

Education Bureau
Curriculum Support Division
School-based Curriculum Development (Secondary) Section
School Sharing (2022/23)

Mathematics Education

Promoting e-learning in Mathematics to develop students' ability to inquire
and cater for learner diversity



PUI KIU COLLEGE
培僑書院

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School Background

- Co-educational DSS School
- EMI
- Major Concerns:

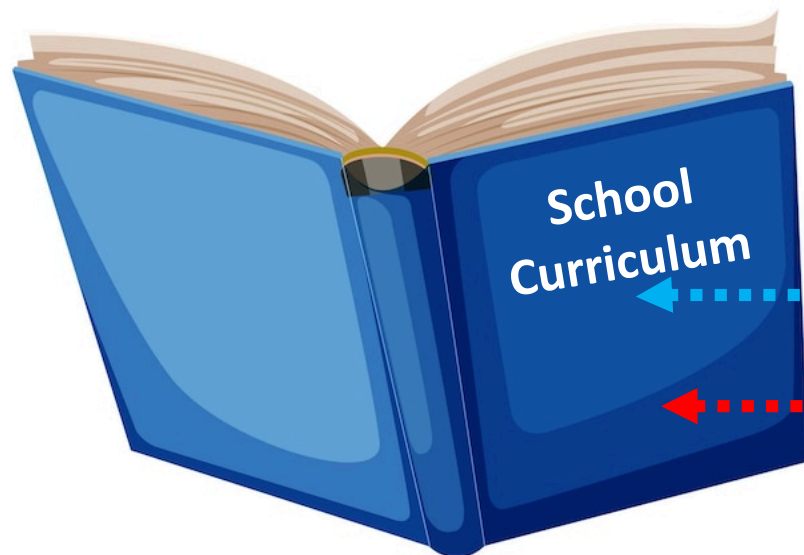
- Through-train School



Curriculum and Enhance the Curriculum Acquaintance of

Differentiated Instruction

Positive Education



D.I. elements

- e-Learning Lesson Plan
- Game Activities
- Origami
- Inquiry and Investigation

Positive Education Elements

- Positive Values
- Linkage between knowledge and daily life
- Appreciation of knowledge and individuals
- ...

Triple E Framework

Engaged Learning (Get students involved)

- Collect students' responses easily
- Provide more possibility for follow-up discussion
- Facilitate teacher-student and student-student interactions

Enhanced Learning (Enhance the learning and teaching effectiveness)

- Understand the concepts through visualization
- Investigate the relationships through dynamic tools
- Make the concepts easier by scaffolding with the help of e-tools

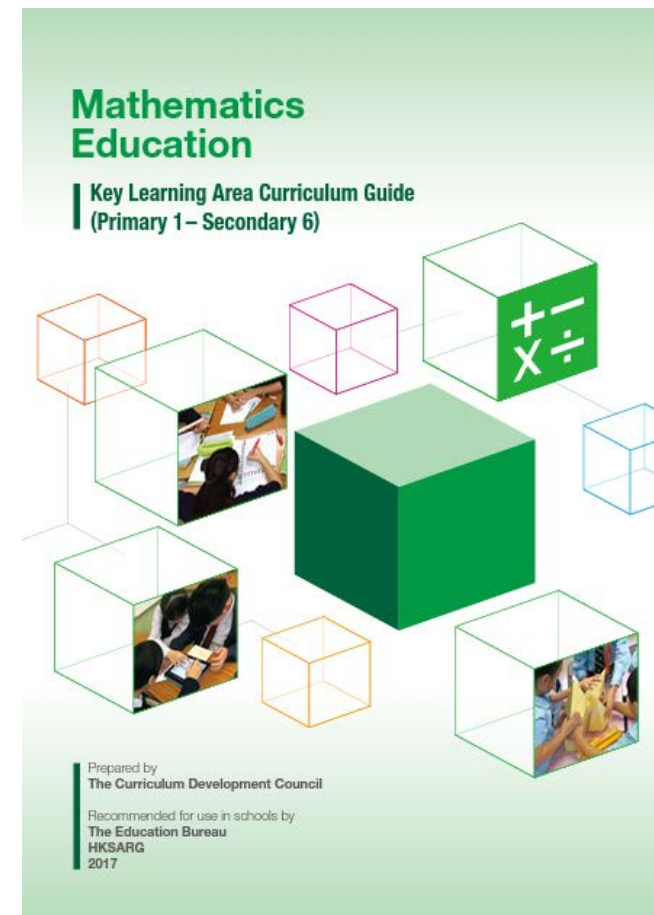
Extended Learning (Create room for further investigation)

- Learn outside the classroom
- Create a bridge between school learning and daily life problems
- Build skills that cannot be learnt in traditional lessons

Use of e-Learning Tools for Inquiry and Investigation

- “The dynamic geometry software facilitates the inquiry and investigation in many geometry topics in the Mathematics curriculum.”
- “With the help of IT tools, students can easily plot graphs for visualising mathematical relations, propose conjectures and make connections between different areas of mathematics.”

Source: Mathematics Education Key Learning Area Curriculum Guide (Primary 1 – Secondary 6) (2017), p.65



S1 Introduction to Geometry

Learning Objectives

- Students are able to perform some basic geometric constructions by using e-tools.
- Students are able to discover geometric properties by using e-tools.

Developing students' ability to inquire

Professional development of Teachers

1. Acquire skills for utilising e-learning tools to:

- enhance student engagement
- help students explore geometric properties
- make good use of electronic assessment data and provide immediate feedback

Developing students' ability to inquire

2. Design lessons to cater for students' diversified learning needs

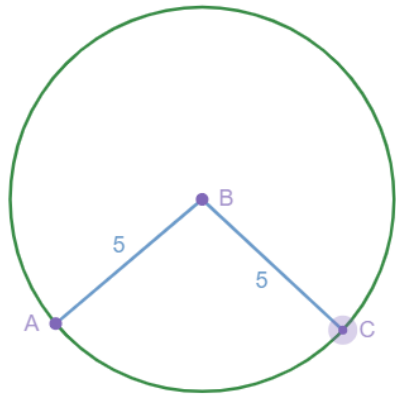
- Tiered tasks
- Multiple solutions to a single problem

Catering for learner diversity

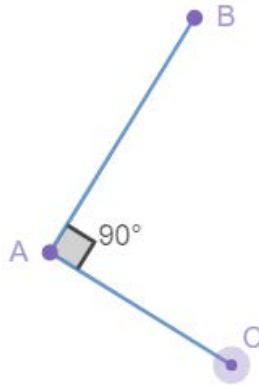
Pre-lesson e-Learning Tasks

- Revision on different types of angles
- Introduction to the use of e-learning tool to inquire and investigate
 - Measuring the size of an angle using the protractor
 - Copying line segments
 - Drawing perpendicular lines
 - Hiding unnecessary lines

