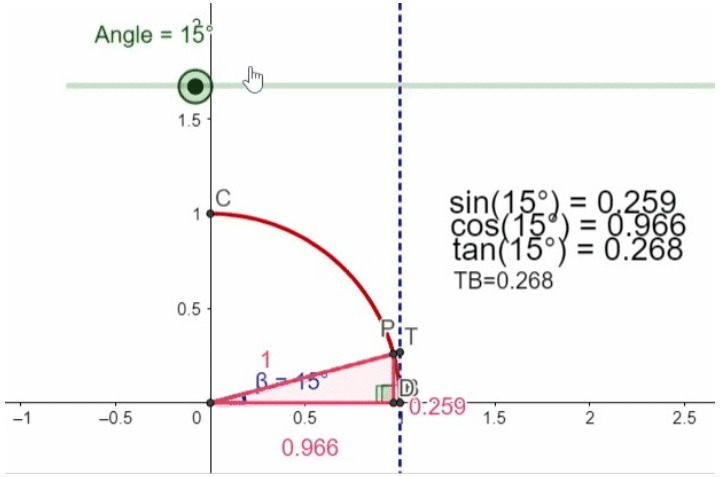
About Target Students

- Students recognised the definition of trigonometric ratios.
- Students were weaker in Mathematics in general.

- Adjust students' learning pace
- Collect/show students' responses and give feedback in a timely manner

Part I: Investigate the Properties of Trigonometric Ratios

- GeoGebra



- Effect of angle size on the ratios
- Range of values of trigonometric ratios

- when I drag the slide
  the opposite side increase
  the adjacent side decrease
- When we drag the slide
  The hypotenuse remain the same
  The opposite side increases
  The adjacent side decreases
- 1 side extend, elongated
  1 side become short short
- H remain unchanged
  A forward=decreases or backward=increases
  O forward=increases or backward=decreases

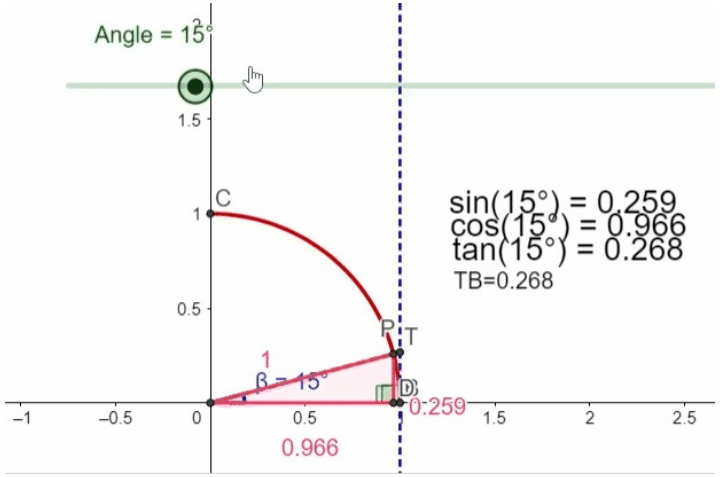
Students **can** point out the change in the length of sides.
Some students **cannot** relate them to the change in the ratios.

Reflections

- Proper questioning?
- Focus more on the lengths instead of the values of the ratios?

Part I: Investigate the Properties of Trigonometric Ratios

- GeoGebra



- Effect of angle size on the ratios
- Range of values of trigonometric ratios

- [ ] Cos cannot be larger than one based on the app
- [ ] Sine and cosine data should be less than 1
- [ ] Sine<1
  Cosine<1
  Tangent (unlimited)
- [ ] The range of the angle is 1-89
- [ ] ratio of sin must be positive and less then one
  ratio of cos must be positive and less then one
  ration of tan must be positive
  ^-----^
  * *

Students can partly identify the range of the ratios (i.e. sin $\theta < 1$).

By observations on the numerical values only

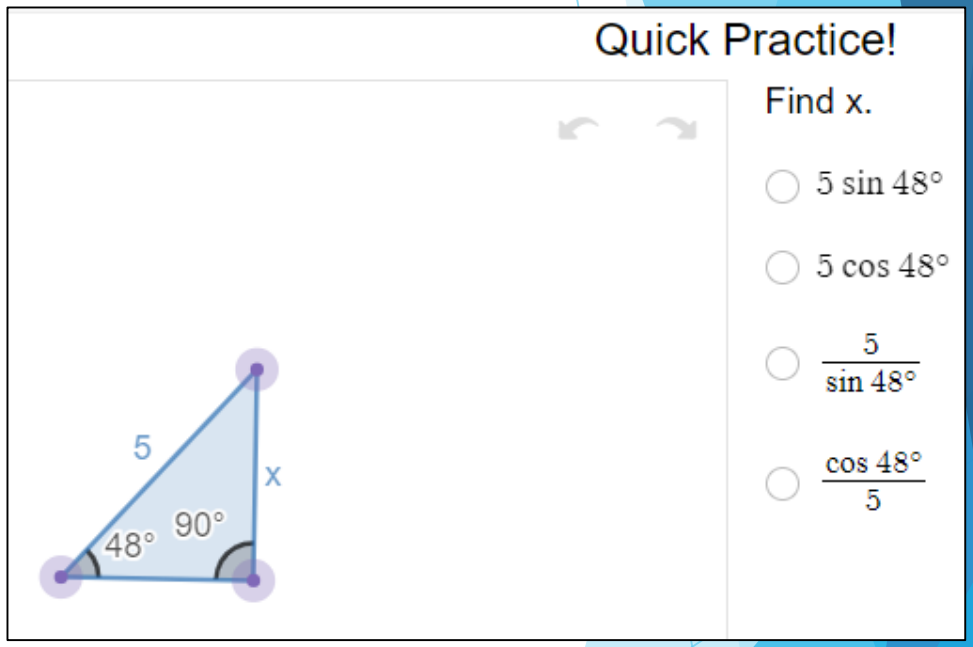
Geometric interpretation?

- Explanation
- Follow-up Task

Part II: Finding length of sides

- Finding the length of a side by observation
  - Identifying the correct ratio used
  - Using $0 < \sin\theta < 1$ and $0 < \cos\theta < 1$ to determine whether the operation $\times/\div$ should be used

- Considering the angle "48°":
  - The hypotenuse is given and the opposite side is to be found. Thus the sine ratio should be used.
  - Note that $\sin 48° < 1$. Thus $5\sin 48° < 5$ and $\frac{5}{\sin 48°} > 5$. As the opposite side is shorter than the hypotenuse, the correct answer should be $5\sin 48°$.