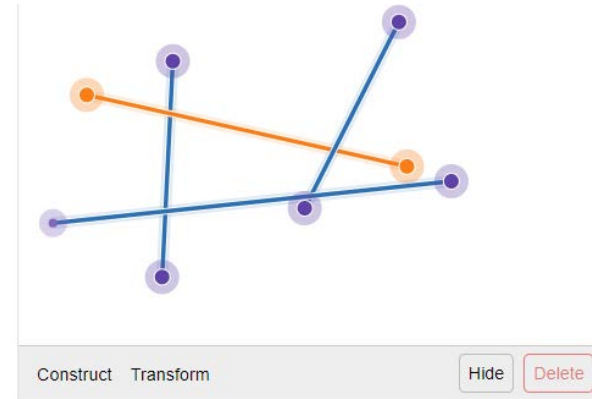
Developing students' ability to inquire before the lesson



Copying line segments



Drawing perpendicular lines



Hiding unnecessary lines

e-Learning Tasks During Lesson

Revision on types of angles (1) Drag the purple points to sort the angles into their proper places.

The task interface consists of a 2x2 grid of boxes. The top-left box is labeled "Straight Angles" and contains three angles: a straight angle, an obtuse angle, and an acute angle. The top-right box is labeled "Acute Angles" and contains three acute angles. The bottom-left box is labeled "Obtuse Angles" and contains two obtuse angles. The bottom-right box is labeled "Right Angles" and contains two right angles. Each angle has a purple dot at its vertex, which is intended to be dragged into the correct category box.

Grouping

Assessing students' prior knowledge by engaging them in diversified e-learning tasks

e-Learning Tasks During Lesson

Sort the cards into 6 piles so that the picture and definition match the words on the blue cards.

Definitions (Red cards):

- An angle that measures exactly 90 degrees.
- An angle that measures between 0 and 90 degrees.
- An angle that measures between 90 and 180 degrees.
- An angle that measures exactly 180 degrees.
- Lines that intersect to form right angles.
- Lines in the same plane that do not intersect.

Terms (Blue cards):

- Straight Angle
- Acute Angle
- Obtuse Angle
- Right Angle
- Perpendicular Lines
- Parallel Lines

Diagrams (White cards):

- 135° angle
- 62° angle
- 90° angle
- 180° angle
- Parallel Lines
- Intersecting Lines

Card Sort

Check Yourself

Press the button to see how you did on the Card Sort.

[Check My Work](#)

Self-checking

e-Learning Tasks During Lesson

Problem-solving tasks

- Draw an angle with specific size (e.g. 30° , 60° , 90°).
- Draw triangles satisfying specific conditions.

😊😊 Congratulations! Now you can draw a 30 degree, 60 degree or 90 degree angle using the Desmos tools!

1. An obtuse-angled Δ with an interior angle 120°
2. An obtuse-angled Δ with an interior angle 60°
3. A right-angled Δ with an interior angle 60°
4. An isosceles Δ with an interior angle 120°

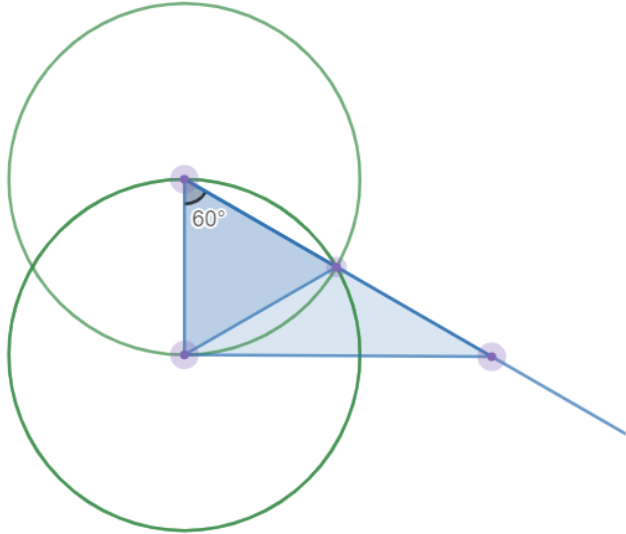
Catering for learner diversity

- Tasks with different levels of difficulty
- Two tasks chosen by students

Students' Learning Outcome

7B 31 [redacted]
Student 30 of 38

Task 1.4: 3. A right-angled Δ with an interior angle 60°



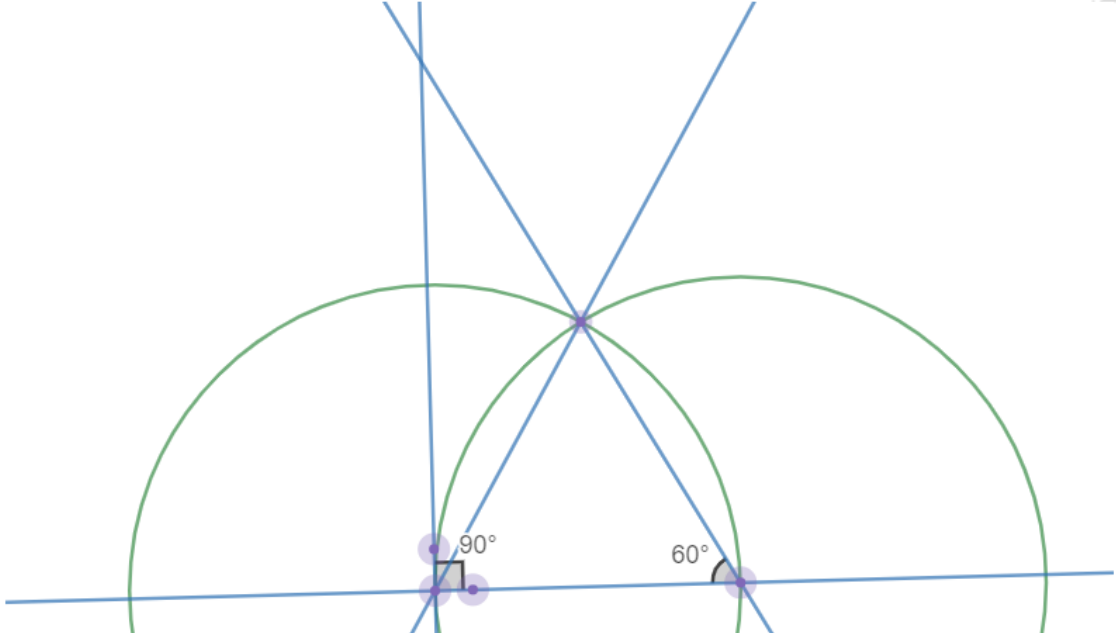
The diagram shows a right-angled triangle with a 60° angle. The vertices are marked with purple dots. Three green circles are drawn: one centered at the 60° vertex, one centered at the right-angle vertex, and one centered at the third vertex. The circles intersect at the vertices of the triangle.