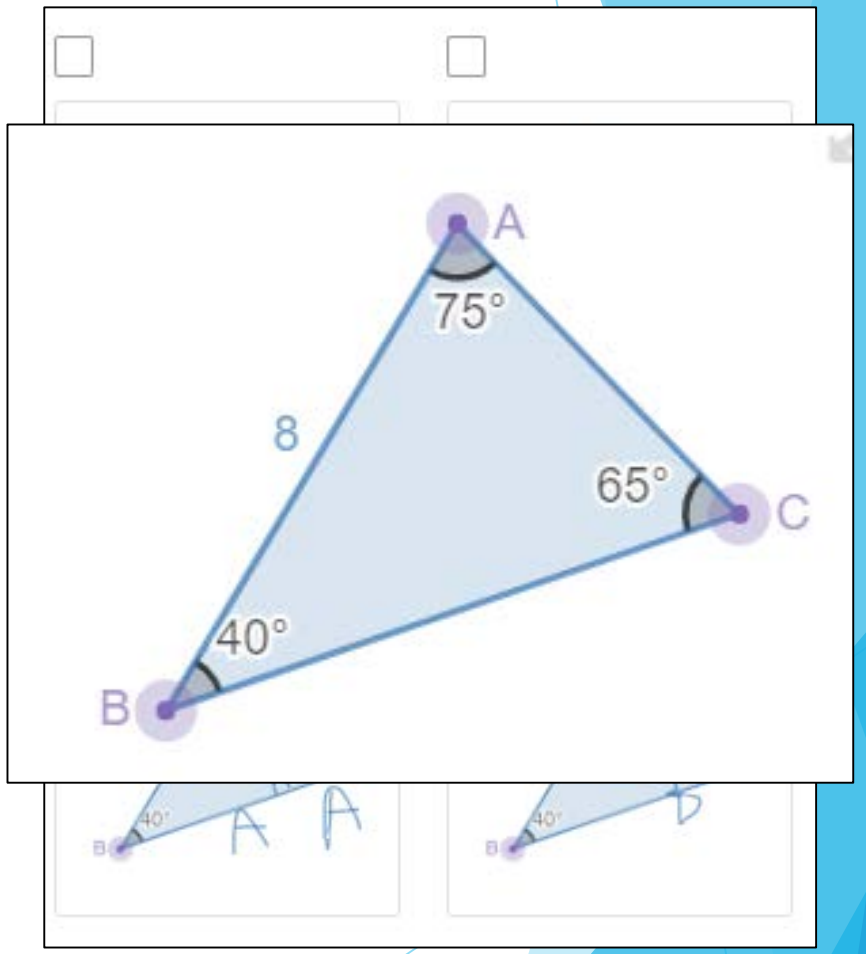


Application of the properties of trigonometric ratios to solve problems

Part II: Find the length of sides

- Finding the length of a side by observation
  - Identifying the correct ratio used
  - Using $0 < \sin\theta < 1$ and $0 < \cos\theta < 1$ to determine whether the operation $\times/\div$ should be used

- Solving harder problems by adding auxiliary lines to form right-angled triangles
  - Acute-angled triangle
  - Obtuse-angled triangle



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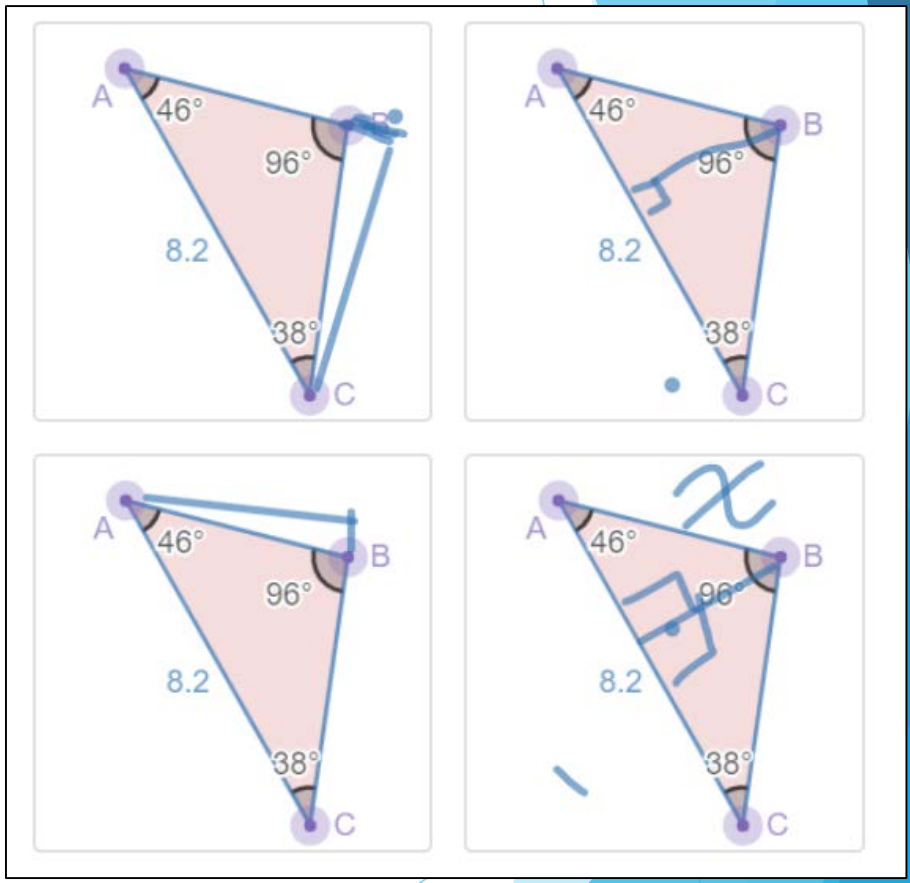
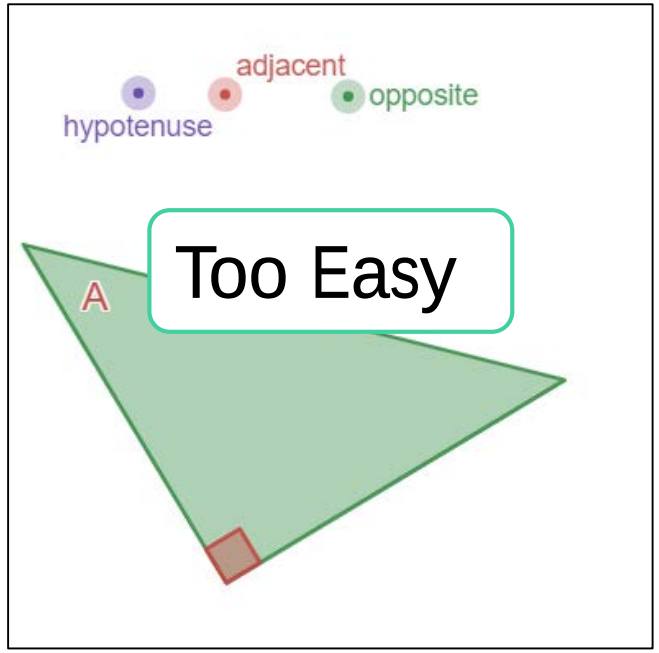


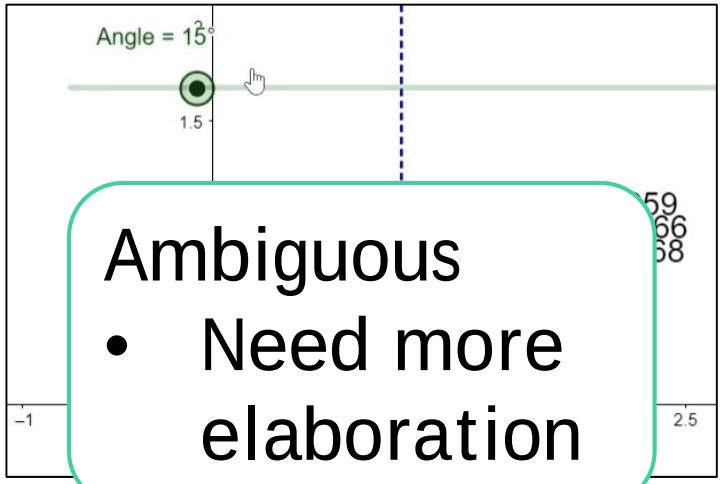
Evaluation - Engagement

Matching



Too Easy

Self-exploration

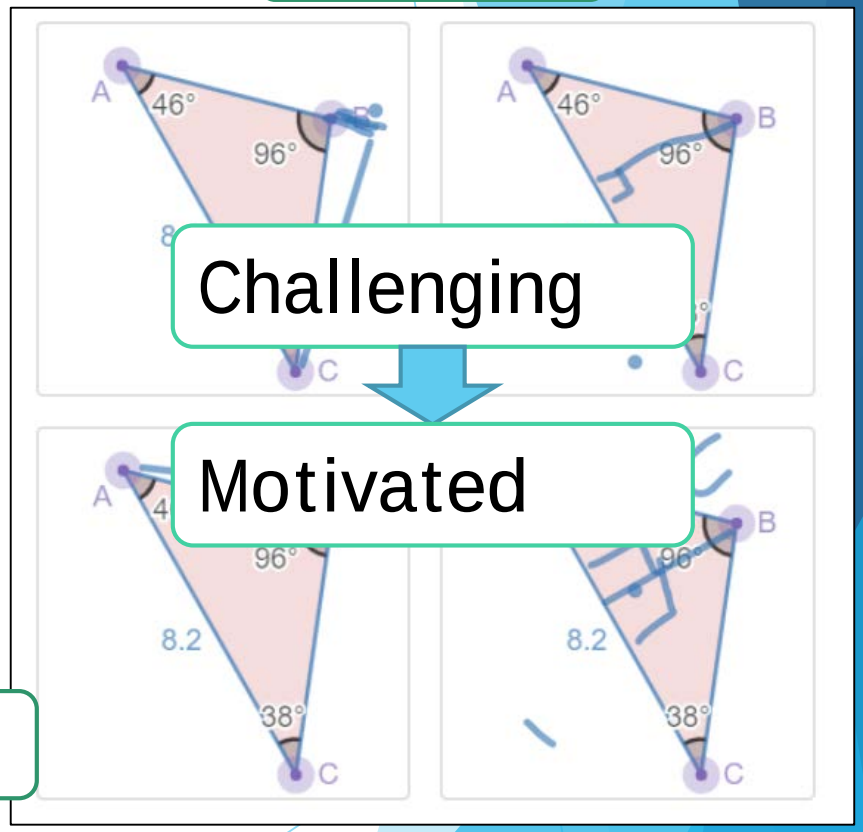


Ambiguous

- Need more elaboration

- Hands-on Tasks

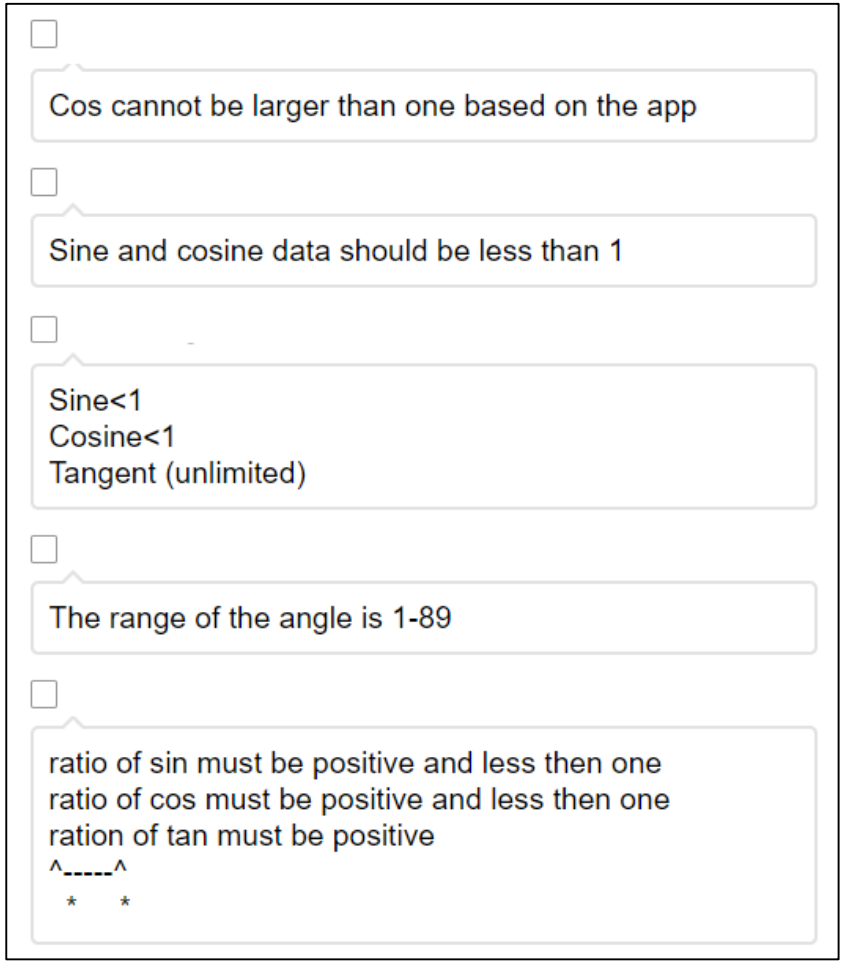
Self-practice



Challenging

Motivated

Evaluation - Engagement



Areas for improvement

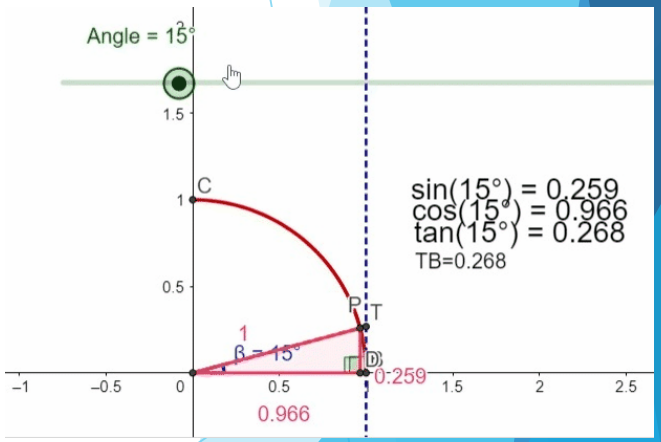
- Peer-peer interaction
- Discussion

Evaluation - Enhancement

- Understand the properties of trigonometric ratios
  - effect of angle size on the ratios ($x \uparrow, \sin x \uparrow$)
  - range of values of trigonometric ratios($0 < \sin x < 1$ for $0° < x < 90°$)

- Simple
- More elaboration on the geometric meaning of the trigonometric ratios

- Use trigonometric ratios to solve problems involving plane figures
  - Find the length of sides of a right-angled triangle using trigonometric ratios