Problem-solving skills

- Students can construct triangles using different strategies.

7B03 [redacted]
Student 30 of 38

Task 1.4: 3. A right-angled Δ with an interior angle 60°



The diagram shows a right-angled triangle with a 60° angle. The vertices are marked with purple dots. A vertical blue line is drawn. Two green semi-circles are drawn: one centered at the right-angle vertex and one centered at the 60° vertex. The semi-circles intersect at the third vertex of the triangle. A right-angle symbol is shown at the right-angle vertex.

Students' Learning Outcome

7B03 Student 38 of 53

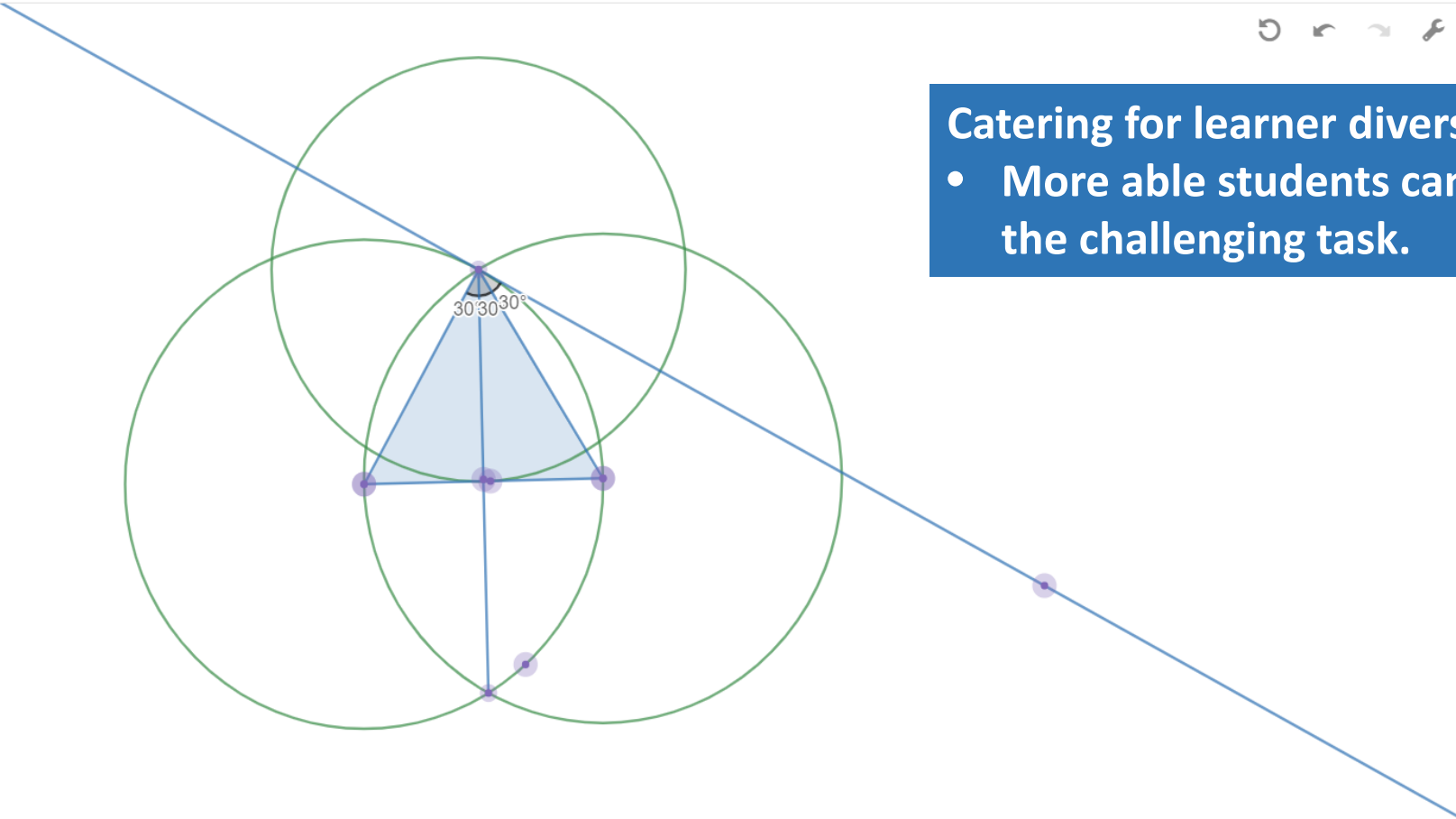
Task 1.3: Try to "trisect" a Right Angle (Divide the right angle into 3 equal parts)

Construct Transform

Select Point Circle Polygon Angle

Segment Line Ray Vector Arc

More Tools



The diagram shows a right angle being trisected into three 30-degree angles. A central blue triangle is formed by the intersections of three green circles. The top vertex of the triangle is at the vertex of the right angle. The bottom two vertices are on the two rays of the right angle. The angle at the top vertex is labeled as three 30-degree angles. The circles are centered at the vertices of the triangle and at the midpoint of its base.

27 of 48 Next >

30° 30° 30°

Catering for learner diversity

- More able students can complete the challenging task.