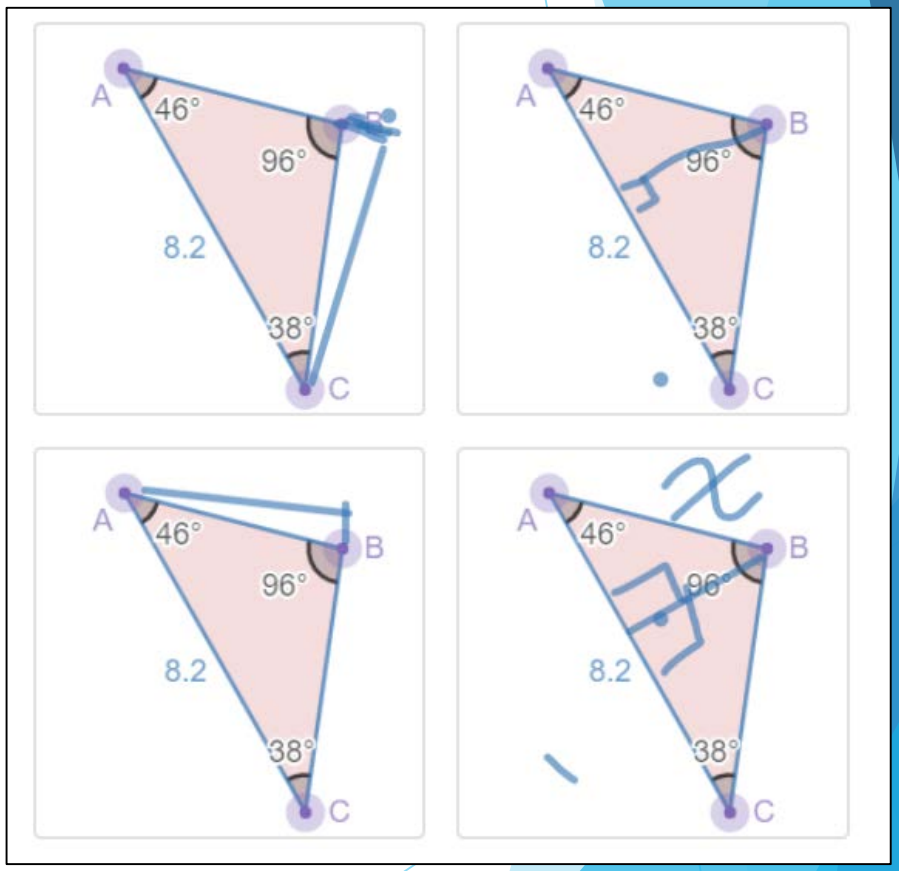
Student CAN choose the correct step.

Students CANNOT choose the correct step **quickly.**

Connection between Part I and Part II could be strengthened to enhance students' understanding about the properties of trigonometric ratios.

Evaluation - Extension

- Finding height of obtuse-angled triangles
  - Collect feedback instantly using e-platform
  - Choosing a proper vertex is crucial to problem-solving
    - May add a simple matching task
- More properties/identities of trigonometric ratios
  - By observations only
  - Elaboration on geometric meaning
  - Create platform for students to share their idea
- Other possibilities?
  - Trigonometric ratios when $\theta = 0°$ or $\theta = 90°$



Evaluation - Extension

- Finding height of obtuse-angled triangle
  - Collect feedback instantly
  - Choosing a proper vertex is crucial to problem-solving
    - May add a simple matching task
- More properties/identities of trigonometric ratios
  - By observations numerical values only
  - More elaboration using the geometric interpretation
  - Create a platform for students to share their idea
- Other possibilities?
  - Values of trigonometric ratios when $\theta = 0°$ or $\theta = 90°$

Check the following boxes if the equation is true.

(Select all that apply.)

- ☐ $\sin(\theta + 30°) = \sin\theta + \sin 30°$
- ☐ $\sin^2\theta + \cos^2\theta = 1$
- ☐ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
- ☐ $\sin\theta + \cos\theta = \tan\theta$
- ☐ $\sin 2\theta = 2\sin\theta$

Catering for learner diversity

- Adjust the learning pacing in e-platform
  - Students have the **freedom** to choose the suitable question according to their level
- Assessing students' learning using e-platform
  - Respond/follow up timely
- Sharing students' work using e-platform
  - Engagement

- Student-centered
- Peer-peer interaction

Reflections

- Mixed use of diversified learning tools
  - "Think out of" the iPad
- Consider the learning needs of users(students)

Way forward

To develop students' ability to inquire and cater for learner diversity, teachers could capitalise their efforts on

- designing diversified learning tasks to engage, enhance and extend student learning;
- making further use of e-learning tools to promote interactive learning and foster student engagement and participation; and
- strengthening knowledge management for disseminating good practices of learning and teaching strategies.

End