e-Learning Tasks During Lesson

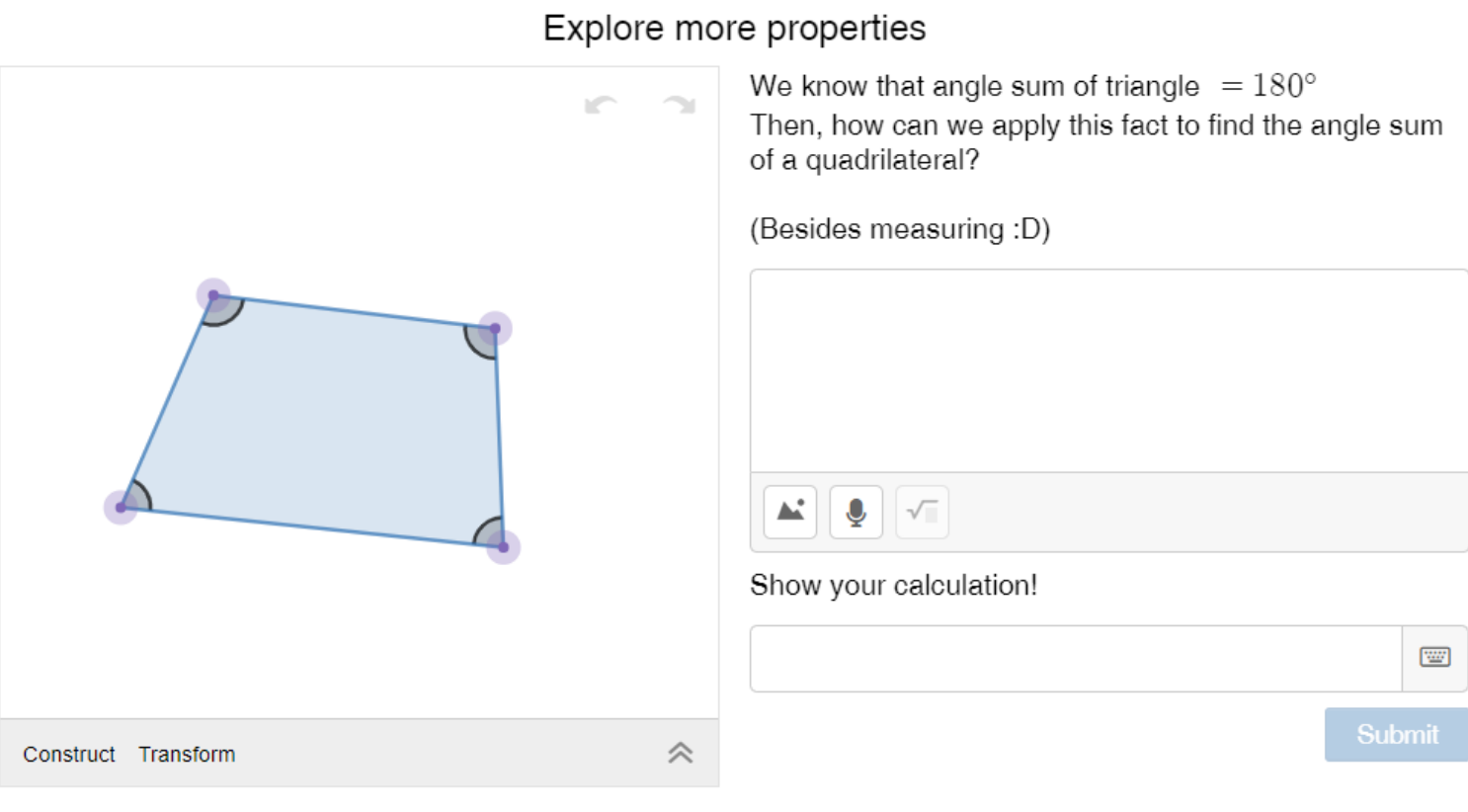
Inquiry and Investigation

- Angle sum of “reflex angles” of a triangle
- Angle sum of a quadrilateral
- Angle sum of “reflex angles” of a quadrilateral

Catering for learner diversity

- Tasks with different levels of difficulty

Explore more properties



We know that angle sum of triangle = 180°
Then, how can we apply this fact to find the angle sum of a quadrilateral?
(Besides measuring :D)

Show your calculation!

Submit

Developing students' ability to inquire

- Students may
 - measure the size of each angle to find the angle sum of a specific quadrilateral;
 - move the vertices to observe the angle sum of a quadrilateral;
 - draw auxiliary line segments and use the fact about angle sum of triangle to find the angle sum of a quadrilateral.

Evaluation using Triple E Framework

	Evaluation
Engagement	<ul style="list-style-type: none">• This can be achieved through the use of interactive e-learning tasks, as well as through collaborative activities that encourage communication and teamwork among students.
Enhancement	<ul style="list-style-type: none">• Through the problem solving tasks, students develop a deeper understanding of the properties of equilateral triangles.• Students are allowed to demonstrate their understanding of geometry using the platform of the e-learning tool.
Extension	<ul style="list-style-type: none">• Using the platform, students can further investigate the properties about the angle sum of a polygon.

Teacher's Reflection (Catering for learner diversity)

Student engagement:

- Students use e-learning platforms to complete diversified e-learning tasks.
- Students are allowed to express their reflections on the learning process in different forms.

Tiered tasks:

- Learning tasks are designed with different levels of difficulty (including challenging questions).
- Students are allowed to choose the problems they want to solve in the e-platform according to their interests/abilities.

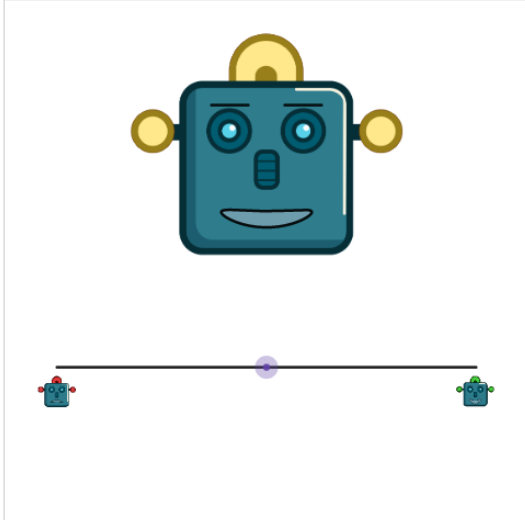
Multiple solutions:

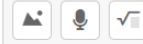
- Students are allowed to use different methods to draw angles.
- Students have chances to present their way of thinking through the e-platform. Teachers can show students' works easily and facilitate further discussion among students.

How are you feeling about your ability to apply the properties using regular polygon?

Slide to the right if you feel confident.
Slide to the left if you don't feel confident.

If you'd like, say more about your response below.





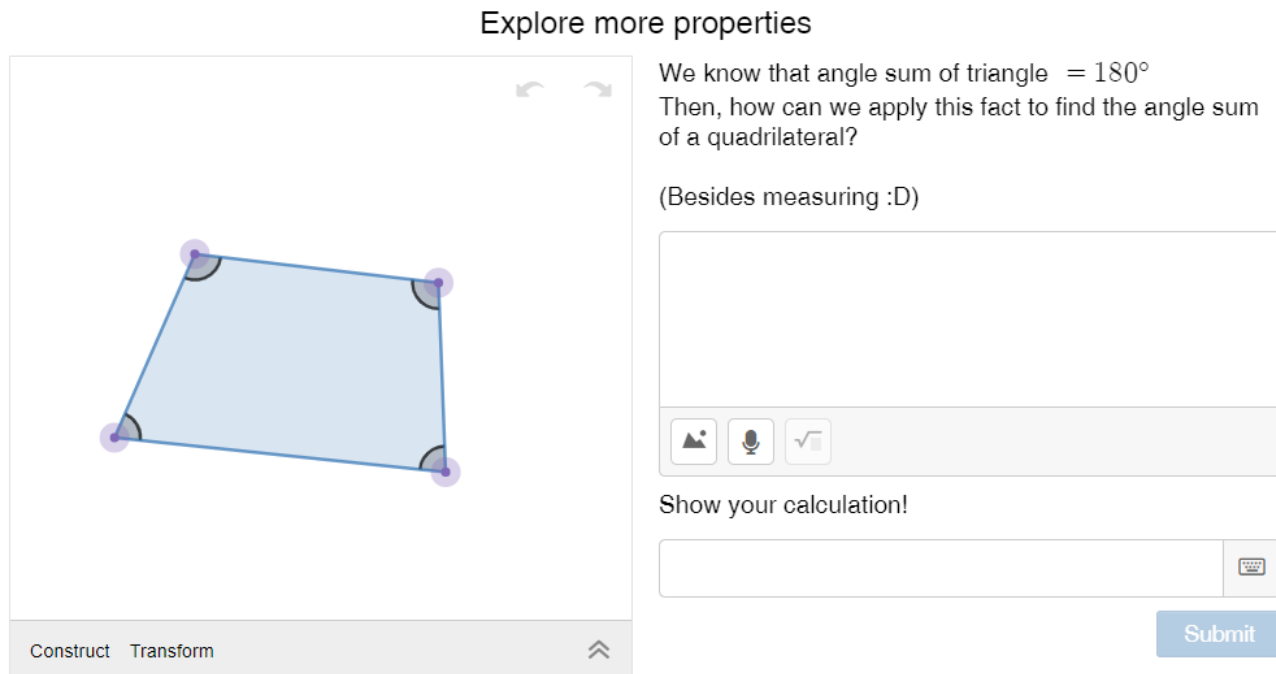
Proper use of e-learning tools and careful design of e-learning tasks can cater for learner diversity effectively.

Teacher's Reflection (Inquiry and investigation)

Helping students explore geometric properties using e-learning tools:

- Students use the e-learning tools to observe patterns and make conjectures about the geometric properties.
- Students use the e-platform to explain their conjectures.

Explore more properties



We know that angle sum of triangle = 180°
Then, how can we apply this fact to find the angle sum of a quadrilateral?
(Besides measuring :D)

Show your calculation!

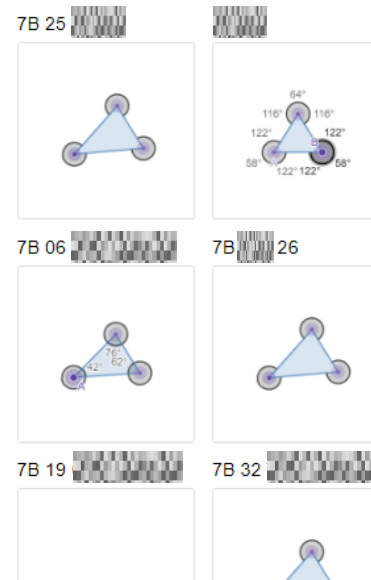
Submit

The dynamic geometry software facilitates the inquiry and investigation in many geometry topics in the Mathematics curriculum.

Teacher's Reflection (Effectiveness of e-learning strategies)

Utilising electronic assessment data to provide immediate feedback:

- Teacher uses the data in the pre-lesson e-learning task to understand whether students have mastered the drawing skills.
- Based on students' performance in the e-learning activities, teacher understands how students think mathematically and provides immediate feedback (asking and probing questions).



Explore more properties

According to the triangle on the left, what is the sum of the marked angle? You may use the "Select" the angle and see the label. And "add" all them together below the Response Box.

Responses Summary

7B 25

900

$116 + 64 + 116 + 122 + 58 + 122 + 122 + 58 + 122$

7B 06

$(360 - 44) + (360 - 63) + (360 - 73)$

7B 19

900

The use of e-assessment can enhance learning and teaching by providing instant feedback to both students and teachers.

S2 Trigonometric Ratios

Learning Objectives

- ▶ Understand the properties of trigonometric ratios
 - ▶ effect of angle size on the ratios ($x \uparrow, \sin x \uparrow$)
 - ▶ range of values of trigonometric ratios ($0 < \sin x < 1$ for $0^\circ < x < 90^\circ$)
- ▶ Use trigonometric ratios to solve problems involving plane figures
 - ▶ Find the length of sides of a right-angled triangle using trigonometric ratios

- Suitable resources found

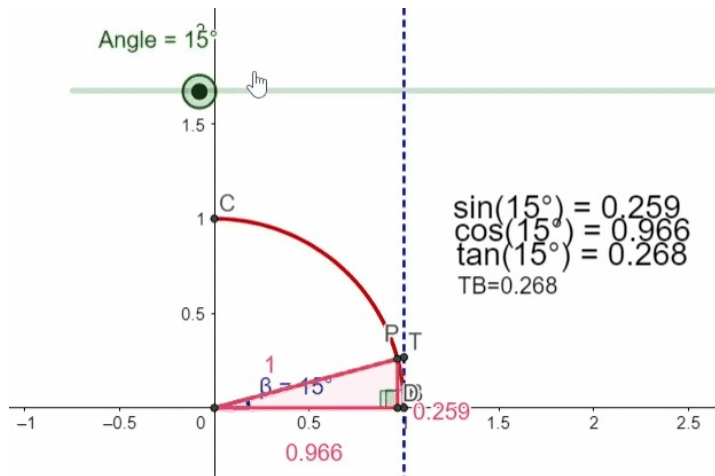
About Target Students

- ▶ Students recognised the definition of trigonometric ratios.
- ▶ Students were weaker in Mathematics in general.

- Adjust students' learning pace
- Collect/show students' responses and give feedback in a timely manner

Part I: Investigate the Properties of Trigonometric Ratios

▶ GeoGebra



- ▶ Effect of angle size on the ratios
- ▶ Range of values of trigonometric ratios

- when I drag the slide
the opposite side increase
the adjacent side decrease
- When we drag the slide
The hypotenuse remain the same
The opposite side increases
The adjacent side decreases
- 1 side extend, elongated
1 side become short short
- H remain unchanged
A forward=decreases or backward=increases
O forward=increases or backward=decreases

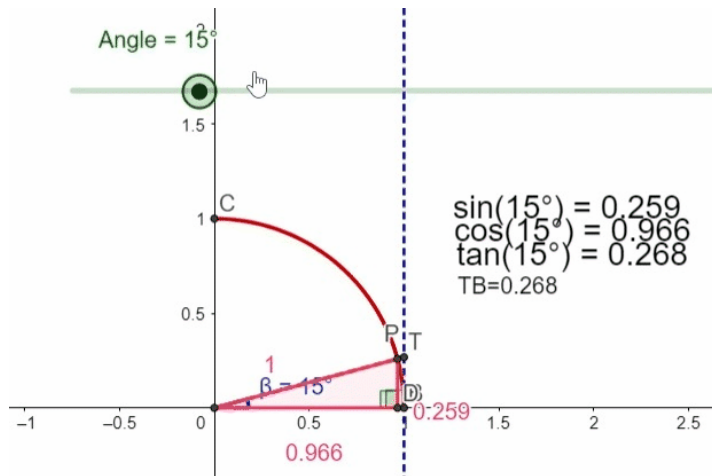
Students can point out the change in the length of sides. Some students cannot relate them to the change in the ratios.

Reflections

- Proper questioning?
- Focus more on the lengths instead of the values of the ratios?

Part I: Investigate the Properties of Trigonometric Ratios

▶ GeoGebra



- ▶ Effect of angle size on the ratios
- ▶ Range of values of trigonometric ratios

Cos cannot be larger than one based on the app
 Sine and cosine data should be less than 1
 Sine < 1
 Cosine < 1
 Tangent (unlimited)
 The range of the angle is 1-89
 ratio of sin must be positive and less than one
 ratio of cos must be positive and less than one
 ratio of tan must be positive
 $\wedge \text{---} \wedge$
 * *

Students can partly identify the range of the ratios (i.e. $\sin \theta < 1$).

By observations on the numerical values only

Geometric interpretation?

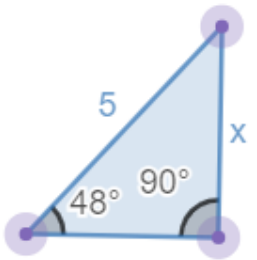
- Explanation
- Follow-up Task

Part II: Finding length of sides

- ▶ Finding the length of a side by observation
 - ▶ Identifying the correct ratio used
 - ▶ Using $0 < \sin \theta < 1$ and $0 < \cos \theta < 1$ to determine whether the operation \times/\div should be used

- Considering the angle "48°":
 - The hypotenuse is given and the opposite side is to be found. Thus the sine ratio should be used.
 - Note that $\sin 48^\circ < 1$. Thus $5 \sin 48^\circ < 5$ and $\frac{5}{\sin 48^\circ} > 5$. As the opposite side is shorter than the hypotenuse, the correct answer should be $5 \sin 48^\circ$.

Quick Practice!




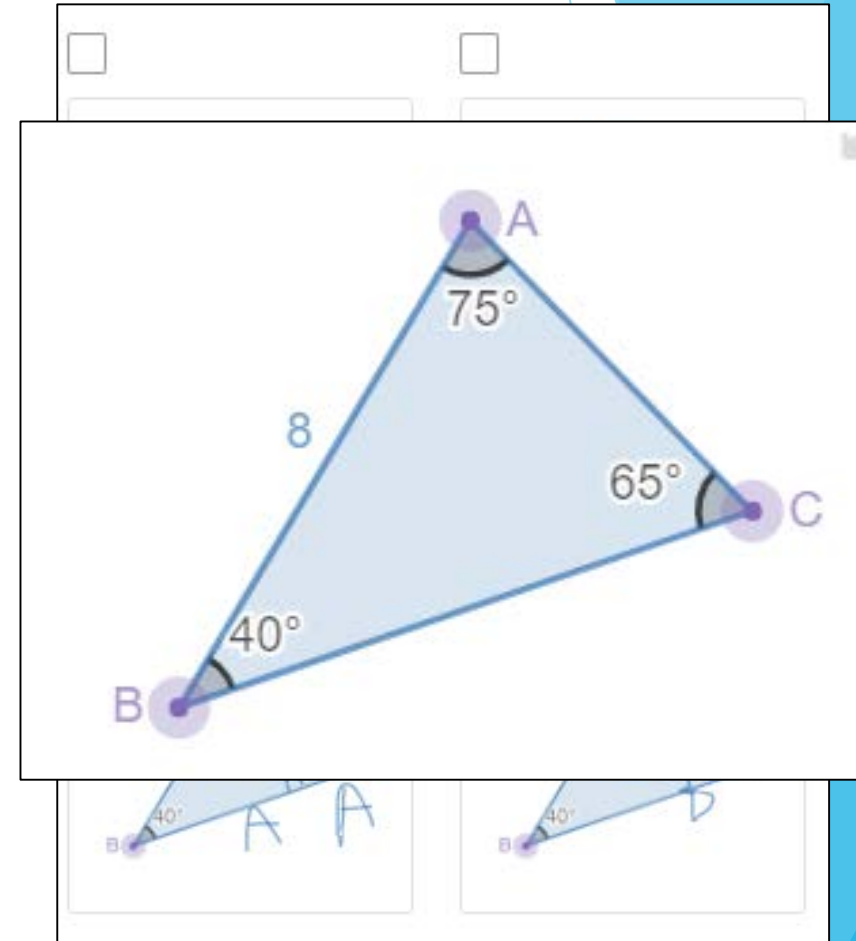
Find x.

- $5 \sin 48^\circ$
- $5 \cos 48^\circ$
- $\frac{5}{\sin 48^\circ}$
- $\frac{\cos 48^\circ}{5}$



Application of the properties of trigonometric ratios to solve problems

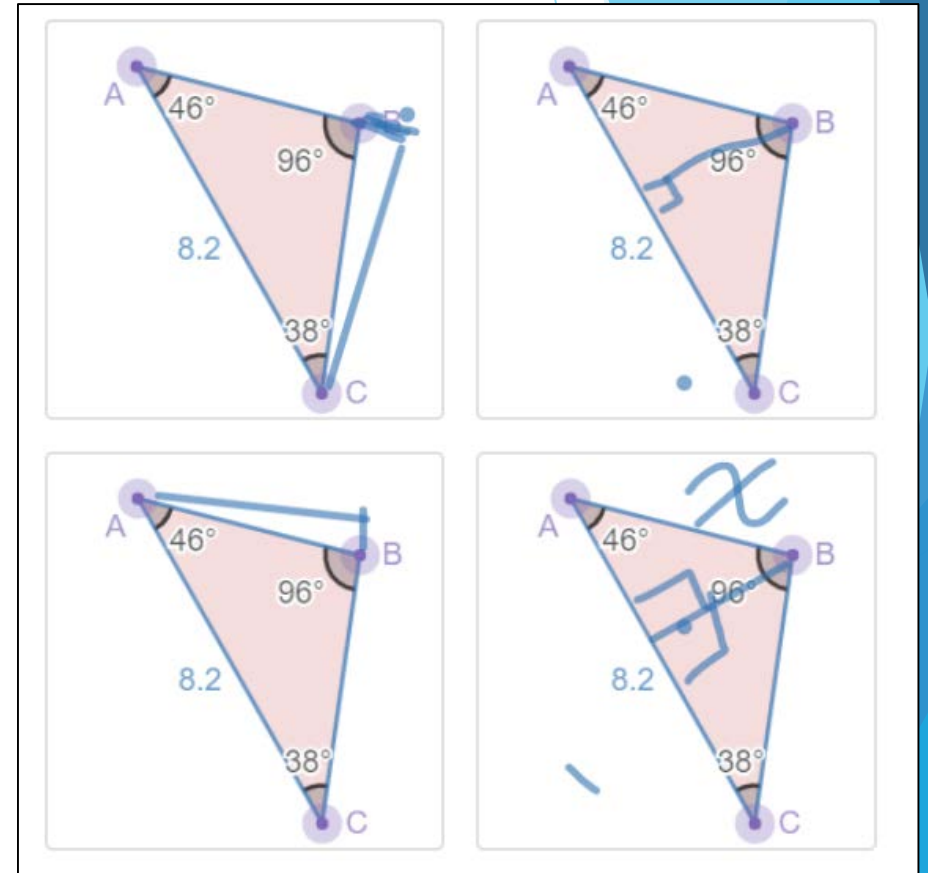
Part II: Find the length of sides

- ▶ Finding the length of a side by observation
 - ▶ Identifying the correct ratio used
 - ▶ Using $0 < \sin \theta < 1$ and $0 < \cos \theta < 1$ to determine whether the operation \times/\div should be used
- ▶ Solving harder problems by adding auxiliary lines to form right-angled triangles
 - ▶ Acute-angled triangle 
 - ▶ Obtuse-angled triangle



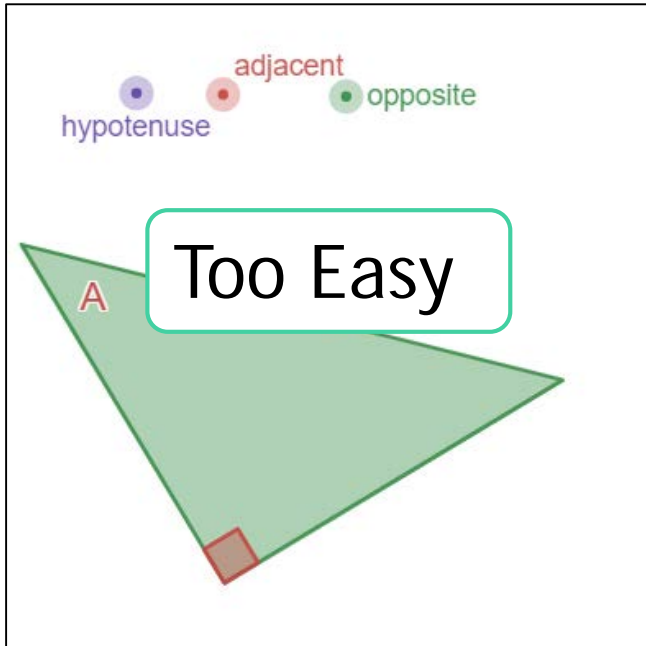
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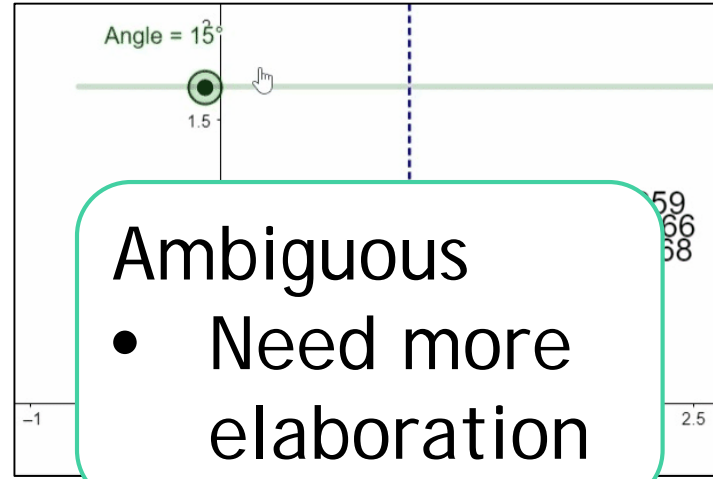


Evaluation - Engagement

Matching

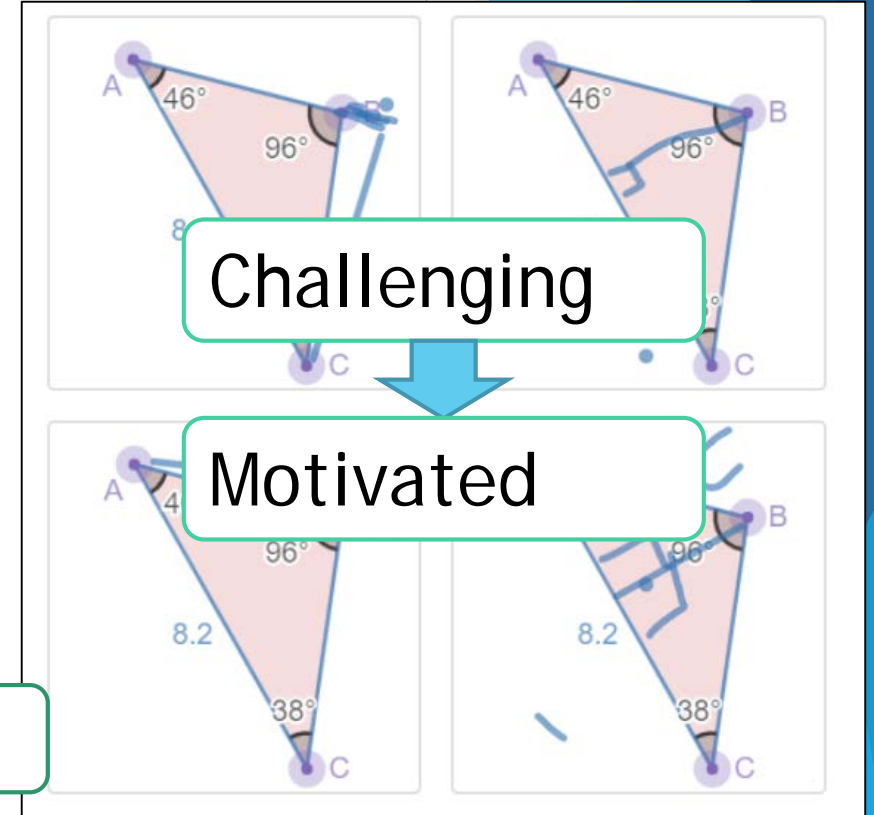


Self-exploration



• Hands-on Tasks

Self-practice



Evaluation - Engagement

Cos cannot be larger than one based on the app

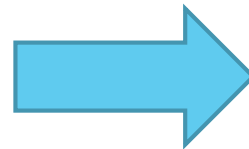
Sine and cosine data should be less than 1

Sine<1
Cosine<1
Tangent (unlimited)

The range of the angle is 1-89

ratio of sin must be positive and less then one
ratio of cos must be positive and less then one
ration of tan must be positive

^-----^
* *
* *



Areas for improvement

- Peer-peer interaction
- Discussion

Evaluation - Enhancement

- ▶ Understand the properties of trigonometric ratios

- ▶ effect of angle size on the ratios ($x \uparrow, \sin x \uparrow$)
- ▶ range of values of trigonometric ratios ($0 < \sin x < 1$ for $0^\circ < x < 90^\circ$)



- Simple
- More elaboration on the geometric meaning of the trigonometric ratios

- ▶ Use trigonometric ratios to solve problems involving plane figures

- ▶ Find the length of sides of a right-angled triangle using trigonometric ratios

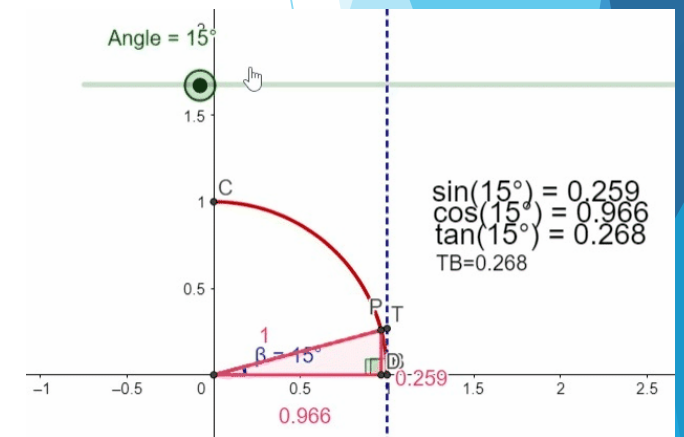


Student **CAN** choose the correct step.

Students **CANNOT** choose the correct step quickly.

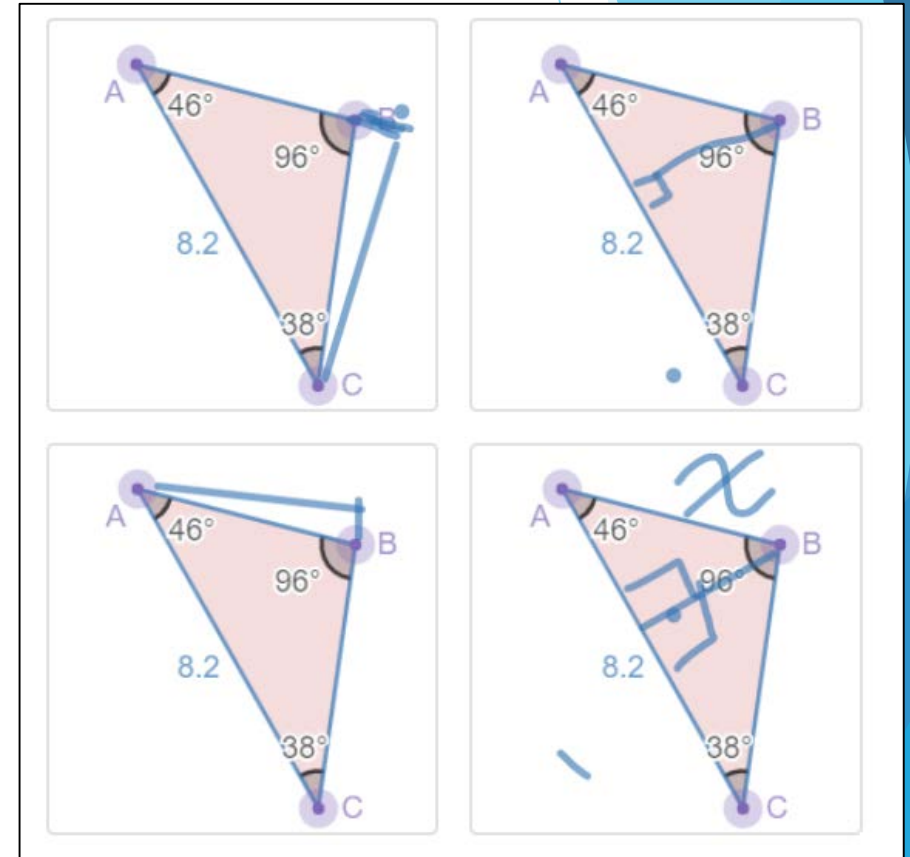


Connection between Part I and Part II could be strengthened to enhance students' understanding about the properties of trigonometric ratios.



Evaluation - Extension

- ▶ Finding height of obtuse-angled triangles
 - ▶ Collect feedback instantly using e-platform
 - ▶ Choosing a proper vertex is crucial to problem-solving
 - ▶ May add a simple matching task
- ▶ More properties/identities of trigonometric ratios
 - ▶ By observations only
 - ▶ Elaboration on geometric meaning
 - ▶ Create platform for students to share their idea
- ▶ Other possibilities?
 - ▶ Trigonometric ratios when $\theta = 0^\circ$ or $\theta = 90^\circ$



Evaluation - Extension

- ▶ Finding height of obtuse-angled triangle
 - ▶ Collect feedback instantly
 - ▶ Choosing a proper vertex is crucial to problem-solving
 - ▶ May add a simple matching task
- ▶ More properties/identities of trigonometric ratios
 - ▶ By observations numerical values only
 - ▶ More elaboration using the geometric interpretation
 - ▶ Create a platform for students to share their idea
- ▶ Other possibilities?
 - ▶ Values of trigonometric ratios when $\theta = 0^\circ$ or $\theta = 90^\circ$

Check the following boxes if the equation is true.

(Select all that apply.)

$\sin(\theta + 30^\circ) = \sin \theta + \sin 30^\circ$

$\sin^2 \theta + \cos^2 \theta = 1$

$\tan \theta = \frac{\sin \theta}{\cos \theta}$

$\sin \theta + \cos \theta = \tan \theta$

$\sin 2\theta = 2 \sin \theta$

Catering for learner diversity

- ▶ Adjust the learning pacing in e-platform
 - ▶ Students have the freedom to choose the suitable question according to their level
- ▶ Assessing students' learning using e-platform
 - ▶ Respond/follow up timely
- ▶ Sharing students' work using e-platform
 - ▶ Engagement

- Student-centered
- Peer-peer interaction

Reflections

- ▶ Mixed use of diversified learning tools
 - ▶ "Think out of" the iPad
- ▶ Consider the learning needs of users(students)

Way forward

To develop students' ability to inquire and cater for learner diversity, teachers could capitalise their efforts on

- designing diversified learning tasks to engage, enhance and extend student learning;
- making further use of e-learning tools to promote interactive learning and foster student engagement and participation; and
- strengthening knowledge management for disseminating good practices of learning and teaching strategies.

End